Supersymmetric Insulating phases of topo-superconductors

Theory and realizations

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המחלקה לחומר מעובה

Collaborators

- Eran Sagi
- Hiromi Ebisu
- Yukio Tanaka
- Ady Stern

1. arXiv:1806.03304 Spin liquids from Majorana Zero Modes in a Cooper Box-] PRB
Authors: Eran Sagi, Hiromi Ebisu, Yukio Tanaka, Ady Stern, Yuval Oreg

2. arXiv:1811.04474 Emergent supersymmetry in a chain of Majorana Cooper pair boxes -] PRL
Authors: Hiromi Ebisu, Eran Sagi, Yuval Oreg

Array of conventional Josephson junctions

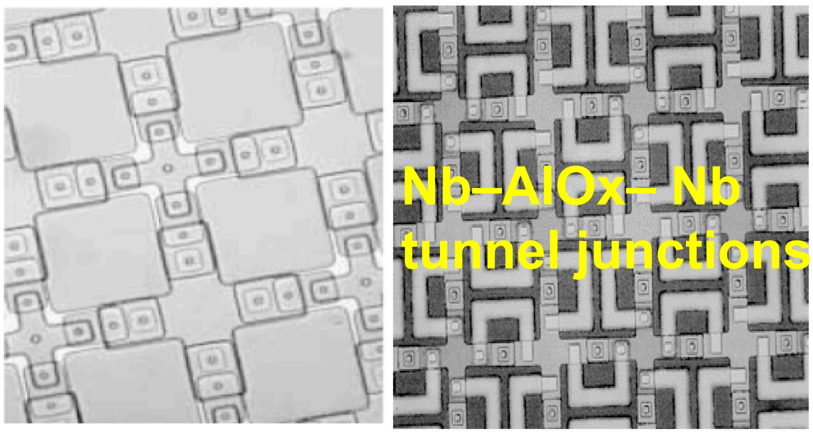

Figure 1. Photograph of unshunted (left) and shunted (right) Josephson junction arrays.

New Developments in Josephson Junctions Research, 2010: 25-44
ISBN: 978-81-7895-328-1 Editor: Sergei Sergeenkov

2 **Experimental and theoretical study on 2D ordered and 3D disordered SIS-type arrays of Josephson junctions**

Fernando M. Araujo-Moreira[1] and Sergei Sergeenkov[2]
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Topological

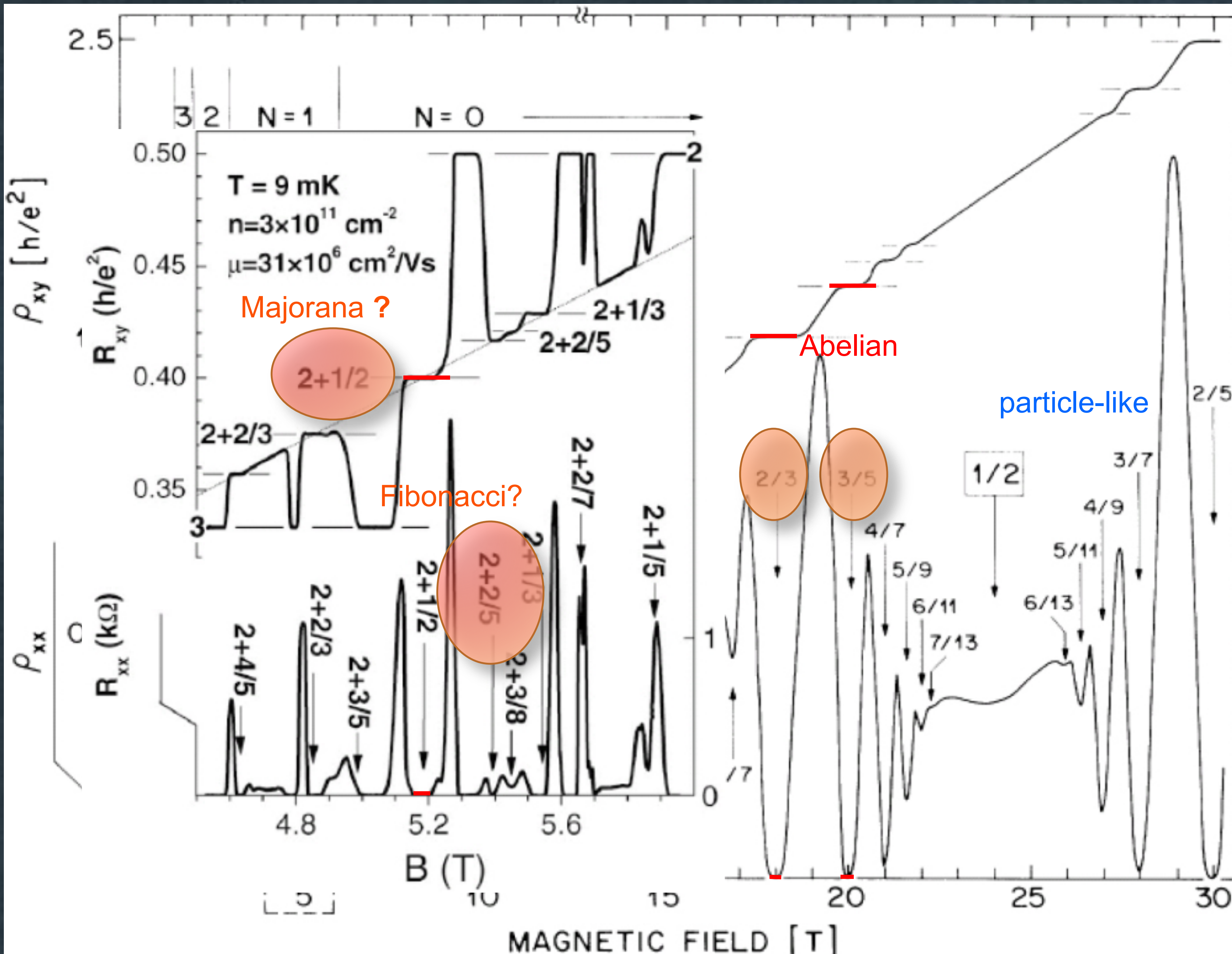
$E_J \gg E_c$ Superconductor $\quad t \gg U$ Metal

$E_c \gg E_J$ Insulator $\quad U \gg t$ Insulator

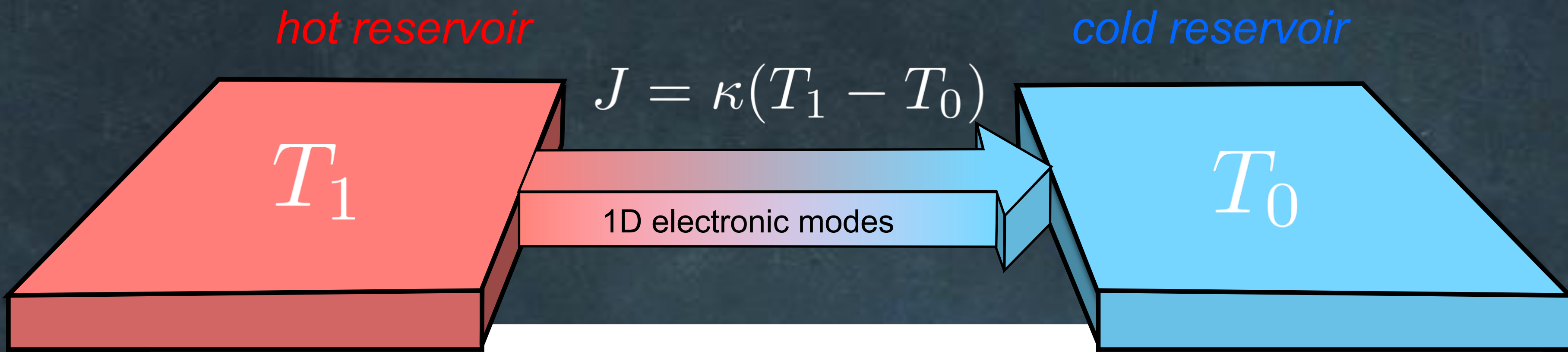
Has internal degrees: MZM

Outline

- Short intro to central charge, heat conductance, and Fibonacci particles
- Superconductor and insulator phases of an array of Josephson junctions
- Equivalence between Majo Zero Modes in a Cooper box and Spins
- 1D models and Experimental signatures
- SUSY Tri-critical Ising transition in the insulating phase with central charge 7/10
- Extension to 2D?



T = 9 mK
n=3×10^11 cm^-2
μ=31×10^6 cm^2/Vs
Majorana ?
Fibonacci?
Abelian
particle-like
2+1/2
2+2/5
2+1/3
2+2/3
2+4/5
2+3/5
2+2/7
2+1/5
2+3/8
2/3
3/5
4/7
5/9
6/11
7/13
1/2
6/13
5/11
4/9
3/7
2/5
N = 1
N = 0
ρxy [h/e²]
Rxy (h/e²)
ρxx
Rxx (kΩ)
B (T)
MAGNETIC FIELD [T]



- Edge modes in general have quantized heat conductance:

$$\kappa = c\frac{k_B^2 \pi^2}{3h} T$$

- C: Is by definition the central charge.
-     Integer for integer QHE and abelian FQHE [et al. Heiblum]
-     Fractional for the non abelian states
-     For $\nu = 5/2$ [MR] c = integer + 1/2
-     For $\nu = 12/5$ [RR] c = integer + 4/5

For a topological SC wire exactly at the transition there are two counter propagating Majorana modes with c = 1/2.

Also True for the transverse field Ising model

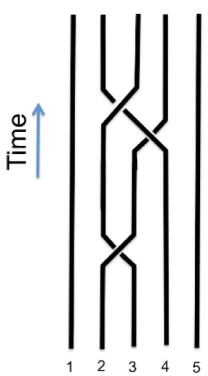
Fusion Rules and Ground states degeneracy

$\nu =$ 5/2 [MR] has MZMs $\quad \sigma \times \sigma = 1 + \psi$

12/5 [RR] has Fibonacci $\quad \tau \times \tau = 1 + \tau$

5/2 [MR] with 2N MZM $\quad D = 2^N$

12/5 [RR] with N Fibonacci $\quad D = F_N$



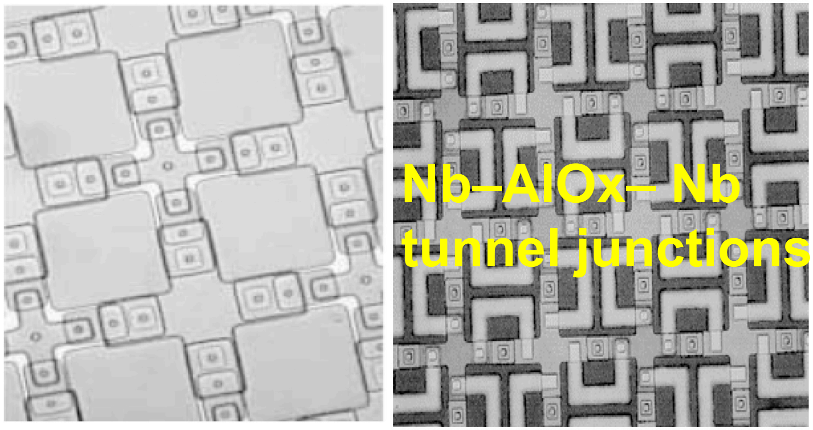
Braiding particles -- matrix manipulations

in the degenerate ground state space.

MZM's do not give universal TQC

While Fibonacci's do.

Array of conventional Josephson junctions



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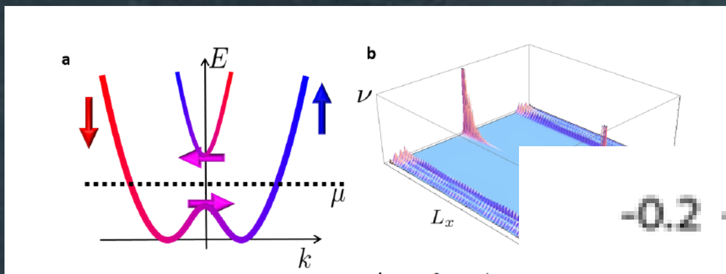
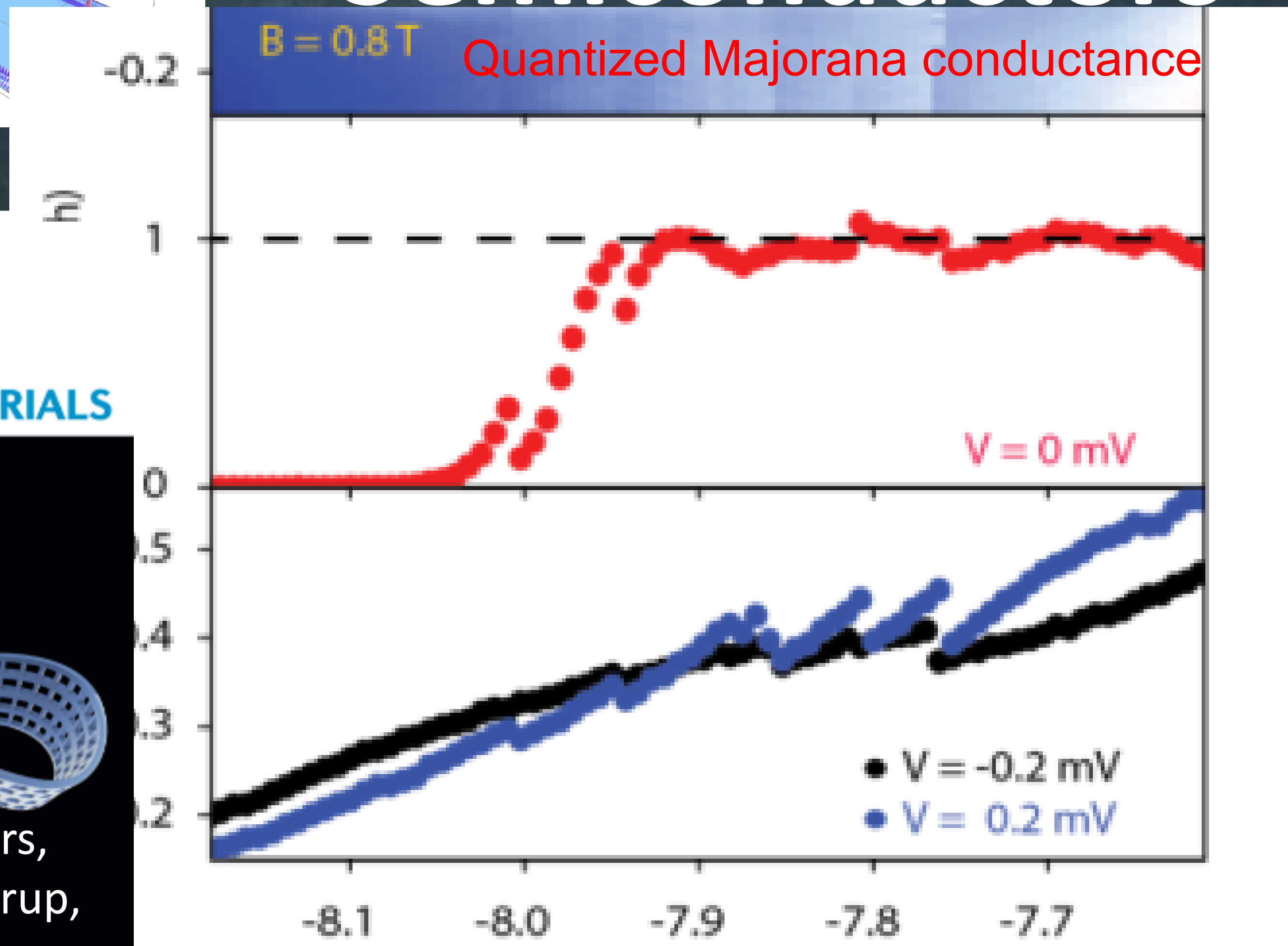
- We know how to create MZM in an alternative simpler system SC + SOC + Magnetic field [non universal]
- Can we create a system with fractional central charge that carry Fibonacci particles using TSC?
- Our conjecture: yes – the insulating phase of TSC may have exotic states supporting Fibonacci
- I will show today an explicit critical system with c=7/10 in 1D with possible future extension to 2D
- The basic ingredients are MZMs in a Cooper Box
- The interaction between them creates a TSC insulating phases with the required properties

MZM can be found in Semiconductors



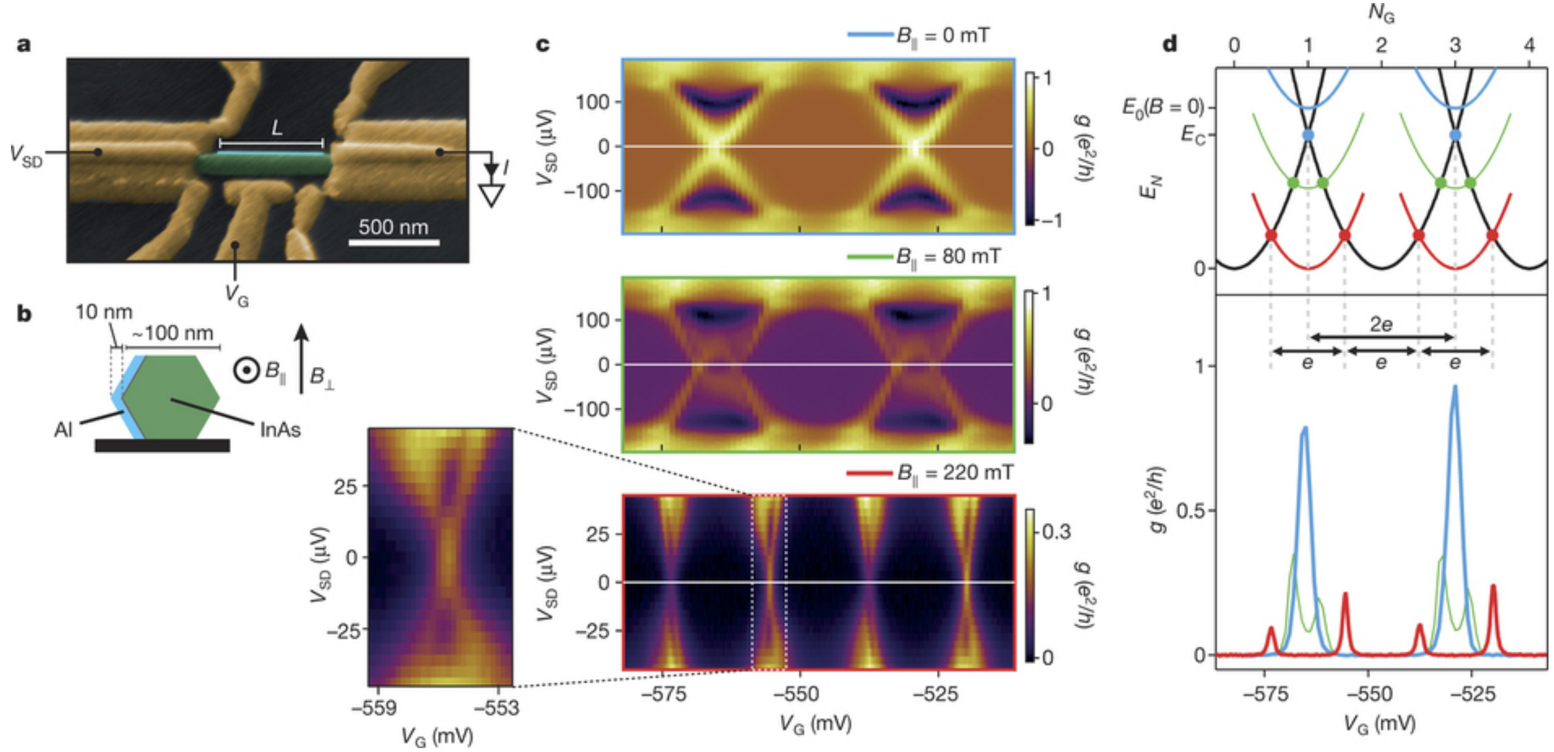


2018: Lutchyn, Bakkers, Kouwenhoven Krogstrup, Marcus, YO

nature International journal of science 2018 :Zhang,…, 20 authors, Leo P. Kouwenhoven

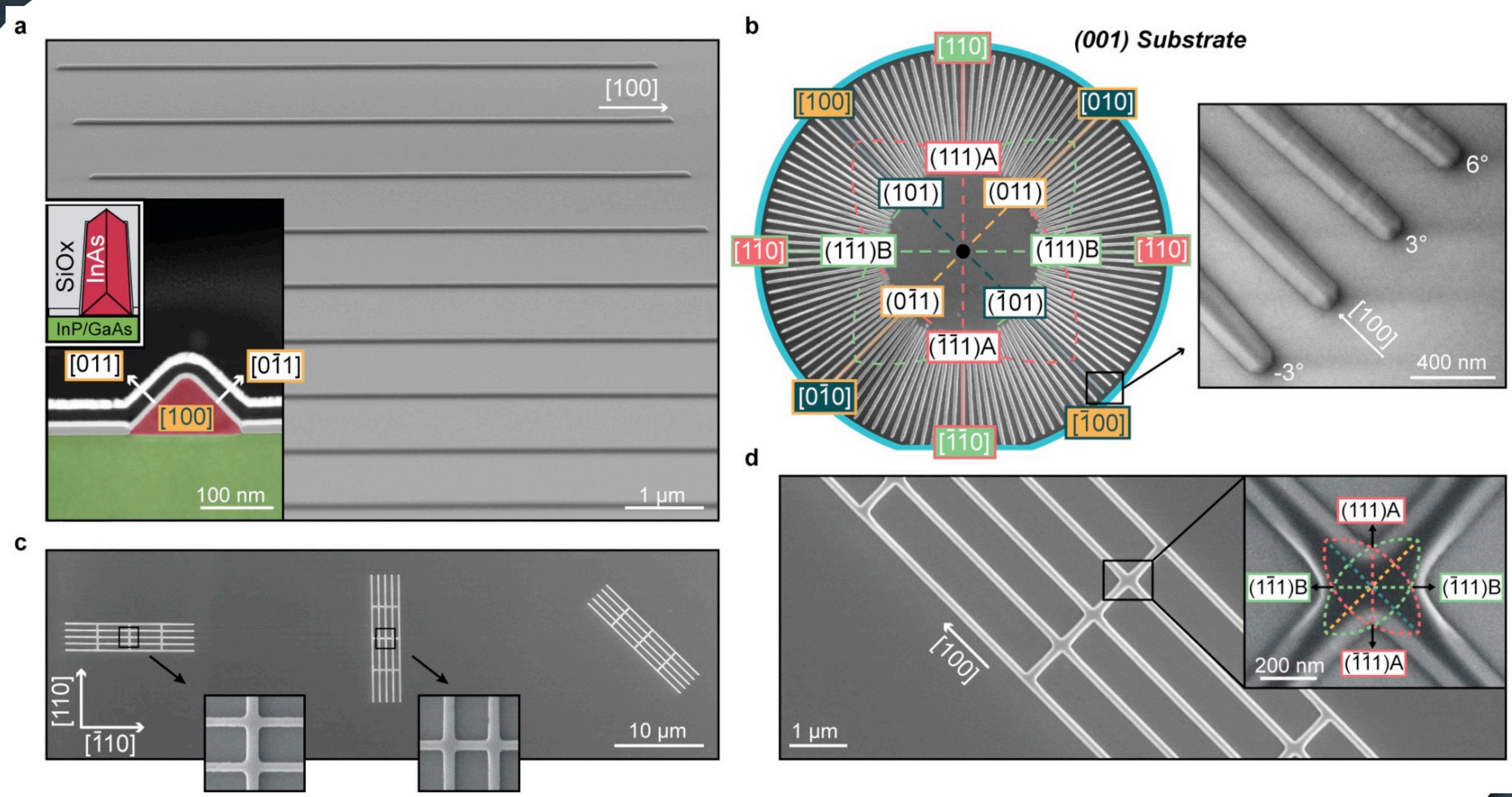
Copenhagen Experiment

the Majorana Cooper-pair Box



Albrecht et al, Nature 531, 206 (2016)
Copenhagen

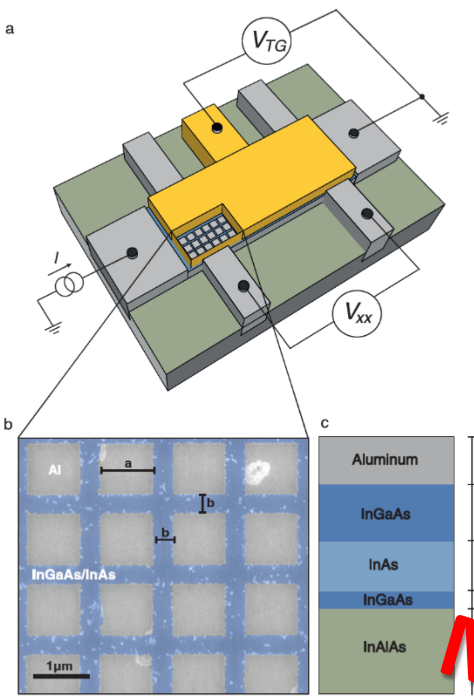
Selective-Area-Growth

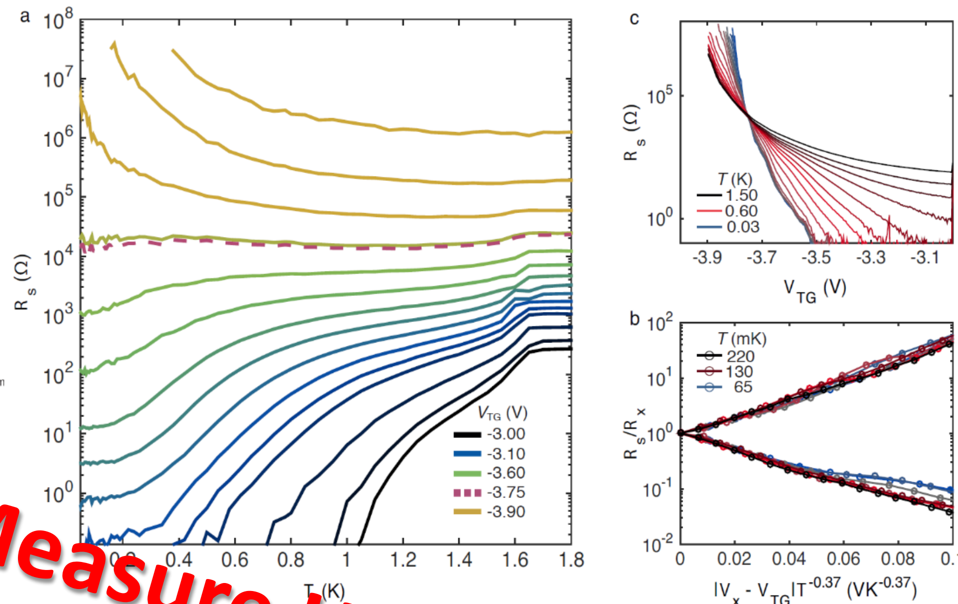


2D Al-InAs

Charlotte Bøttcher
Fabrizio Nichele
Charles Marcus 2018

Array of superconducting islands connected by 2DEG:
A new SIT system





400 x 100 array

a = 1 µm
b = 0.15 µm

$\nu z = 2.7$

Measure Heat Transport

What happens when the SC is topological?

Senthil, Potter, Fisher and Balents and Nayak, Qi and Barkeshli:
Conventional SC- Insulator proliferation of [h/2e] vortexes
Topological SC has MZM in the core so only pairs of vortexes [h/e] proliferate
→ leading to Kitaev's Spin Liquid [KSL]

Parton construction:
[slave-bosons]

Charged Boson
[Chargon]

Electron

$$c_\sigma = b f_\sigma$$

Neutral Spin-full Fermion
[Spinon]

SC: c forms a p wave topological superconductor [TSC]
Insulator: b is in the Mott phase and f forms a TSC [KSL]

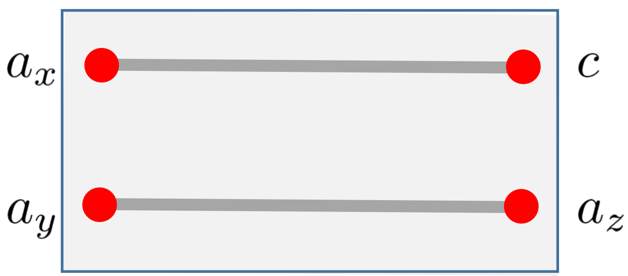
What are the insulating phases when the SC is topological?

Today:

1. Construct insulating models by mapping to 1D spin chains
2. Super Symmetric [SUSY] 1+1 field theoretical models with central charge 7/10
   - 2+1 has Fibonacci particles, wanted for Universal TQC
   - Measured by heat conductance

All based on Majorana-Cooper pair Boxes [*local* interactions] and *local* tunneling between the Majorana zero modes

Tetron-Spin 1/2



$$\hat{f}_1 = \frac{1}{2}(\hat{c} + i\hat{a}_x)$$

$$\hat{f}_2 = \frac{1}{2}(\hat{a}_y + i\hat{a}_z)$$

$$E_1 = E_2 = 0$$

$$\begin{aligned} a_x^2 &= a_y^2 = 1 \\ a_z^2 &= c^2 = 1 \end{aligned}$$

In a Cooper Box [Quantum dot made of superc0nductor in the Coulomb blockade regime]:

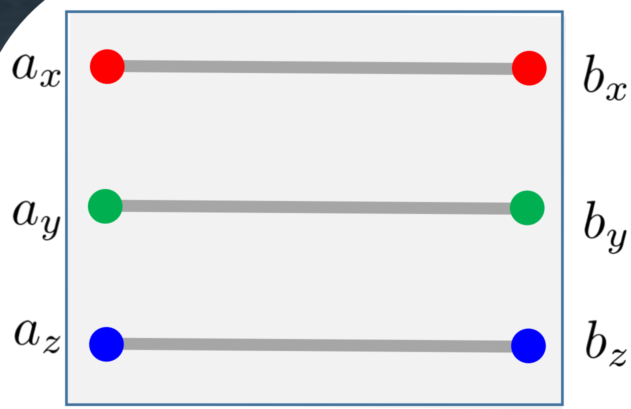
$$\mathcal{H}_c = U\left(\hat{n}_m - 2\hat{N}_c - \mathcal{N}_g\right)^2$$

- U: Charging Energy
- $\hat{n}_m$: $\hat{f}_1^\dagger \hat{f}_1 + \hat{f}_2^\dagger \hat{f}_2 = \hat{n}_1 + \hat{n}_2$
- $\hat{N}_c$: Cooper pairs
- $\mathcal{N}_g$: Gate

$$\mathcal{N}_g = 0$$

Nc	nm	n1	n2
0	0	0	0
1	2	1	1

Hexon- 2xSpin 1/2



$$\mathcal{P} = (ia_x b_x)(ia_y b_y)(ia_z b_z) = ia_x a_y a_z b_x b_y b_z$$

$$S_a^x = ia_y a_z, \quad S_a^y = ia_x a_z, \quad S_a^z = ia_x a_y$$

$$S_b^x = ib_y b_z, \quad S_b^y = ib_x b_z, \quad S_b^z = ib_x b_y.$$

Hilbert space [4 states]
Operator Algebra
Constraint

$\mathcal{P}$

$$H_2 = iJ_x a_x b_x = J_x S_a^x S_b^x$$

$$H_1 = it_{xy} a_x a_y = B_z S_z^a.$$

1D Spin models

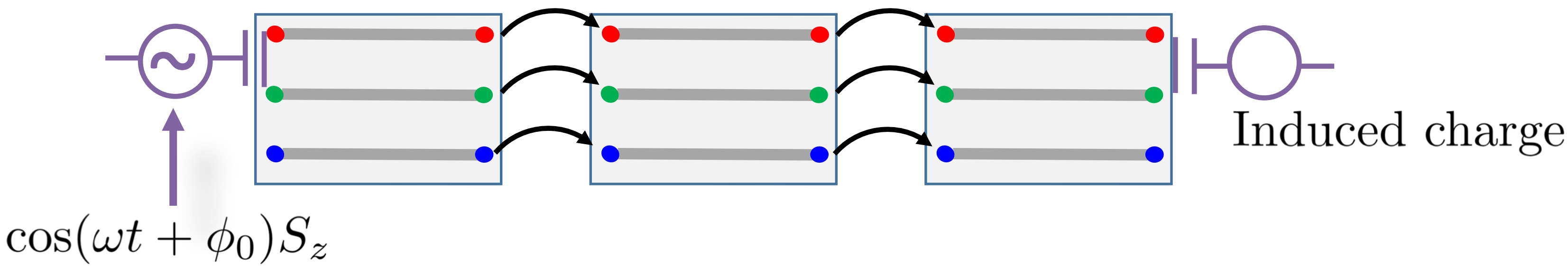
- ✓ XXZ Chain
- ✓ Transvers Field Ising
- ✓ AKLT model –[Haldane Gap]

Gates manipulate all Coupling Consts.

- ✓ Tune to a critical point
- ✓ Apply perturbation
- ✓ Measure

$$H_{\text{pert}} = \cos(\omega t + \phi_0) S^z(x_0)$$

1D-Proposed Measurements



Linear response: $\langle S^z(x,t)\rangle = \int dt' \cos(\omega t')\chi(t-t', x-x_0)$

$$\chi(t-t', x-x_0) = i\,\langle [S^z(x,t), S^z(x',t')]\rangle\, \Theta(t-t')$$

At criticality: $G \sim 1/(x^2 - v^2 t^2)^{2h}$

Modified Bessel

$$\langle S^z(x,t)\rangle = \frac{V_0 \alpha^{4h}}{v^{2h+\frac{1}{2}}} \left(\frac{\omega}{|\Delta x|}\right)^{2h-\frac{1}{2}} \times \Re\left\{ B e^{i(\omega t+\phi_0)} K_{\frac{1}{2}-2h}\left(i\frac{\omega|\Delta x|}{v}\right)\right\}$$

1D SUSY Models

- M. Blume, Phys. Rev. 141, 517 (1966).
- H. Capel, Physica 32, 966 (1966).
- E. Fradkin and L. Susskind, Phys. Rev. D 17, 2637 (1978).
- E. Witten, Nuclear Physics B 142, 285 (1978)
- D. Friedan, Z. Qiu, and S. Shenker, Physics Letters B 151, 37 (1985).
- A. B. Zamolodchikov, Sov. J. Nucl. Phys. 44, 529 (1986).
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- X. Zhu and M. Franz, Phys. Rev. B 93, 195118 (2016).
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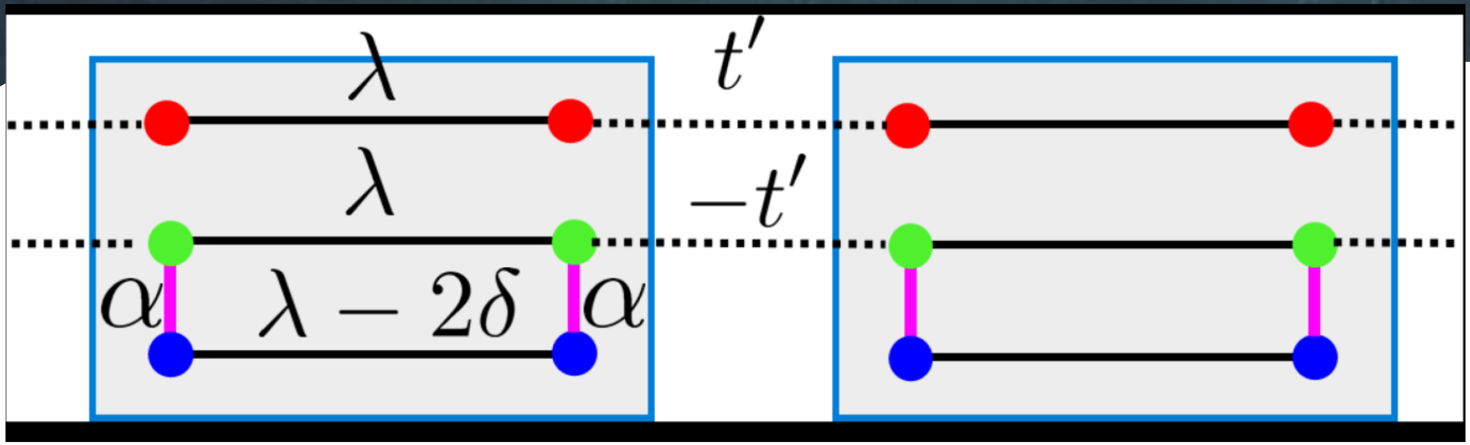
1D SUSY Models



Blume Capel model Spin 1!

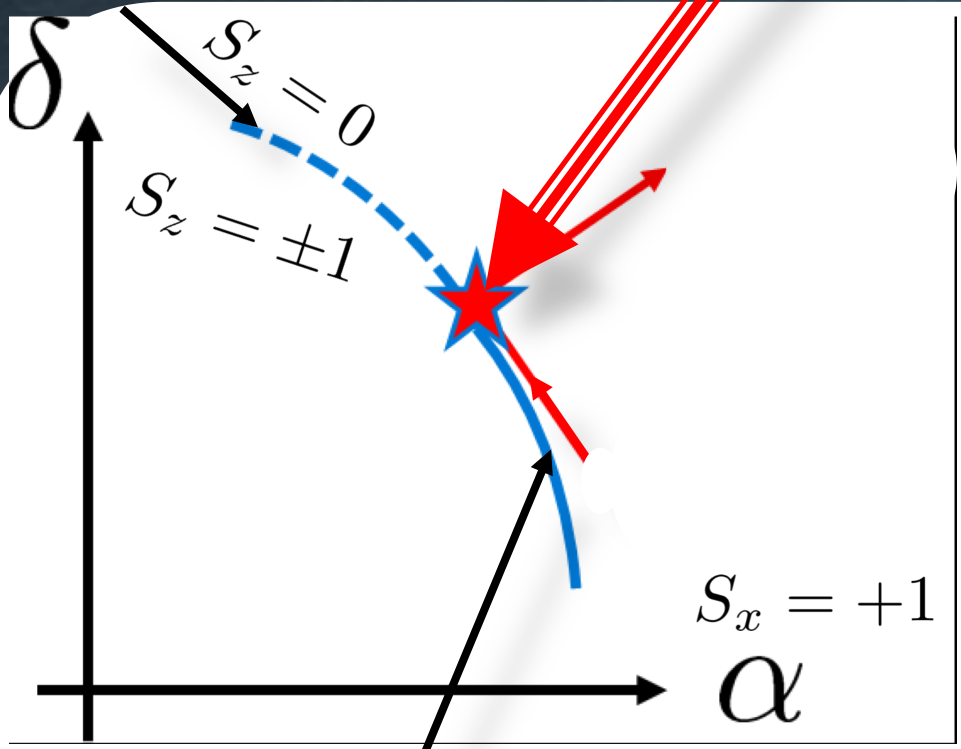
$$H_{\mathrm{BC}} = \sum_j \alpha S_x^j + \delta (S_z^j)^2 - J S_z^j S_z^{j+1}$$

$\lambda > 0$ projects singlet out

$J \sim t'^2/U$

Tri Critical Ising

First order transition

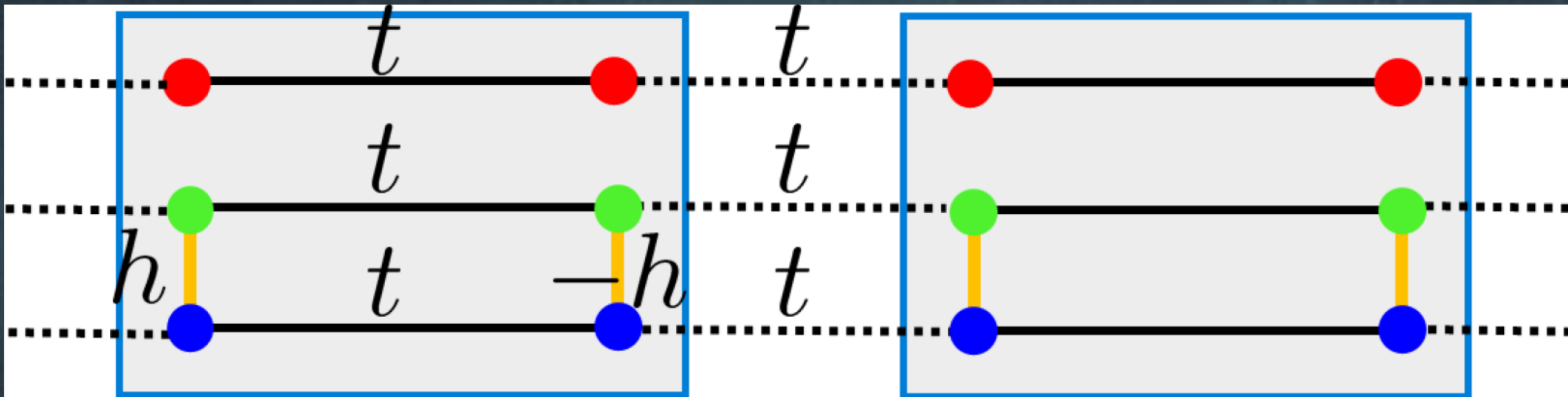


Central charge 7/10

Transverse field Ising

$$H_{\mathrm{BC}} = \sum_j \alpha S_x^j + \delta (S_z^j)^2 - J S_z^j S_z^{j+1}$$

Susy Field Theory



$$H_t = it \sum_{j,p=x,y,z} b_p^j (a_p^{j+1} - a_p^j), \qquad = \frac{i}{2} \sum_p t(\eta_{pR}\partial_r \eta_{pR} - \eta_{pL}\partial_r \eta_{pL}),$$

$$H_h = ih \sum_j (a_y^j a_z^j - b_y^j b_z^j). \qquad = -ih(\eta_{yR}\eta_{zL} + \eta_{yL}\eta_{zR}).$$

Villain Trans [t=0]

$$\mathcal{H}_c = U \sum_j \left(\hat{n}_m^j - 2\hat{N}_c^j - \mathcal{N}_g^j\right)^2 \longrightarrow \frac{U}{\pi^2} \sum_j \cos(\pi(\hat{n}_M^j + \mathcal{N}_g^j))$$

$$\sum_{\{\hat{N}_c^j\}} \sum{}' e^{-H_U} e^{-H_0} = \sum_{\{\hat{N}_c^j\}} \sum{}' e^{-\sum_j \frac{U}{\pi^2}(2\hat{N}_c^j\pi + \hat{n}_M^j\pi + n_g^j\pi)^2} e^{-H_0} \simeq \sum{}' e^{-\sum_j \frac{U}{\pi^2}\cos(\pi(\hat{n}_M^j + n_g^j))} e^{-H_0}$$

Susy Field Theory

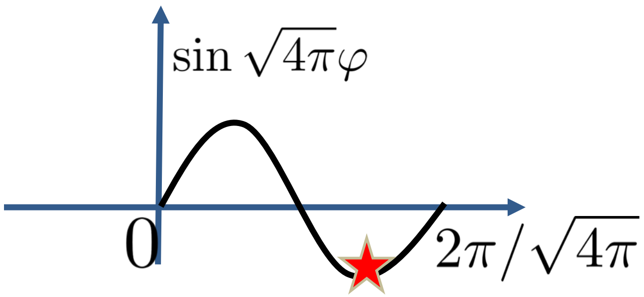
$\psi_p = (\eta_{pR}, \eta_{pL})^T \quad \psi_x \to \psi,\ e^{i\varphi} = \psi_y + i\psi_z, t \to 1, d(\text{Boxsize}) \to 1$

$g = U/(d\pi)^2, h \to h/d^2$

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}(\partial_\mu \varphi)^2 + \frac{1}{2}\bar{\psi} i \partial\!\!\!/ \psi \\ &- g(\sin^2\sqrt{4\pi}\varphi - 2\cos\sqrt{4\pi}\varphi\bar{\psi}\psi) \\ &+ h\sin\sqrt{4\pi}\varphi \end{aligned}$$

With $h = 0$ generalized G-N model

Saddle point for large $h \gg 1$



Redefining $\tilde{\sigma} = \sqrt{K}\tilde{\varphi}$, $u = 4g\sqrt{\frac{4\pi}{K}}$, $K = 1-\frac{4g}{\pi}\rho$, $\rho = \frac{2g/\pi-1}{8g^2/\pi^2-1}$, $h = 2(1-\rho)g$, the saddle point theory takes the form

$$\mathcal{L} \simeq \frac{1}{2}(\partial_\mu\tilde{\sigma})^2 + \frac{1}{2}\bar{\psi}i\not{\partial}\psi - \frac{1}{2}u\tilde{\sigma}\bar{\psi}\psi - \frac{1}{8}u^2\tilde{\sigma}^4,$$

[Notice that the expression for ρ requires $g/t > \pi/(2\sqrt{2})$, which is consistent with our initial assumption that $U \gg t$.]

The field theory we obtain is $\mathcal{N} = 1$ super LG action. The relation to the super LG action can be obtained explicitly by considering the SUSY model

$$S_{\text{SUSY}} = \int dxdt\, d\theta^2 \left[\frac{1}{4}(\bar{D}\Phi)(D\Phi) + W(\Phi)\right], \tag{1}$$

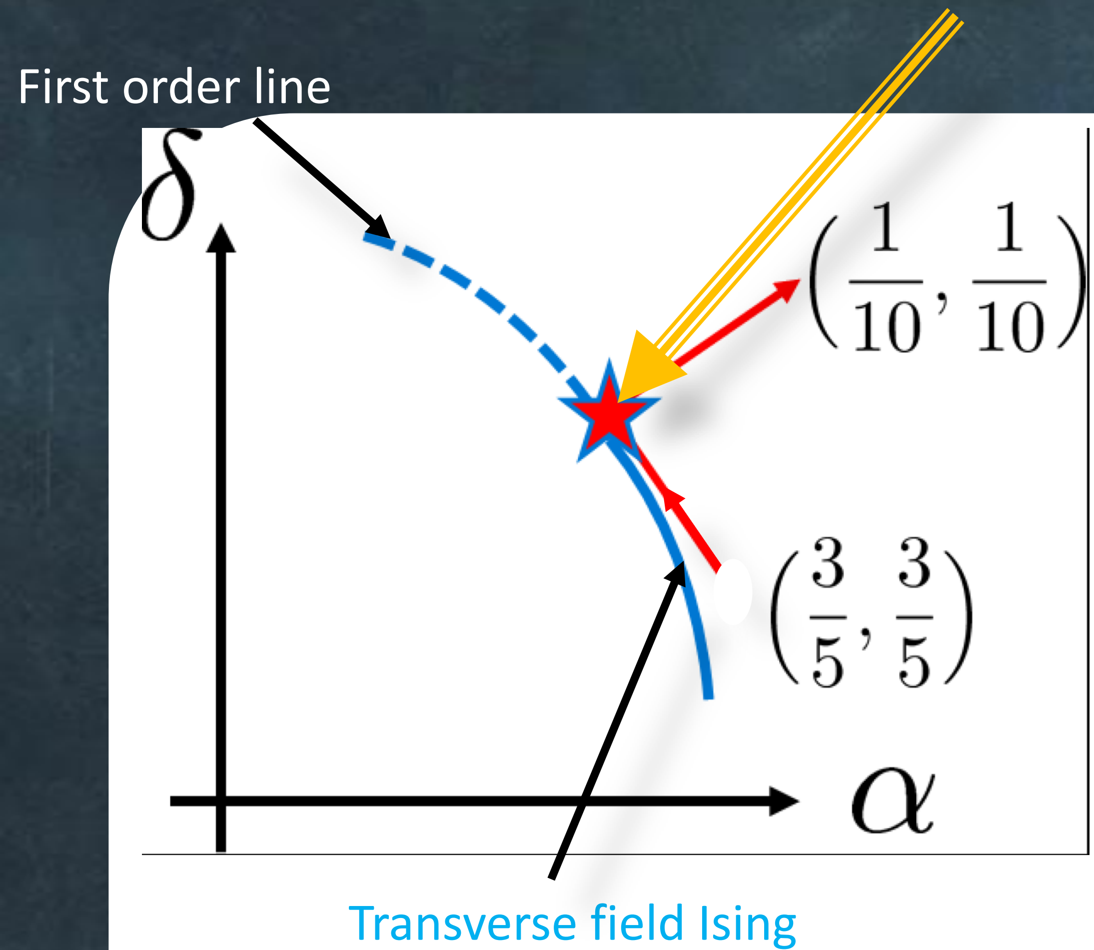
where Φ is the superfield defined by $\Phi = \tilde{\varphi} + \bar{\theta}\psi + \frac{1}{2}\bar{\theta}\theta F$, D represents covariant derivative in superspace, and $W(\Phi)$ describes superpotential which is a polynomial function of Φ. In our case, $W(\Phi)$ is given by $W(\Phi) = \frac{v}{6}\Phi^3$

Zamolodchikov [1986] showed that at long distances, the super LG action with a super potential $W(\Phi) \simeq \Phi^m$ $(m = 2, 3, \cdots)$ exhibits a supersymmetric analog of the minimal models, characterized by central charge $c = \frac{3}{2} - \frac{12}{m(m+2)}$. Since our case corresponds to $m = 3$, the theory given in Eq. (1) effectively manifests an emergent SUSY described by a SCFT with $c = 7/10$.

[1+1]D

Very sensitive to disorder

Tri Critical Ising SUSY

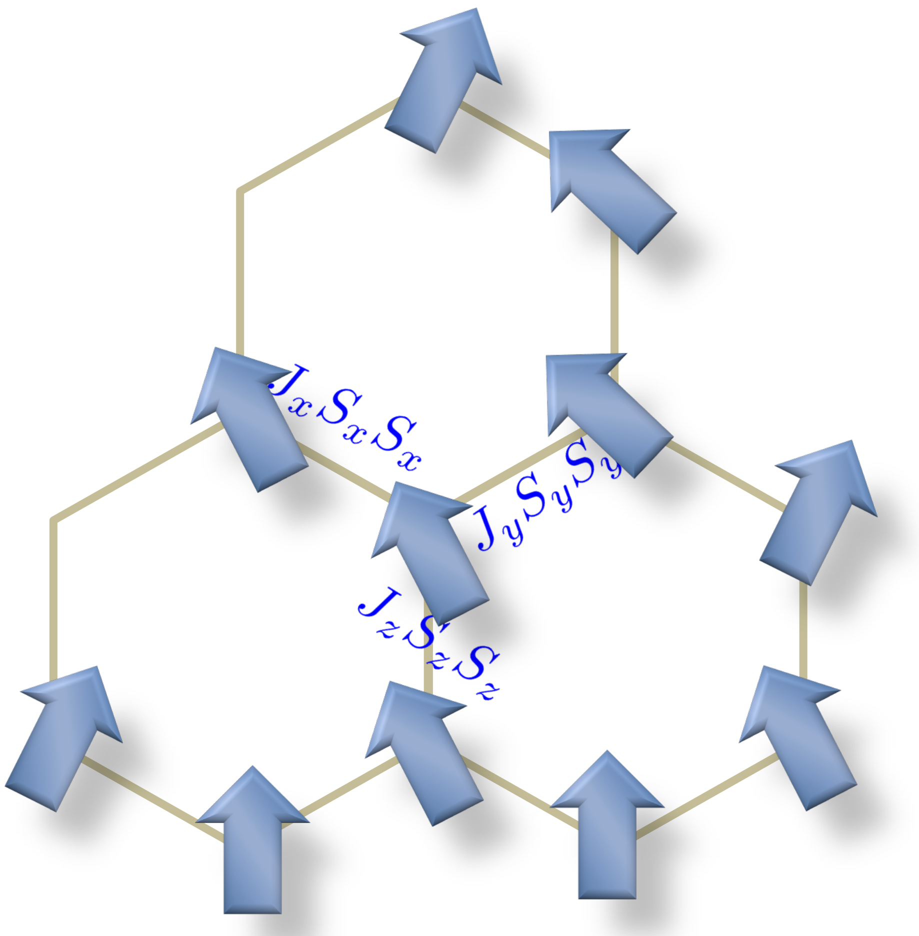


Central charge 7/10

Intermediate Summary

- Insulating phases of TSC
- Majo zero mode as a building block for spins and fermions
- 6 Majo-Zero Modes in a Cooper Box is a convenient building block
- Controlled by Gates Only
- A Chain with 7/10 cc [in 2+1 Fibonacci particles]
- local measurements
- 2D?

2D Spin Liquids/ Kitaev's Honeycomb Model

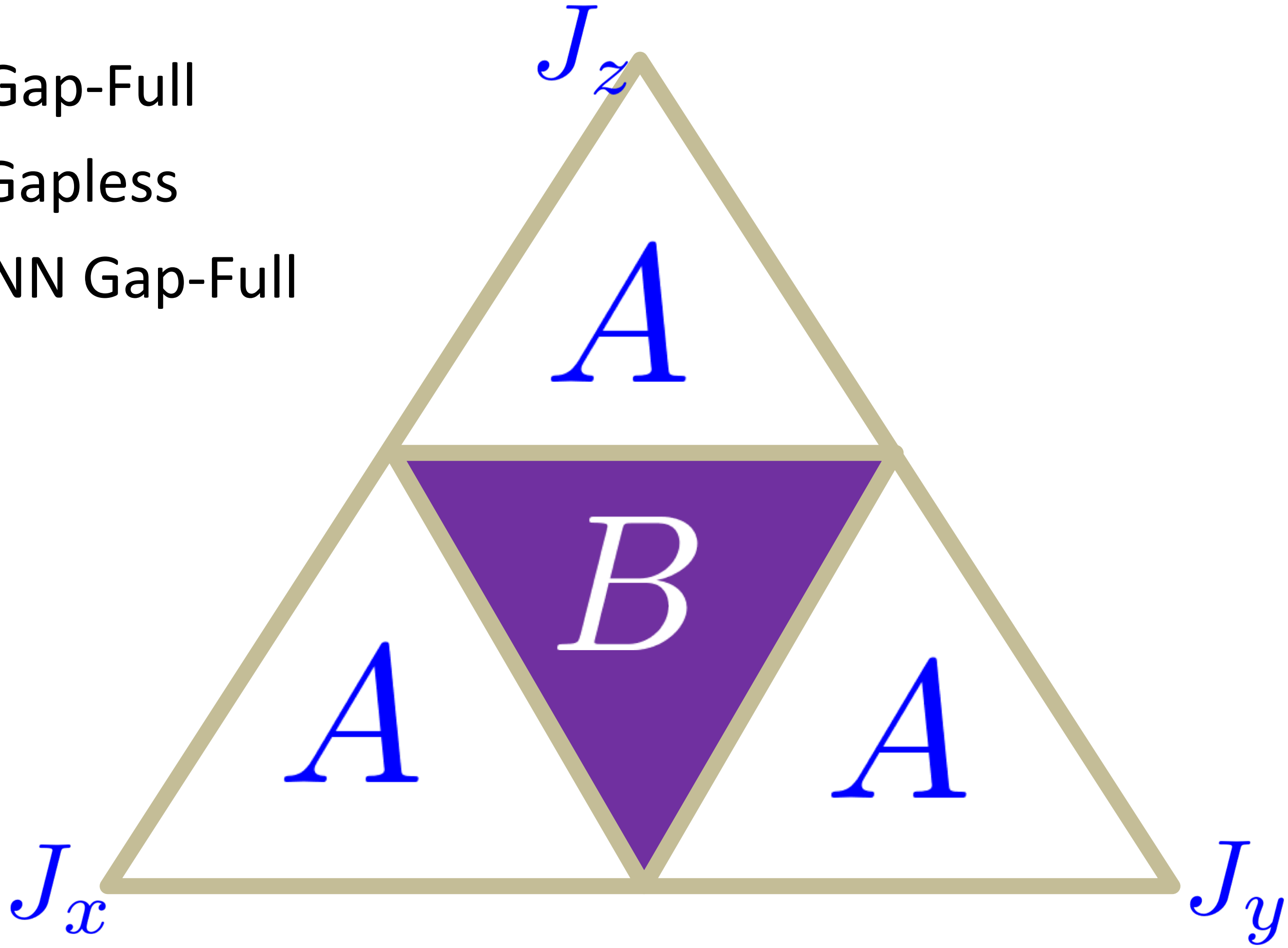


Phase Diagram

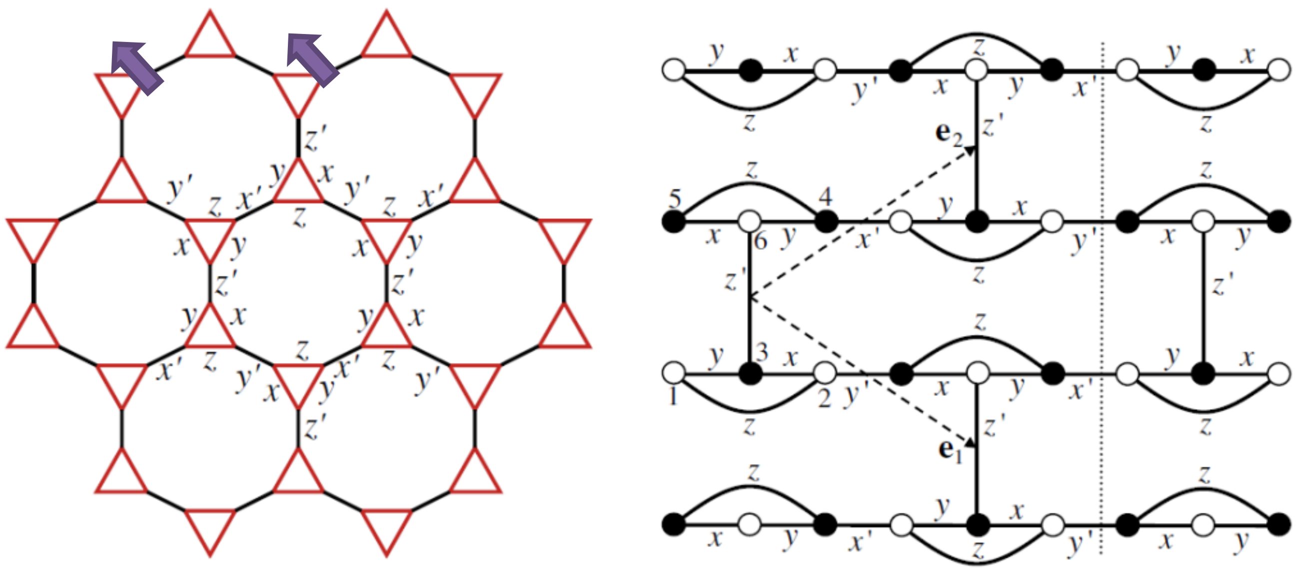
A: Gap-Full

B: Gapless

+NNN Gap-Full



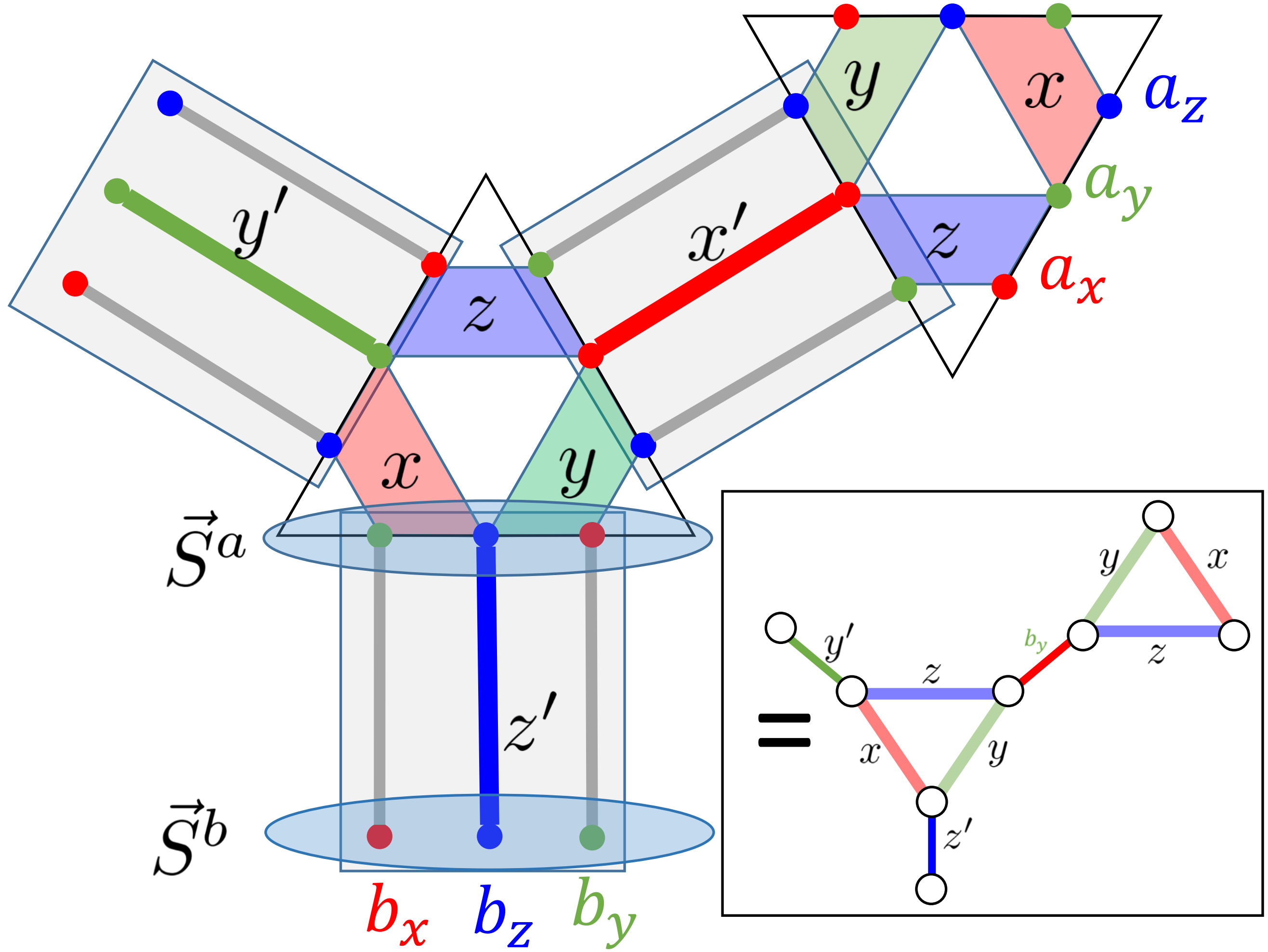
Kitaev's Honeycomb model/ Yao Kivelson Model



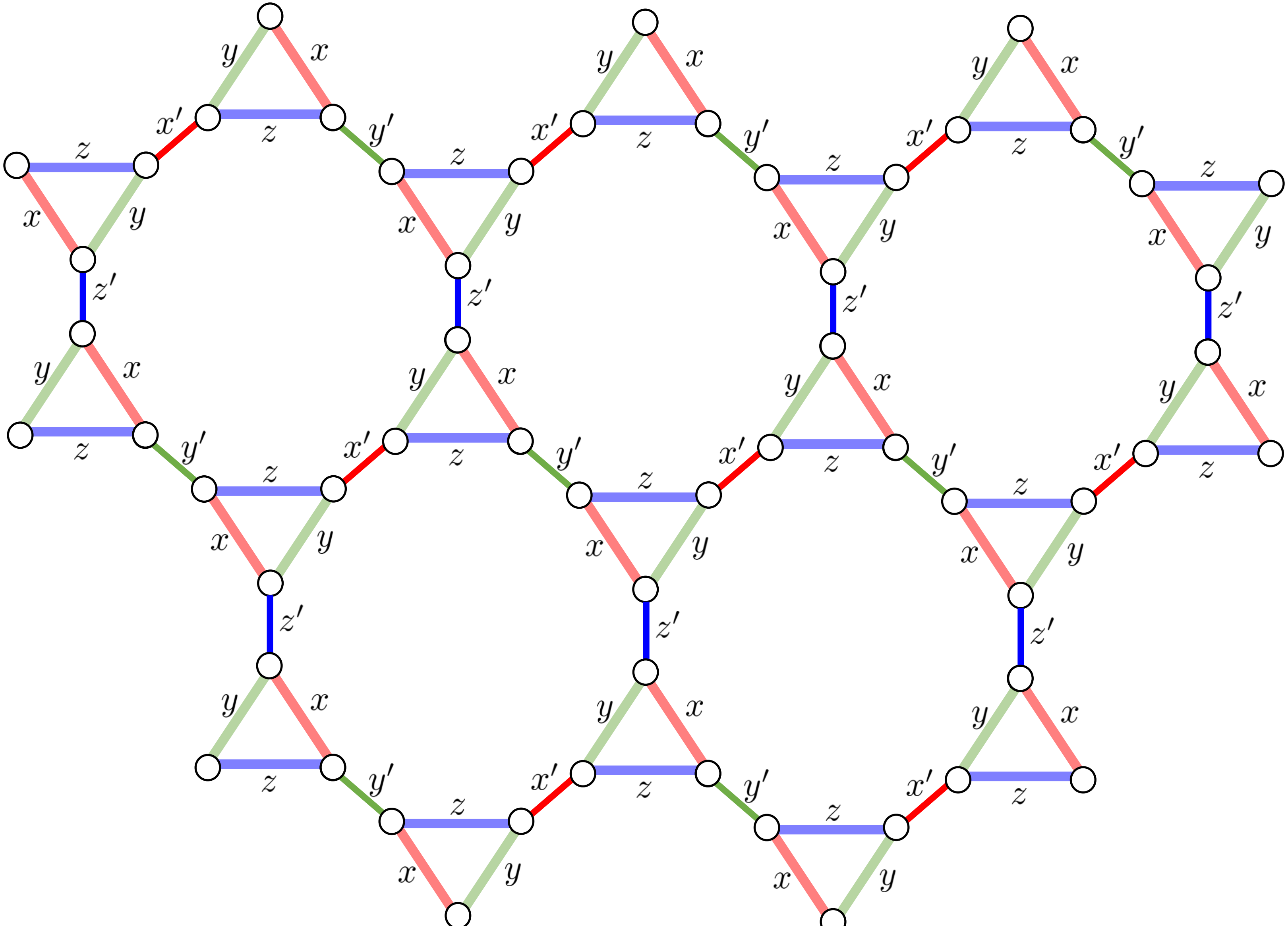
(a) (b)

$$\mathcal{H} = \sum_{x-\text{link}} J_x \sigma_i^x \sigma_j^x + \sum_{y-\text{link}} J_y \sigma_i^y \sigma_j^y + \sum_{z-\text{link}} J_z \sigma_i^z \sigma_j^z + \sum_{x'-\text{link}} J'_x \sigma_i^x \sigma_j^x + \sum_{y'-\text{link}} J'_y \sigma_i^y \sigma_j^y + \sum_{z'-\text{link}} J'_z \sigma_i^z \sigma_j^z, \quad (1)$$

Realizations [Color Code]

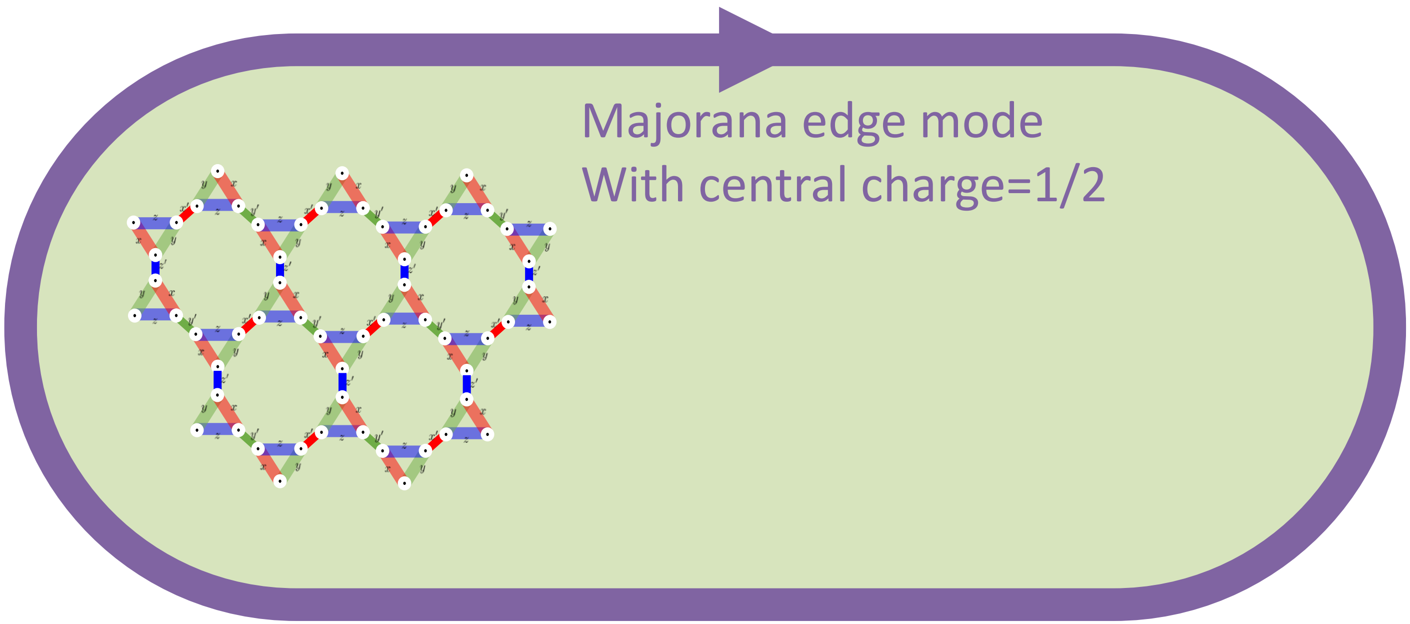


See also
Barkeshli Sau



Collective spin edge mode

Loops with odd number of sites SxSySz – spontaneous chirality breakings



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