Nano-Scale `Dark State' Optical Potentials for Cold Atoms

M. Łącki, M. Baranov, H. Pichler, P. Zoller, arXiv:1607.07338

Manipulation of atoms by coherent optical forces on scale of $\gtrsim$ 10 nanometers [sub-wavelength scale]

… with far field optics

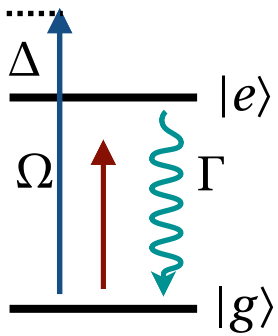
✓ ‘geometric potentials’: non-adiabatic corrections to Born-Oppenheimer potentials — *how?*

✓ quantum many-body physics with atoms on ‘nano-scale’ — *why?*

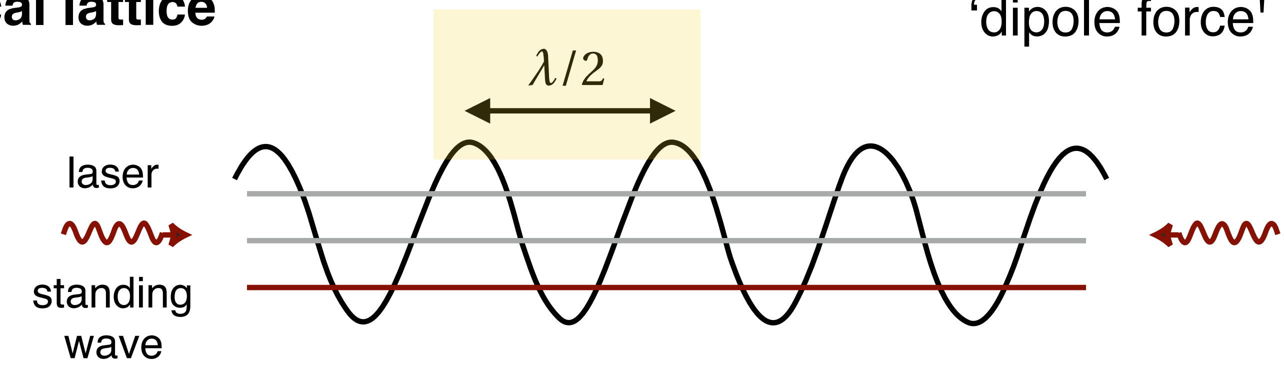
see also: A. V. Gorshkov et al. arXiv:1609.01285

Optical Potential Landscapes for Atoms

- **far off-resonant optical lattice**



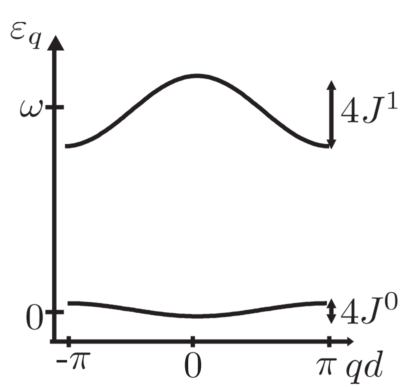
‘dipole force'



$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V_0 \sin^2(kx)$$

optical potential (1D)

- **Bloch bands**



band structure (1D)

✓ AC-Stark shift as optical potential

$$\Delta E_g \sim I(x) \sim V_0 \sin^2(kx)$$

$$\frac{\Omega^2}{4}\,\frac{1}{\Delta - i\frac{1}{2}\Gamma}$$

lattice spacing / energy scale

$\lambda/2$

✓ off-resonant laser

$$|\Delta| \gg \Gamma$$

small dissipation

Optical Potential Landscapes for Atoms

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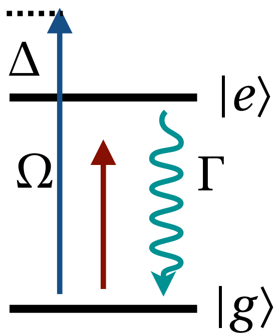
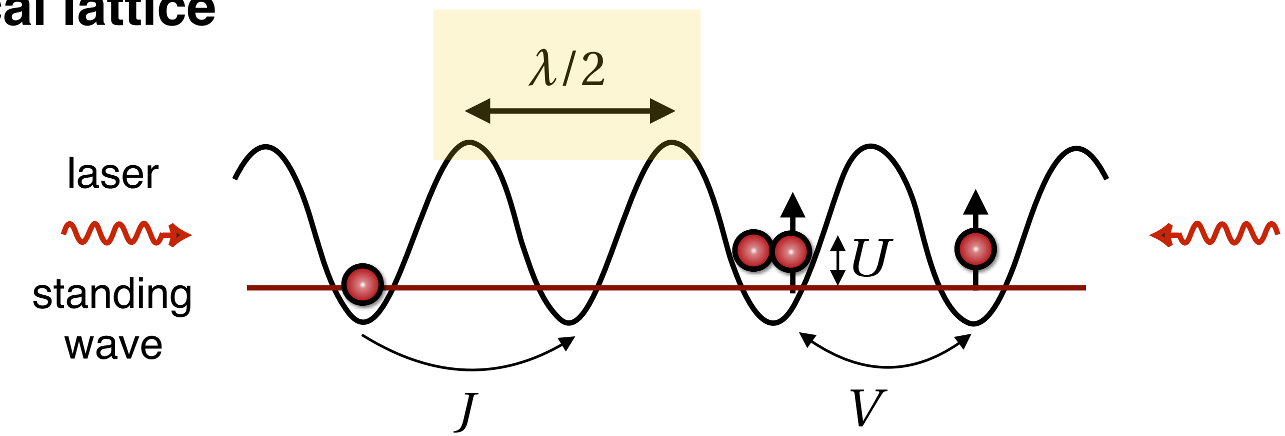
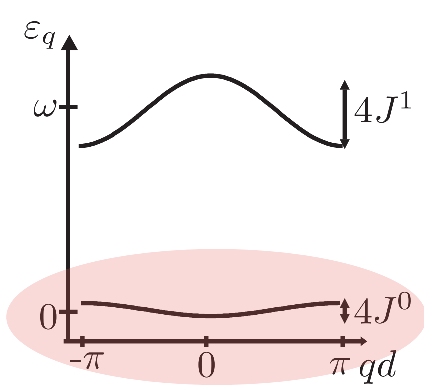




- **Bloch bands**



band structure (1D)

- **many particle physics: Bose / Fermi Hubbard**

$$H = -J \sum_{<i,j>} b_i^\dagger b_j + U \sum_i n_i^2 + V \sum_{ij} n_i n_j$$

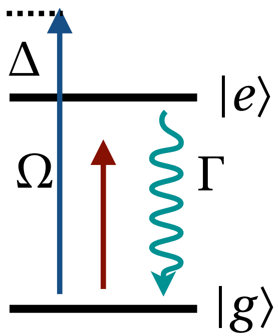
✓ Hubbard toolbox, …

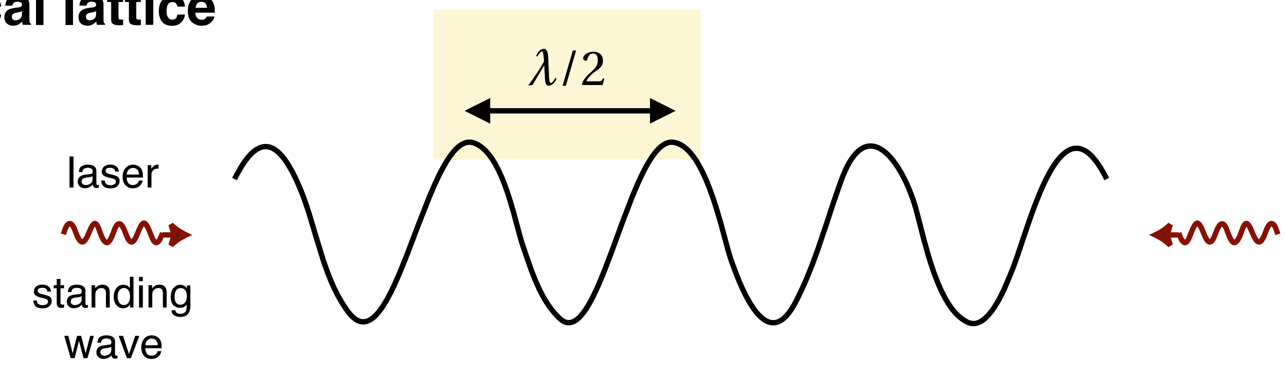
✓ energy scales $J \ll E_R = \frac{\hbar^2 k^2}{2m} \leftrightarrow U, V \leftrightarrow k_B T$ ($k \sim \lambda$)

lattice spacing as fundamental limit for atomic Hubbard models

Optical Potential Landscapes for Atoms

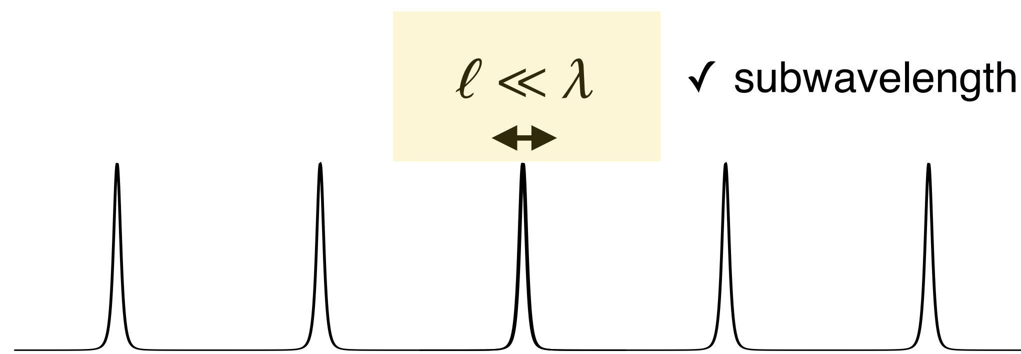
- **far off-resonant optical lattice**





- **landscape of optical potentials *with sub-wavelength resolution* [?]**

how?



✓ subwavelength structure

Kronig-Penney potential

Optical Potential Landscapes for Atoms

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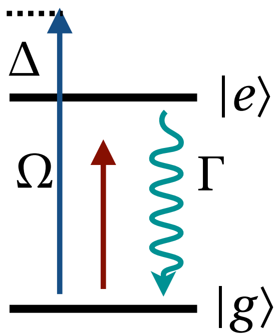

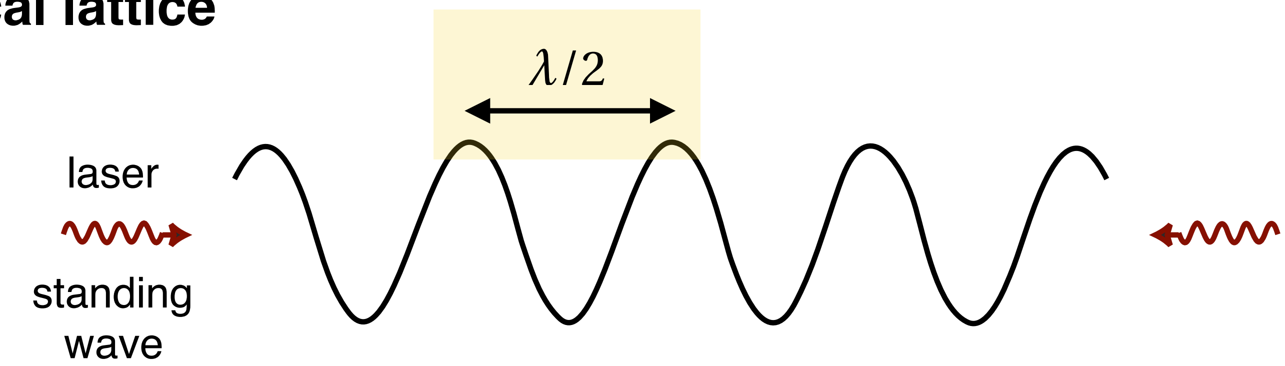

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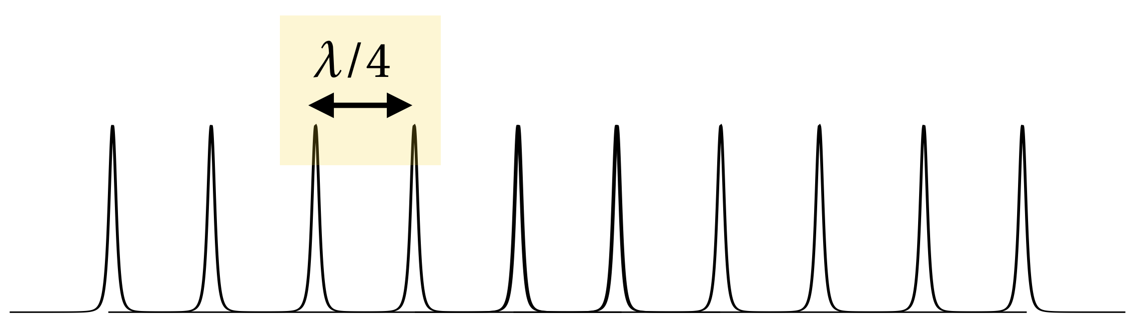

subwavelength lattice

Optical Potential Landscapes for Atoms

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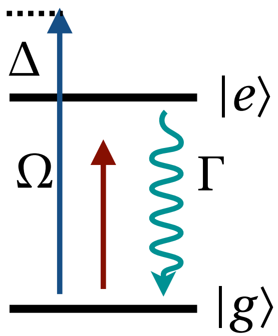

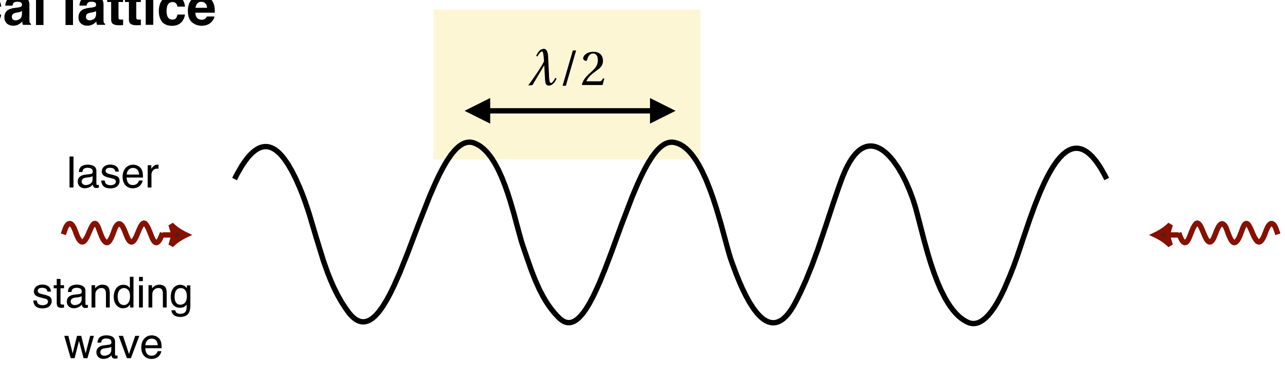

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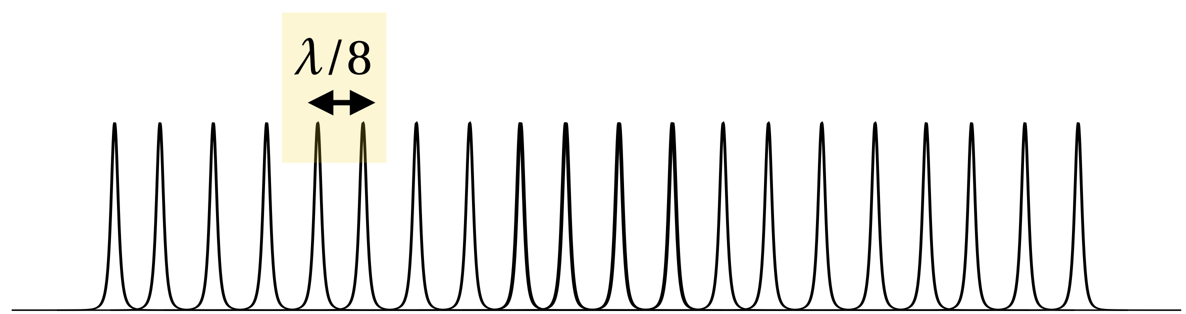


sub-wavelength lattices:

W Yi et al. - NJP 2008

S Nascimbene et al.- PRL 2015

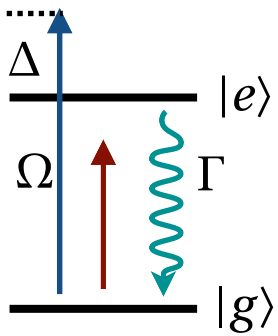
optical forces: atoms go 'nano-scale' ;-)

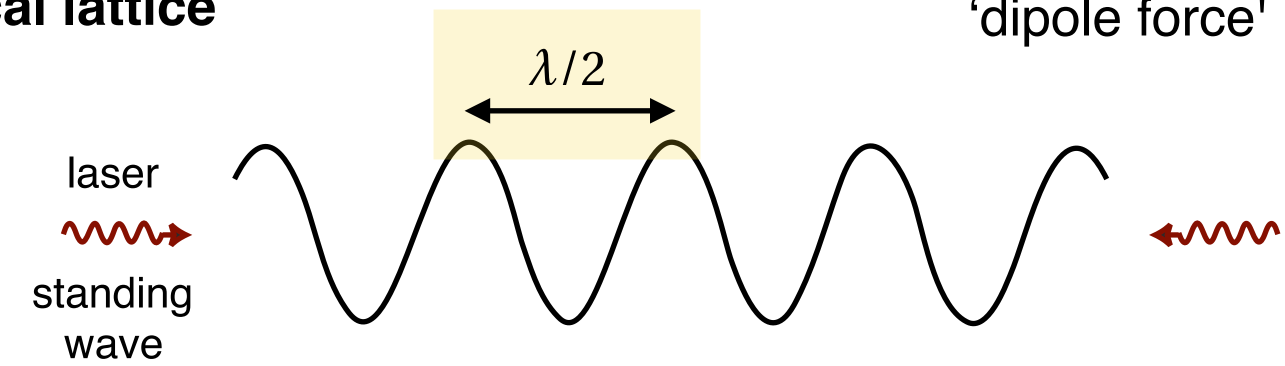
... limit?

Optical Potential Landscapes for Atoms

- **far off-resonant optical lattice**

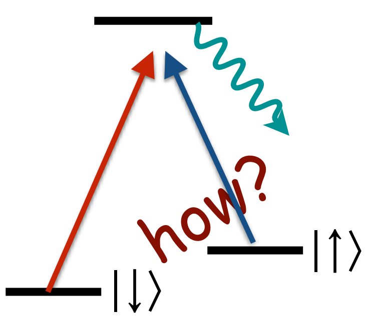
‘dipole force'

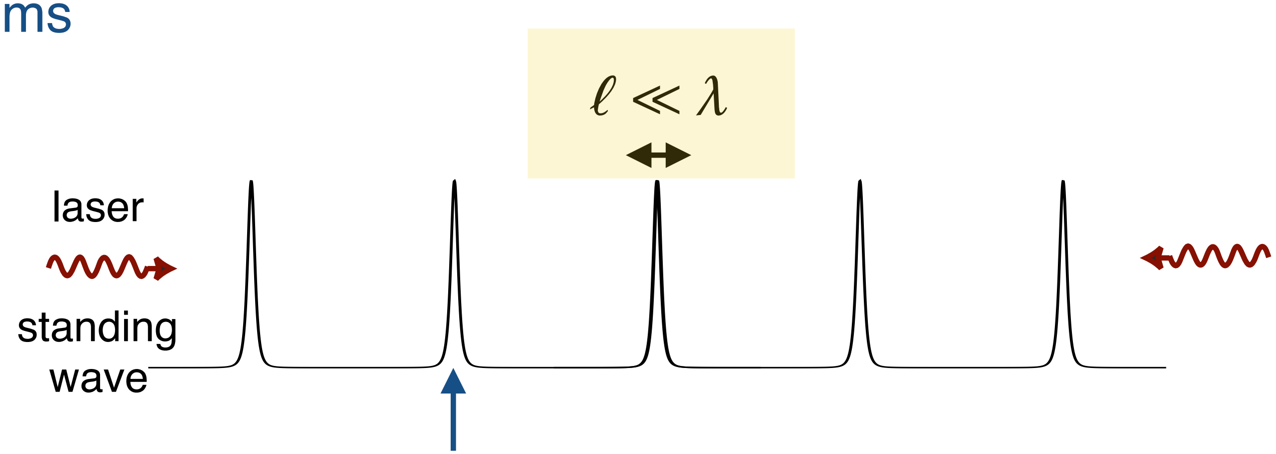




- **landscape of optical potentials *with sub-wavelength resolution* [?]**

(1) dark state in Λ-systems





✓ resonant & off-resonant

(2) … as *conservative* optical potential from *non-adiabatic* correction to atomic motion in dark state

Example & Motivation:

Double Well Potentials …
with Sub-Wavelength Barriers

- **bilayer system: magnetic atoms / polar molecules**

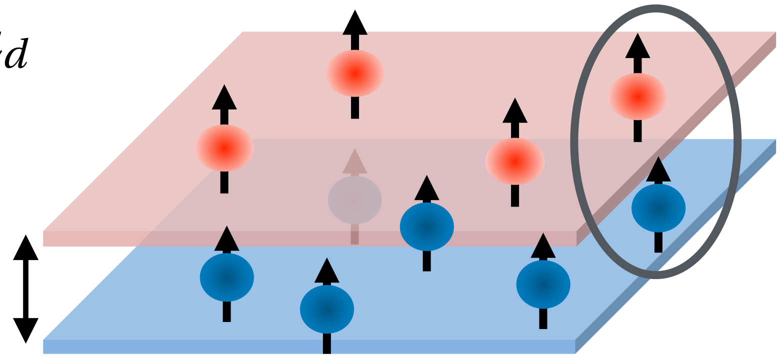
distance vs. dipolar length a_d

✓ optical lattice: $\lambda/2$

✓ subwavelength: $\ell \ll \lambda$

Lanthanides

Dy: $a_d \sim 10\,\mathrm{nm}$



strong interactions:
e.g. pairing

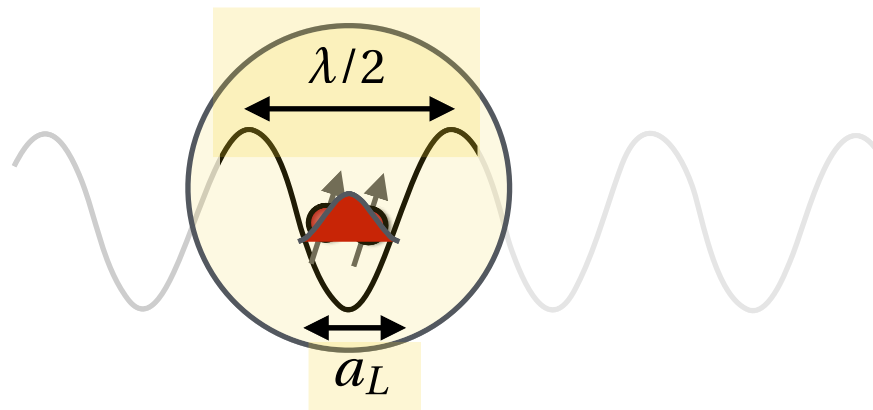
✓ BEC-BCS crossover

✓ bilayer FQH

… atomic quantum many-body physics on ‘nano-scale’

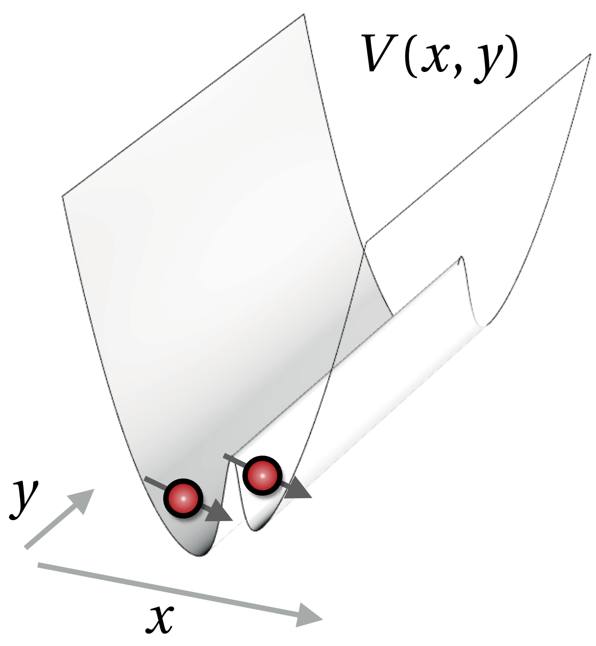
Splitting an Optical Lattice Site

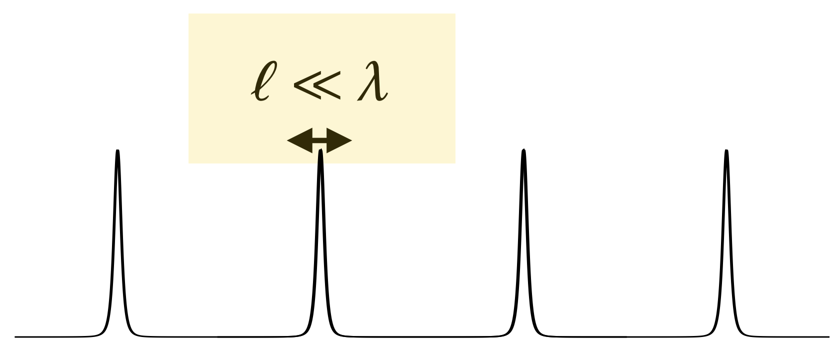
- **far off-resonant optical lattice**



✓ double well / wire / layer
✓ loading from a quantum degenerate gas

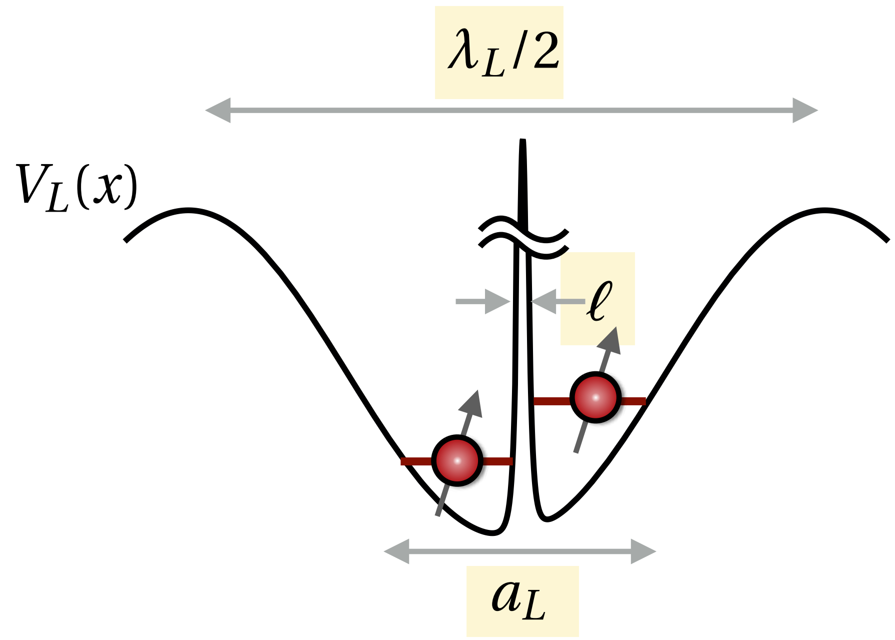
- **tials *with sub-wavelength resolution* [?]**



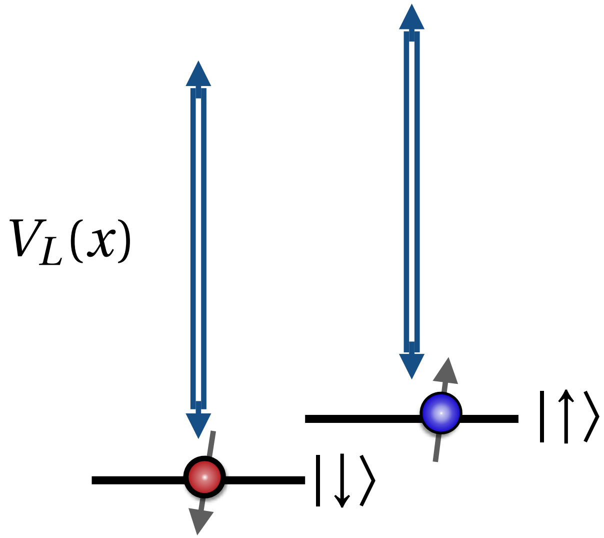


Splitting an Optical Lattice Site: Ingredients (1)

- **optical landscape**



- **atomic & laser configuration**



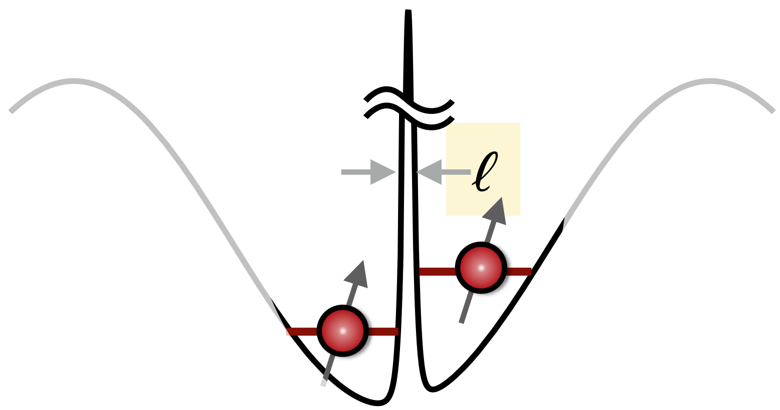
(1) off-resonant optical lattice

$$V_L(x) = V_0 \sin^2(k_L x) \qquad (k_L = 2\pi/\lambda_L)$$

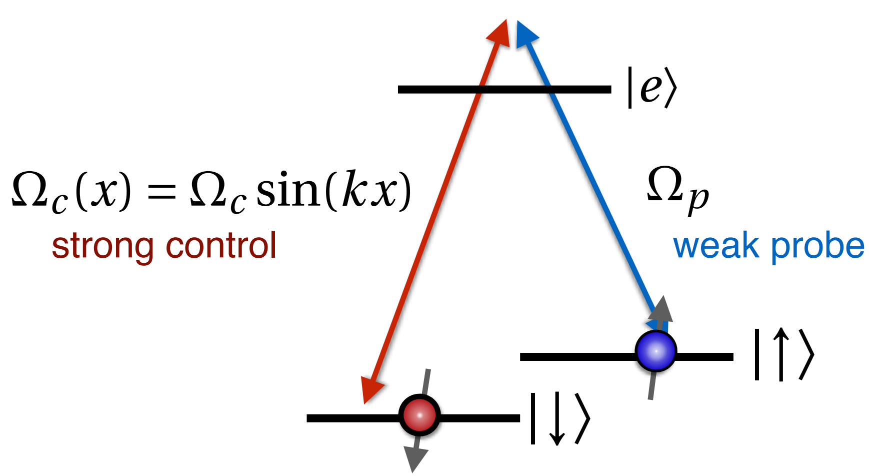
same potential for both ground states

Splitting an Optical Lattice Site: Ingredients (2)

- **optical landscape**



- **atomic & laser configuration**

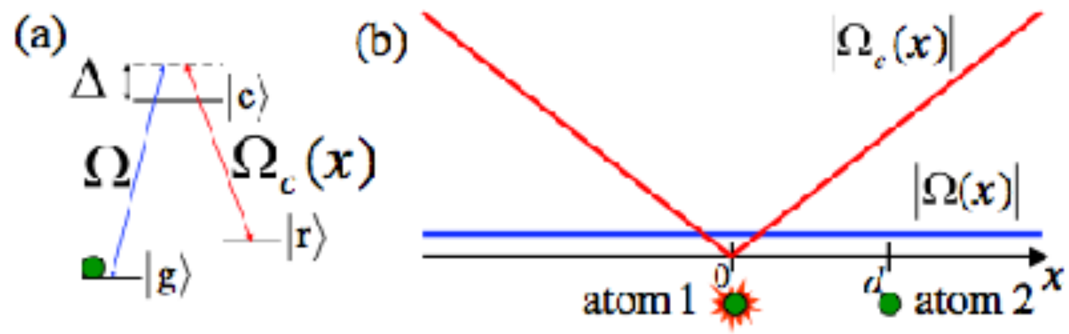


(2) subwavelength barrier via Λ-configuration

PRL 100, 093005 (2008) PHYSICAL REVIEW LETTERS week ending 7 MARCH 2008

Coherent Quantum Optical Control with Subwavelength Resolution

Alexey V. Gorshkov,[1] Liang Jiang,[1] Markus Greiner,[1] Peter Zoller,[2] and Mikhail D. Lukin[1]
[1]Physics Department, Harvard University, Cambridge, Massachusetts 02138, USA
[2]Institute for Quantum Optics and Quantum Information of the Austrian Academy of Sciences, A-6020 Innsbruck, Austria
(Received 11 December 2007; published 7 March 2008)



vs. incoherent
sub-wavelength microscopy:
S. Hell

VOLUME 76, NUMBER 11 PHYSICAL REVIEW LETTERS 11 MARCH 1996

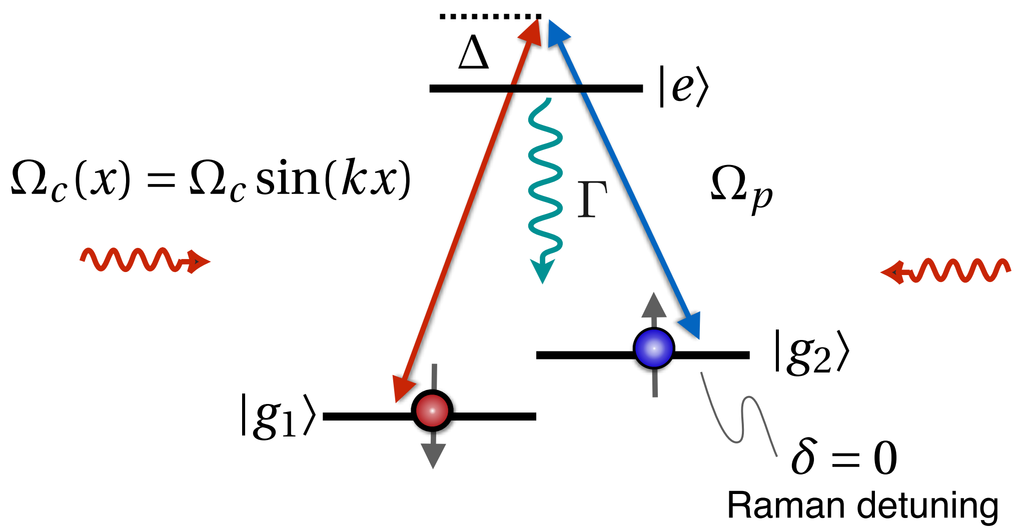
Gauge Structures in Atom-Laser Interaction: Bloch Oscillations in a Dark Lattice

R. Dum and M. Olshanii
Ecole Normale Superieure, Laboratoire Kastler Brossel, 24, Rue Lhomond, F-75231 Paris Cedex 05, France
(Received 6 July 1995)

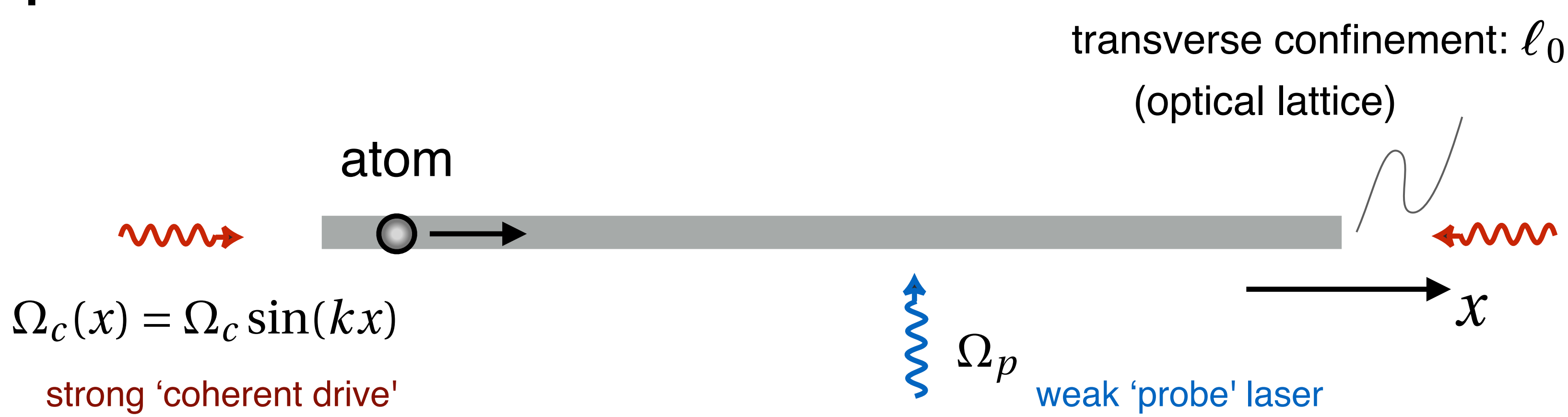
Atoms in dark atomic ground states do not interact with the laser light present and are therefore not susceptible to spontaneous processes. Laser cooling populates these states. Confinement of atoms in dark states in a (periodic) gauge potential allows for a lattice of atoms with a very low decay rate, a *dark lattice*. A dark lattice is a promising system to observe *Bloch oscillations* of neutral atoms, i.e., periodic excursion across the Brillouin zone in a uniform force field.

Atom in Λ-Configuration: 1D Quantum Motion

- **atomic configuration**

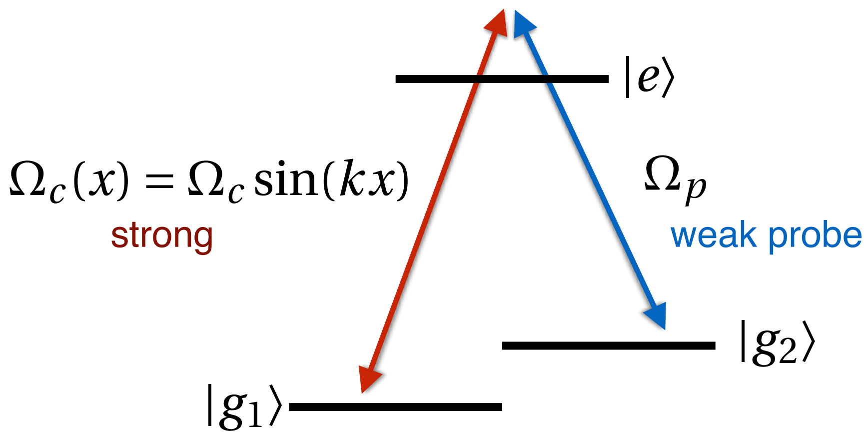


- **quantum motion of atom in 1D**

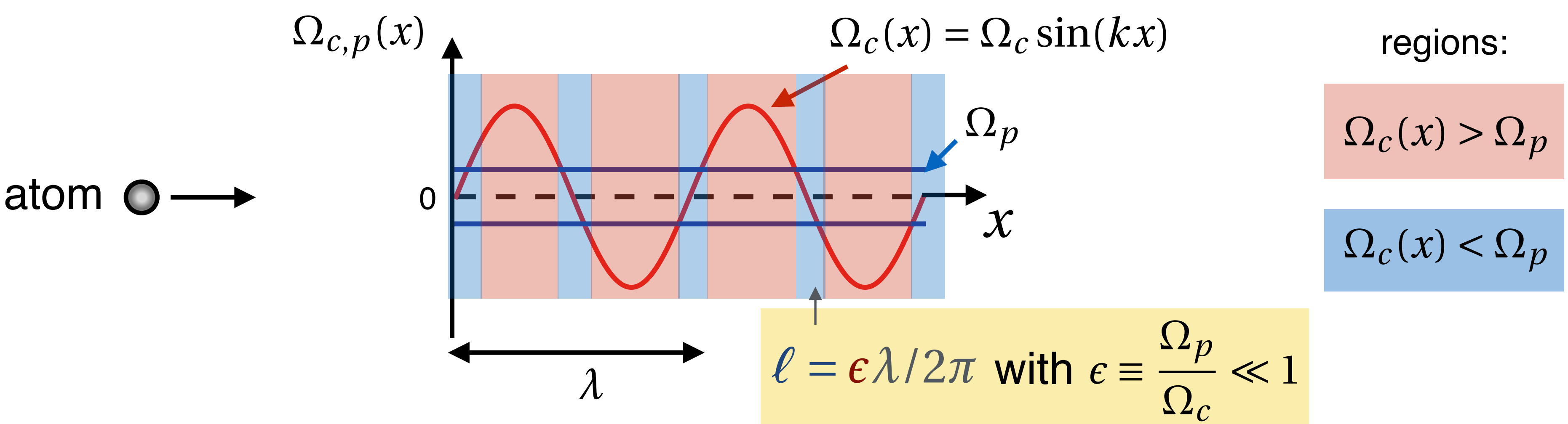


Atom in Λ-Configuration: 1D Quantum Motion

- **atomic configuration**

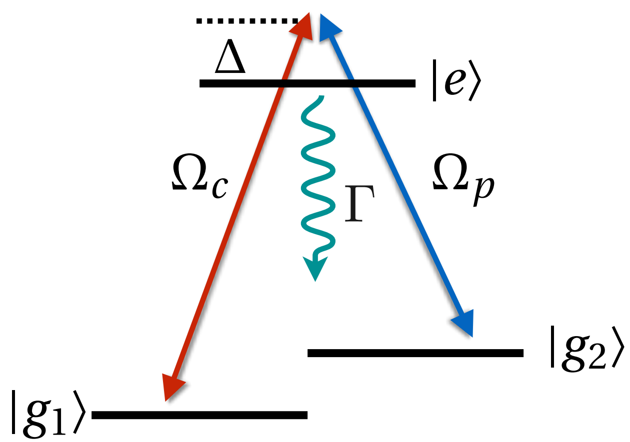


- **Rabi frequencies in space**



Atom in Λ-Configuration: 1D Quantum Motion

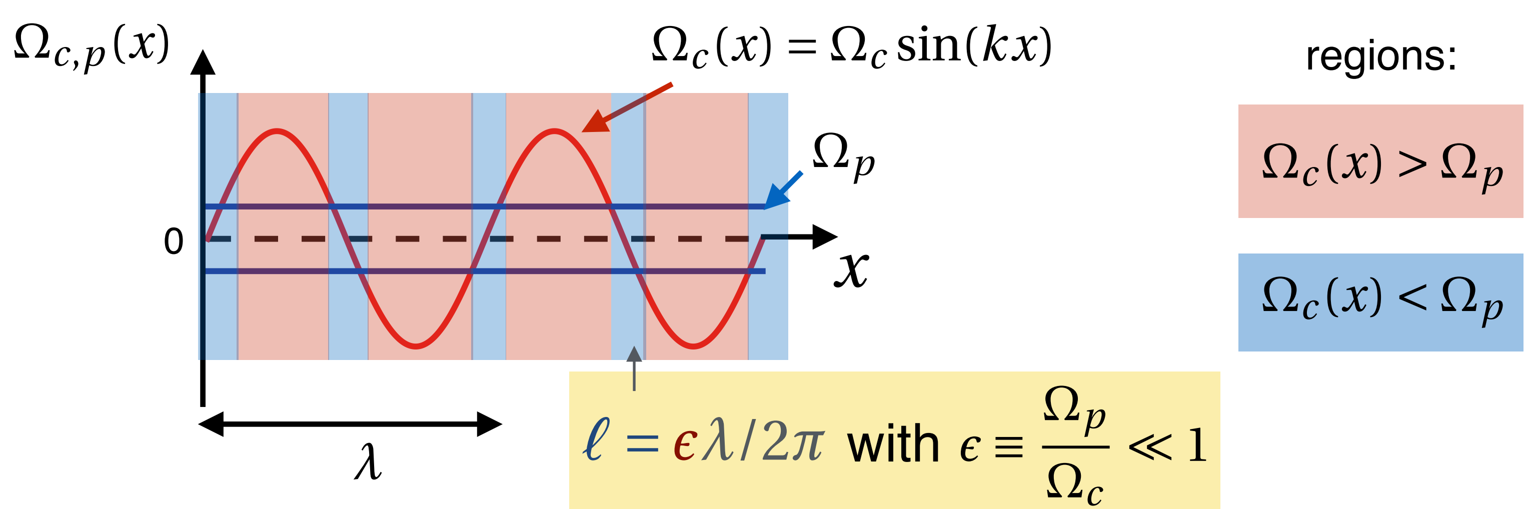
- **Hamiltonian**



$$
\begin{array}{c}
\qquad\qquad\qquad\qquad\qquad\qquad |g_1\rangle \qquad\qquad |e\rangle \qquad\qquad |g_2\rangle \\
H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \begin{pmatrix} 0 & \frac{1}{2}\Omega_c(x) & 0 \\ \frac{1}{2}\Omega_c(x) & -\Delta - i\frac{1}{2}\Gamma & \frac{1}{2}\Omega_p \\ 0 & \frac{1}{2}\Omega_p & 0 \end{pmatrix}
\end{array}
$$

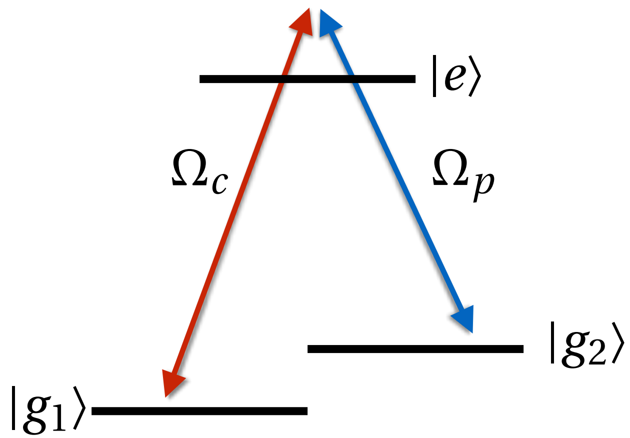
kinetic energy   internal

- **Rabi frequencies in space**



Born-Oppenheimer (Adiabatic) Approximation

- **Hamiltonian**



$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \begin{pmatrix} 0 & \frac{1}{2}\Omega_c(x) & 0 \\ \frac{1}{2}\Omega_c(x) & -\Delta - i\frac{1}{2}\Gamma & \frac{1}{2}\Omega_p \\ 0 & \frac{1}{2}\Omega_p & 0 \end{pmatrix}$$

- **Born-Oppenheimer (adiabatic) approximation**

dark state

$$E_0 = 0$$

$$|0\rangle \quad \sim \quad \Omega_p |g_1\rangle - \Omega_c(x) |g_2\rangle$$

no excited state admixed:

bright states

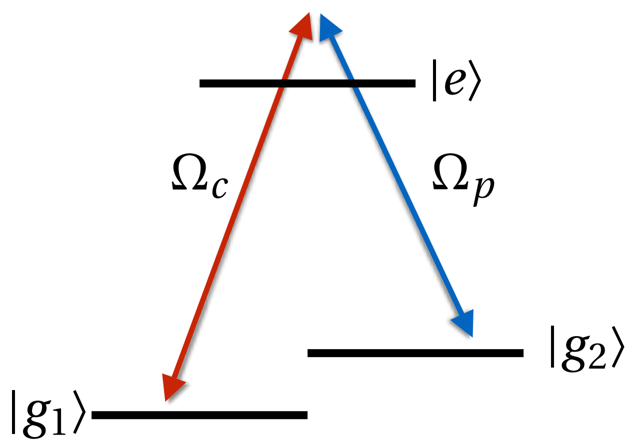
$$E_\pm = \pm E(x) \equiv \pm \frac{1}{2}\sqrt{\Omega_p^2 + \Omega_c(x)^2} - i\frac{1}{4}\Gamma$$

$$|\pm\rangle \quad \sim \quad |e\rangle \pm \frac{1}{E(x)}\left[\Omega_c(x)|g_1\rangle + \Omega_p |g_2\rangle\right]$$

here: $\Omega_{p.c} \gg \Gamma$ and $\Delta = 0$

Born-Oppenheimer (Adiabatic) Approximation

- **Hamiltonian**



$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \begin{pmatrix} 0 & \frac{1}{2}\Omega_c(x) & 0 \\ \frac{1}{2}\Omega_c(x) & -\Delta - i\frac{1}{2}\Gamma & \frac{1}{2}\Omega_p \\ 0 & \frac{1}{2}\Omega_p & 0 \end{pmatrix}$$

- **Born-Oppenheimer (adiabatic) approximation**

dark state

$$E_0 = 0$$

$$|0\rangle = \cos\alpha(x)\,|g_1\rangle - \sin\alpha(x)\,|g_2\rangle$$

$$\tan\alpha(x) = \frac{\Omega_c(x)}{\Omega_p}$$

bright states

$$E_\pm = \pm E(x) \equiv \pm\frac{1}{2}\sqrt{\Omega_p^2 + \Omega_c(x)^2} - i\frac{1}{4}\Gamma$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}\left\{|e\rangle \pm \left[\sin\alpha(x)\,|g_1\rangle + \cos\alpha(x)\,|g_2\rangle\right]\right\}$$

we will expand the atomic wave function in these BO states

Expanding in BO-Channels

- **We expand in (adiabatic) Born-Oppenheimer channels**

$$|\psi(x,t)\rangle = \psi_0(x,t)\,|E_0(x)\rangle + \psi_+(x,t)\,|E_+(x)\rangle + \psi_-(x,t)\,|E_-(x)\rangle$$

'dark' BO channel

'bright' BO channels

spin/*x*-dependent dressed states

- **… to obtain the 3-channel Schrödinger equation**

Hamiltonian

$$\mathscr{H} = \frac{1}{2m}\left(\frac{\hbar}{i}\frac{\partial}{\partial x} - A(x)\right)^2 + V(x)$$

vector potential **BO potential**

$$-i\hbar\partial_x\,|E_\sigma(x)\rangle = \sum_\mu |E_\mu(x)\rangle\, A_{\mu\sigma}(x)$$

$$V_{\mu\sigma}(x) = E_\sigma(x)\delta_{\mu\sigma}$$

Rem.: We will be interested in the parameter regime of 'approximate BO-decoupling' + non-adiabatic corrections.

Expanding in BO-Channels

- **… to obtain the Hamiltonian** for wave functions (ψ_0, ψ_+, ψ_-)

$$H = -\frac{\hbar^2}{2m}\left[\frac{\partial}{\partial x} + \frac{\alpha'}{\sqrt{2}}\begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}\right]^2 + E(x)\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

vector potential (non-adiabatic coupling) **potential**

here:

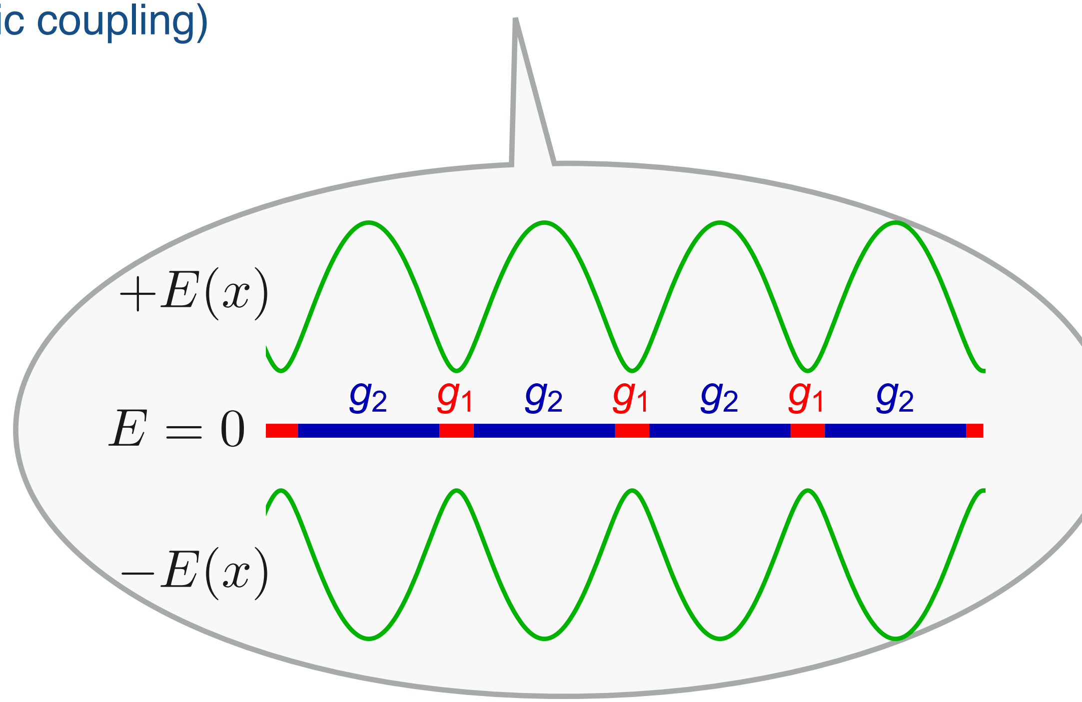
$\Delta = 0$
$\Gamma = 0$

$$\alpha' \equiv \frac{d\alpha}{dx} = k\varepsilon \frac{\cos(kx)}{\varepsilon^2 + \sin^2(kx)}$$

validity of adiabatic approximation:

$$\frac{1}{\epsilon} E_R \equiv \frac{\hbar^2 k^2}{2m} \ll \Omega_c, \Omega_p$$

$$\epsilon \sim \frac{\ell}{\lambda} \ll 1$$



compare Olshanii & Dum, PRL 1996

Expanding in Adiabatic Channels: Version 2

- **… to obtain the Hamiltonian** for wave functions (ψ_0, ψ_+, ψ_-)

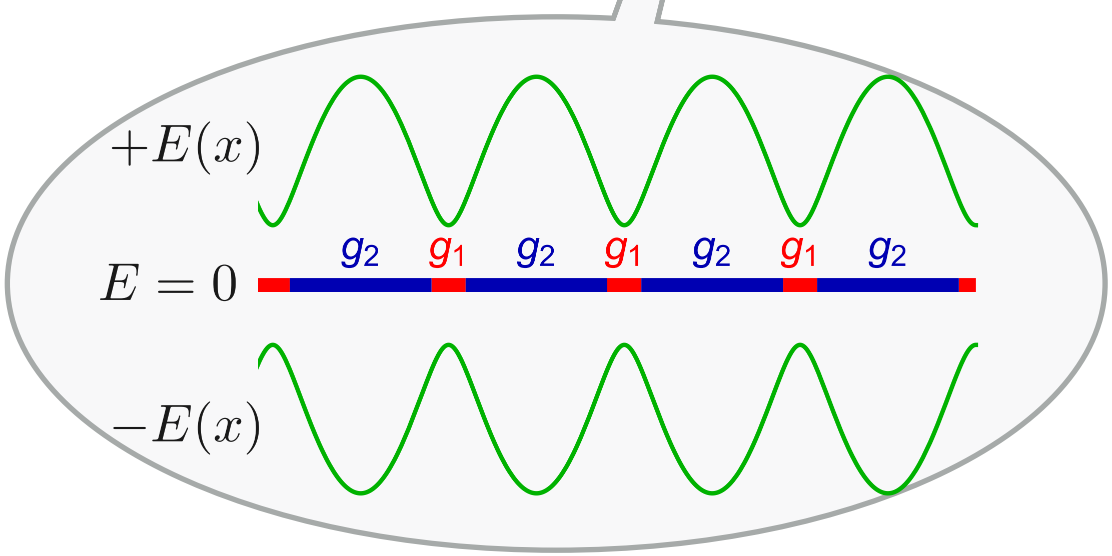
$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + E(x)\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

← **adiabatic**

$$-\frac{\hbar^2}{2m}\left(\frac{\partial}{\partial x}\frac{\alpha'}{\sqrt{2}} + \frac{\alpha'}{\sqrt{2}}\frac{\partial}{\partial x}\right)\begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + \frac{\hbar^2}{2m}\frac{(\alpha')^2}{2}\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

$V_{\text{na}}(x)$

first order correction



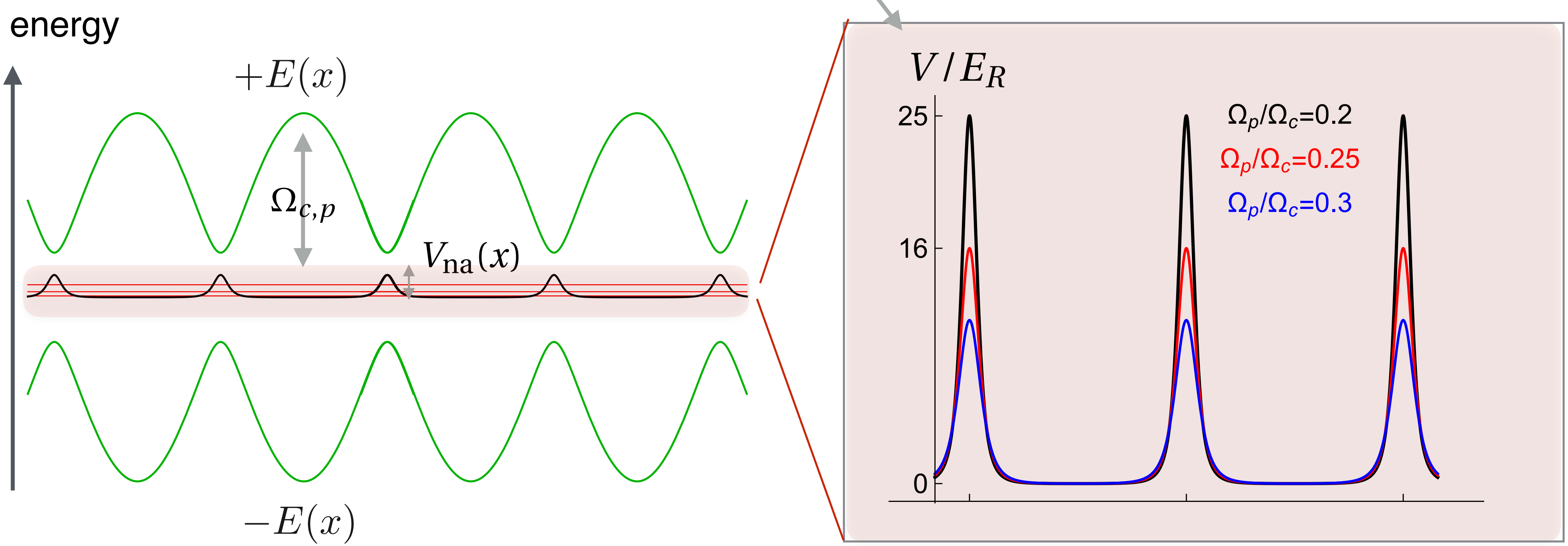
‘Dark State’ Optical Lattice

- **… including the first order non-adiabatic correction**

$$i\hbar \frac{\partial}{\partial t}\psi_0(x,t) = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V_{\rm opt}(x)\right]\psi_0(x,t)$$

‘dark state’ channel

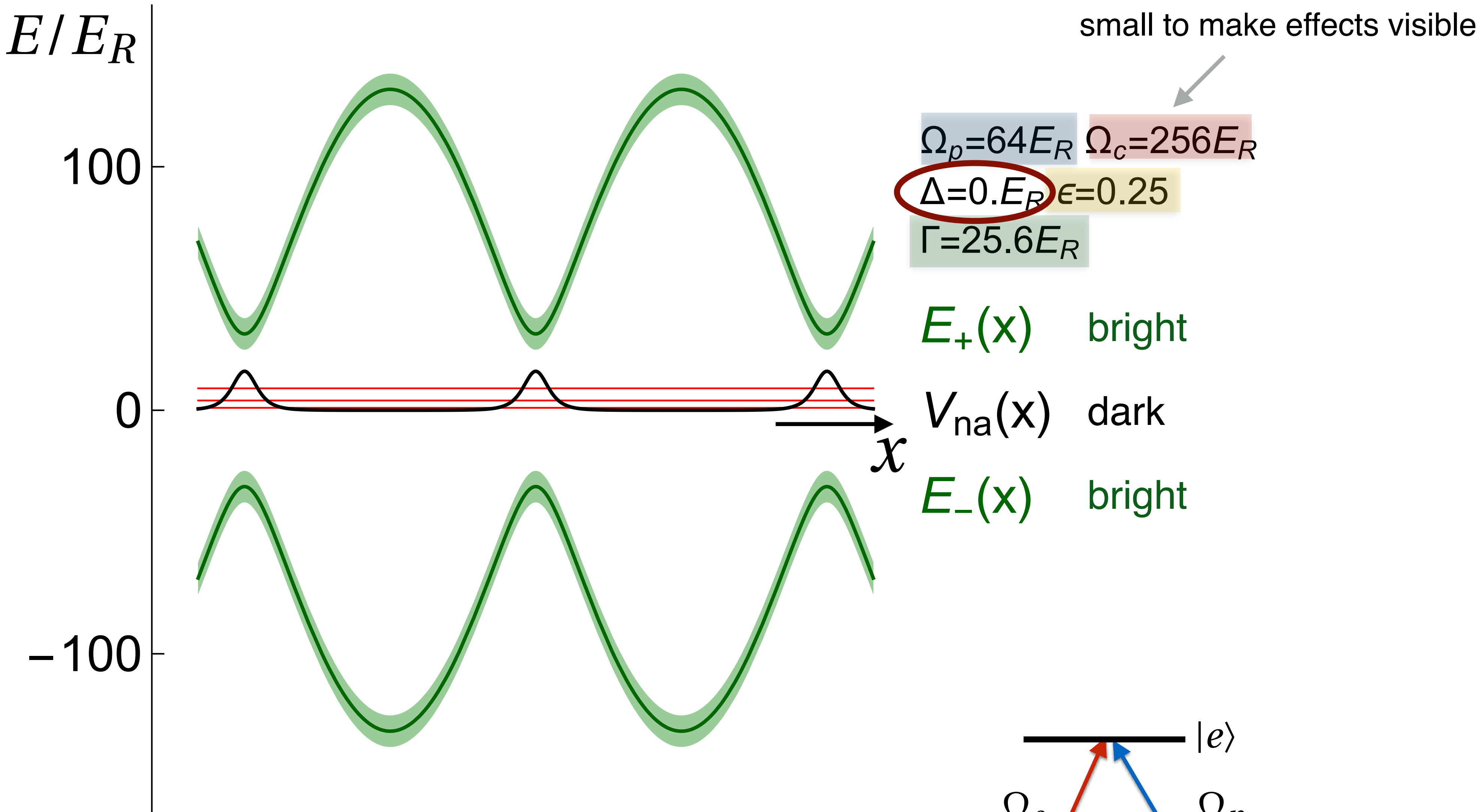
$$V_{\rm opt}(x) \equiv V_{\rm na}(x) = E_R \frac{\varepsilon^2 \cos^2(kx)}{[\varepsilon^2 + \sin^2(kx)]^2}$$

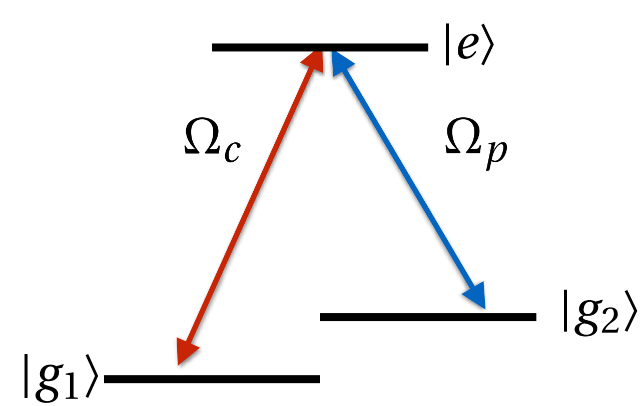


✓ conservative

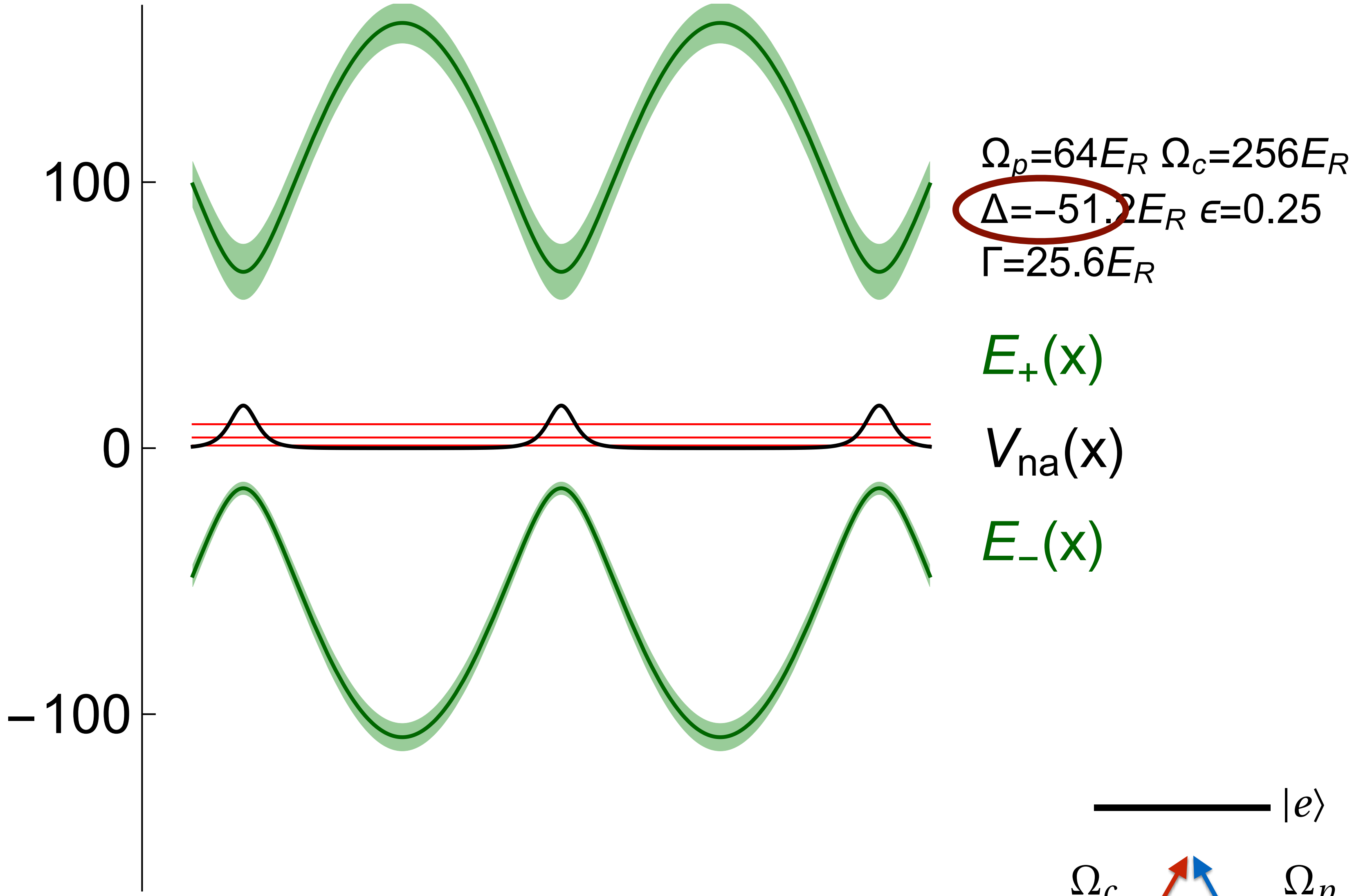
✓ sub-wavelength structures

Visualizing Adiabatic Potentials (1): on resonance

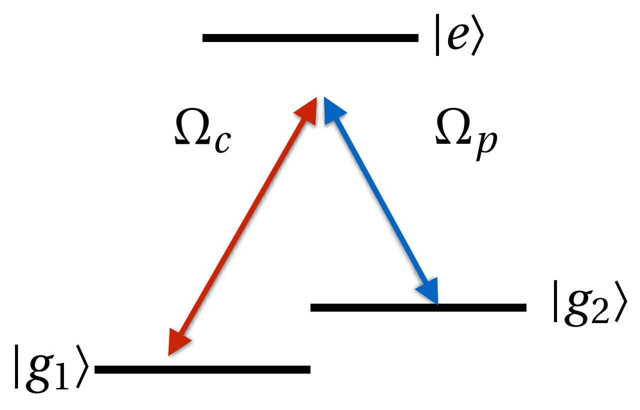




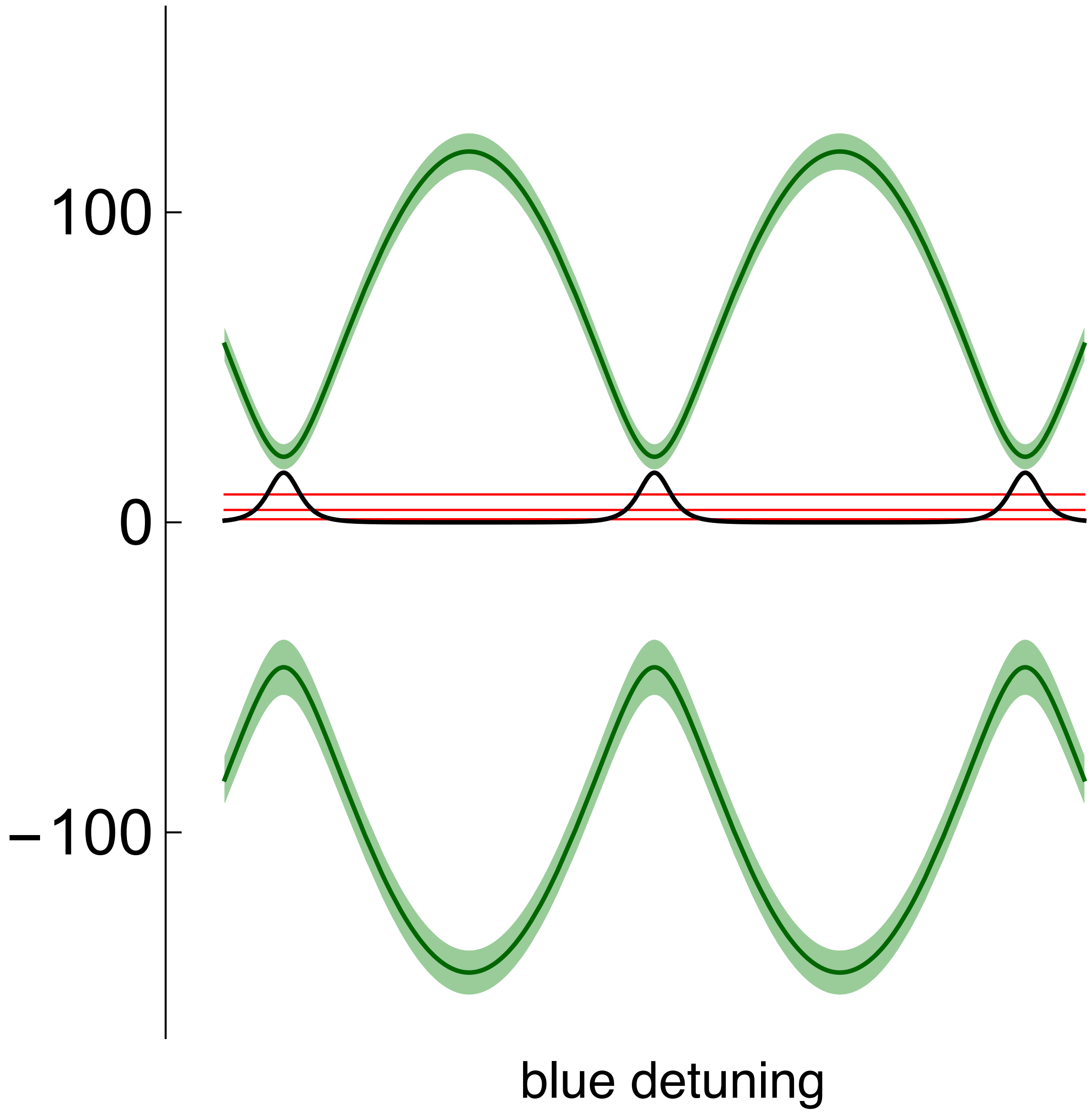
Visualizing Adiabatic Potentials (1): red detuning

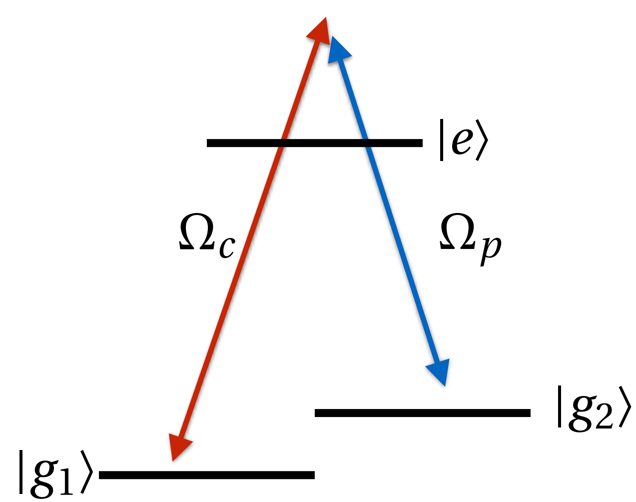


red detuning

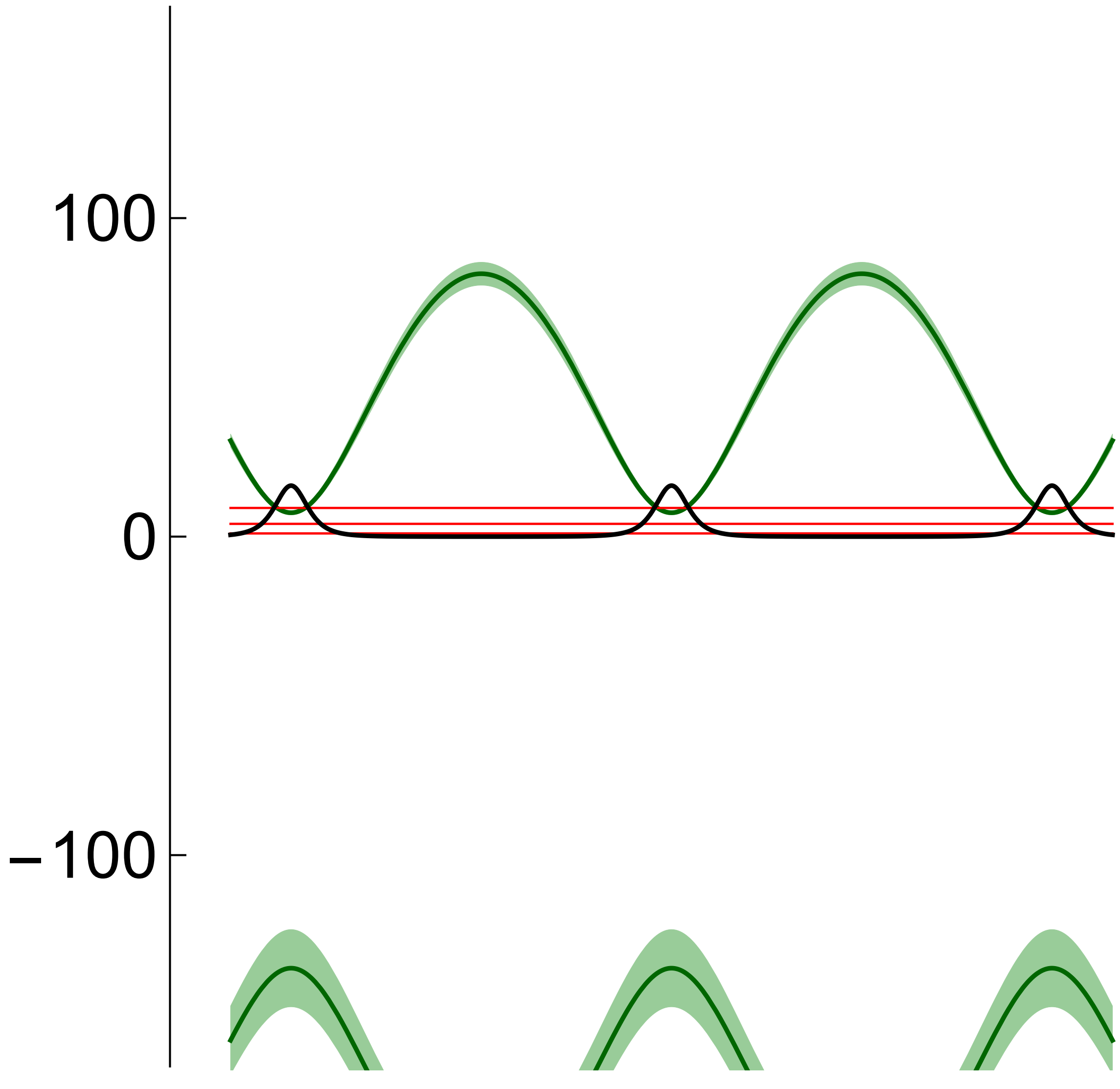


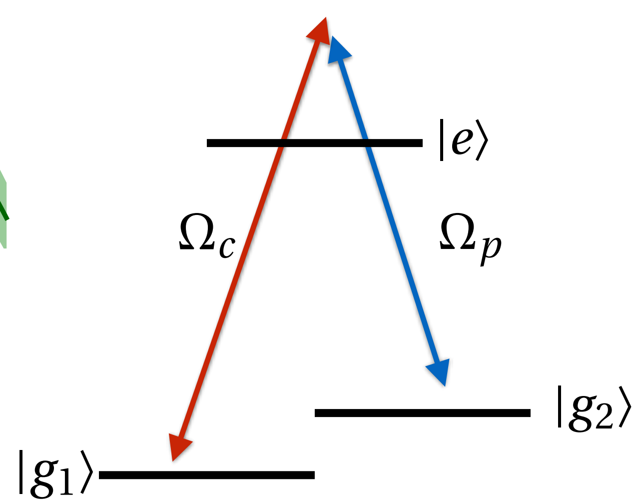
Visualizing Adiabatic Potentials (2): blue detuning



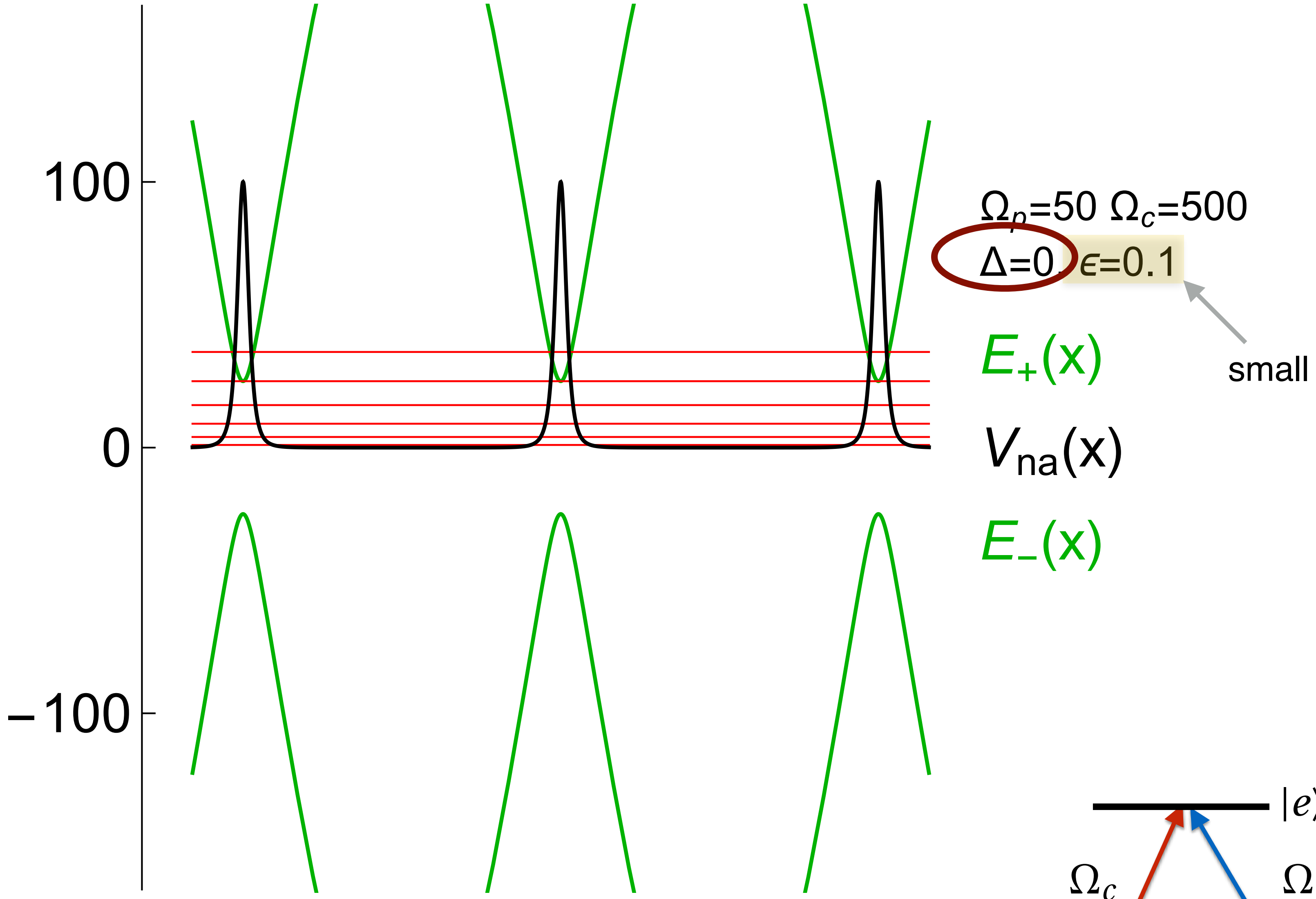


Visualizing Adiabatic Potentials (2): blue detuning

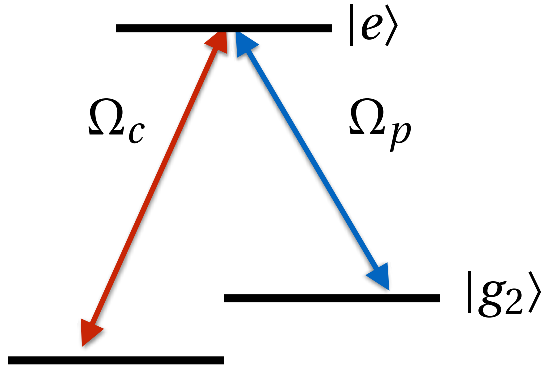




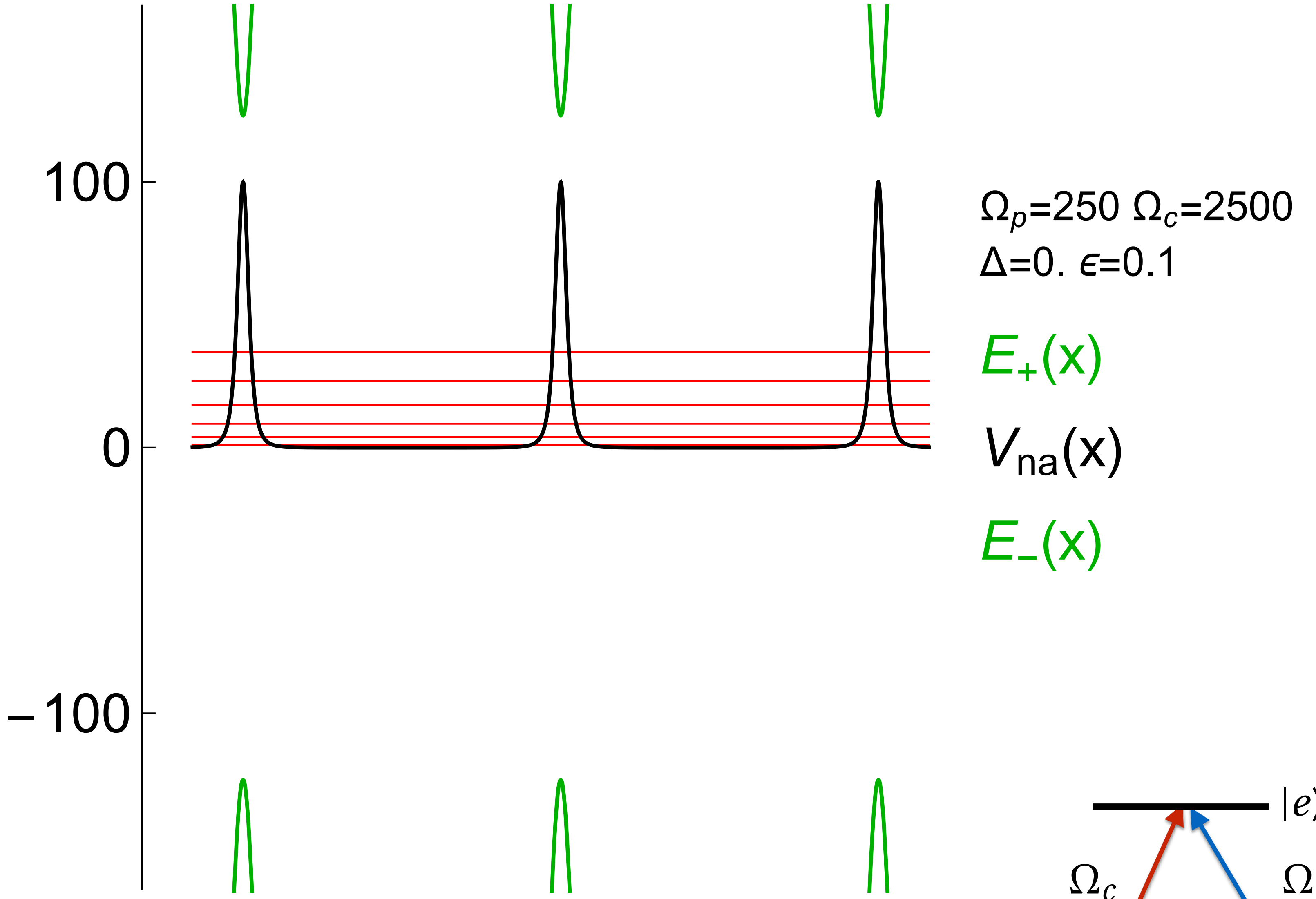
Visualizing Adiabatic Potentials (3): on resonance



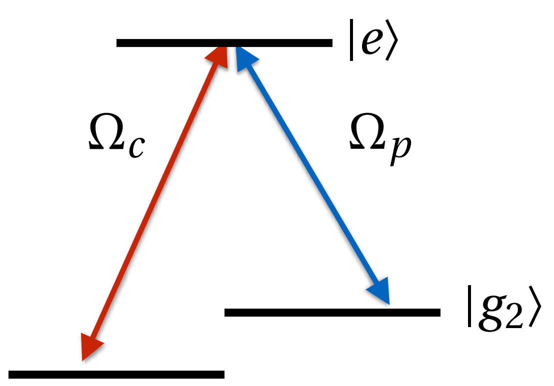
increasing laser power



Visualizing Adiabatic Potentials (3): on resonance

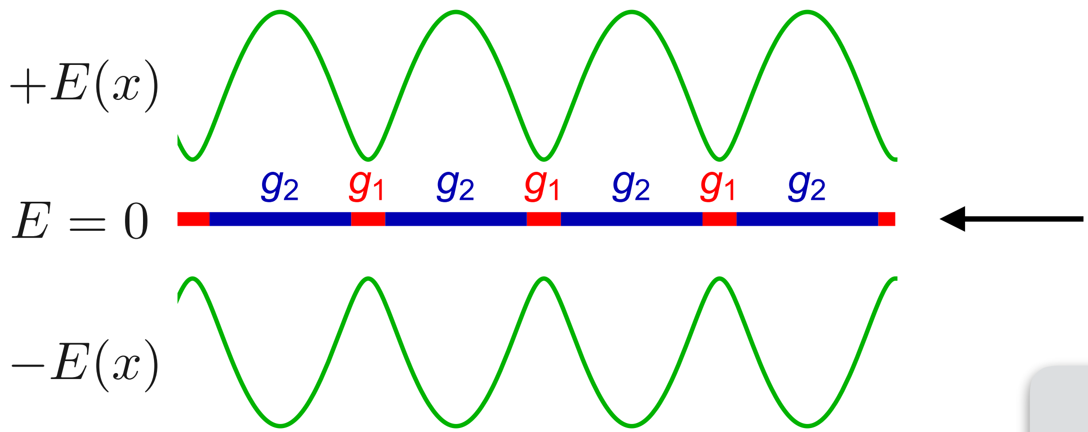


increasing laser power



Discussion

1. Zero order adiabatic approximation

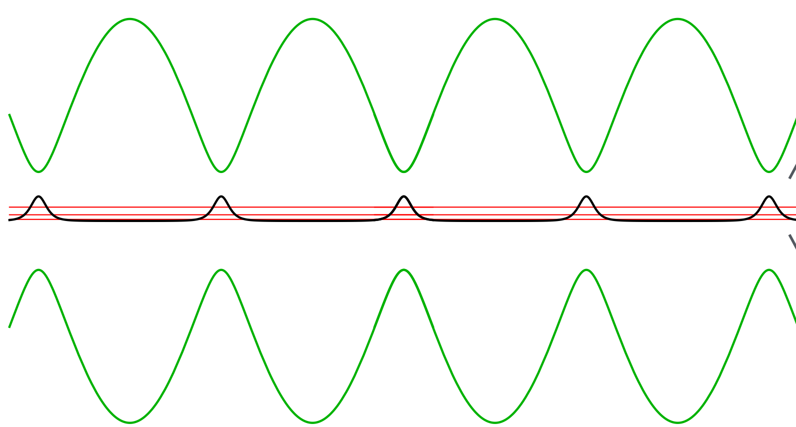


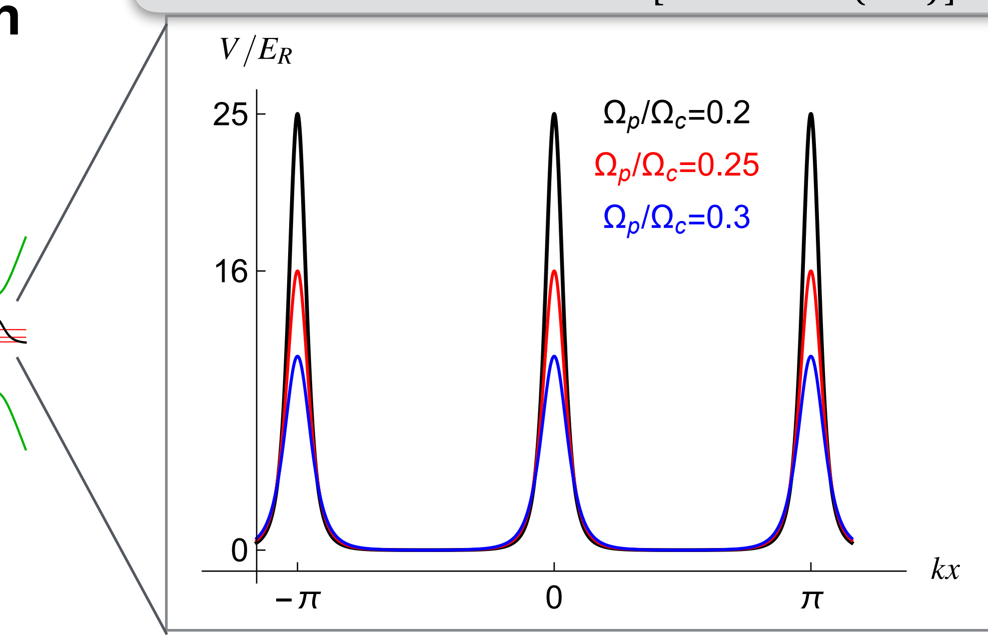
✓ no optical potential

✓ sub-wavelength structure in internal state / interaction

$$V_{\text{opt}}(x) \equiv V_{na}(x) = E_R \frac{\varepsilon^2 \cos^2(kx)}{[\varepsilon^2 + \sin^2(kx)]^2}$$

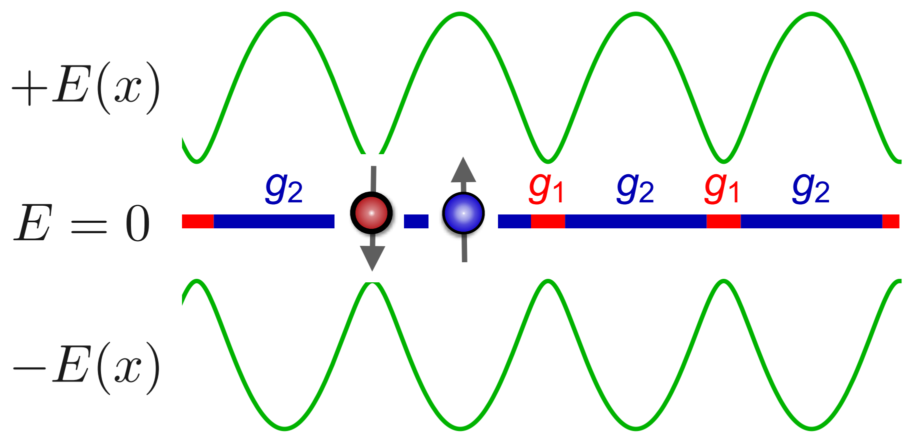
2. First order adiabatic approximation



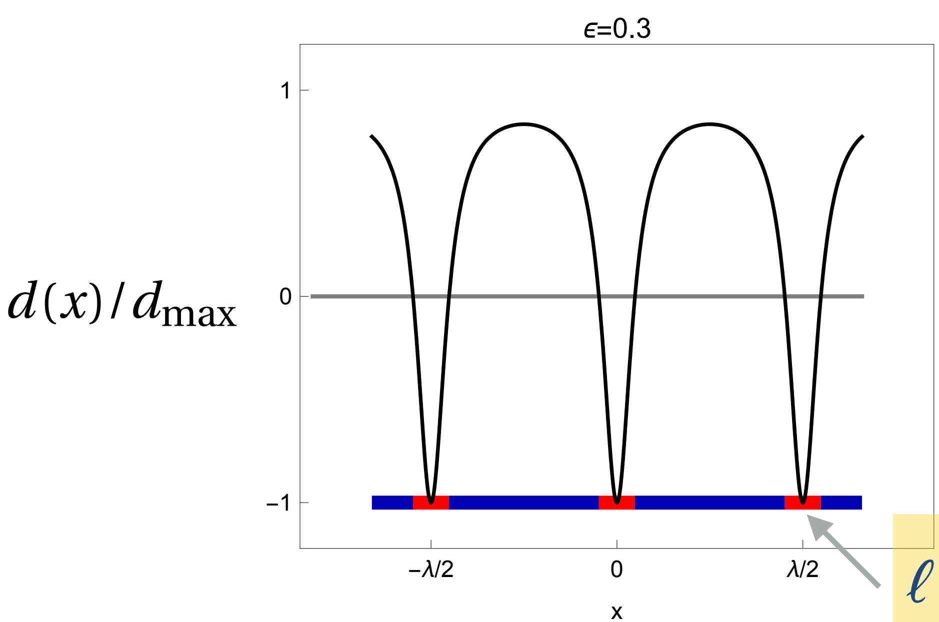


3. **Exact bandstructure:** lifetime due to channel couplings

1. Zero order adiabatic approximation



spatial variation of dipole moment



✓ dipole moment can be any angle

✓ limit to spatial structure:

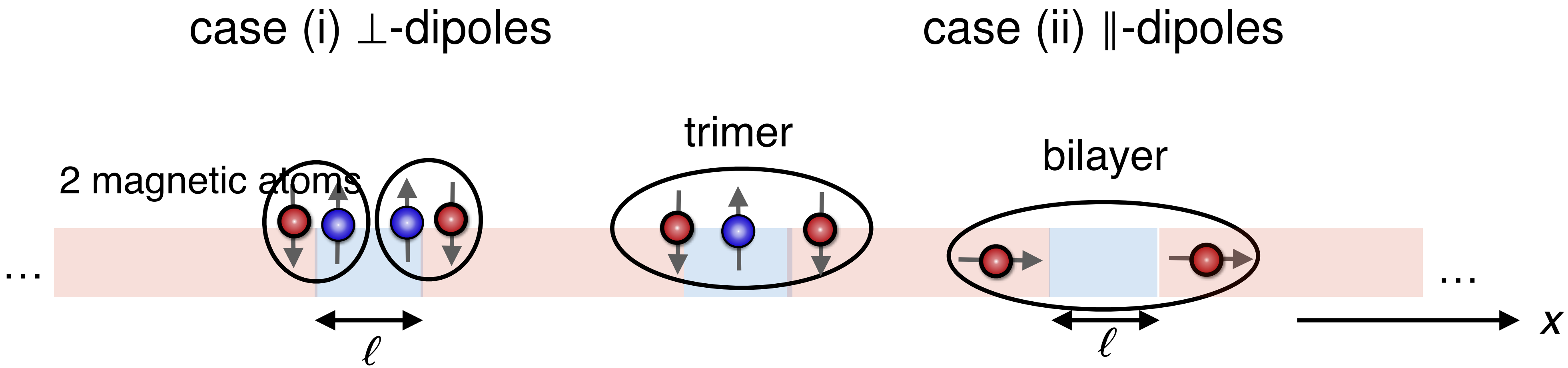
$$\ell > \ell_0$$

(at least in our 1D model)

$\ell = \epsilon\lambda/2\pi$ with $\epsilon \equiv \dfrac{\Omega_p}{\Omega_c} \ll 1$

Quantum Many-Body Physics

- **Two, three etc. particles [bound states]**



'domain wall molecule' as bound state

✓ molecule sees a lattice

energy scales :-)

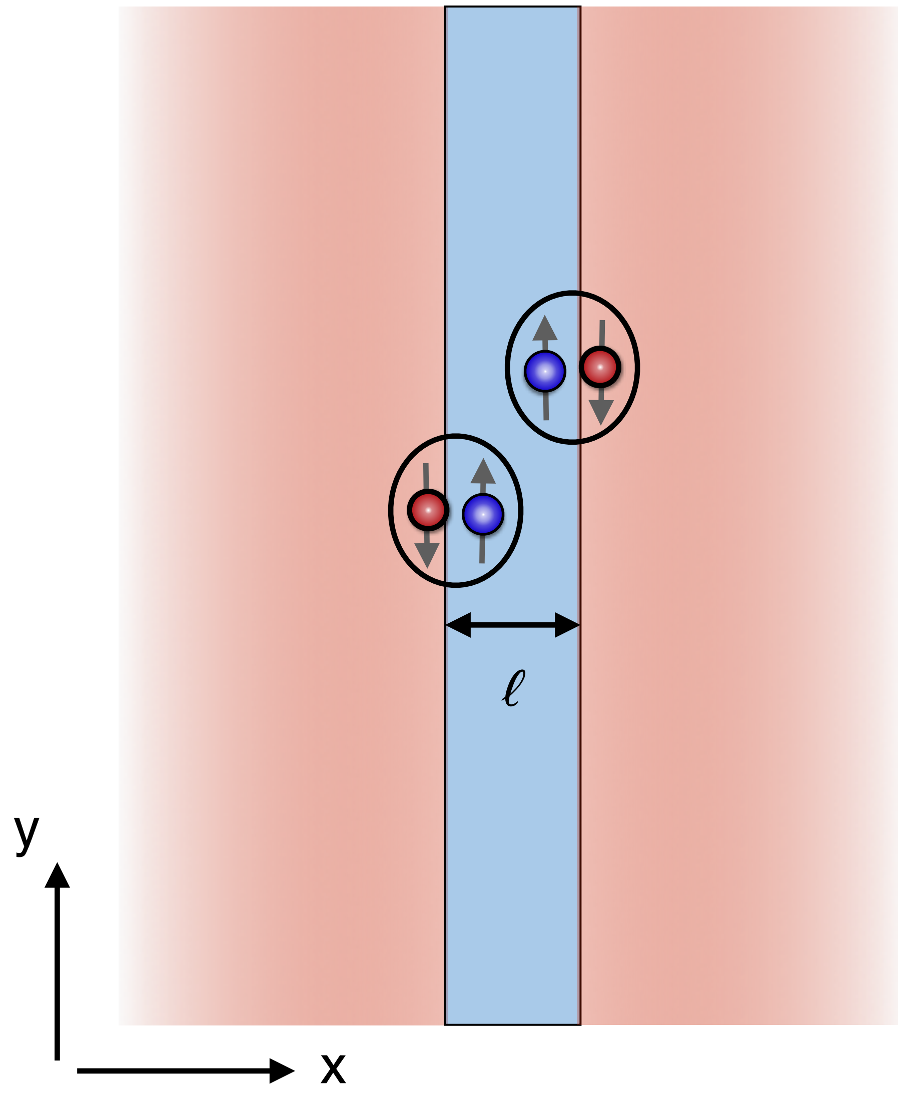
Hamiltonian

$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_1^2} - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_2^2} + \frac{\mu(x_1)\mu(x_2)}{|x_1 - x_2|^3}$$

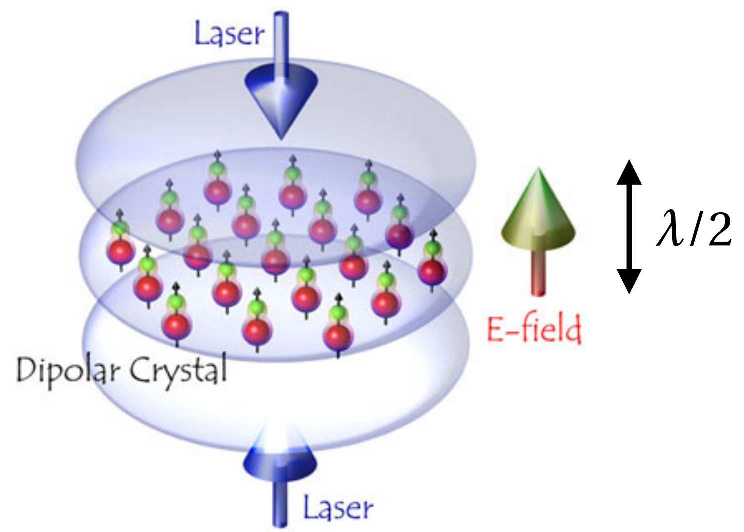
spatial variation of dipole moment

Quantum Many-Body Physics [Preview]

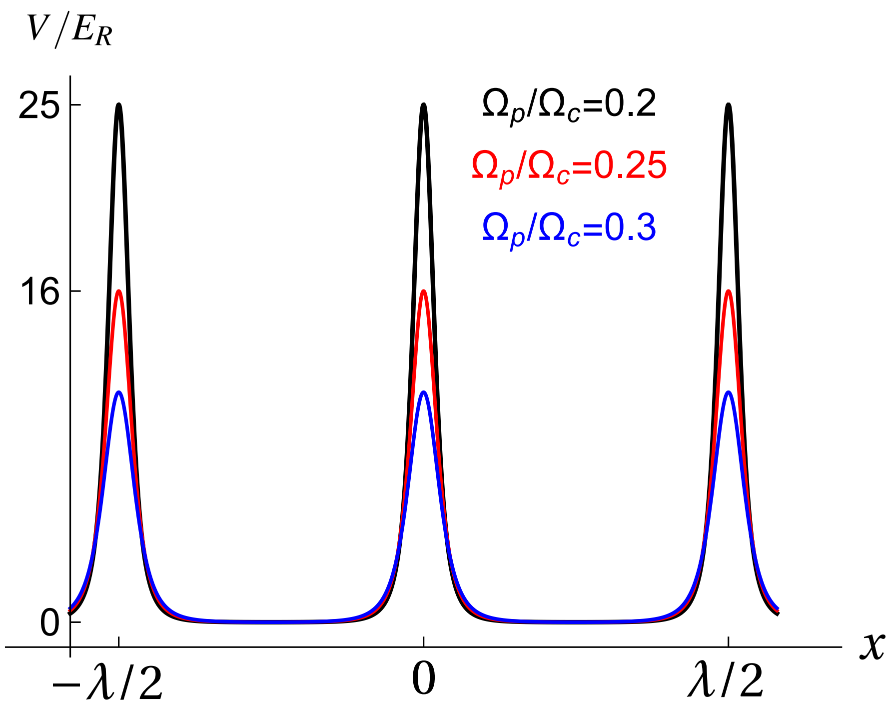
- ***Sub-wavelength*** **bilayer**



- **polar molecules in bilayer from standing light wave**



2. First order adiabatic approximation



$$V_{\rm opt}(x) \equiv V_{na}(x) = E_R \frac{\varepsilon^2 \cos^2(kx)}{[\varepsilon^2 + \sin^2(kx)]^2}$$

Mapping to a Kronig-Penney potential:

$$H_\delta = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{\pi \lambda E_R}{4\epsilon} \sum_n \delta\left(x - \frac{\lambda}{2} n\right)$$

$$\epsilon \ll 1$$

properties:

✓ immune against laser noise

- intensity noise
- laser bandwidth ~ dephasing for Ω_c and Ω_p derived from same laser

✓ independent of detuning

validity:

✓ Born-Oppenheimer

- non-adiabatic coupling vs. separation of dressed states [laser power]

Band Structure

- **Bloch ansatz**

$$\psi_q(x) = e^{iqx} u_q(x)$$

$$u_q(x) = u_q(x + a)$$

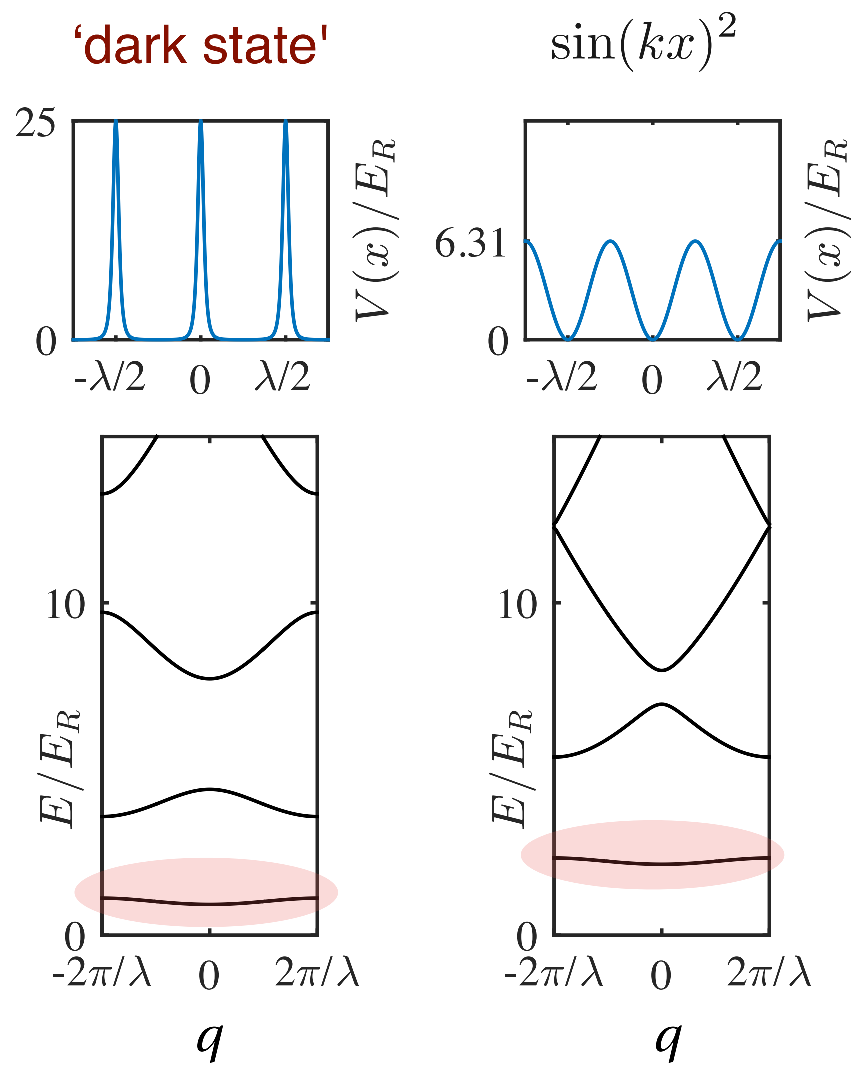
$q \in [-\frac{\pi}{a}, +\frac{\pi}{a})$

Brillouin zone

$a = \lambda/2$

lattice spacing

- **Band structure**



band spacing:

n^2 vs. n →

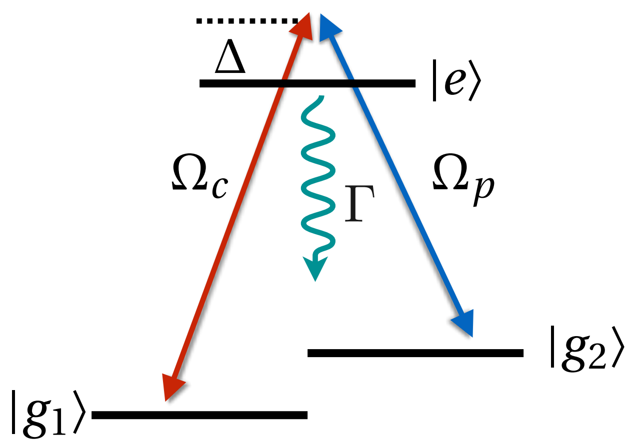
Wannier functions:

~ box vs. harmonic oscillator

same Bloch bandwidth J

3. Band Structure for Coupled BO-Channels

- **Hamiltonian**



- **Multichannel Bloch ansatz**

bare channel functions

$$\psi_q(x) = e^{iqx}\begin{pmatrix} u_{g_1}(x) \\ u_e(x) \\ u_{g_2}(x) \end{pmatrix}, \quad u_\lambda(x+a) = u_\lambda(x)$$

$q \in [-\pi/\lambda, \pi/\lambda)$

quasi-momentum

λ lattice spacing

- **Band structure**

$$\left[\frac{(\frac{\hbar}{i}\frac{\partial}{\partial x}+q)^2}{2m} + \begin{pmatrix} 0 & \frac{1}{2}\Omega_c(x) & 0 \\ \frac{1}{2}\Omega_c(x) & -\Delta - i\frac{1}{2}\Gamma & \frac{1}{2}\Omega_p \\ 0 & \frac{1}{2}\Omega_p & 0 \end{pmatrix}\right]\begin{pmatrix} u_{g_1} \\ u_e \\ u_{g_2} \end{pmatrix} = E(q)\begin{pmatrix} u_{g_1} \\ u_e \\ u_{g_2} \end{pmatrix}$$

complex / lossy band structure

bi-orthogonal set of eigenfunctions

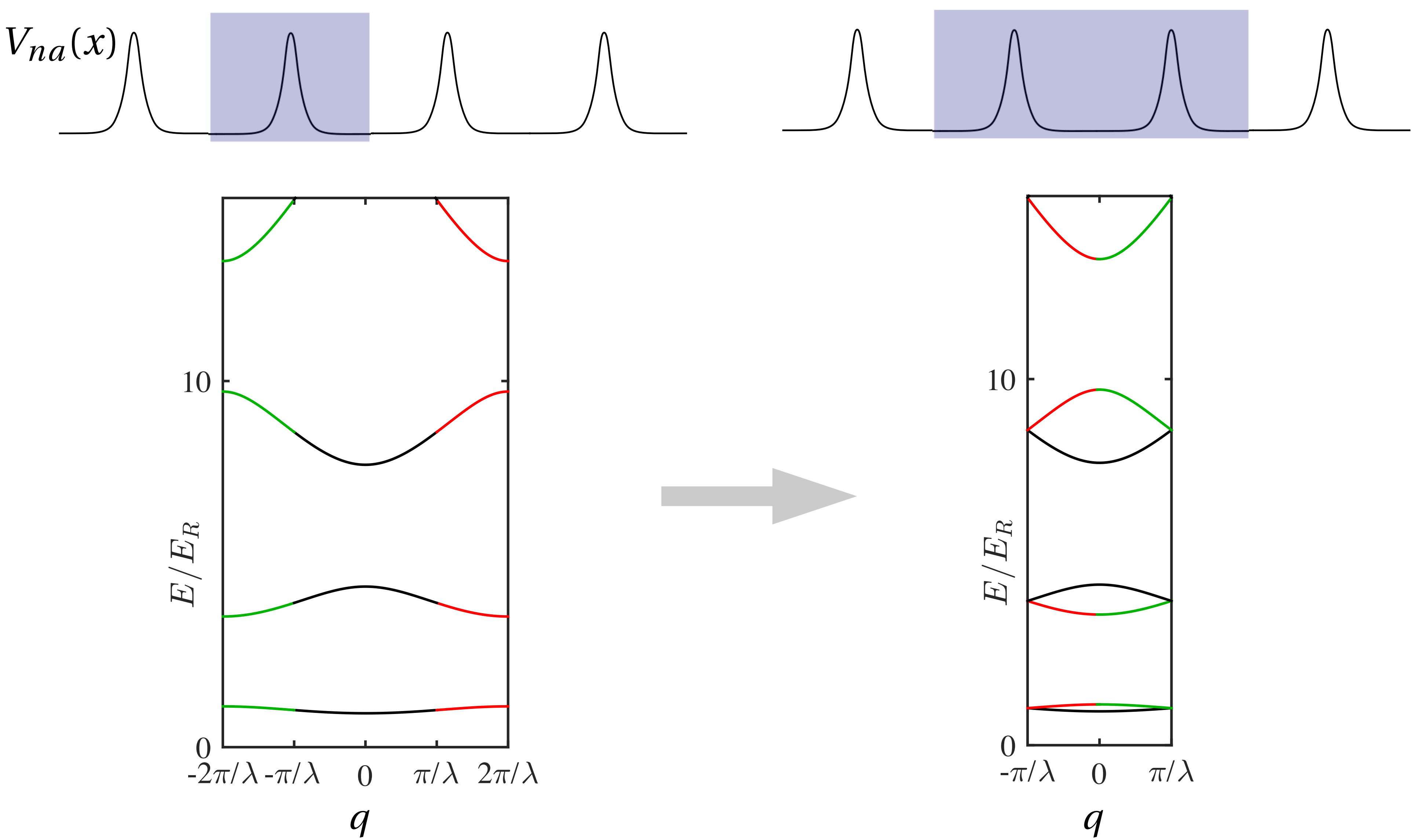
Translation symmetry of …

BO-Hamiltonian

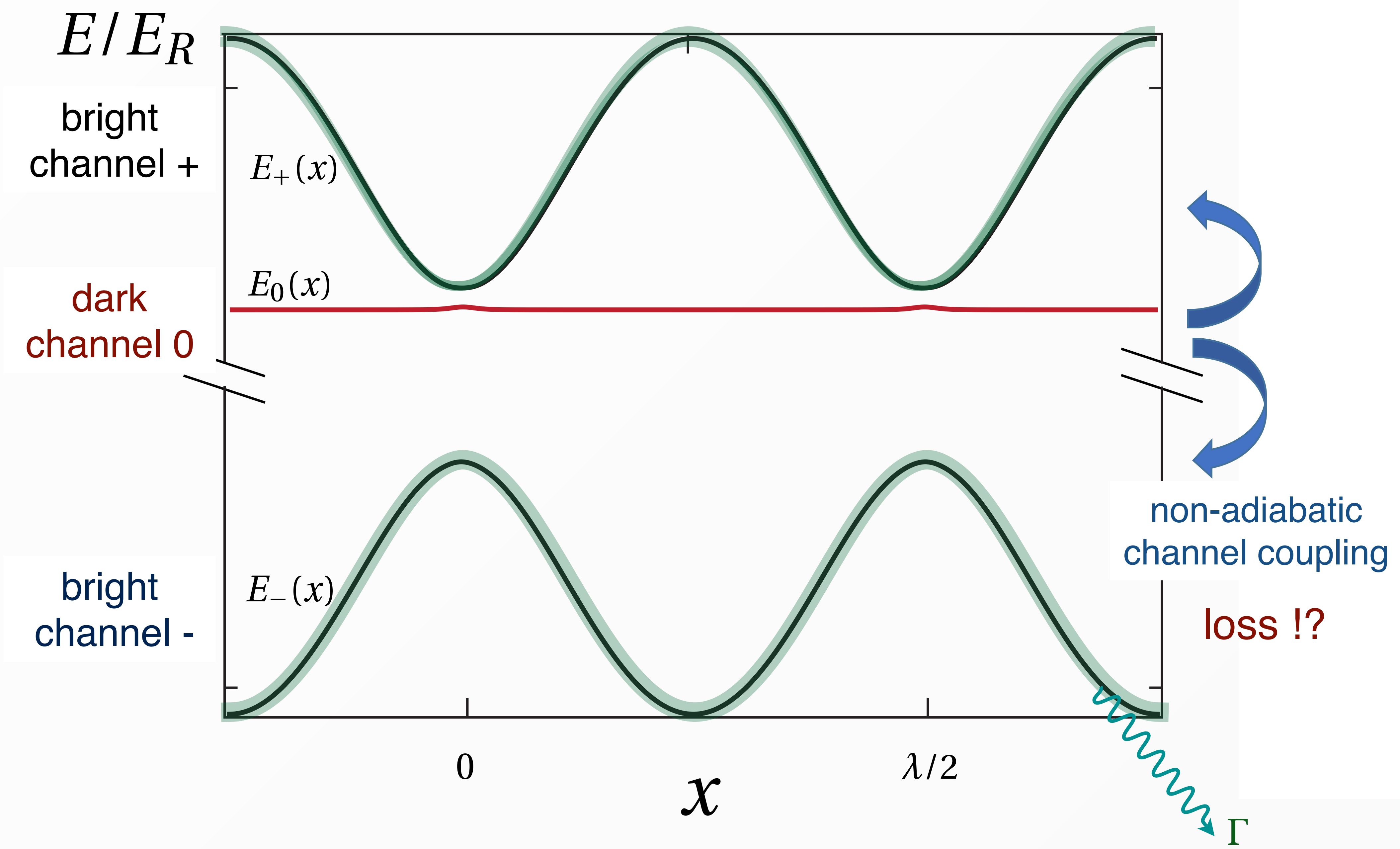
$\lambda/2$ unit cell

Exact Hamiltonian

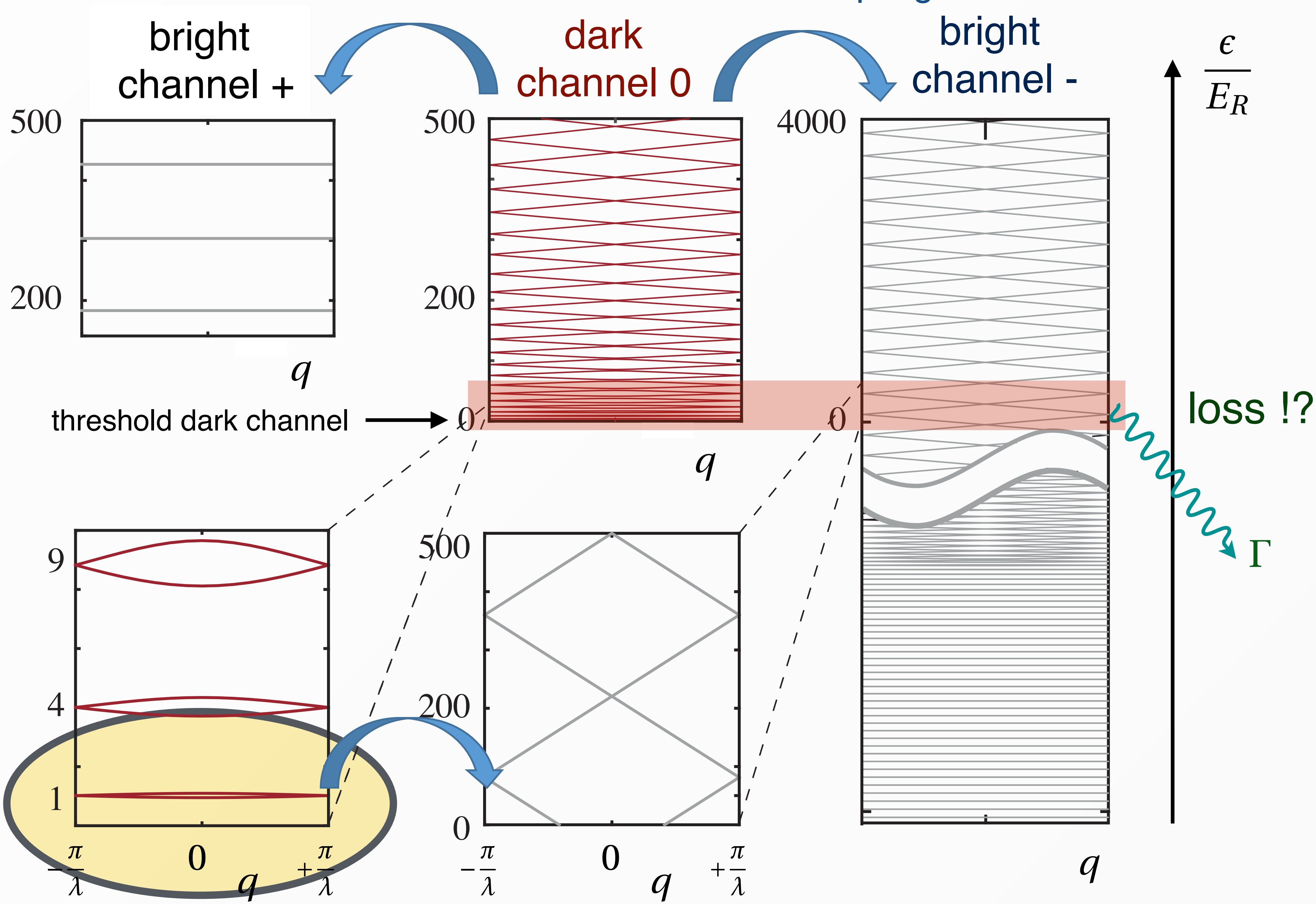
λ unit cell



Coupled BO-Channels



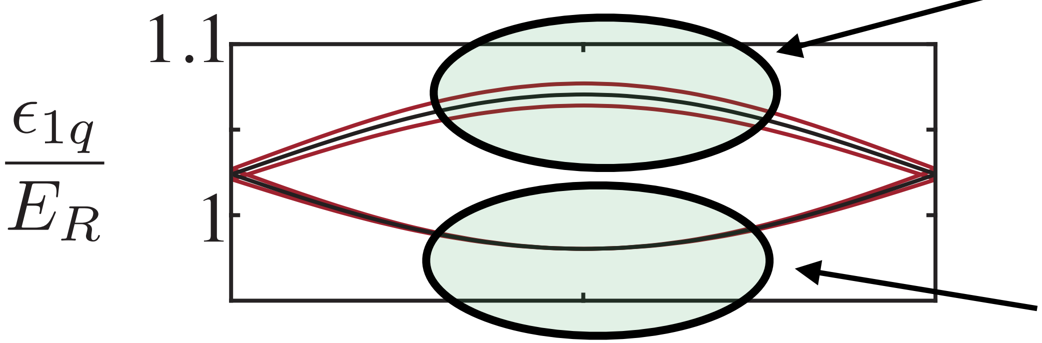
Coupled BO-Channels



decay of the lowest Bloch band (?)

Decay of the 'Lowest Bloch Band': $E(q) = \epsilon(q) - i\frac{1}{2}\gamma(q)$

- **case: non-adiabatic couplings << spontaneous emission**



The faster the atom, the higher the losses

q=0 stable

- energy dispersion

$$\epsilon_{1,q}^{(0)} \approx E_R \left\{ 1 + \frac{4\epsilon}{\pi^2} \left[1 - \cos\frac{\pi q}{k} \right] \right\}$$

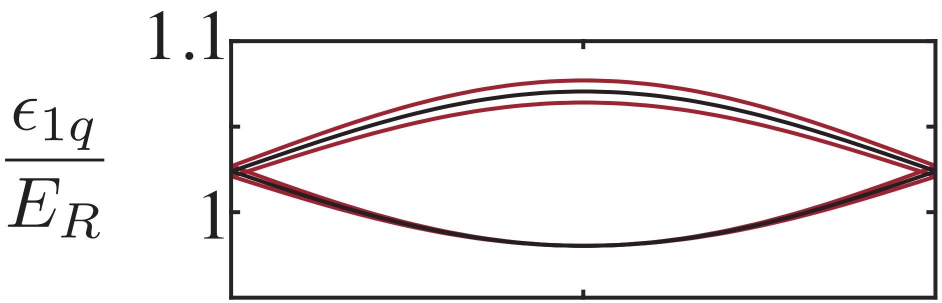
- decay

$$\gamma_{1,q} \approx \gamma_1 \sin^2(\pi q / 2k)$$

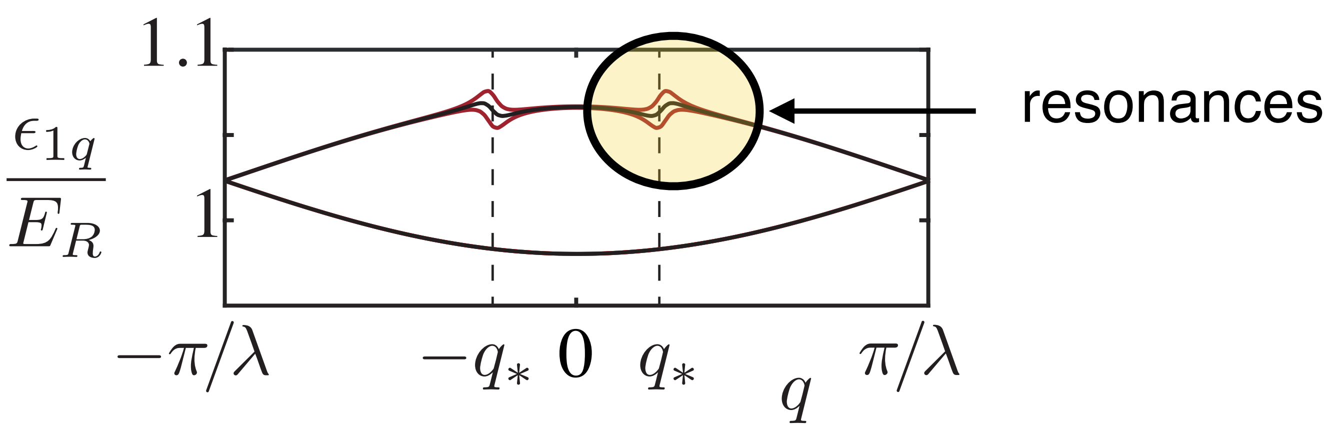
engineering q-dependent dissipation of Bloch bands

Decay of the 'Lowest Bloch Band': $E(q) = \epsilon(q) - i\frac{1}{2}\gamma(q)$

- **case: non-adiabatic couplings << spontaneous emission**



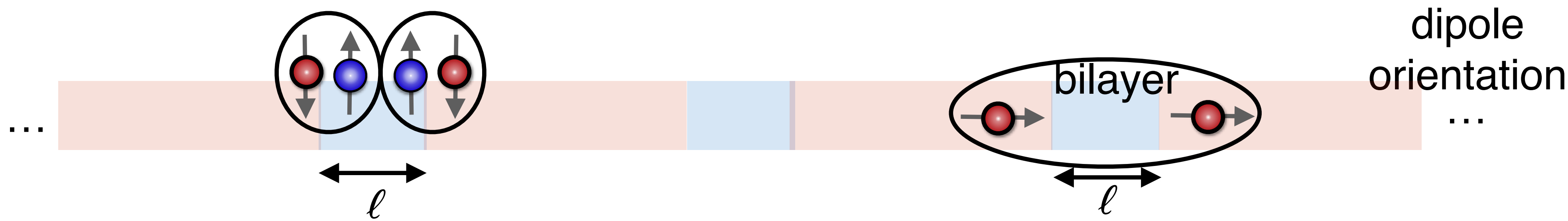
- **case: non-adiabatic couplings >> spontaneous emission**



We can always find a parameter regime, where these losses can be made small.

Returning to quantum many-body physics …

Bound States@Interface: ‘Domain-Wall Molecules'



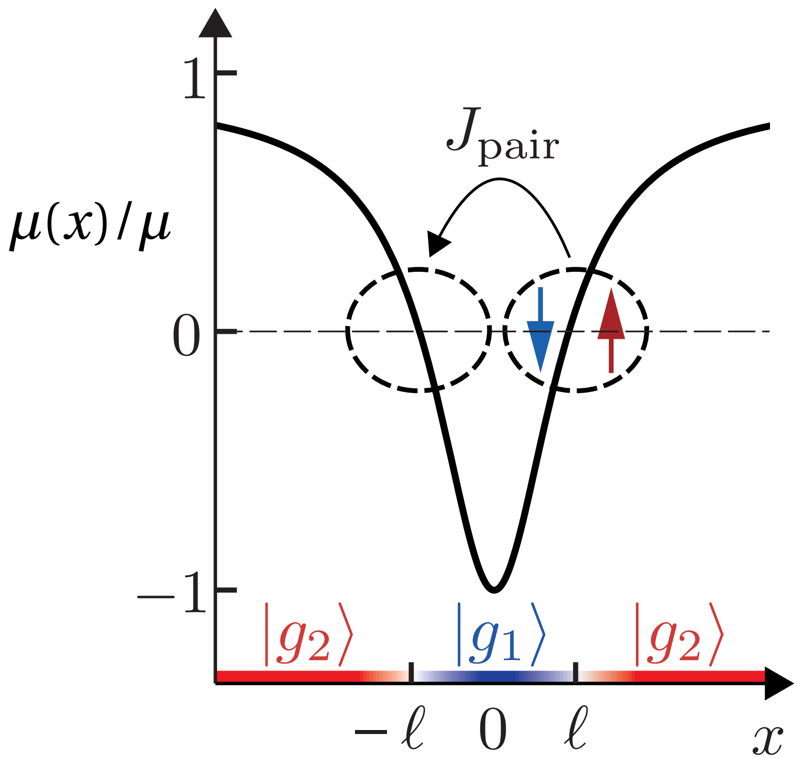
Hamiltonian: for 2 atoms

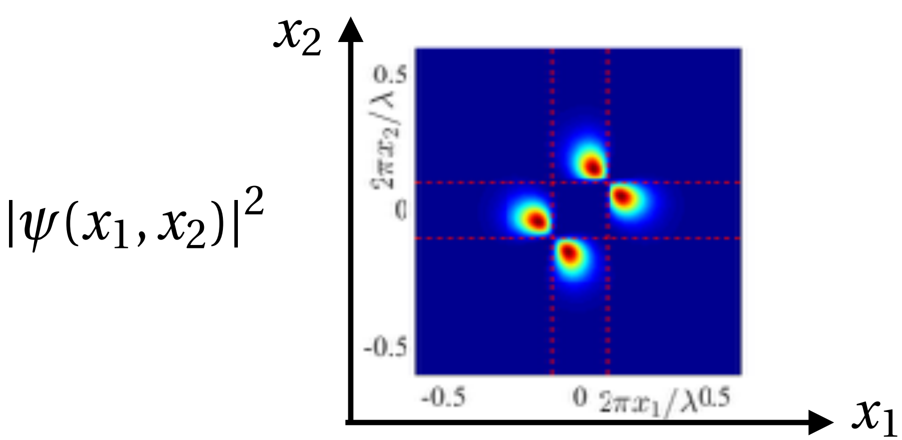
$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_1^2} - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_2^2} + V_{\mathrm{na}}(x_1) + V_{\mathrm{na}}(x_2) + \frac{d(x_1)d(x_2)}{|x_1 - x_2|^3}$$

[Rem.: validity - collisions in presence of light]

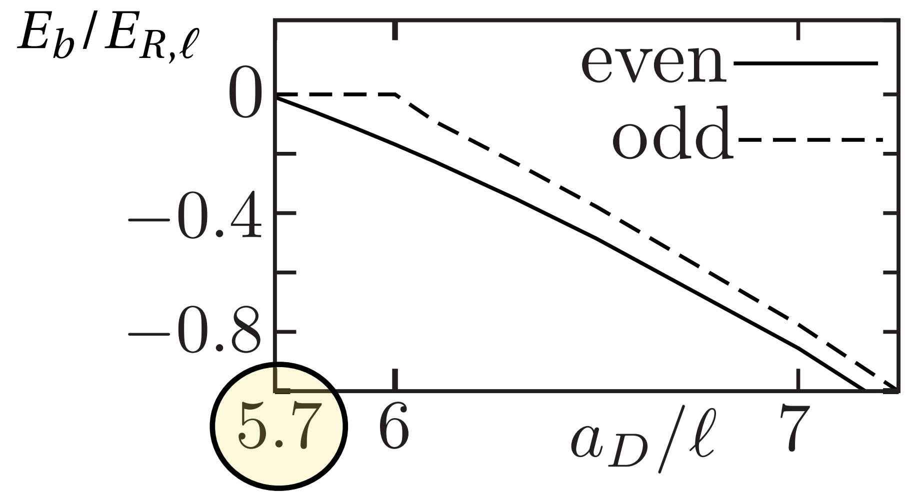
Spatial variation of dipole moment

$$\mu(x) = \mu_1 \sin^2 \alpha(x) + \mu_2 \cos^2 \alpha(x)$$

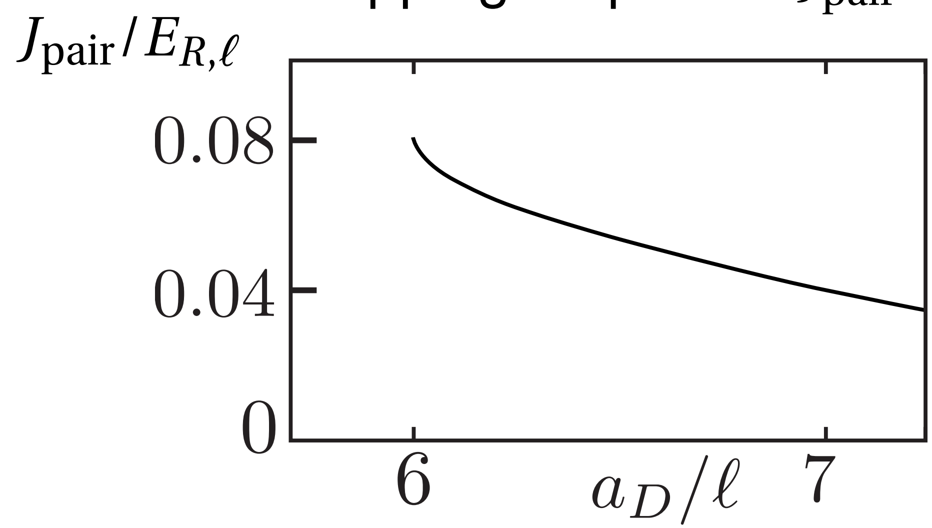




Bound state energies E_b



Hopping amplitude J_{pair}



$E_{R,\ell} \equiv \hbar^2/2m\ell^2 = E_R/\epsilon^2$

$\epsilon = 0.1$

Conclusion / Outlook

Topics we have not talked about

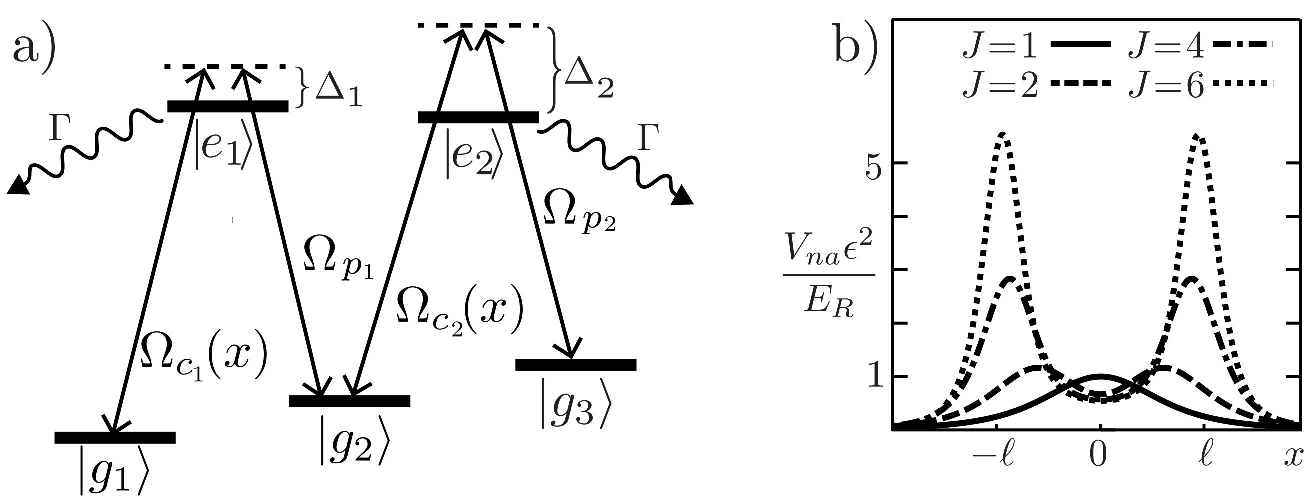
✓ loading

✓ measurement

✓ *loss* for many-atom situation (?)

Other atomic configurations

'atomic quantum dots'



patch-work lattice