Macroscopically degenerate ground state manifold in the SU(3) symmetric Heisenberg model on the kagome lattice

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On the blackboard :

- S=1 chain : AKLT in a nutshell
- Comparing SU(2) and SU(3)
- Construction of the FSS state on kagome lattice: the analog of the AKLT state

Slides :

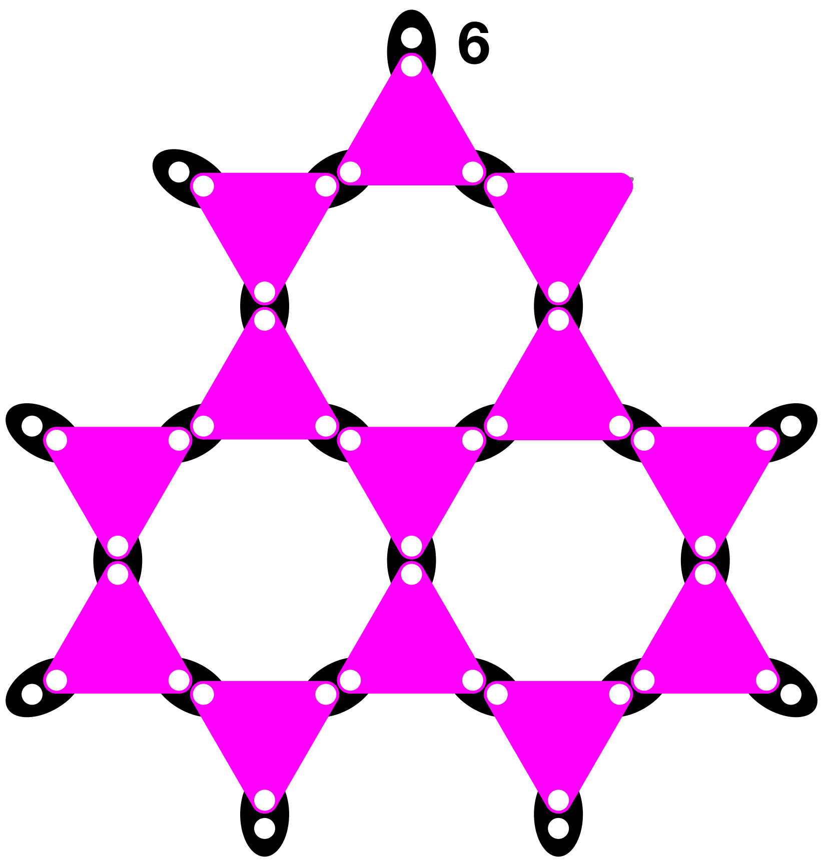
- ED studies of the model
- identification of the origin of the degeneracies
- Penrose graphs, tensor networks, polarization, … etc.

Simplex solid in SU(3) Kagome

D. P. Arovas, Phys. Rev. B **77**, 104404 (2008).

SU(N) singlet on N sites, represented by $b^\dagger_\alpha(i)$ Schwinger bosons:

$$\epsilon^{\alpha_1 \cdots \alpha_N} b^\dagger_{\alpha_1}(i_1) \cdots b^\dagger_{\alpha_N}(i_N)|0\rangle,$$



Addition of two SU(3) spins:

$$\mathbf{3} \times \mathbf{3} = \bar{\mathbf{3}} + \mathbf{6}$$

Each site hosts the symmetric, 6 dimensional irrep because of the bosons (like in the S=1 AKLT wave function case).

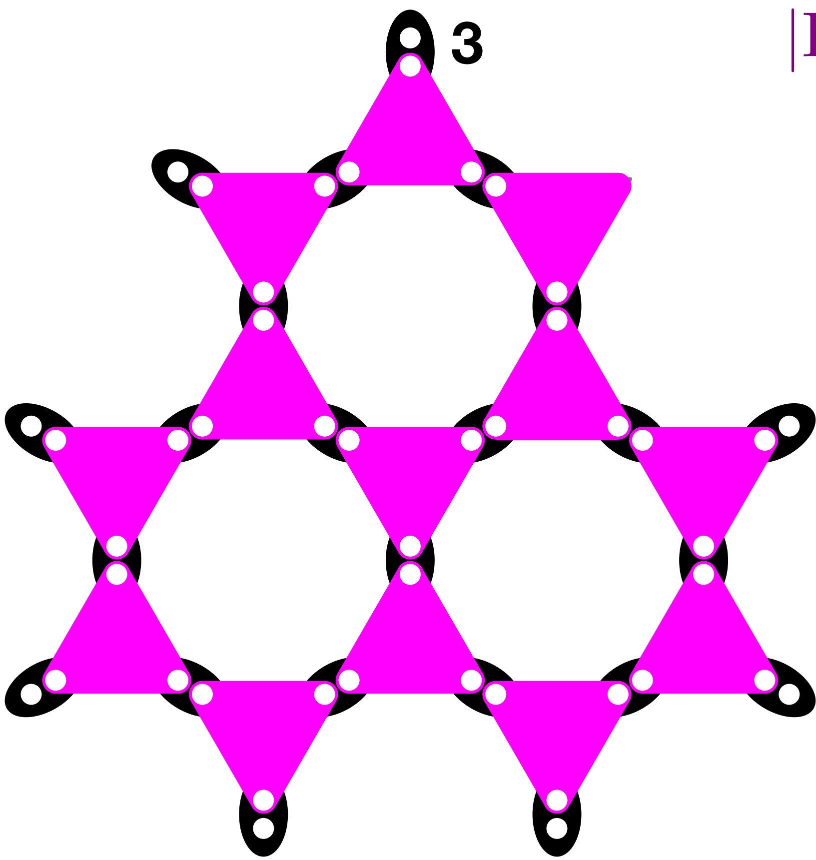
But we can do this with fermions as well !

SU(3) singlet on 3 sites, represented by fermions :

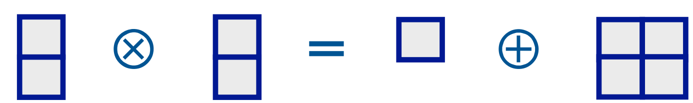
$$|\mathbf{1}(i_1, i_2, i_3)\rangle = \sum_{\alpha,\beta,\gamma} \varepsilon^{\alpha\beta\gamma} f^\dagger_\alpha(i_1) f^\dagger_\beta(i_2) f^\dagger_\gamma(i_3)|0\rangle = \mathcal{F}_{i_1,i_2,i_3}|0\rangle$$

femionic simplex solid wave function:

$$|\mathrm{FSS}\rangle = \prod_{\triangle_i}\prod_{\bigtriangledown_j} \mathcal{F}_{\triangle_i}\mathcal{F}_{\bigtriangledown_j}|0\rangle$$



$$\bar{\mathbf{3}} \times \bar{\mathbf{3}} = \mathbf{3} + \bar{\mathbf{6}}$$



Each site hosts the antisymmetric, 3 dimensional irrep.

Do we know the parent Hamiltonian ?

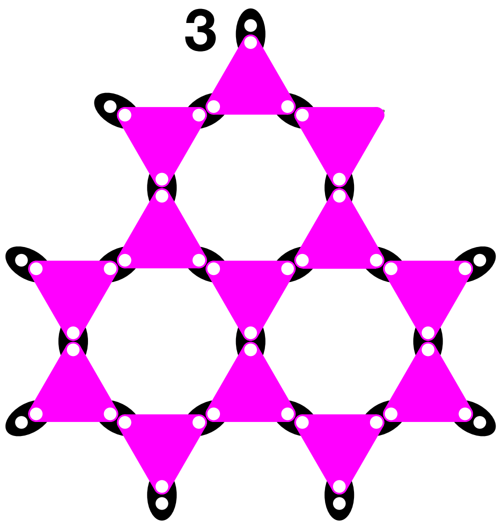
A guess: sum of local projectors, like in the S=1 AKLT model

$$\mathcal{H} = J \sum_{\langle i,j \rangle} \mathcal{P}_{i,j} + K \sum_{\triangle,\triangledown} \left(\mathcal{P}_{i,j,k} + \mathcal{P}_{i,k,j} \right)$$

We may try it on a small cluster:
we generate the FSS, and ask if the condition for being an eigenstate

$$\langle \mathrm{FSS} | \mathcal{H}^2 | \mathrm{FSS} \rangle \langle \mathrm{FSS} | \mathrm{FSS} \rangle = \langle \mathrm{FSS} | \mathcal{H} | \mathrm{FSS} \rangle^2$$

is satisfied with some values of J and K.



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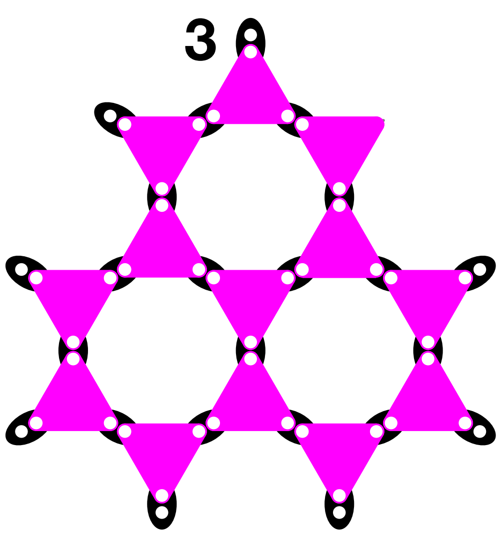
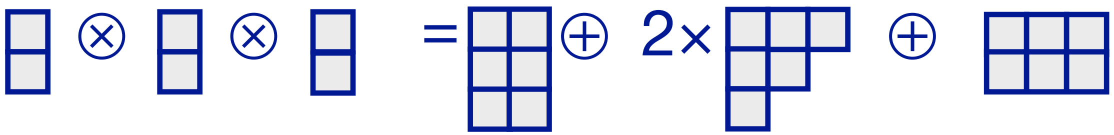


Surprise: it is satisfied for any value of J and K,
the FSS is always an eigenstate of H !
(c.f. AKLT in S=1 chain)

But how does this happen?

The irreps in a triangle

$$\bar{\mathbf{3}} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus 2 \times \mathbf{8} \oplus \overline{\mathbf{10}}$$



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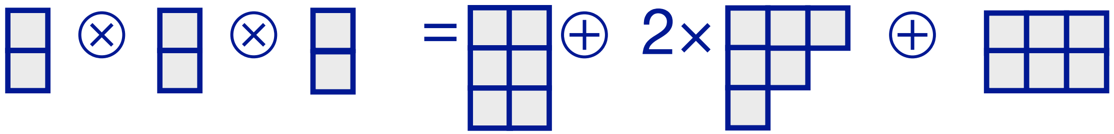
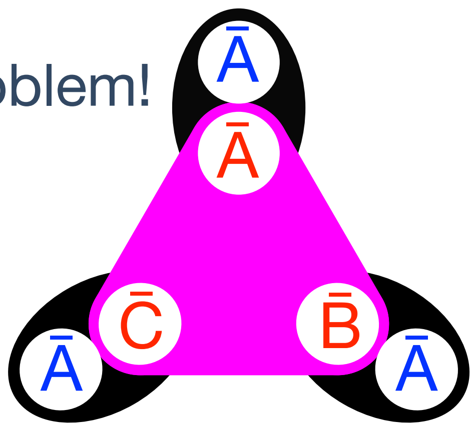


problem!



$$\mathbf{1} \odot \mathbf{10} = 0$$

The irreps in a triangle

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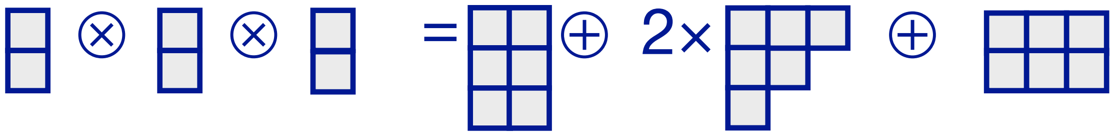

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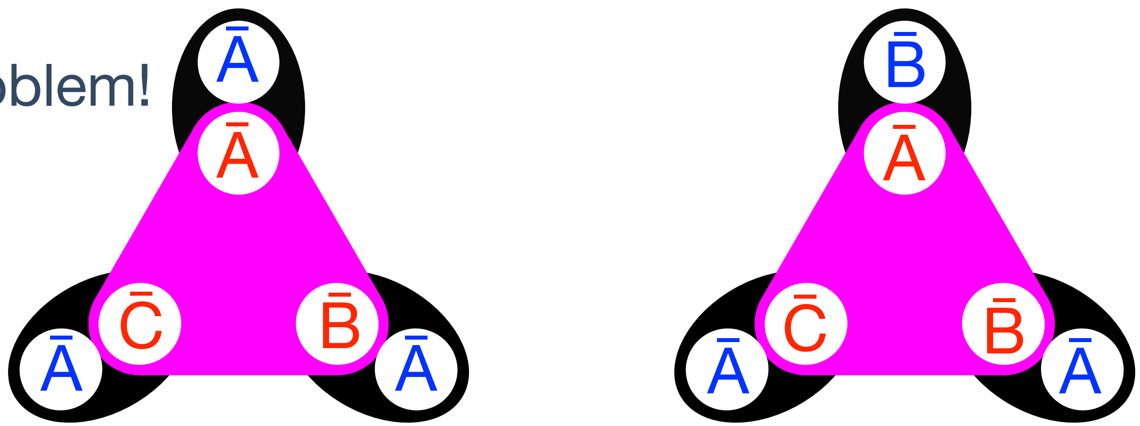
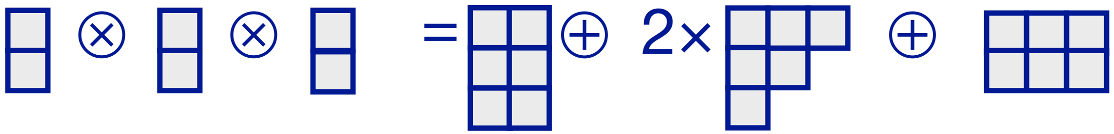


$\mathbf{1} \odot \mathbf{10} = 0$ $\qquad$ $\mathbf{1} \odot \mathbf{8} = \mathbf{8}$

The irreps in a triangle

$$\bar{3} \otimes \bar{3} \otimes \bar{3} = 1 \oplus 2 \times 8 \oplus \overline{10}$$



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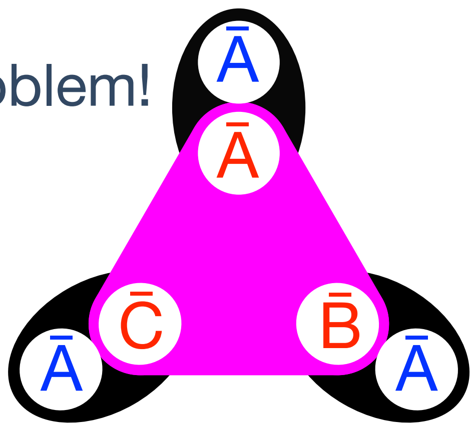
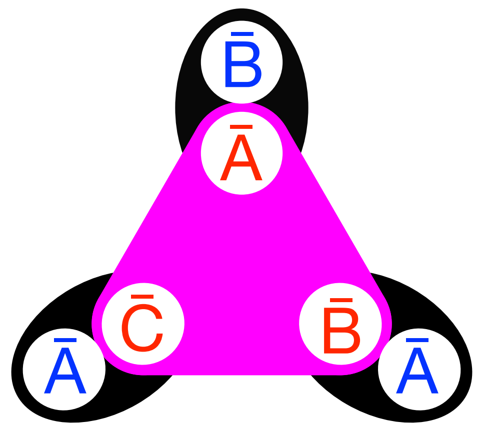
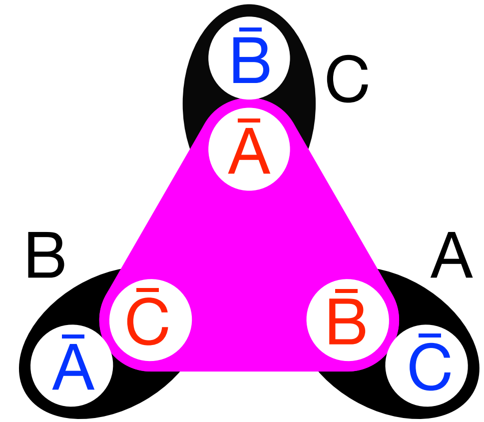


$1 \odot 10 = 0$



$1 \odot 8 = 8$



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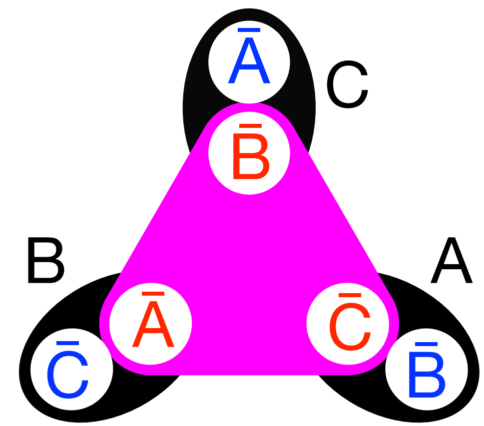
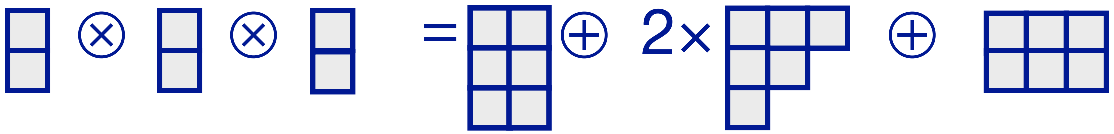


The sum cancels because of odd number of antisymmetrizations: $(-1)^3 = -1$

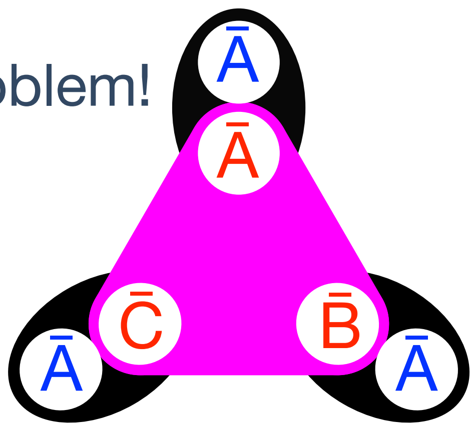
$1 \odot 1 = 0$

The irreps in a triangle

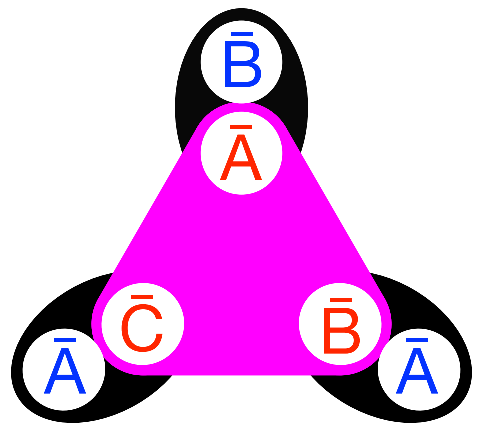
$\bar{\mathbf{3}} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus 2 \times \mathbf{8} \oplus \overline{\mathbf{10}}$



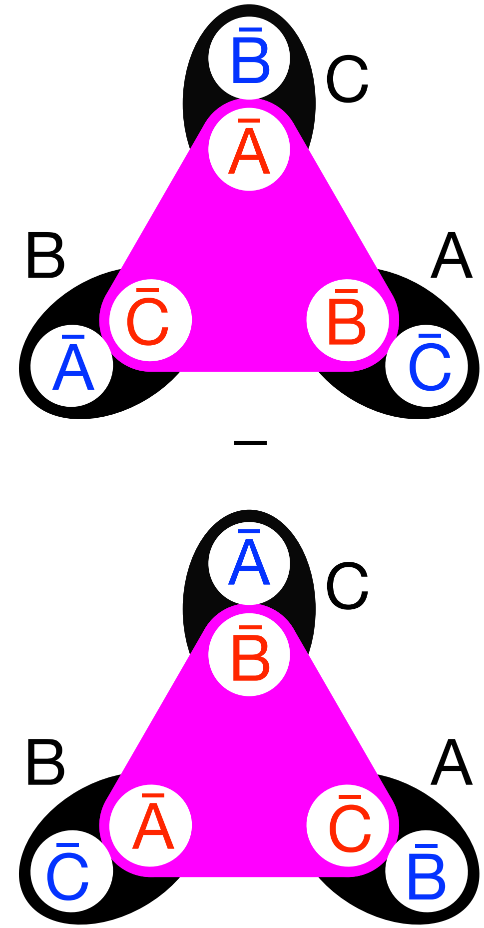
problem!



$\mathbf{1} \odot \mathbf{10} = 0$



$\mathbf{1} \odot \mathbf{8} = \mathbf{8}$



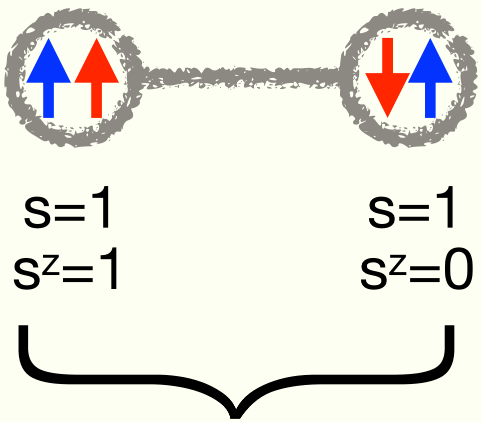
$\odot$	$\mathbf{1}$	$\mathbf{8}^R$	$\mathbf{8}^L$	$\overline{\mathbf{10}}$
$\mathbf{1}$		$\mathbf{8}^R$	$\mathbf{8}^L$	
$\mathbf{8}^R$	$\mathbf{8}^R$	$\mathbf{8}^L$	$\mathbf{1} \oplus \mathbf{10}$	$\mathbf{8}^R$
$\mathbf{8}^L$	$\mathbf{8}^L$	$\mathbf{1} \oplus \mathbf{10}$	$\mathbf{8}^R$	$\mathbf{8}^L$
$\overline{\mathbf{10}}$		$\mathbf{8}^R$	$\mathbf{8}^L$	$\mathbf{10}$

The sum cancels because of odd number of antisymmetrizations: $(-1)^3 = -1$

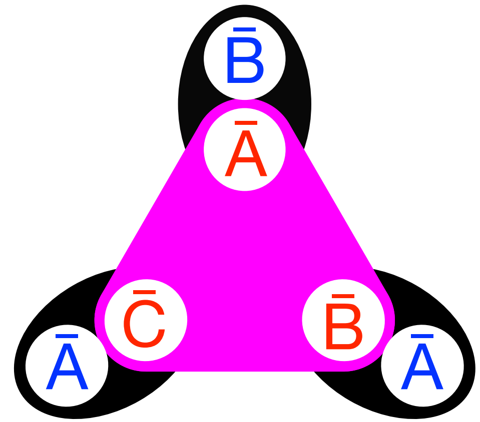
$\mathbf{1} \odot \mathbf{1} = 0$

Comparing the S=1 AKLT chain with FSS

AKLT chain



$$\mathcal{H}^{\mathrm{AKLT}} = \sum_{\mathrm{bonds}} |S=2\rangle\langle S=2|$$



$$\mathbf{1} \odot \mathbf{8} = \mathbf{8}$$

Fermionic simplex solid is an eigenstate of the Hamiltonian

$$\mathcal{H}^{\mathrm{FSS}} = \sum_{\triangle,\triangledown} \left(c_{\mathbf{1}} |\mathbf{1}\rangle\langle\mathbf{1}| + c_{\mathbf{10}} |\mathbf{10}\rangle\langle\mathbf{10}| \right)$$

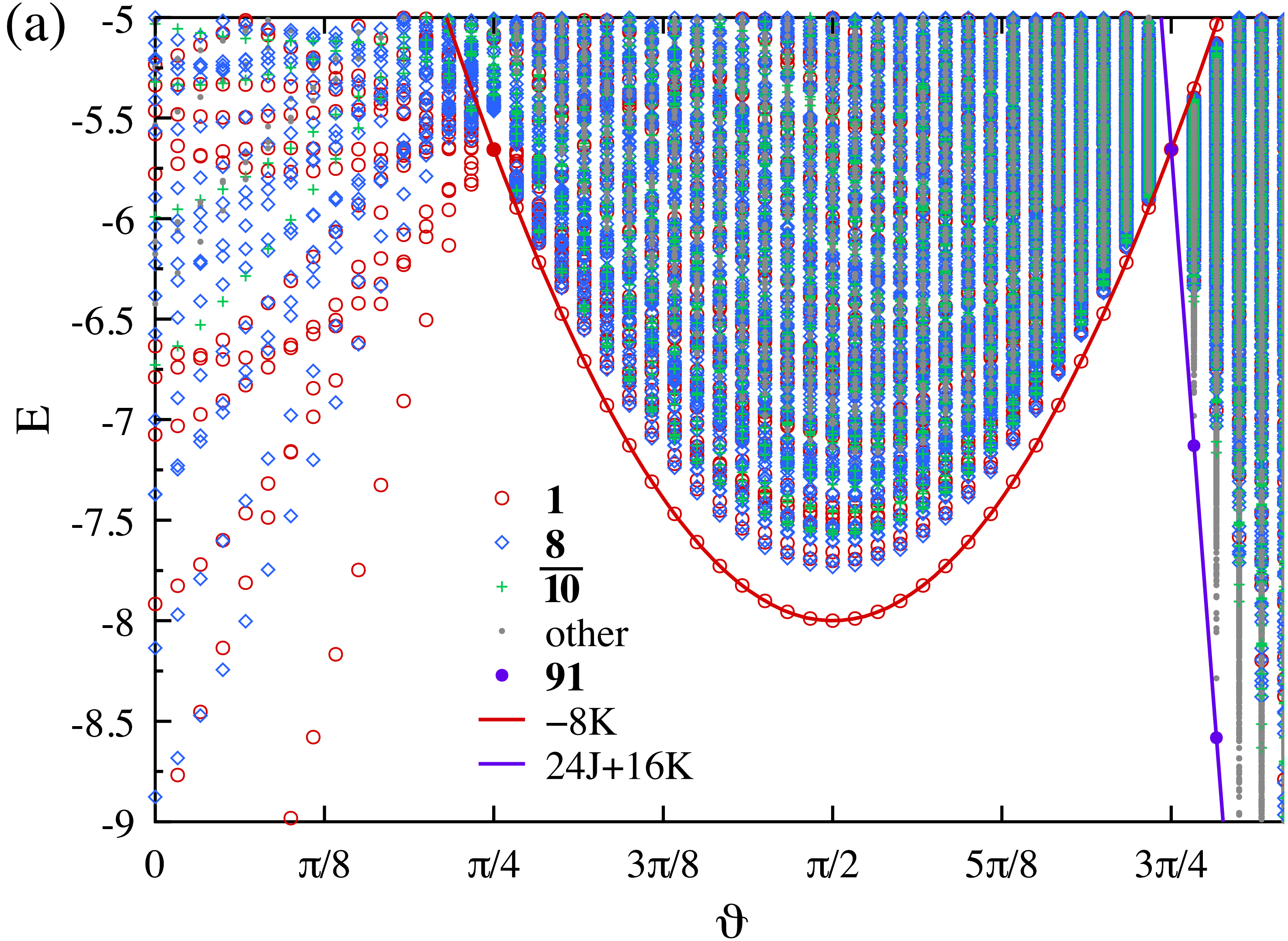
and ground state when $c_1>0$ and $c_{10}>0$.

$$J = \frac{1}{6}\left(c_{\mathbf{10}} - c_{\mathbf{1}}\right), \quad K = \frac{1}{6}\left(c_{\mathbf{10}} + c_{\mathbf{1}}\right)$$

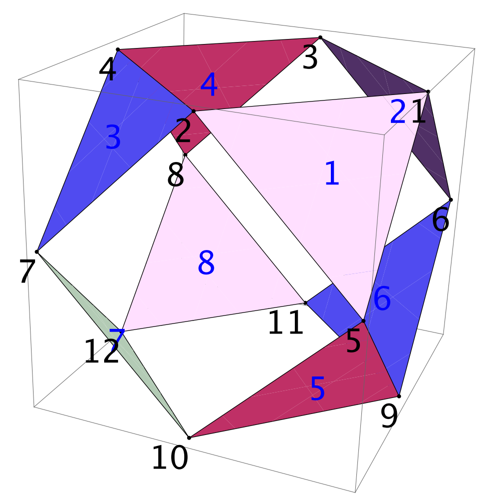
$$\mathcal{H} = J \sum_{\langle i,j\rangle} \mathcal{P}_{i,j} + K \sum_{\triangle,\triangledown} \left(\mathcal{P}_{i,j,k} + \mathcal{P}_{i,k,j}\right)$$

full ED for small system (12 sites)

$$\mathcal{H} = J \sum_{\langle i,j \rangle} \mathcal{P}_{i,j} + K \sum_{\triangle,\triangledown} (\mathcal{P}_{i,j,k} + \mathcal{P}_{i,k,j}) \qquad J = \cos\vartheta, \quad K = \sin\vartheta$$



34650 states in the singlet sector, but the symmetry group is large



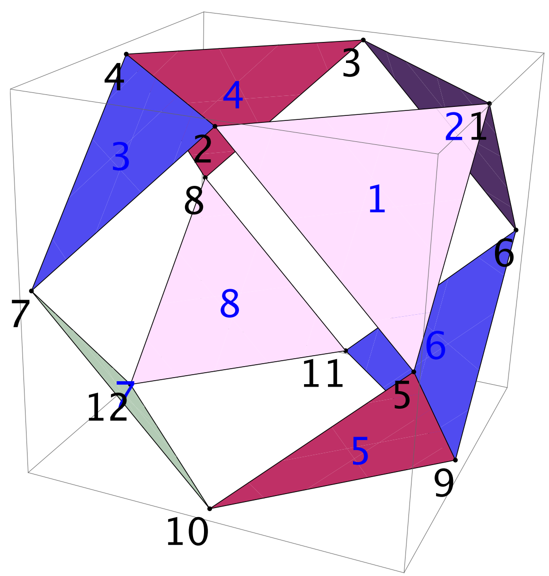
full ED for small system (12 sites)

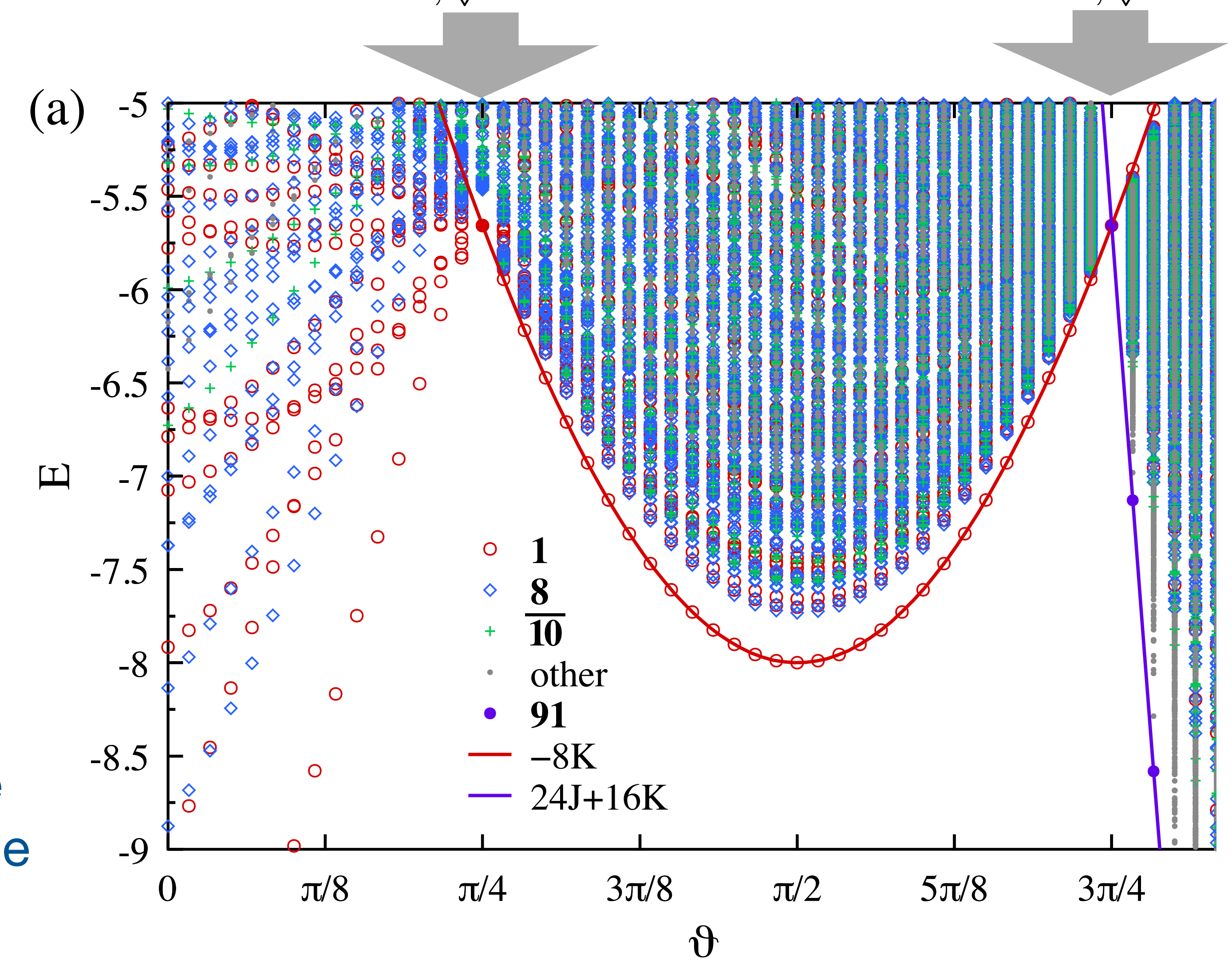
$$\mathcal{H} = J \sum_{\langle i,j \rangle} \mathcal{P}_{i,j} + K \sum_{\triangle,\nabla} (\mathcal{P}_{i,j,k} + \mathcal{P}_{i,k,j}) \qquad J = \cos\vartheta, \quad K = \sin\vartheta$$

$$c_{\mathbf{1}} = 3(K - J)$$

$$c_{\mathbf{10}} = 3(K + J)$$

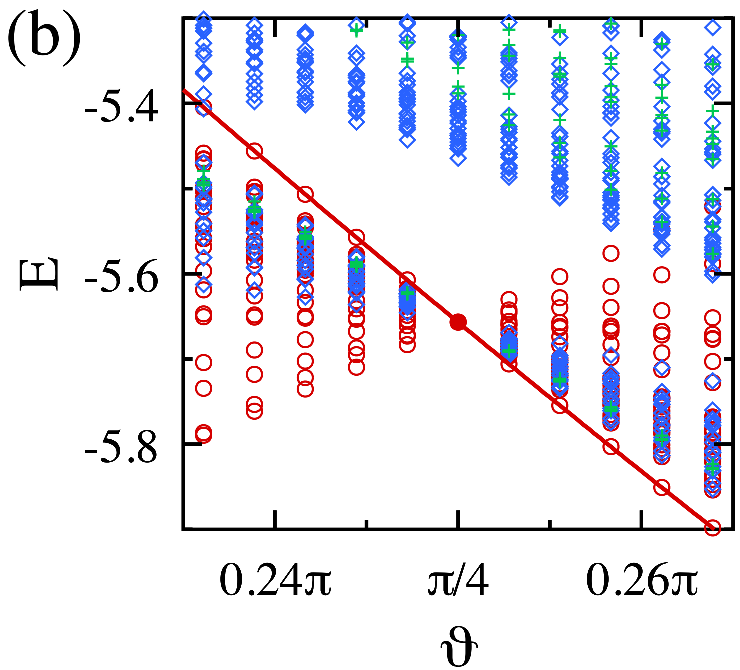
$$\mathcal{H} = \sum_{\triangle,\nabla} |\mathbf{10}\rangle\langle\mathbf{10}| \qquad \mathcal{H} = \sum_{\triangle,\nabla} |\mathbf{1}\rangle\langle\mathbf{1}|$$

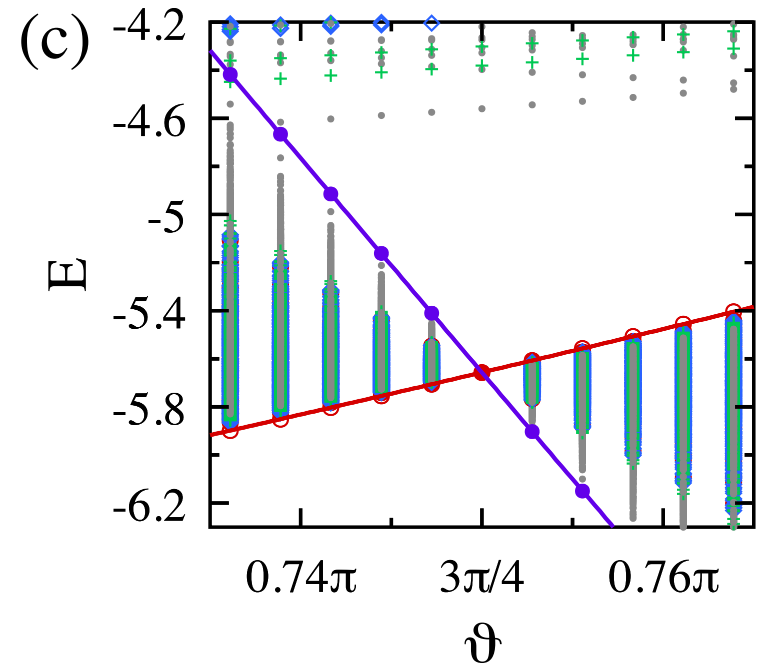




34650 states in the singlet sector, but the symmetry group is large

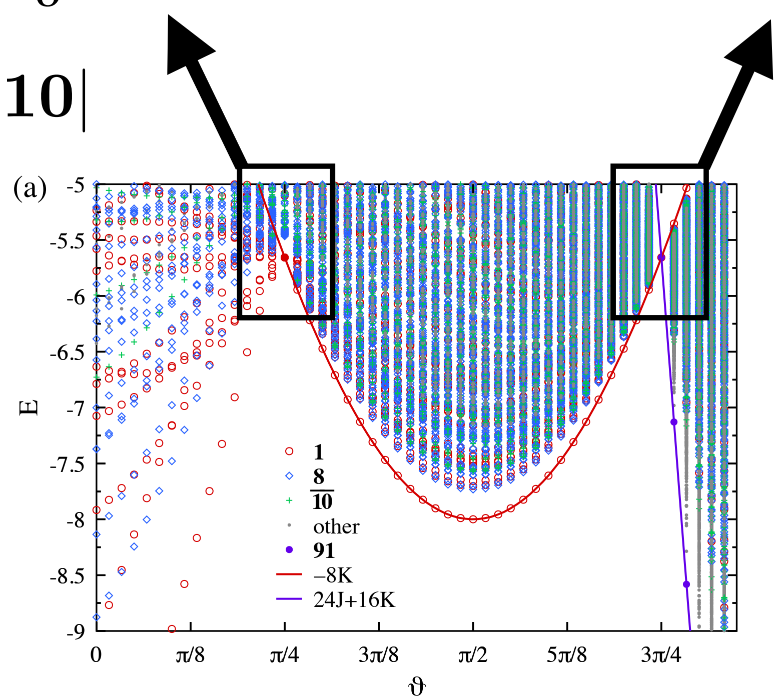
full ED for small system (12 sites) - degenerate GS





$$\mathcal{H} = \sum_{\triangle,\triangledown} |\mathbf{10}\rangle\langle\mathbf{10}|$$

$$J = K$$



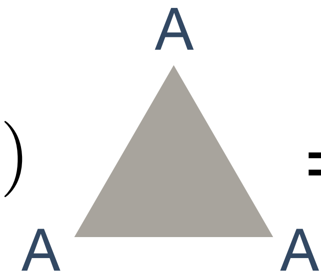
$$\mathcal{H} = \sum_{\triangle,\triangledown} |\mathbf{1}\rangle\langle\mathbf{1}|$$

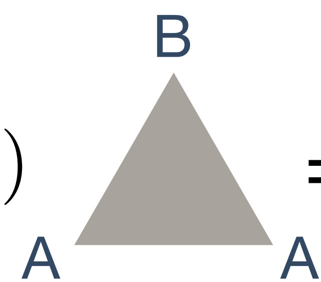
$$J = -K$$

The ϑ=3π/4 (J = –K) case

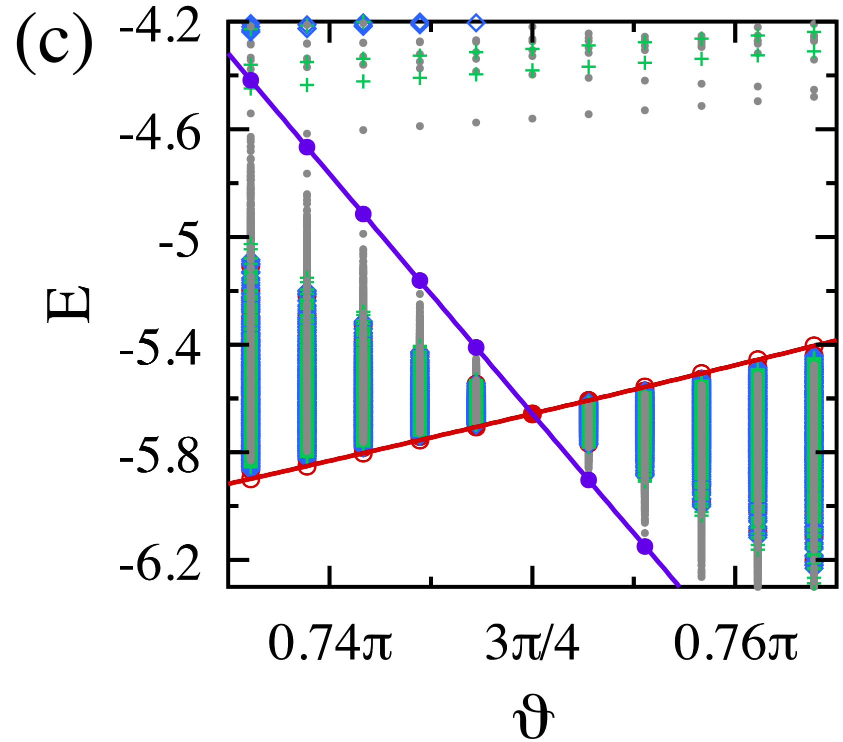
$$\mathcal{H} = \sum_{\triangle,\triangledown} |\mathbf{1}\rangle\langle\mathbf{1}|$$

$$\mathcal{H}_\triangle = \mathcal{P}_{i,j,k} + \mathcal{P}_{i,k,j} - \mathcal{P}_{i,j} - \mathcal{P}_{i,k} - \mathcal{P}_{j,k}$$

$(\mathcal{H}_\triangle + 1)$  = 0

$(\mathcal{H}_\triangle + 1)$  = 0

triangles having no more than two colors are degenerate eigenstates

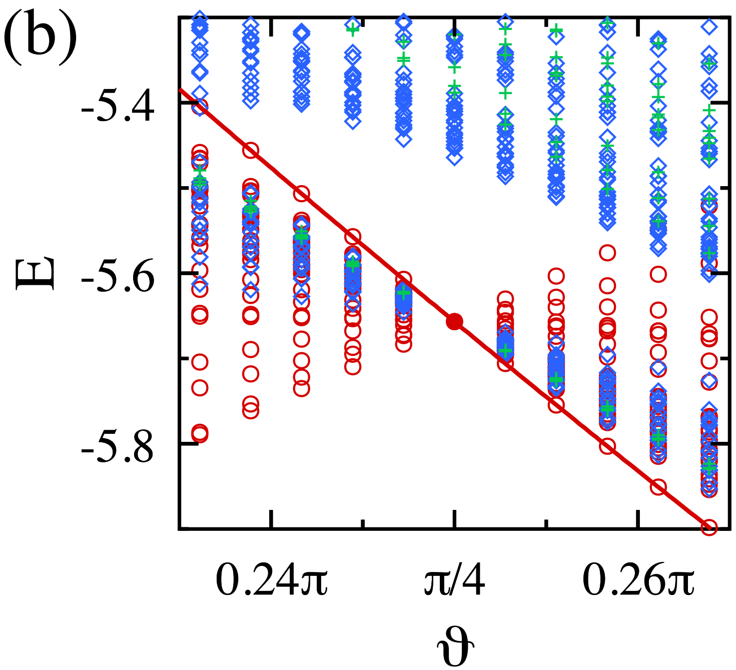


385427 states are degenerate

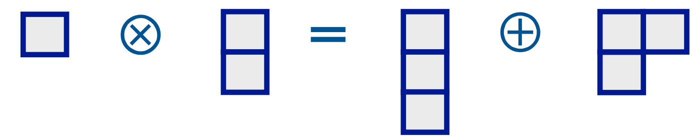
3^{12}=531441 is the total number of states

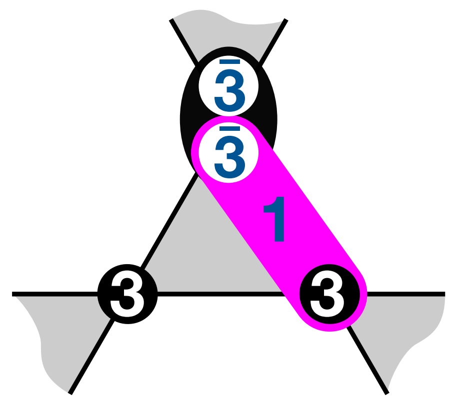
The $\vartheta=\pi/4$ (J = K) case

$$\mathcal{H} = \sum_{\triangle,\nabla} |\mathbf{10}\rangle\langle\mathbf{10}|$$



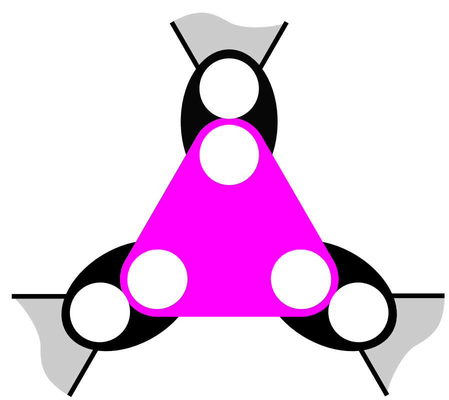
$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$$

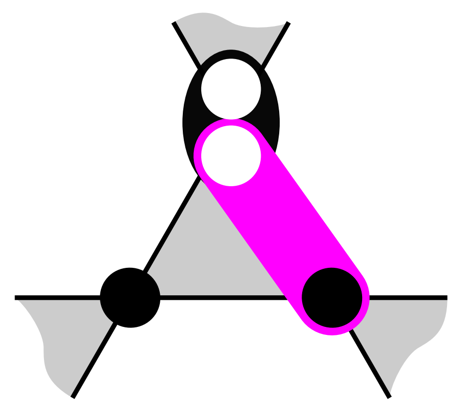




The irepps of 3 spins in the triangle contain **1** and **8,** but no **10.**

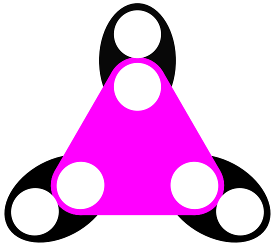
the building blocks are:





The J = K case: Lego time!

$$\mathcal{H} = \sum_{\triangle,\triangledown} |\mathbf{10}\rangle\langle\mathbf{10}|$$



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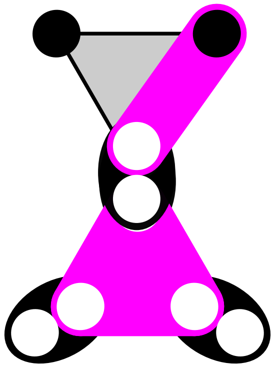

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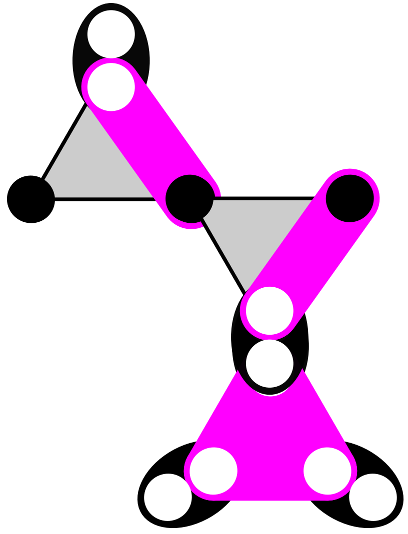

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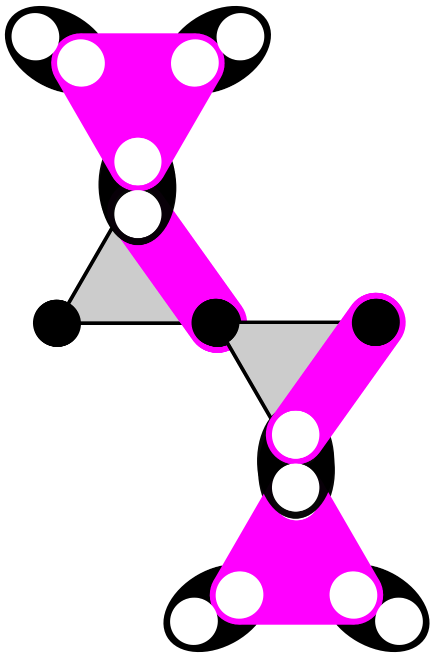

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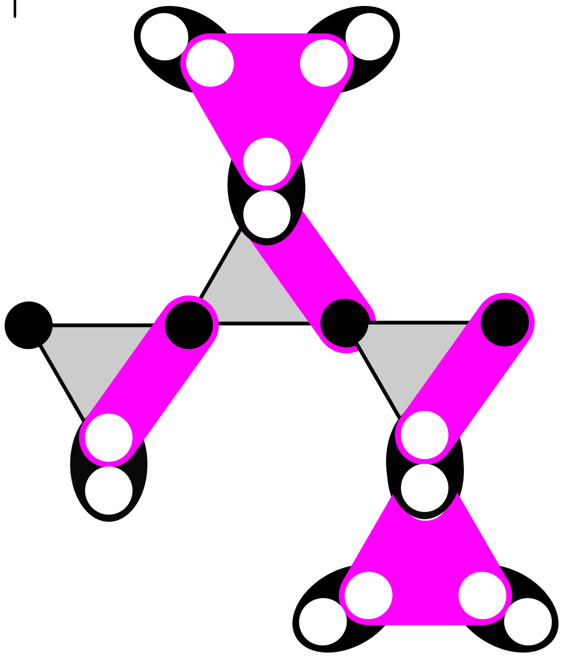

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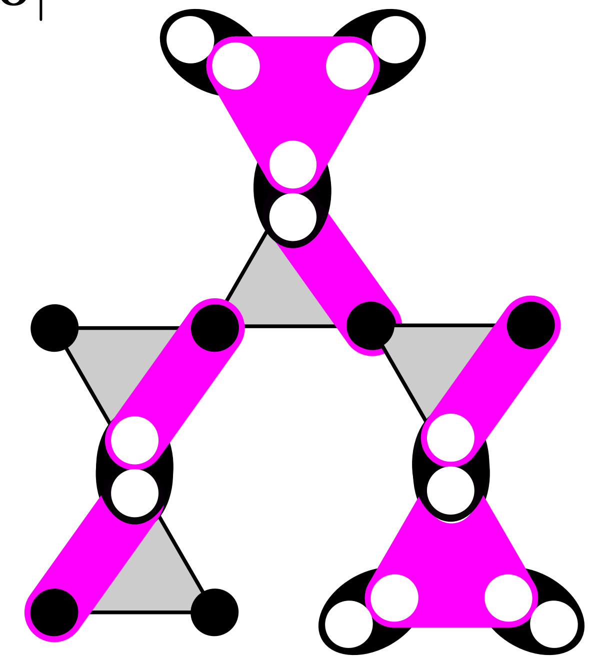

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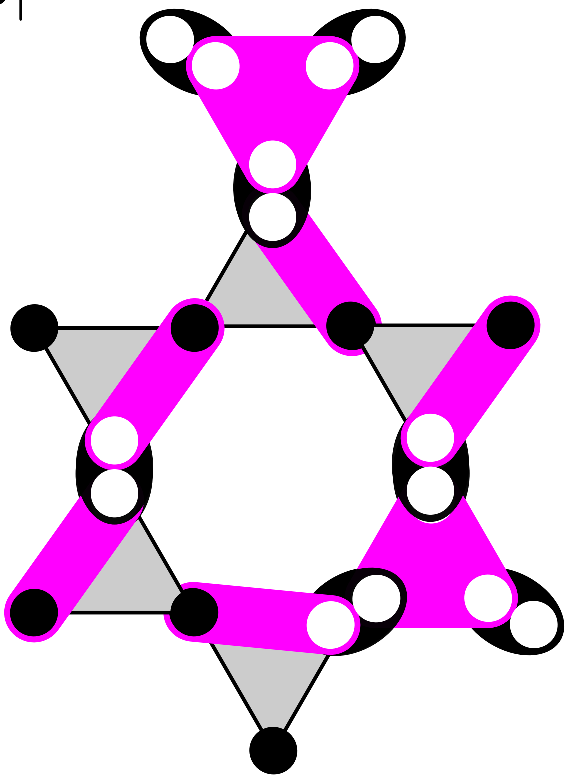

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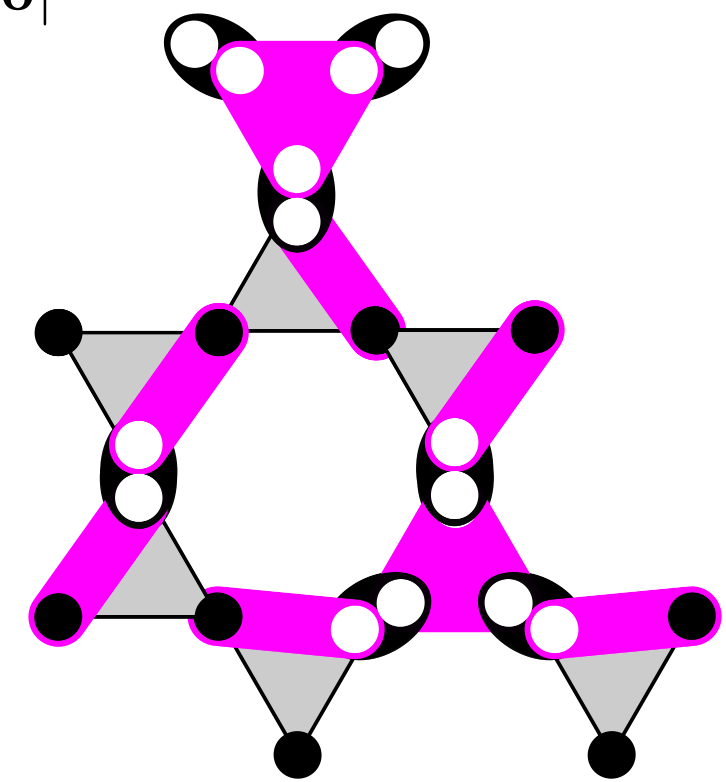

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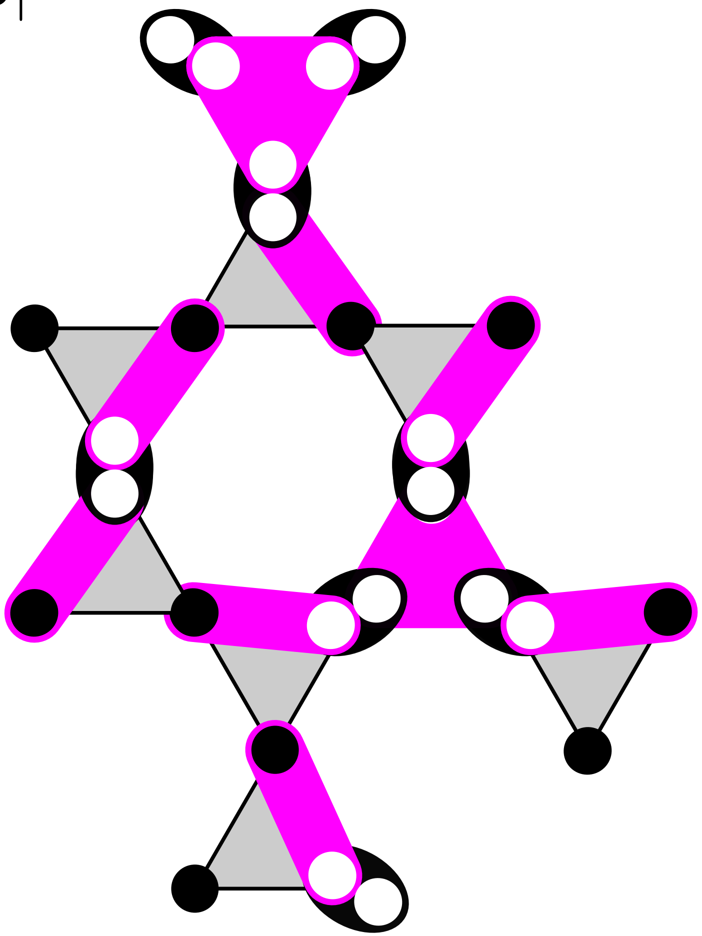

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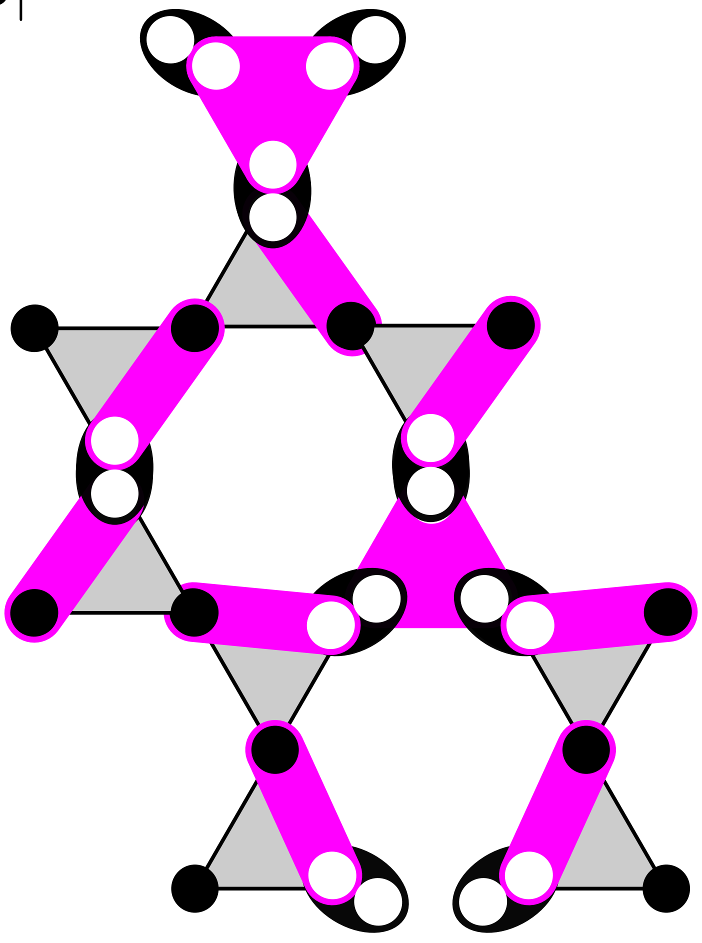

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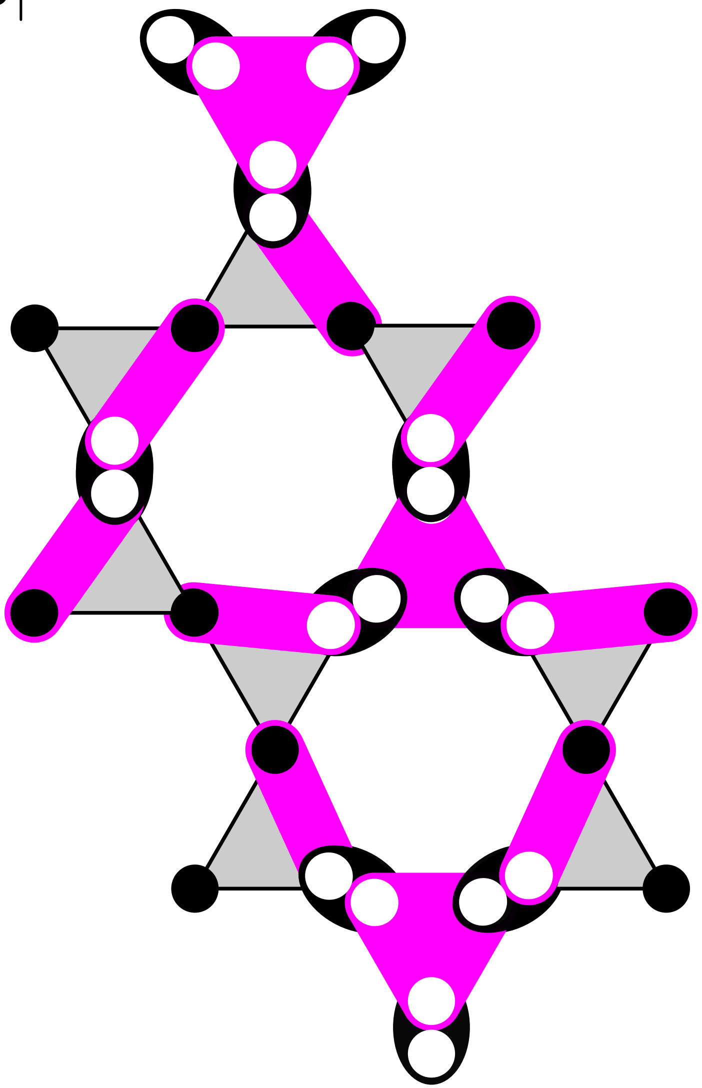
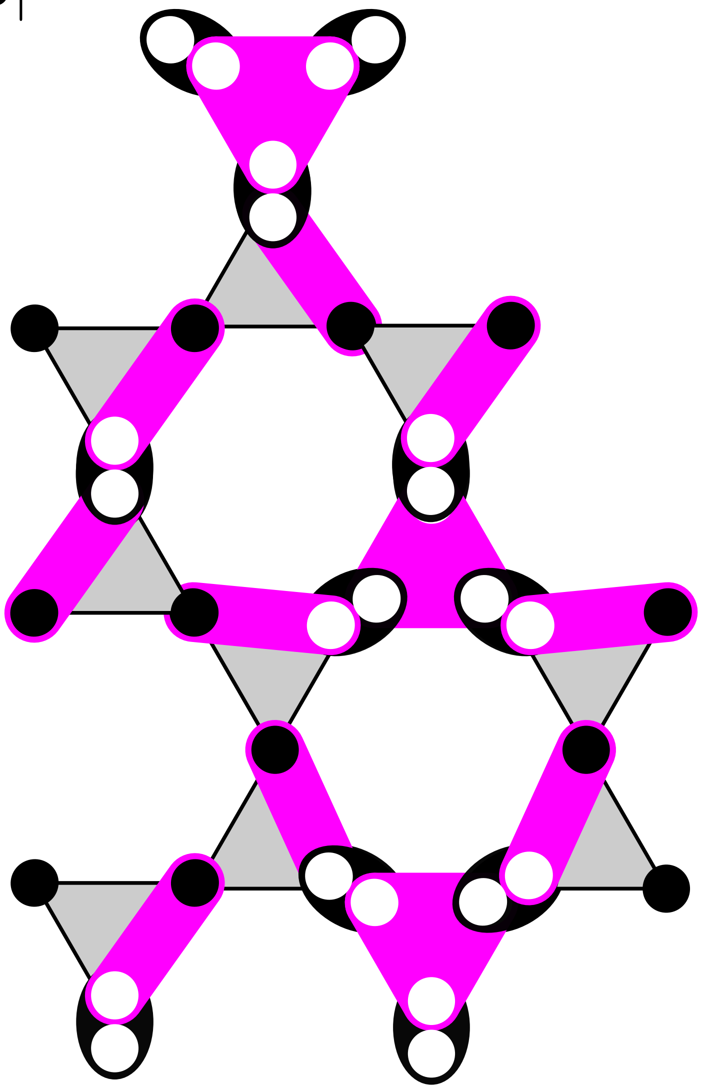


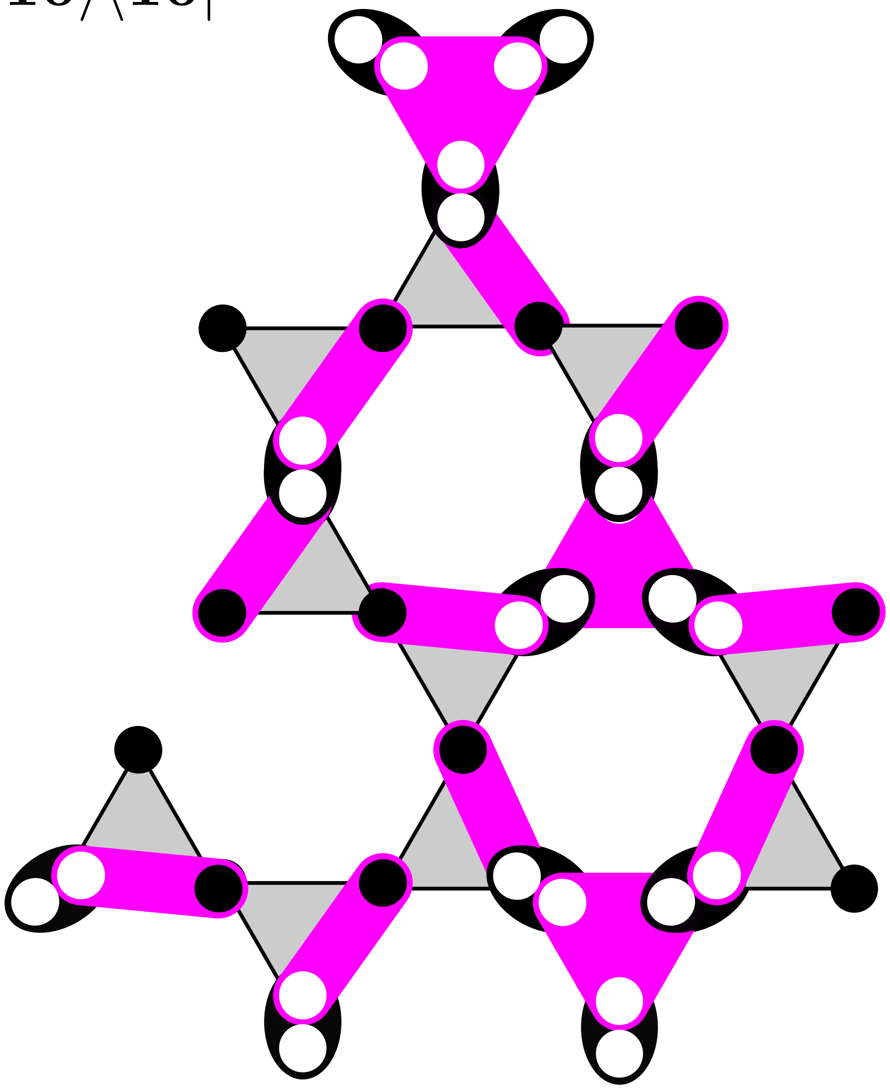
The J = K case: Lego time!

$$\mathcal{H} = \sum_{\triangle,\triangledown} |\mathbf{10}\rangle\langle\mathbf{10}|$$



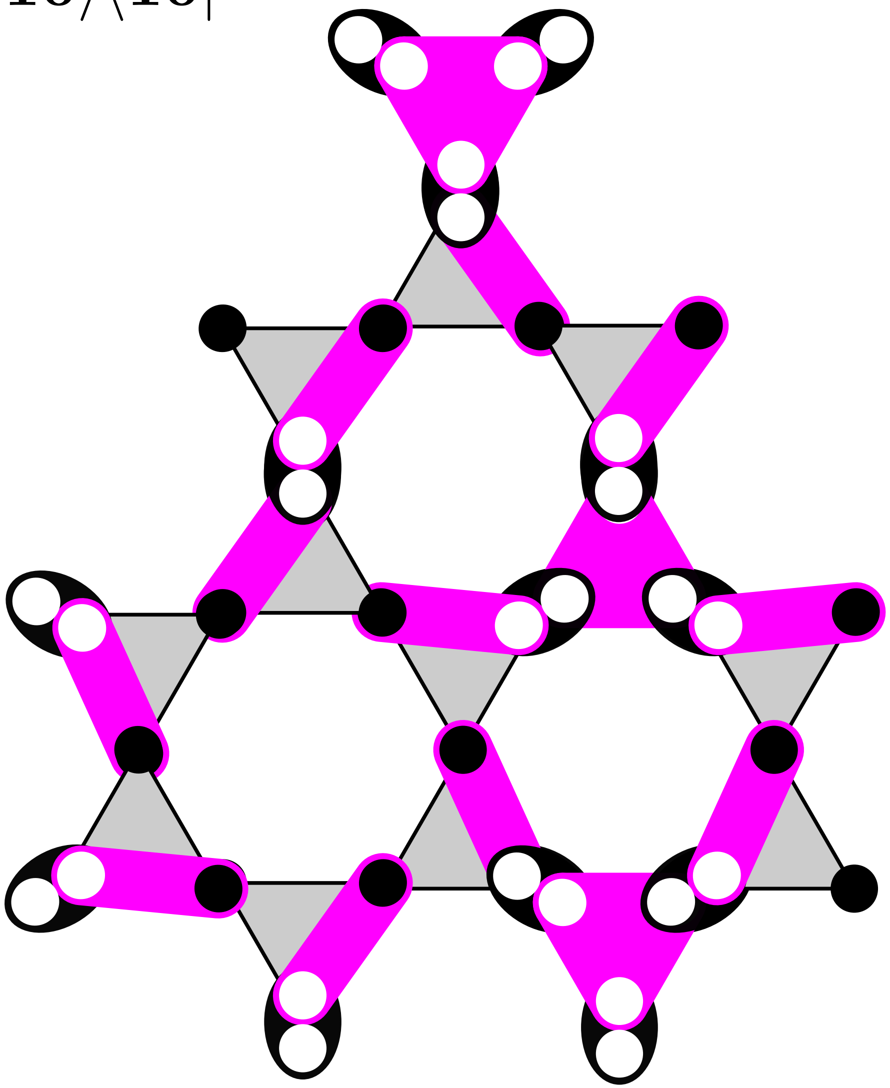
The J = K case: Lego time!

$$\mathcal{H} = \sum_{\triangle,\bigtriangledown} |\mathbf{10}\rangle\langle\mathbf{10}|$$



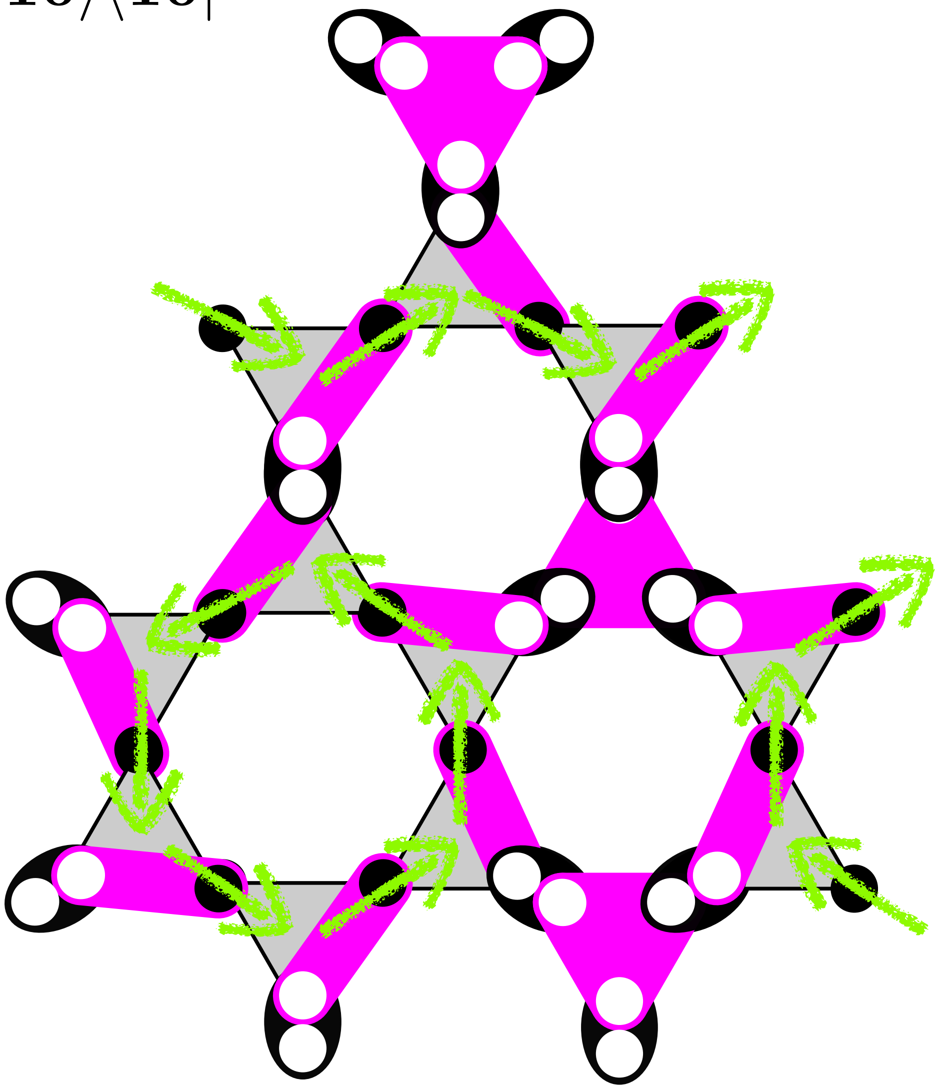
The J = K case: Lego time!

$$\mathcal{H} = \sum_{\triangle,\triangledown} |\mathbf{10}\rangle\langle\mathbf{10}|$$



The J = K case: Lego time!

$$\mathcal{H} = \sum_{\triangle,\nabla} |\mathbf{10}\rangle\langle\mathbf{10}|$$



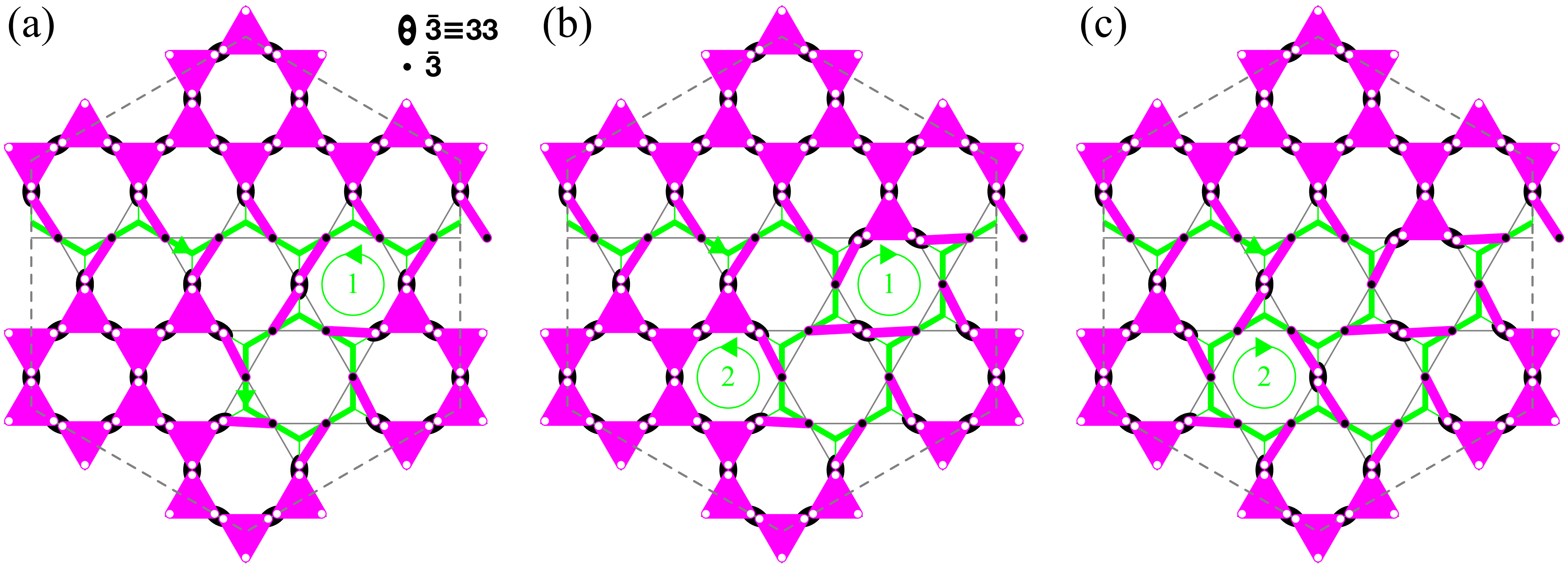
"current conservation" - some kind of a Coulomb liquid ?

On each bond 3 possibilities: 2 directions of arrow and absence of an arrow.

Z3 degrees of freedom

topological sectors (definition not obvious because of overlap and non-orthogonality)

The J = K case: singlet states characterized by directed loops on honeycomb lattice



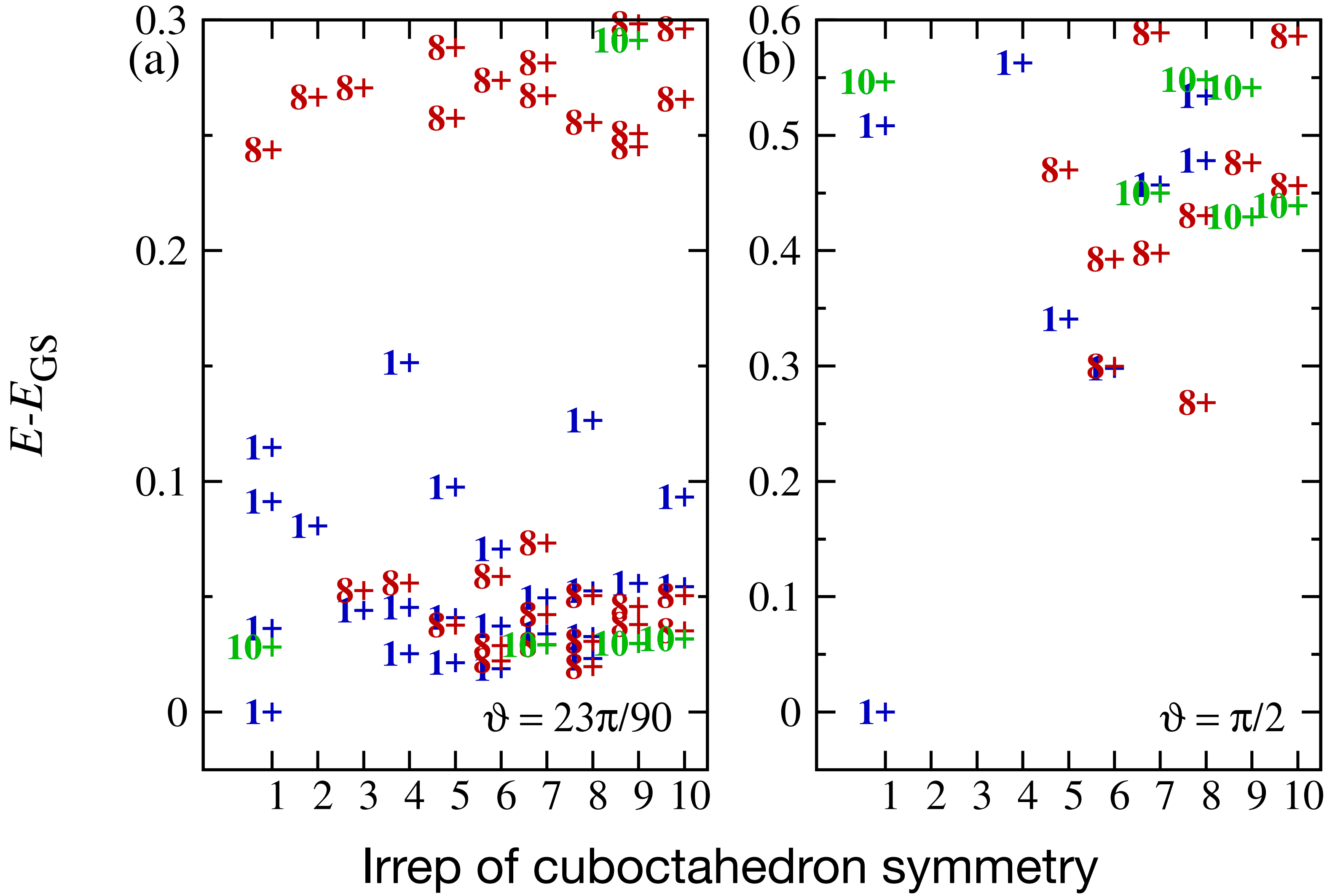
local loops -> extensive number of loops

for 12 sites they span the singlet GS manifold

number of undirected loops = $2\times2\times2^{(Nhex-1)}$

N	undirected	directed	lin. ind.
12	32	69	48
27	1024	2551	2485
36	8192	22437	

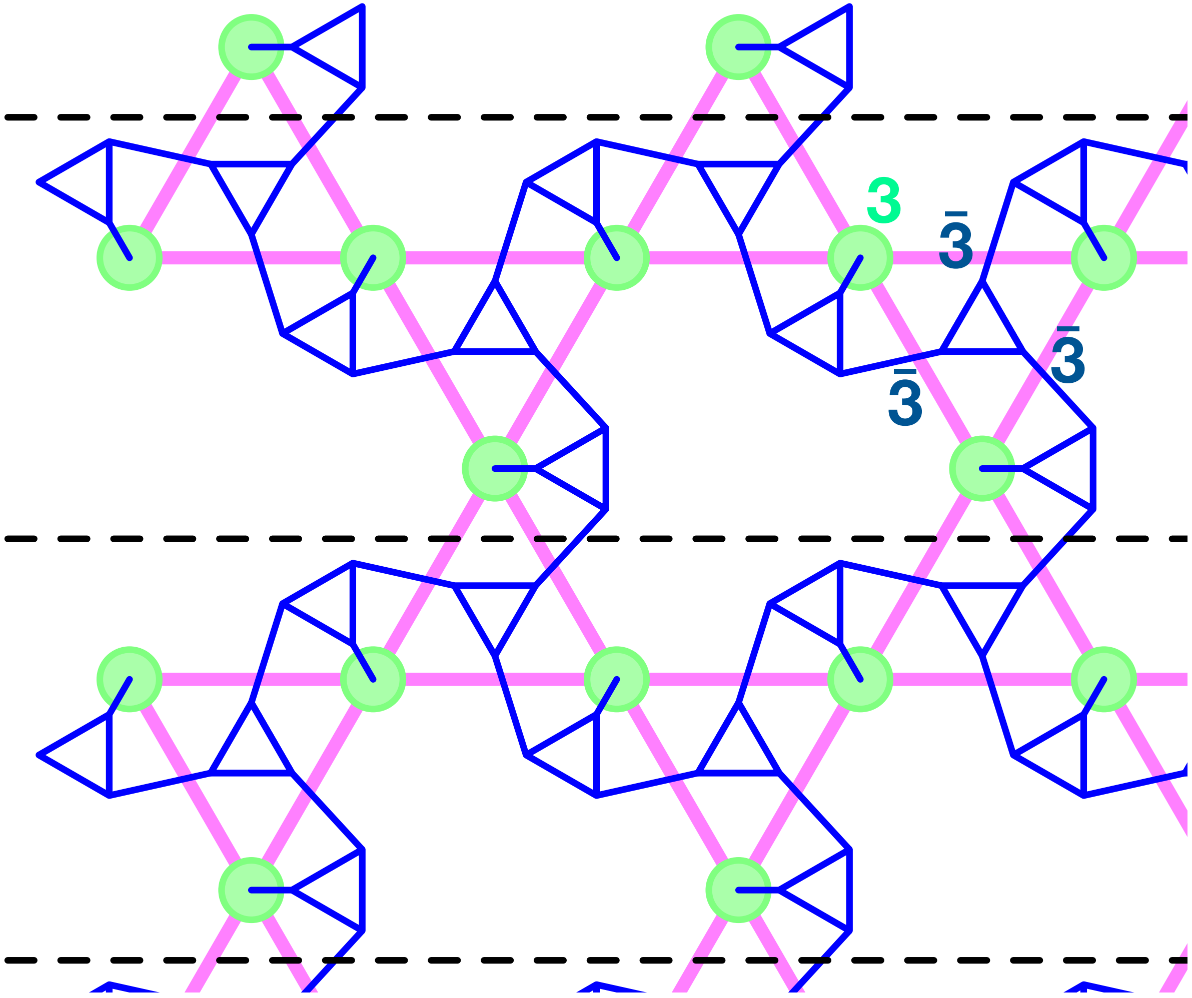
The J = K case: other irreps also appear



degeneracy at $\vartheta = \pi/4$: 468 = (48)×**1** + (40)×**8** + (10)×$\overline{\mathbf{10}}$

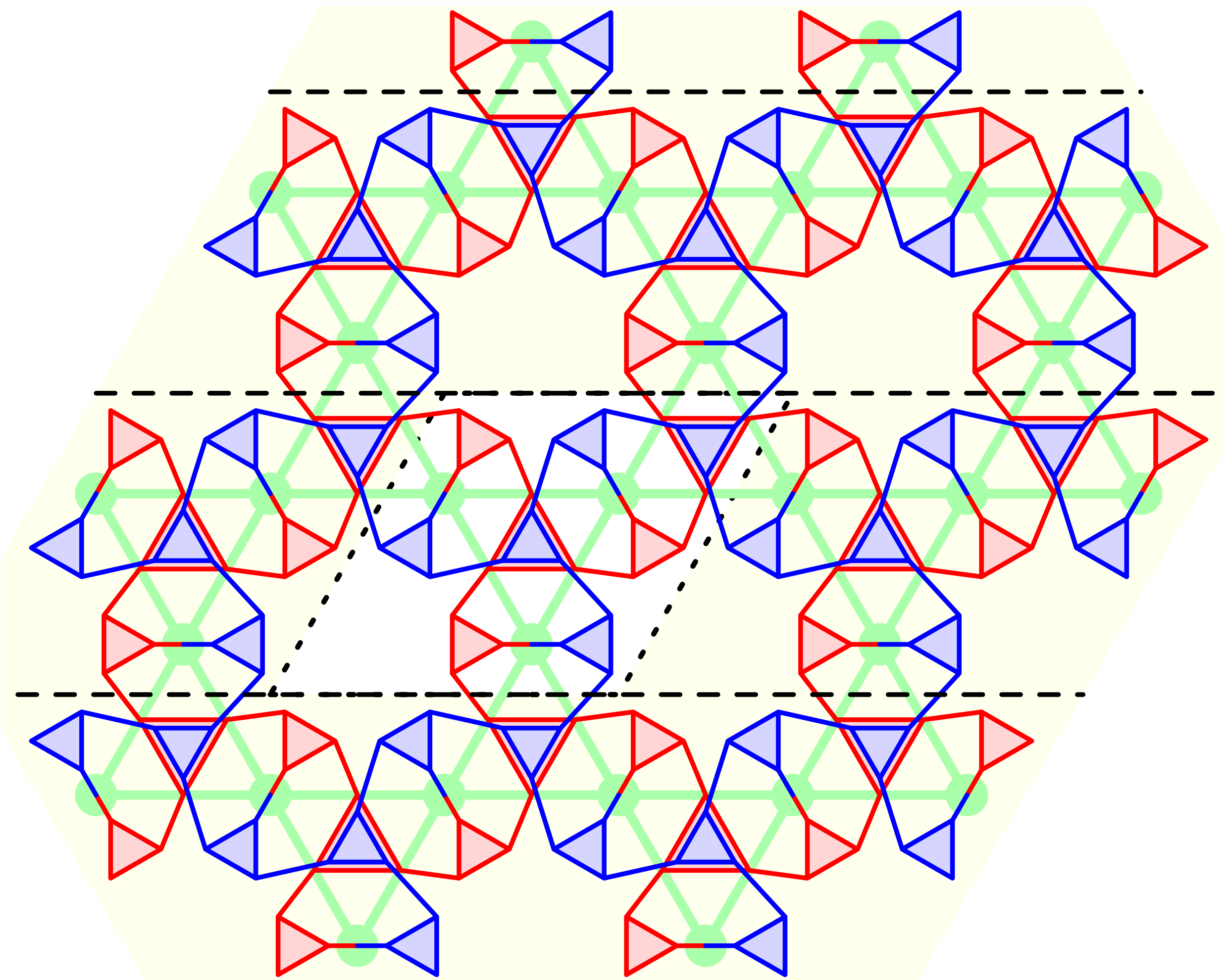
What is the origin of the higher SU(3) irreps ???

Tensor network: the wave function



each triangle represents the antisymmetrizing Levi-Civita symbol

Tensor network: the overlap



graph of contracted
Levi-Civita symbols

R. Penrose,
Applications of
negative dimensional
tensors, 1971

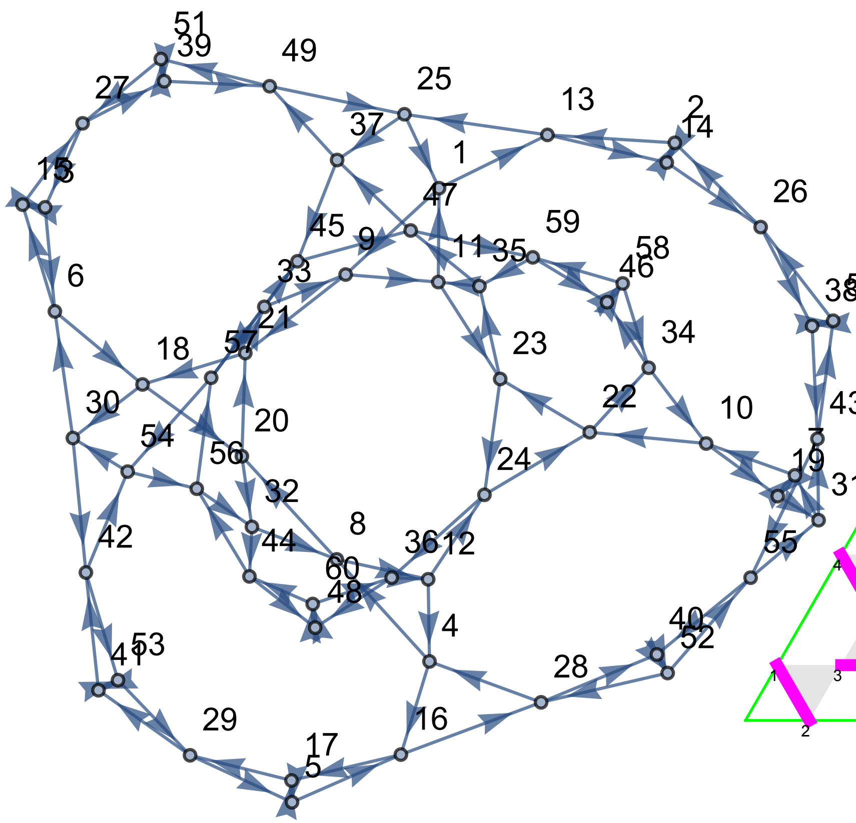
Penrose polynomial,
defined for plane graphs

12: 13392
27: 1828256832
36: 2220531642144

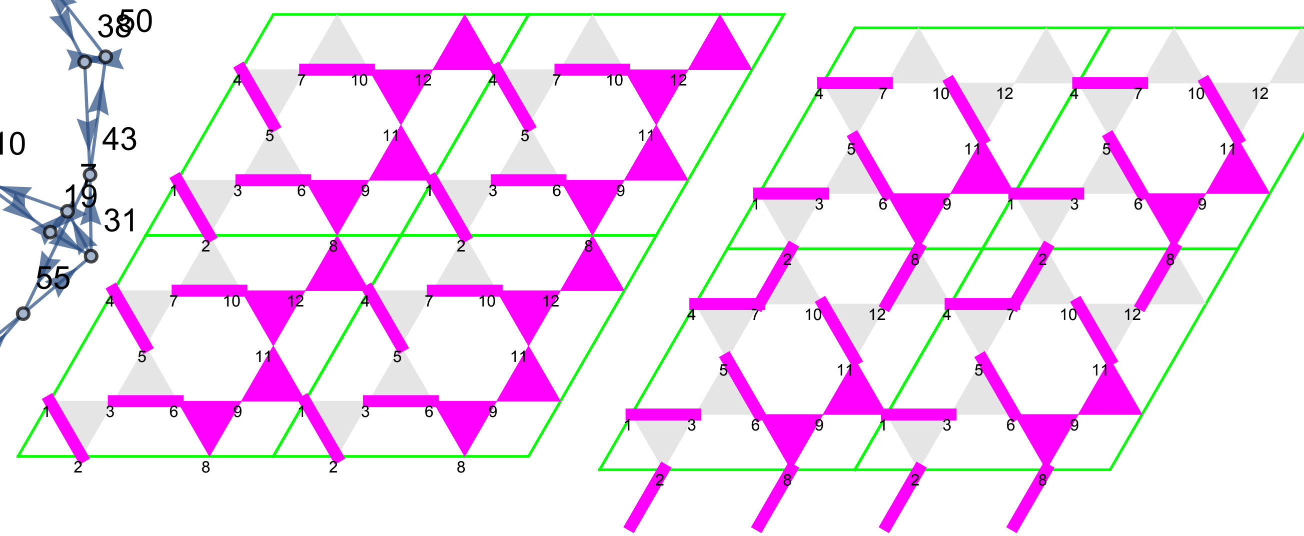
gfortran has 128-bit long
integer type:-)

Example for overlap (12 sites)

$$\varepsilon^{1,9,11}\varepsilon^{2,13,14}\varepsilon^{3,6,15}\varepsilon^{4,8,12}\varepsilon^{5,16,17}\varepsilon^{7,10,19}\varepsilon^{18,20,21}\varepsilon^{22,23,24}\varepsilon^{25,37,49}\varepsilon^{26,38,50}$$
$$\varepsilon^{27,39,51}\varepsilon^{28,40,52}\varepsilon^{29,41,53}\varepsilon^{30,42,54}\varepsilon^{31,43,55}\varepsilon^{32,44,56}\varepsilon^{33,45,57}\varepsilon^{34,46,58}\varepsilon^{35,47,59}\varepsilon^{36,48,60}$$
$$\varepsilon_{1,13,25}\varepsilon_{2,14,26}\varepsilon_{3,15,27}\varepsilon_{4,16,28}\varepsilon_{5,17,29}\varepsilon_{6,18,30}\varepsilon_{7,19,31}\varepsilon_{8,20,32}\varepsilon_{9,21,33}\varepsilon_{10,22,34}$$
$$\varepsilon_{11,23,35}\varepsilon_{12,24,36}\varepsilon_{37,45,47}\varepsilon_{38,43,50}\varepsilon_{39,49,51}\varepsilon_{40,52,55}\varepsilon_{41,42,53}\varepsilon_{44,48,60}\varepsilon_{46,58,59}\varepsilon_{54,56,57}$$
$$= 49152$$



The graphs are “bipartite” (median graph for degree 3 regular bipartite graph)



Penrose graph

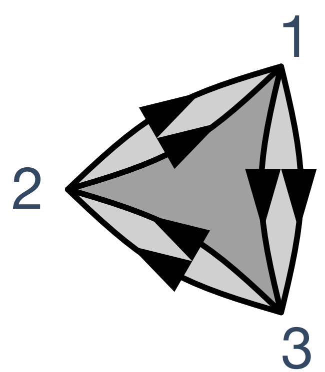
Evaluating Penrose graphs

$$\varepsilon_{i,j,k}\,\varepsilon^{i,j,k} = 6$$

$$\varepsilon_{i,j,k}\,\varepsilon^{i,j,l} = 2\delta^l_k$$

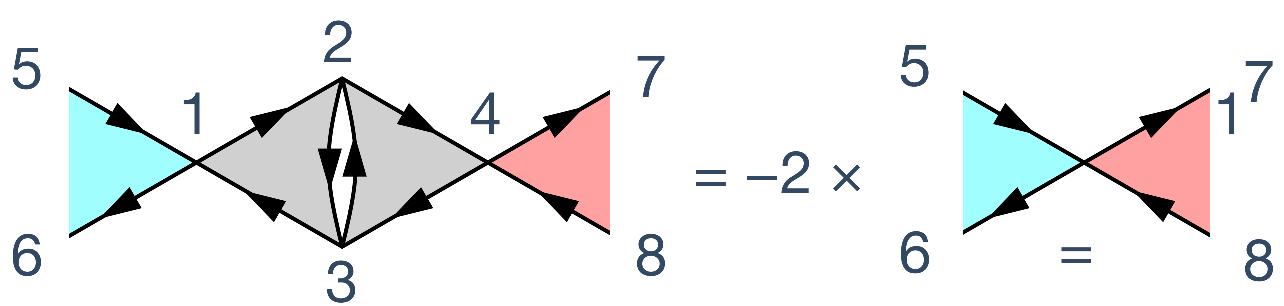
$$\varepsilon_{i,j,k}\,\varepsilon^{i,l,m} = \delta^l_j\delta^m_k - \delta^m_j\delta^l_k$$

implied sum over repeated indices



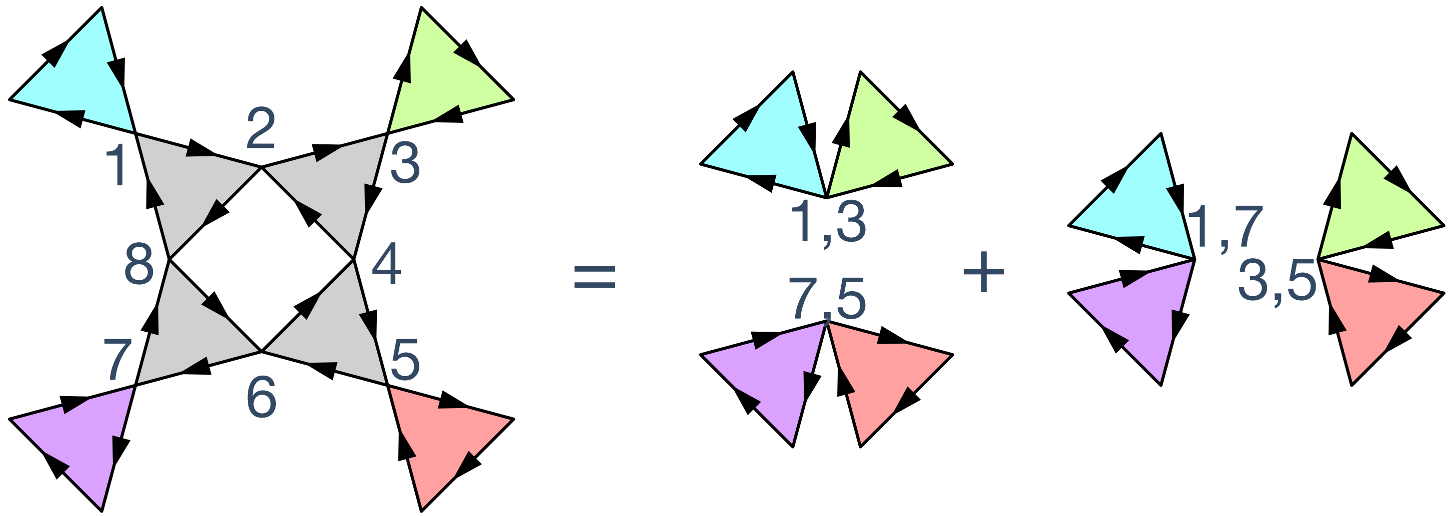
$$\varepsilon_{1,2,3}\,\varepsilon^{1,2,3} = 6$$

We can define a recursive procedure to evaluate the Penrose graph:

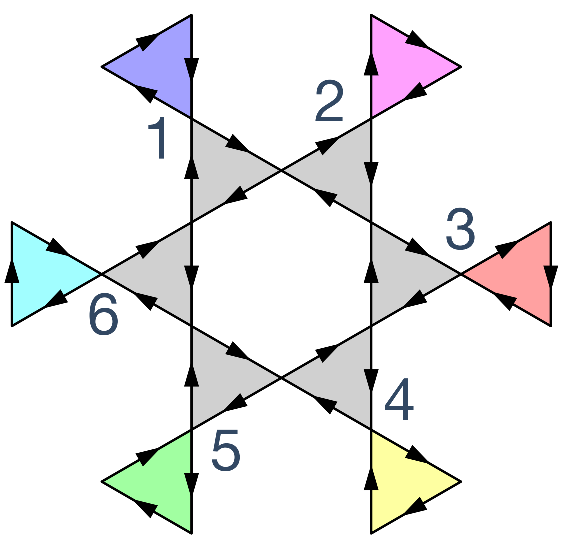


$$\ldots\varepsilon_{5,1,6}\,\varepsilon^{1,2,3}\,\varepsilon_{2,4,3}\,\varepsilon^{4,7,8}\ldots = -2\times\ldots\varepsilon_{5,1,6}\,\varepsilon^{1,7,8}\ldots$$

Evaluating Penrose graphs



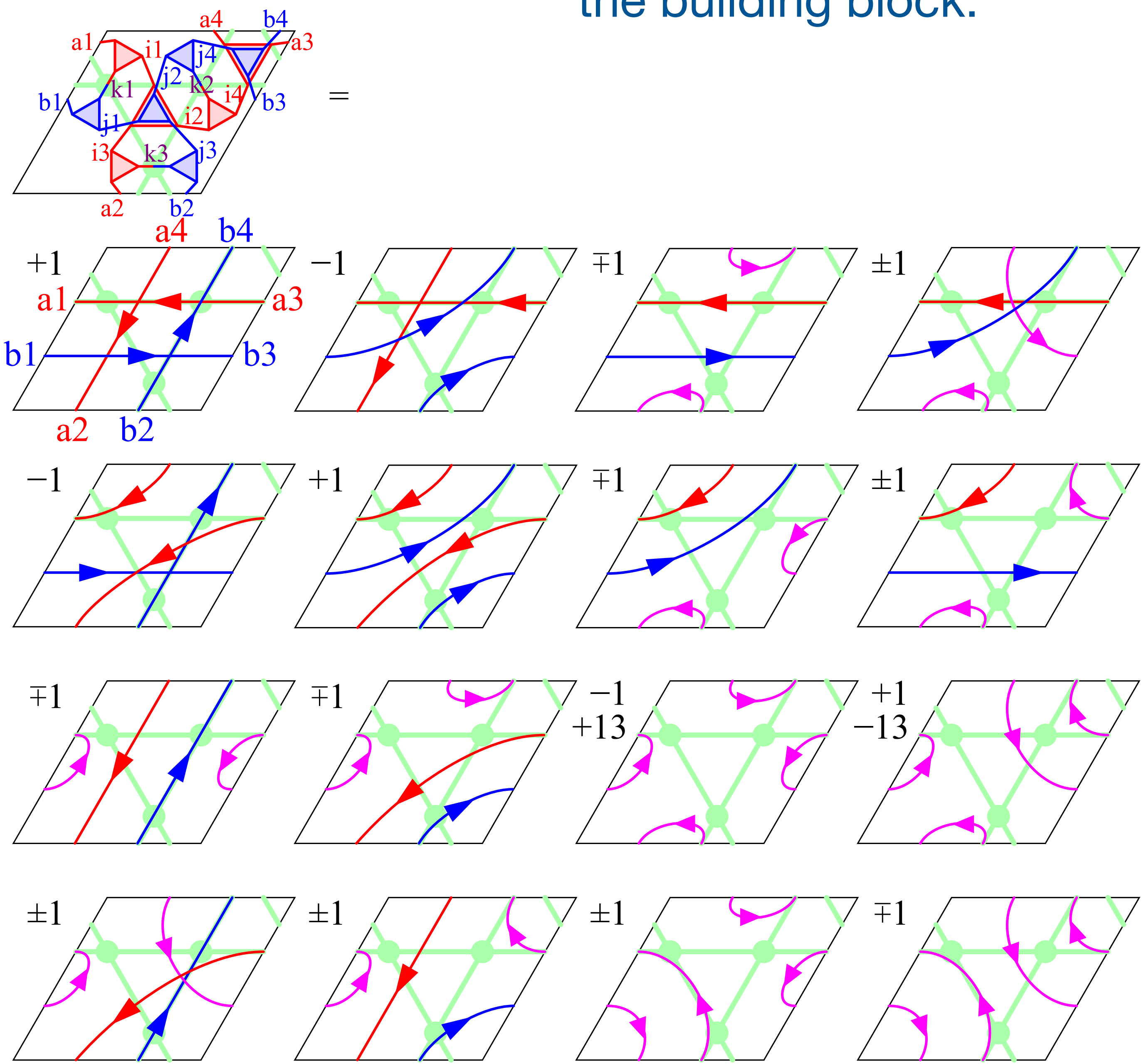
$$\ldots\varepsilon_{1,2,8}\,\varepsilon^{2,3,4}\,\varepsilon_{4,5,6}\,\varepsilon^{6,7,8}\ldots = \delta_1^3\delta_5^7 + \delta_1^7\delta_5^3$$



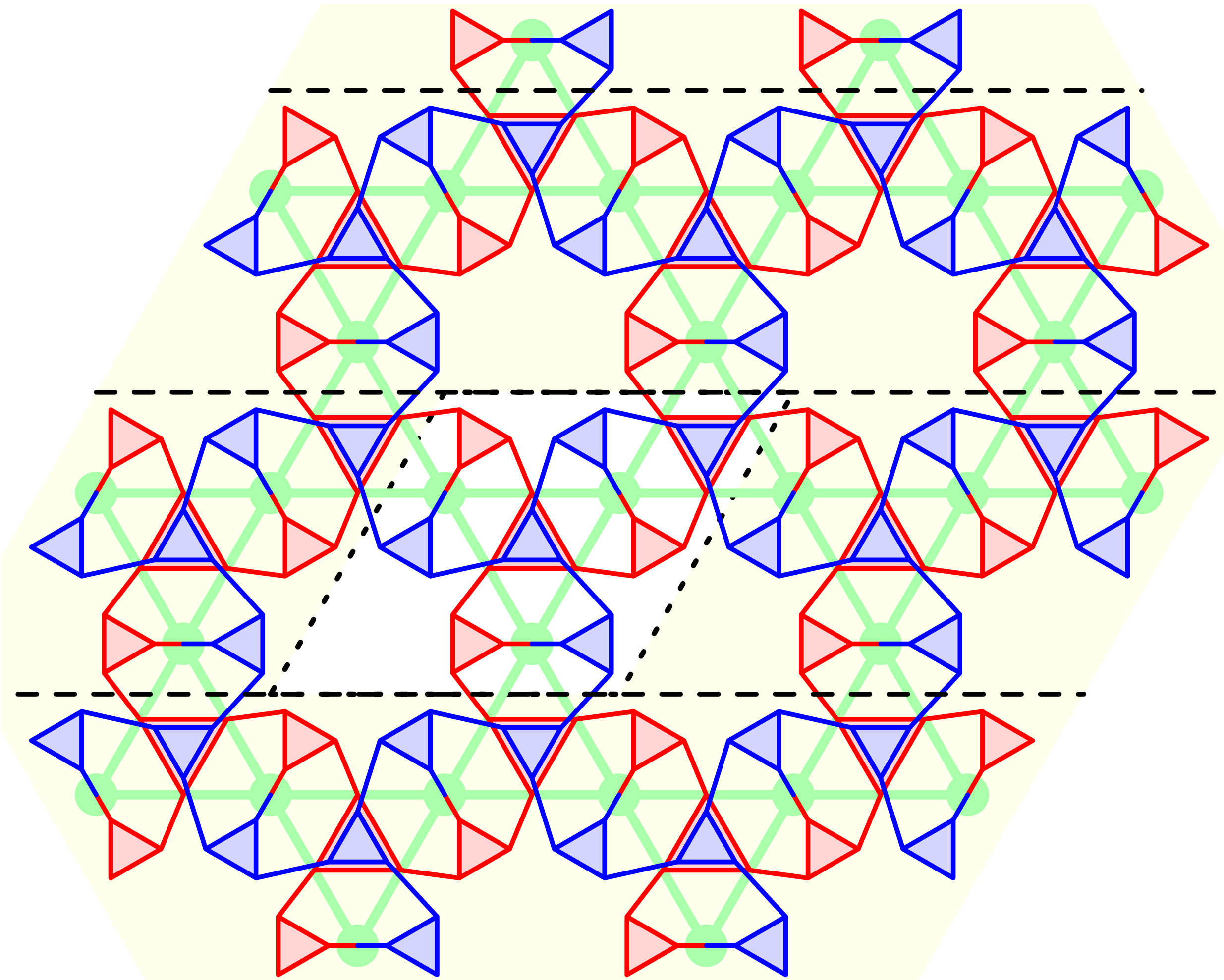
$$= -\,\delta_1^2\delta_3^6\delta_5^4 - \delta_1^6\delta_3^4\delta_5^2 - \delta_1^4\delta_3^2\delta_5^6 + \delta_1^4\delta_3^6\delta_5^2$$

Evaluating using tensor network

the building block:



Tensor network: the overlap



is simply a product of matrices

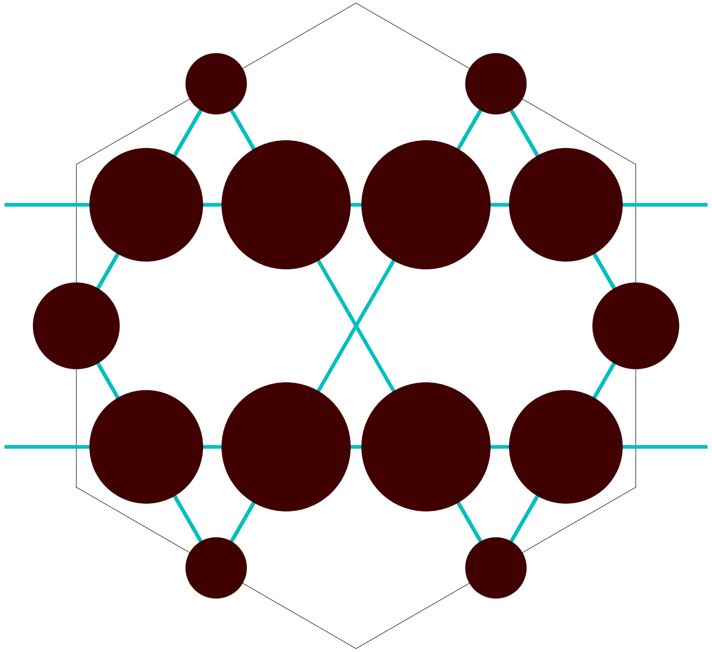
Spin-spin correlation function

calculated using "tensor network"

12 sites

27 sites

48 sites



Spin-spin correlation function

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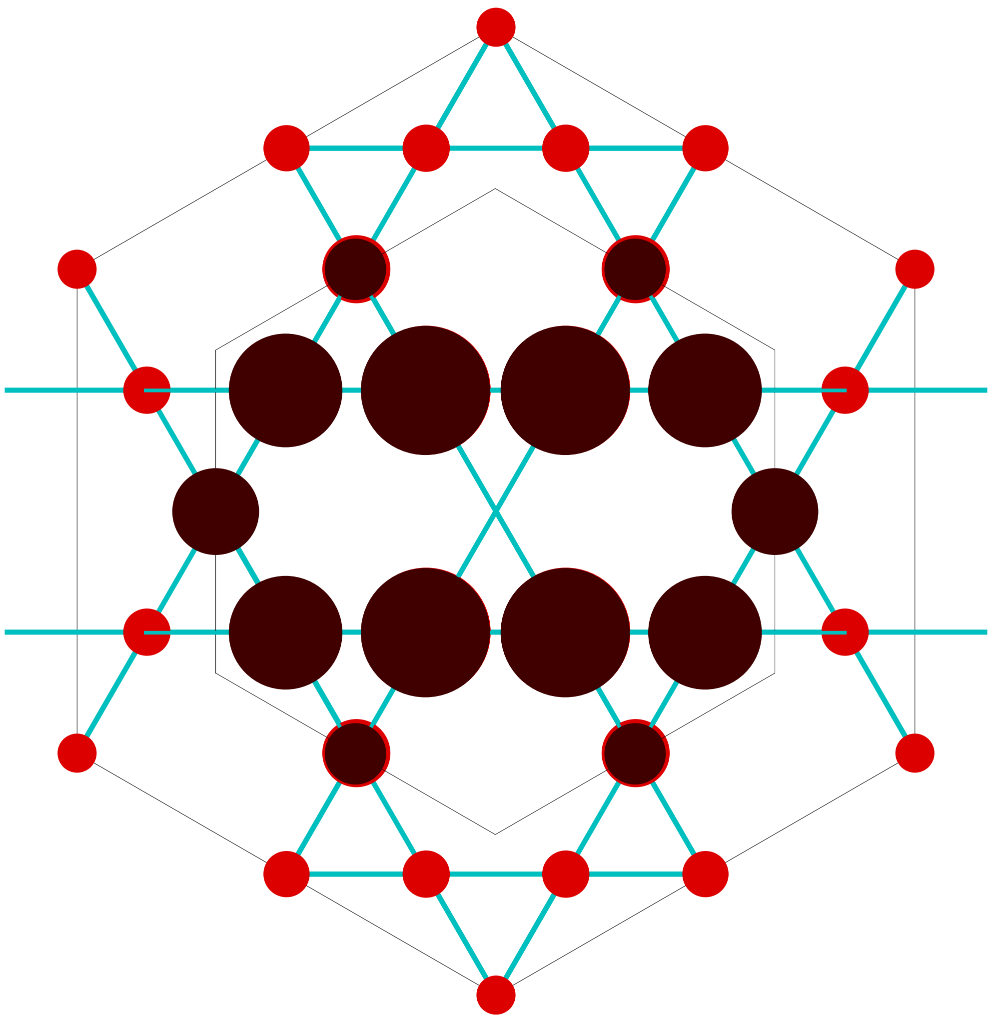

Spin-spin correlation function

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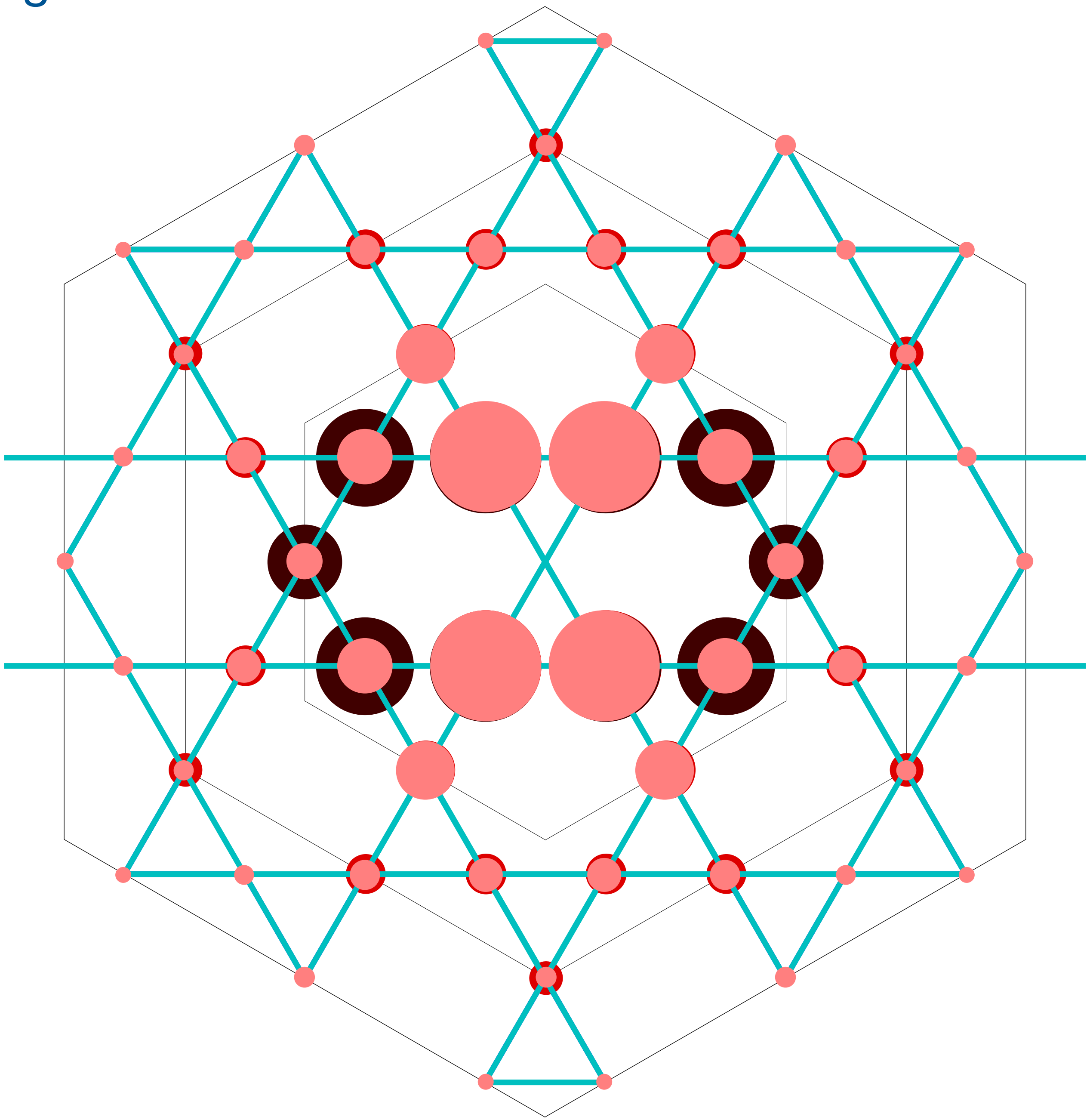
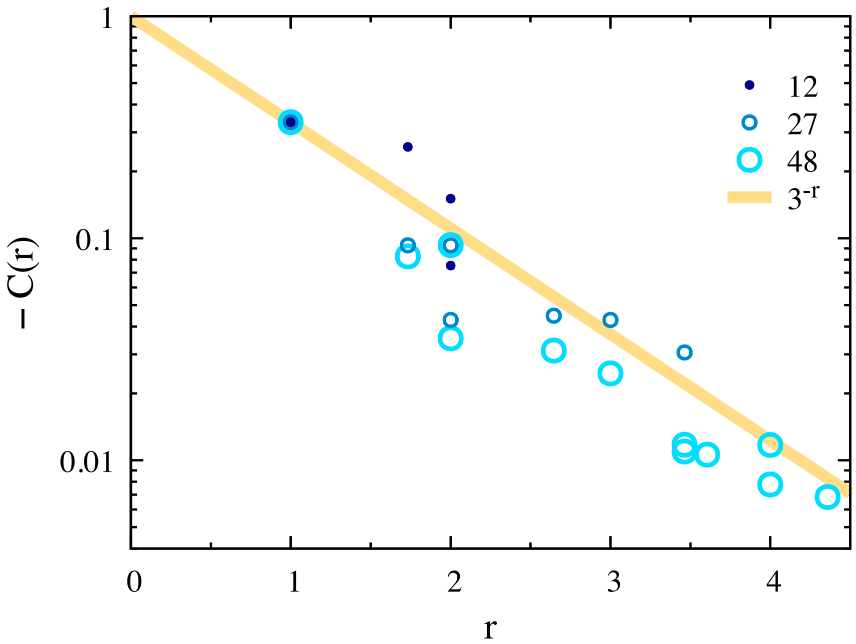
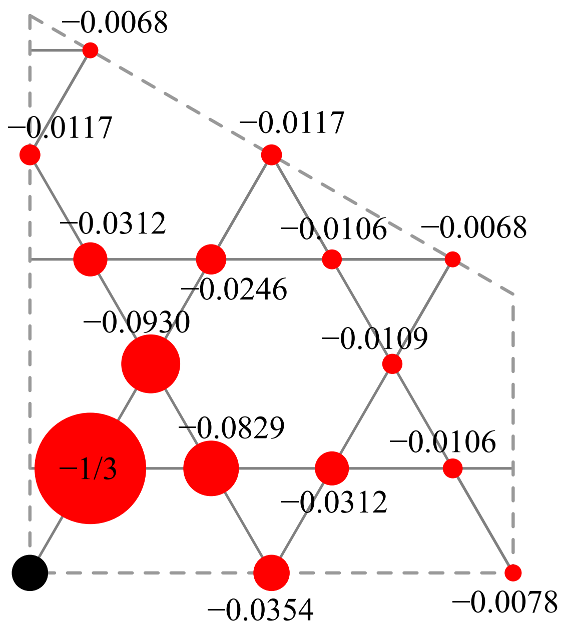
48 sites



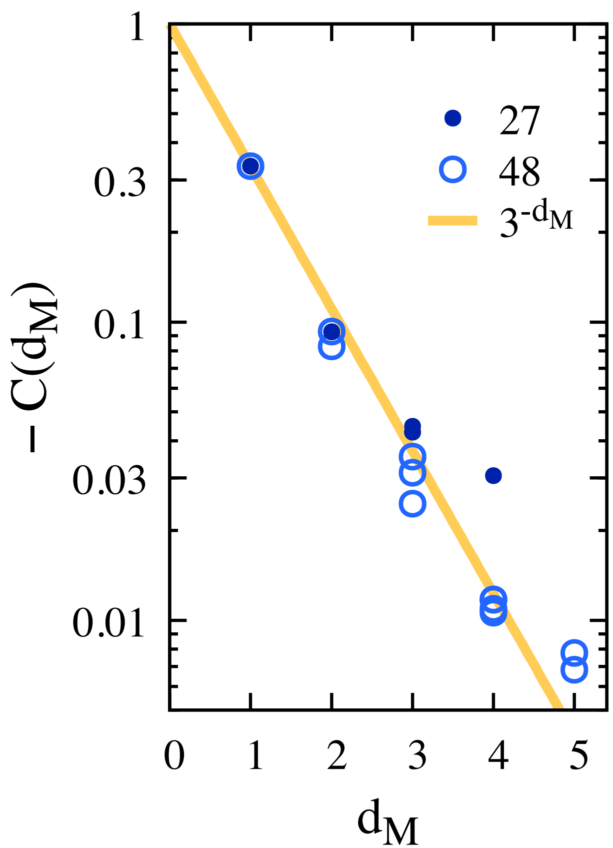
Spin-spin correlation function





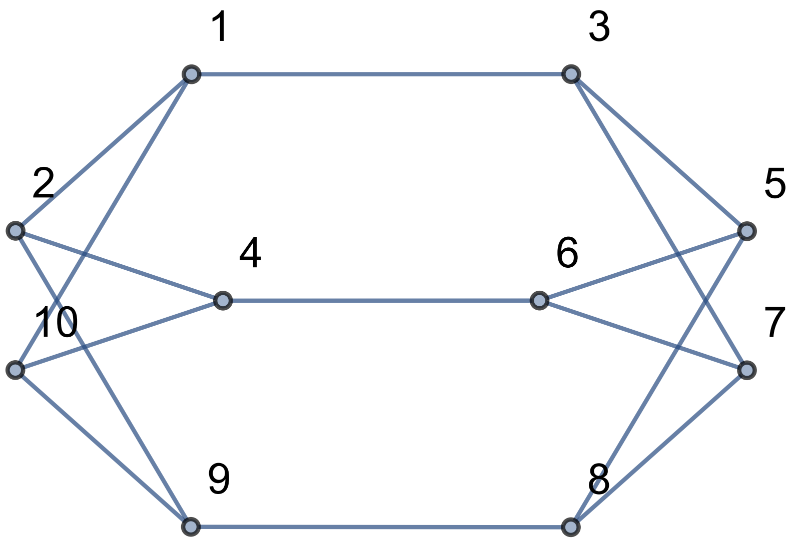
decays exponentially,

$$C(r) = \langle \mathrm{FSS}|A^{\mu}A_{\mu}|\mathrm{FSS}\rangle$$
$$= \langle \mathrm{FSS}|\left(P_{0,r} - 1/3\right)|\mathrm{FSS}\rangle$$
$$\approx 3^{-r}$$



Manhattan distance

Triviality ?



Regular graph of degree 3 (cubic graph).

The medial graph is a “kagome” lattice (corner sharing triangles), FSS is a ground state.

The transformations of the FSS wave function under the generators of isomorphism group are

{–1,1,–1,–1,1}

For the trivial state they are all 1.

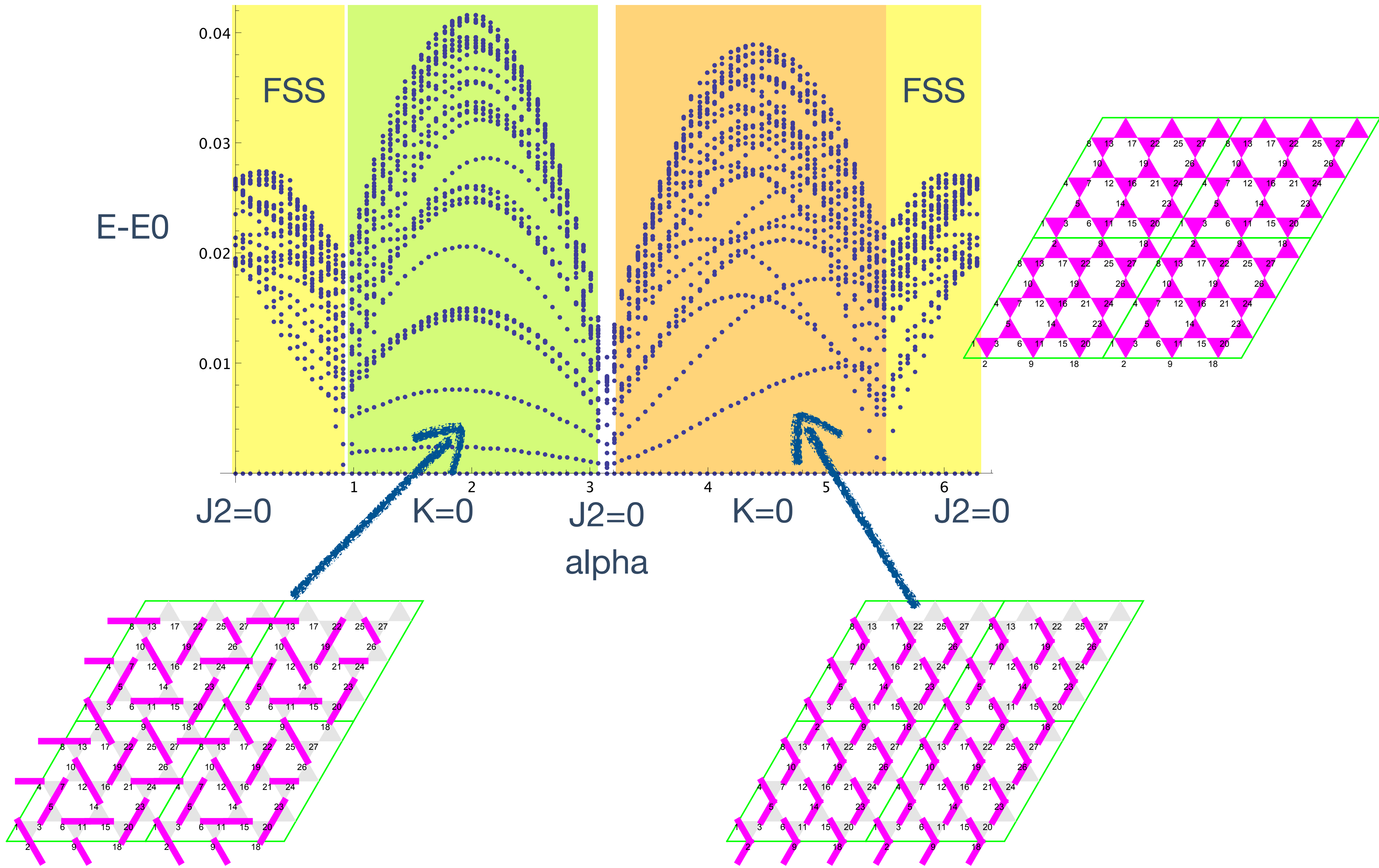
1	{1, 2, 3}
2	{1, 4, 5}
3	{2, 6, 7}
4	{4, 8, 9}
5	{6, 10, 11}
6	{8, 10, 12}
7	{7, 12, 13}
8	{11, 13, 14}
9	{5, 14, 15}
10	{3, 9, 15}

{{1 → 1, 2 → 2, 3 → 3, 4 → 4, 5 → 5, 6 → 7, 7 → 6, 8 → 8, 9 → 9, 10 → 12, 11 → 13, 12 → 10, 13 → 11, 14 → 14, 15 → 15},
{1 → 1, 2 → 2, 3 → 3, 4 → 5, 5 → 4, 6 → 6, 7 → 7, 8 → 14, 9 → 15, 10 → 11, 11 → 10, 12 → 13, 13 → 12, 14 → 8, 15 → 9},
{1 → 3, 2 → 2, 3 → 1, 4 → 9, 5 → 15, 6 → 6, 7 → 7, 8 → 8, 9 → 4, 10 → 10, 11 → 11, 12 → 12, 13 → 13, 14 → 14, 15 → 5},
{1 → 6, 2 → 2, 3 → 7, 4 → 10, 5 → 11, 6 → 1, 7 → 3, 8 → 8, 9 → 12, 10 → 4, 11 → 5, 12 → 9, 13 → 15, 14 → 14, 15 → 13},
{1 → 4, 2 → 8, 3 → 9, 4 → 1, 5 → 5, 6 → 10, 7 → 12, 8 → 2, 9 → 3, 10 → 6, 11 → 11, 12 → 7, 13 → 13, 14 → 14, 15 → 15}}

Lifting the degeneracy: K - J2 model

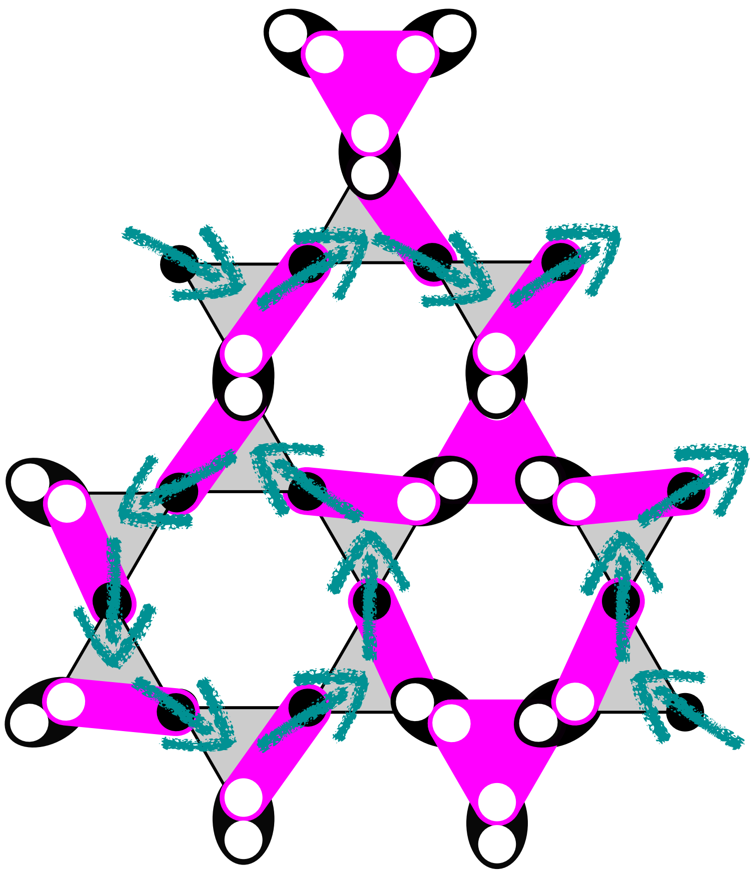
$$K = \cos\alpha$$
$$J_2 = \sin\alpha$$

ED in the Hilbert space spanned by singlets, 27 sites

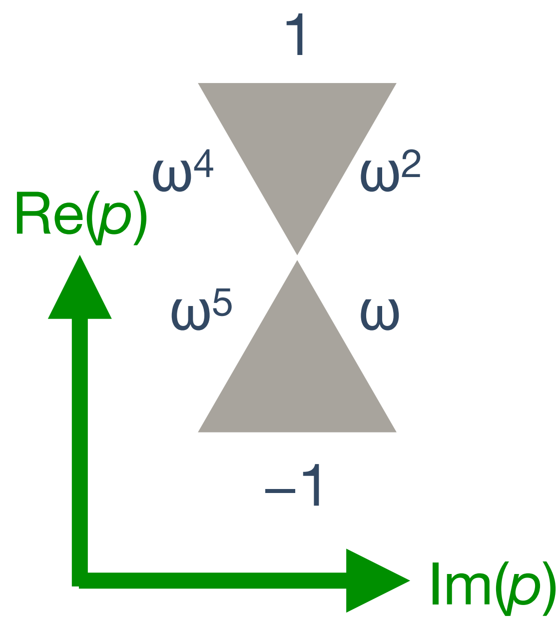


Topological sectors (polarizability)

following Bulaevskii, Batista, Mostovoy, and Khomskii,
Phys. Rev. B **78**, 024402 (2008).



$$\omega = \exp \frac{2\pi i}{6}$$

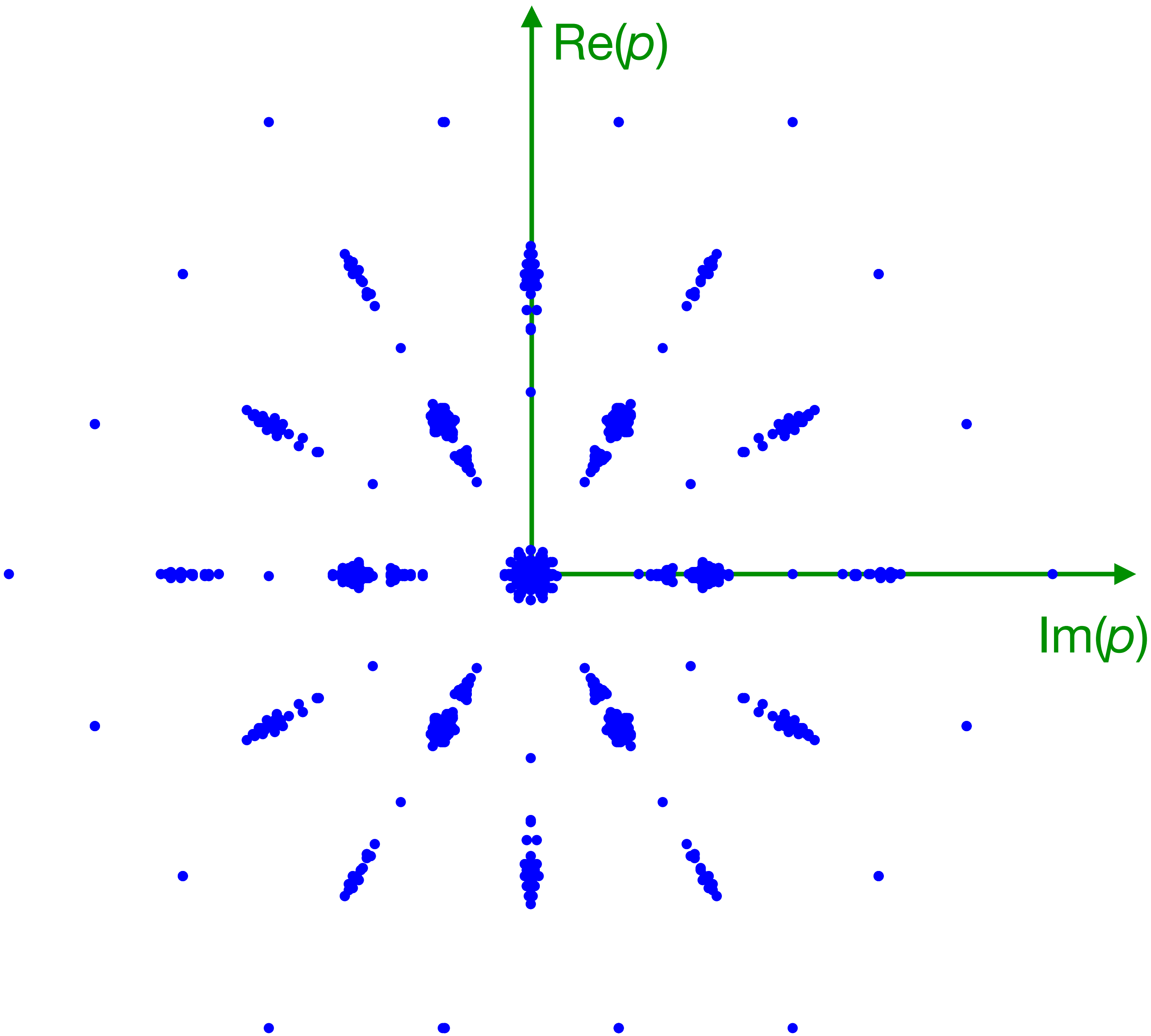


we calculate the eigenvalues of the polarization operator p:

$$p = \sum_{j \in \text{bonds}} \omega^{l(j)} \langle \mathscr{P}_j \rangle$$

where $\langle \mathscr{P}_j \rangle$ is the expectation value of the spin correlation on the bond.

Topological sectors (polarizability)



we calculate the eigenvalues of the polarization operator:

$$p = \sum_{j \in \text{bonds}} \omega^{l(j)} \mathscr{P}_j$$

27 sites, 2485 states

Tensor networks: Z_3 topological order

I. Kurecic, L. Vanderstraeten, N. Schuch, A gapped SU(3) spin liquid with Z_3 topological order, Phys. Rev. B **99**, 045116 (2019)

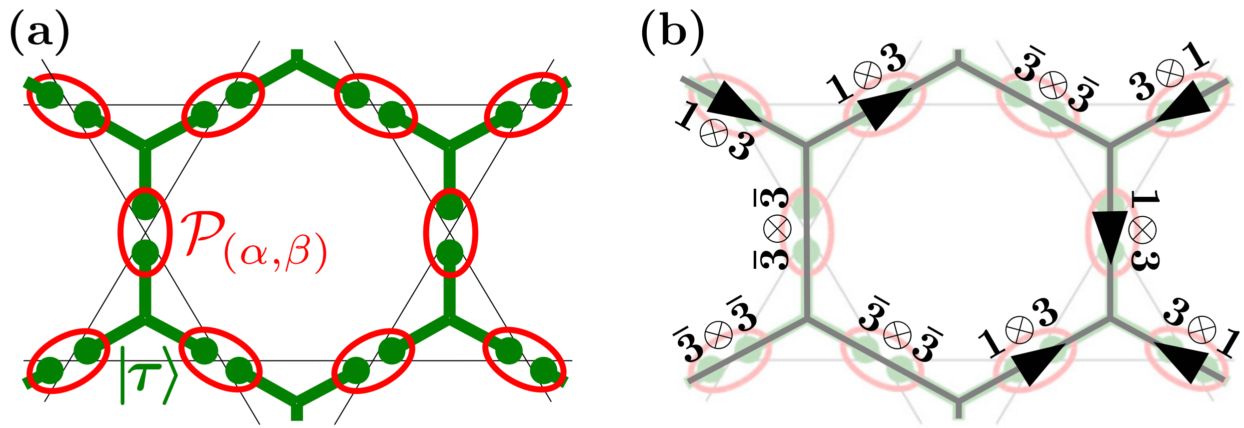
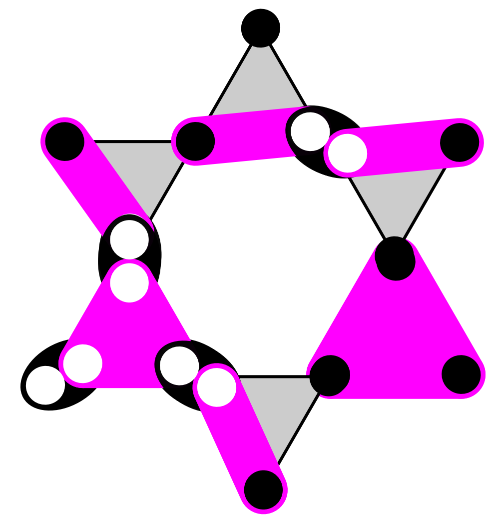
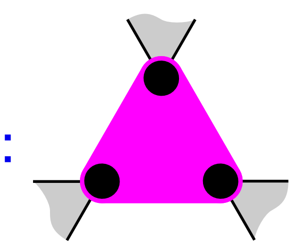


FIG. 1. (a) The model is constructed from trimers $|\tau\rangle$ which are in a singlet state with representation $\mathcal{H}_v \equiv \mathbf{1} \oplus \mathbf{3} \oplus \bar{\mathbf{3}}$ at each site (green dots), to which a map $\mathcal{P}_\bullet$ is applied which selects the physical degrees of freedom from $\mathcal{H}_v \otimes \mathcal{H}_v$. (b) Mapping to a $\mathbb{Z}_3$ topological model: Each site holds a $\mathbb{Z}_3$ degree of freedom: one of two arrows or no arrow. The arrows are pointing towards the $\mathbf{3}$ representation and satisfy a Gauss law across each vertex due to the fusion rules of the SU(3) irreps.



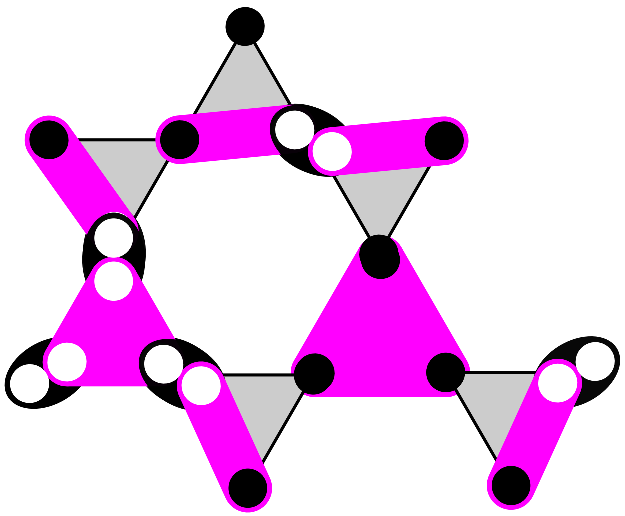
The trimer singlet is new:



parent Hamiltonian has 17 (?) sites, not shown in the papers

Tensor networks: Z_3 topological order

I. Kurecic, L. Vanderstraeten, N. Schuch, A gapped SU(3) spin liquid with Z_3 topological order, Phys. Rev. B **99**, 045116 (2019)



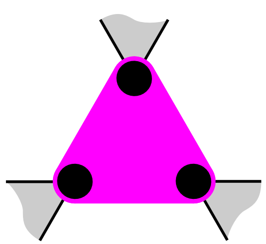
$$N_{\text{sites}} = \frac{3}{2} N_{\bar{\mathbf{3}}\bar{\mathbf{3}}\bar{\mathbf{3}}} + 3N_{\mathbf{333}} + \frac{3}{2} N_{\bar{\mathbf{3}}\mathbf{3}}$$

$$N_{\text{tris}} = N_{\bar{\mathbf{3}}\bar{\mathbf{3}}\bar{\mathbf{3}}} + N_{\mathbf{333}} + N_{\bar{\mathbf{3}}\mathbf{3}}$$

$$3N_{\text{tris}} = 2N_{\text{sites}}$$

from these equations: $N_{\mathbf{333}} = 0$

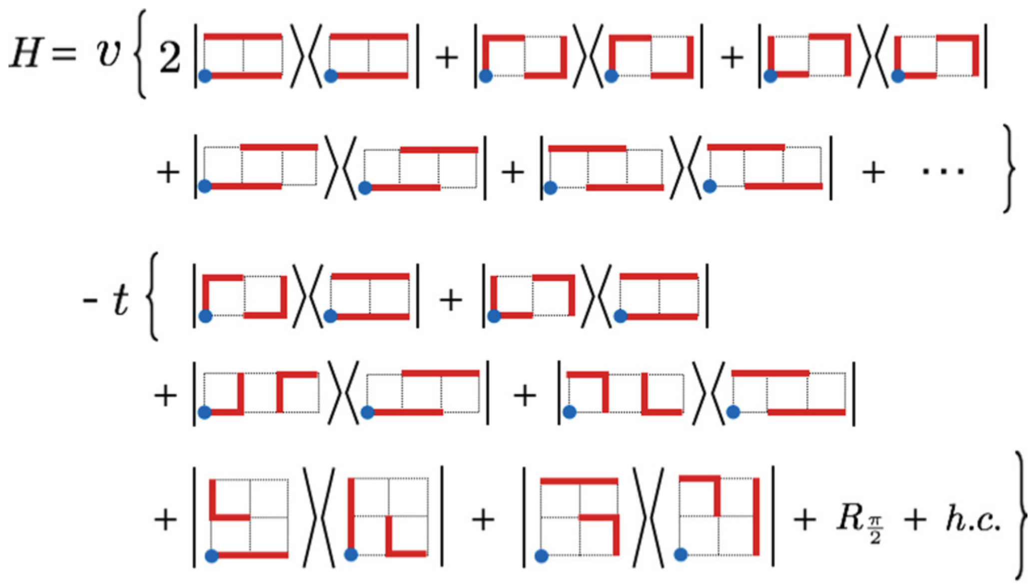
a single



creates an unhappy triangle elsewhere (unless saved by non-orthogonality)

Tensor networks: Z_3 topological order

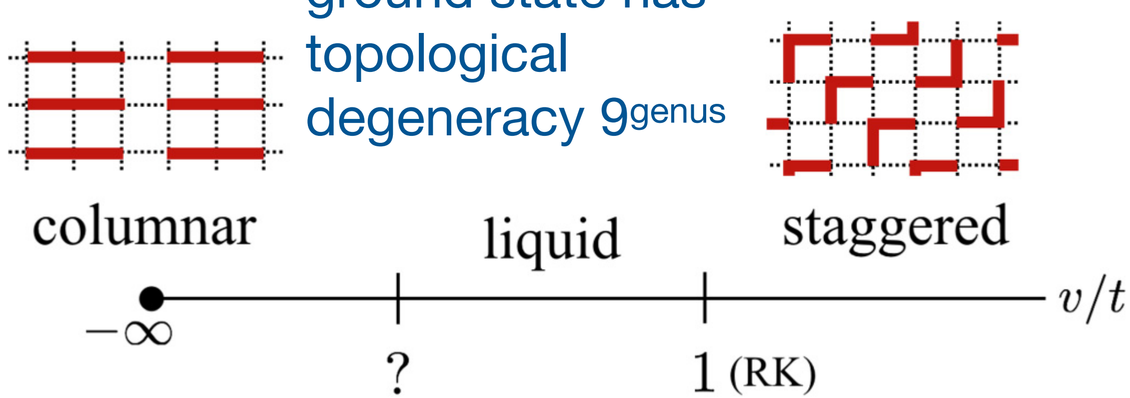
H. Lee, Y. Oh, J. H. Han, and H. Katsura
Resonating valence bond states with trimer motifs
Phys Rev B **95**, 060413(R) (2017)



Trimers are not the singlets of an SU(3) models (antisymmetry missing).

They defined winding numbers, leading to 3 topological sectors along both direction (Z3 vs Z2 in dimer coverings).

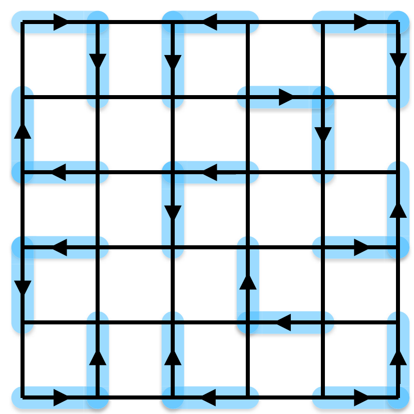
ground state has topological degeneracy 9^{genus}

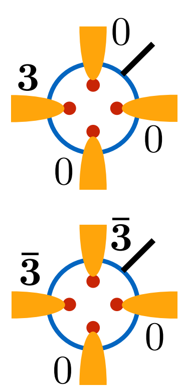


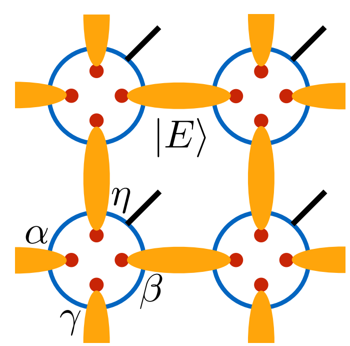
Tensor networks: Z_3 topological order

Xiao-Yu Dong, Ji-Yao Chen, Hong-Hao Tu
SU(3) trimer resonating-valence-bond state on the square lattice
Phys. Rev. B 98, 205117 (2018).

Trimers are now singlets of an SU(3) models (antisymmetry denoted by arrows).







Conclusions

- Designed an exact AKLT-like ground state with a simple parent Hamiltonian.
- For special cases, a macroscopically large number of states become degenerate.
- Gauss law, states characterized by topological (?) quantum numbers (sectors)
- Point separating different phases
- many open questions: Coulomb phase, fractional excitations, origin of non-singlet states,…