Collective modes of a magnetized U(1) spin liquid

Oleg Starykh, Univ of Utah

in collaboration with Leon Balents, KITP

Topological Quantum Matter: Concepts and Realizations, Oct 17, 2019

HFM-2020, May 17-22, 2020, Shanghai

Welcome

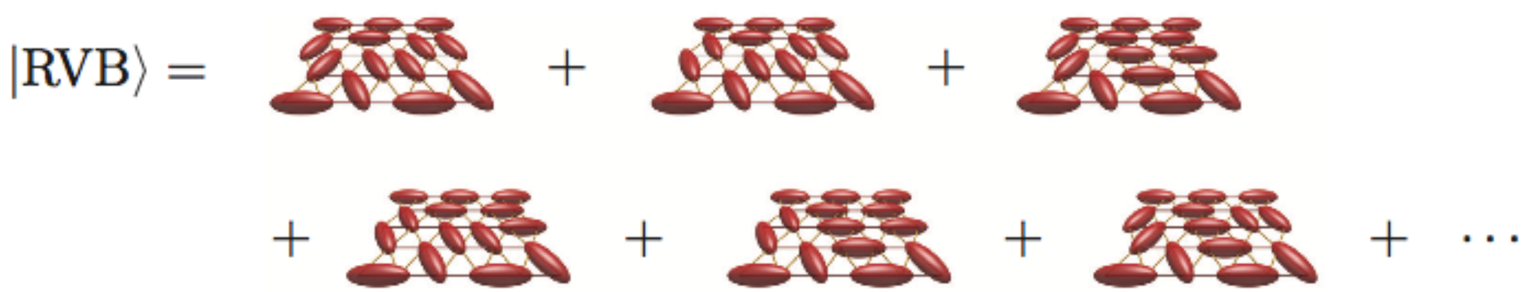
On behalf of the organizing committee, it is our honor to invite you to attend the 10th International Conference on Highly Frustrated Magnetism 2020 (HFM 2020) which will be held at Shanghai Jiao Tong University in Shanghai, China from the 17th to the 22nd of May, 2020.

Links

Highly Frustrated Magnetism

The big question(s)

What is quantum spin liquid?



No broken symmetries.
Quantum entangled state:
fractionalized excitations = spinons
emergent gauge fields

Figure 1. A 'resonating valence bond' (RVB) state. Ellipsoids indicate spin-zero singlet states of two $S = 1/2$ spins.

Savary, Balents 2017

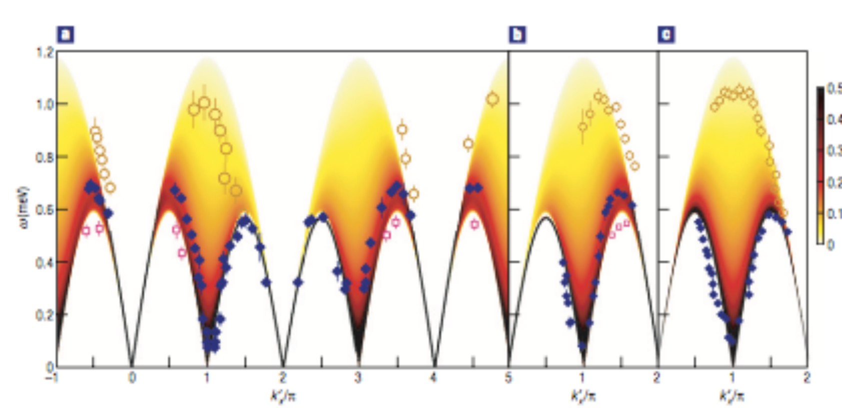
Which materials realize it?

Past candidates: Cs2CuCl4, kagome volborthite…

Current candidates: kagome herbertsmithite, α-RuCl3, YbMgGaO4, organic Mott insulators

How to detect/observe it?

Neutrons (if good single crystals are available), RIXS, NMR, thermal transport, terahertz optics, ESR

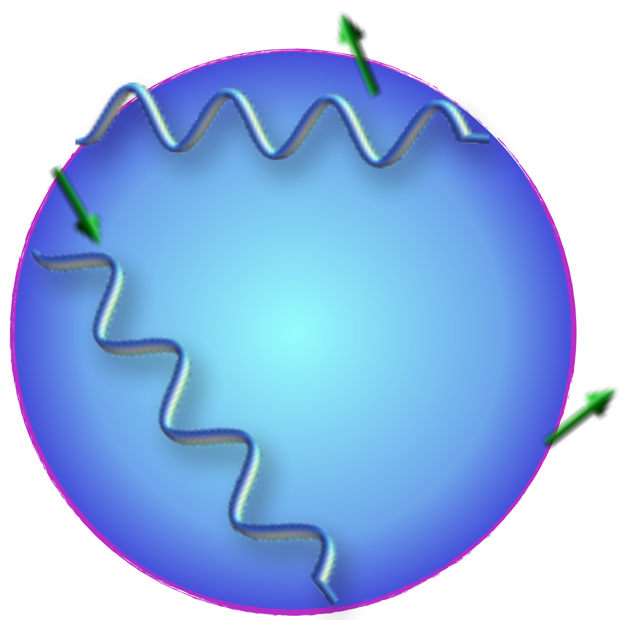


Outline

- QSL, spinon Fermi surface
  - Some history
  - Candidate materials
- Spinon continuum in magnetic field
- Spin-orbit interactions
  - Weak spin-orbit: ESR linewidth due to gauge fluctuations
  - Strong spin-orbit: spinon resonance
- Conclusions

Spin liquid with spinon Fermi surface

$$|\Psi\rangle = \prod_i \hat{n}_i(2-\hat{n}_i) \prod_{k<k_F} c^\dagger_{k\uparrow}c^\dagger_{k\downarrow}|0\rangle$$



- The most gapless/highly entangled QSL state
- Like a "metal" of neutral fermions w/ a U(1) gauge field
- Prototype "non-Fermi liquid" state of great theoretical interest

Quantum Spin Liquid

Resonating **V**alence **B**ond state

P. W. Anderson *Mater. Res. Bull.* ***8***, *153 (1973)*

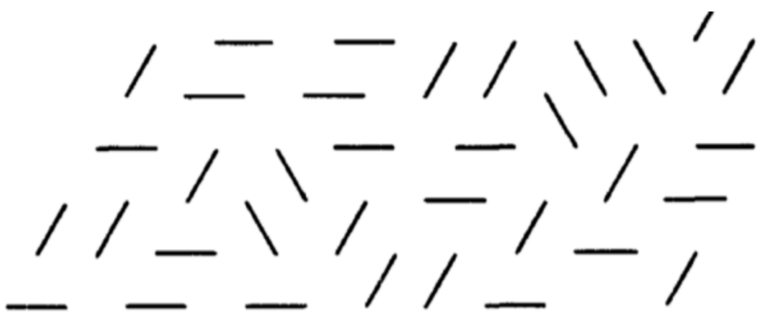
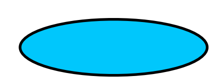


FIG. 3

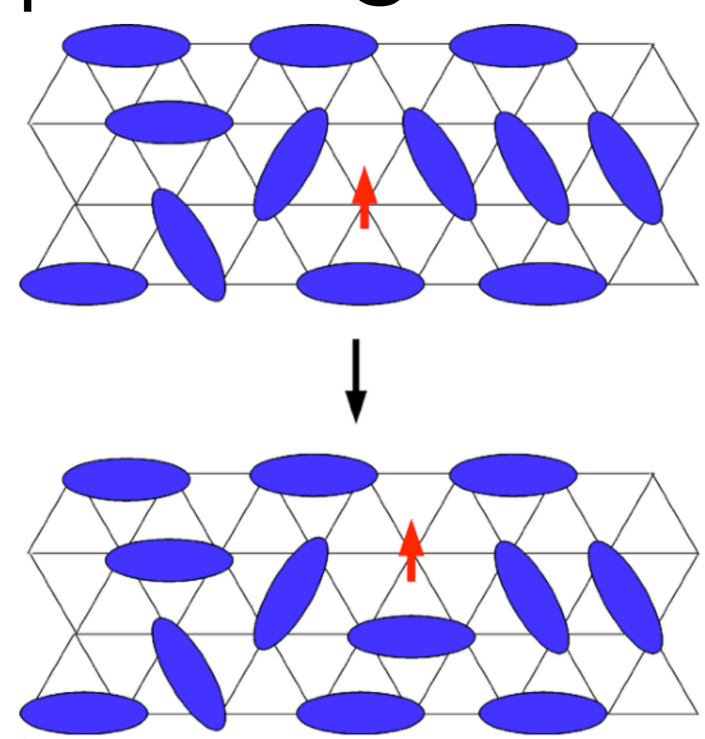
Random arrangements of pair bonds on a triangle lattice. (a) Shows a regular arrangement with $2^{N/4}$ alternative distinct pairings ("rhombus" approximation). (b) An arbitrary arrangement.

a "quantum liquid" of spins

 $= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$

Quantum superposition of spin-singlet coverings.

Excitations are obtained by breaking the singlet bond -- they are *spin-1/2 spinons*!



High Tc era

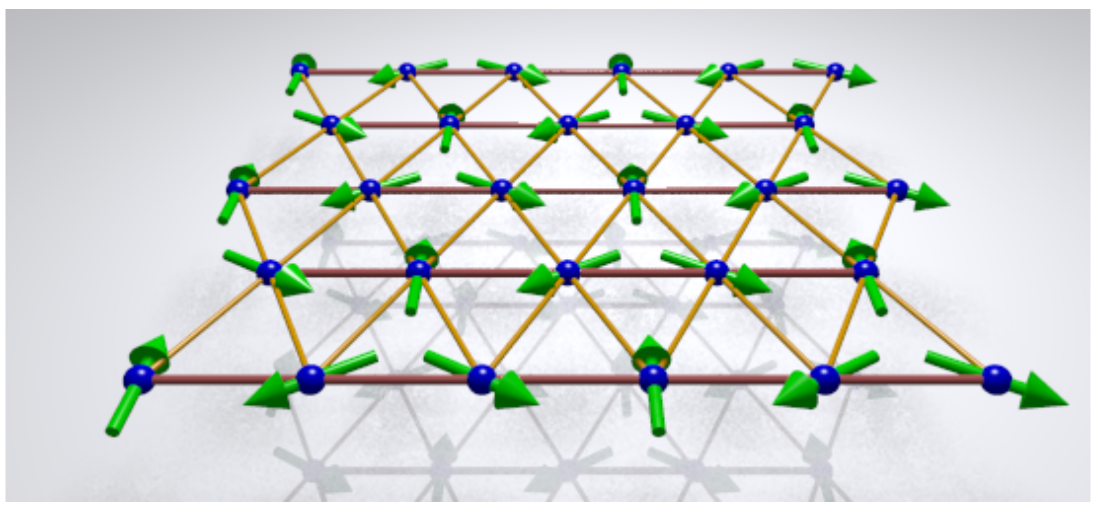
The Resonating Valence Bond State in La_2CuO_4 and Superconductivity

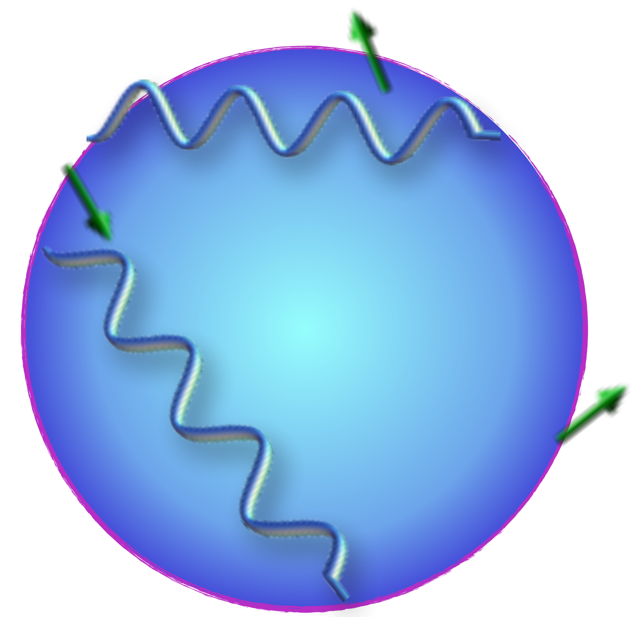
P. W. ANDERSON *Science*, ***235***, *1196–1198 (1987)*

The oxide superconductors, particularly those recently discovered that are based on La_2CuO_4, have a set of peculiarities that suggest a common, unique mechanism: they tend in every case to occur near a metal-insulator transition into an odd-electron insulator with peculiar magnetic properties. This insulating phase is proposed to be the long-sought "resonating-valence-bond" state or "quantum spin liquid" hypothesized in 1973. This insulating magnetic phase is favored by low spin, low dimensionality, and magnetic frustration. The preexisting magnetic singlet pairs of the insulating state become charged superconducting pairs when the insulator is doped sufficiently strongly. The mechanism for superconductivity is hence predominantly electronic and magnetic, although weak phonon interactions may favor the state. Many unusual properties are predicted, especially of the insulating state.

The existence of a pseudo-Fermi surface, I believe, is real and the spin excitations may resemble those of a real Fermi liquid. This would explain the experimental observation of a Fermi-like susceptibility.

Triangular lattice w/ ring exchange





- Motrunich (2005): ring exchange stabilizes a spin liquid

Spin Bose-metal phase in a spin-$\frac{1}{2}$ model with ring exchange on a two-leg triangular strip

D. N. Sheng, Olexei I. Motrunich, and Matthew P. A. Fisher
Phys. Rev. B **79**, 205112 – Published 20 May 2009

DMRG

- Motrunich, Lee/Lee: spin liquid state favored by ring exchange is the "spinon Fermi sea" state

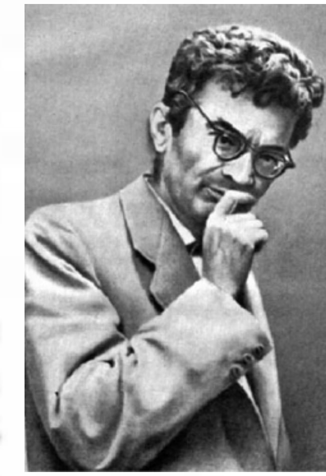
And some pre-history

Fermions in antiferromagnets

THE THERMAL CONDUCTIVITY OF THE PARAMAGNETIC DIELECTRICS AT LOW TEMPERATURES

By I. POMERANCHUK

(Received October 25, 1940)



In the paramagnetic dielectrics the paramagnetic spectrum can take part in the heat transfer. Besides the mutual collision (as in the ordinary dielectrics) the free path of the phonons essentially depends on this spectrum. At low temperatures the main part in the conduction of heat is played in some cases by the phonons, in the others by the paramagnetic spectrum. The run of the curve of termal conductivity as function of temperature has a complicated character with a number of maxima and minima, and is quite different as compared with the corresponding situation in ordinary dielectrics. An experimental check of the theory could give opportunity to find out the character of the paramagnetic exchange spectrum.

spinwave $\neq$ magnon

Bloch (4) had examined the magnetic energy levels (the ferromagnetic spectrum) in the case when the exchange integral has the sign leading to the ferromagnetism.

• Bloch (4) has obtained the Bose statistics for the spin waves. This is due to the fact that his model of a ferromagnetic body has a particular state at $T=0$ (a total saturation), which is of an extreme *mechanical* kind as compared with all other states.

broken symmetry state

magnetic excited levels correspond to the deviations from the normal distribution of the magnetic moments which are propagating through the whole crystal and are not localized in a definite place of the lattice. Such magnetic excitations will be called in the following «magnons» (this name was suggested by L. Landau).

the notion of magnon is introduced

Fermions in antiferromagnets

Regarding statistics of spin excitations:

"The experimental facts available suggest that the magnons are submitted to the Fermi statistics; namely, when $T \ll T_{CW}$ the susceptibility tends to a constant limit, which is of the order of const/T_{CW} (5) [for $T > T_{CW}$, χ=const/($T + T_{CW}$)]. Evidently we have here to deal with the Pauli paramagnetism which can be directly obtained from the Fermi distribution. Therefore, we shall assume the Fermi statistics for the magnons."

(5) A. Perrier and Kamerlingh Onnes, Leiden Comm. No.139 (1914)

Yu.A. Freiman, H.J. Jodl / Physics Reports 401 (2004) 1–228

ALBERT PERRIER and H. KAMERLINGH ONNES. Magnetic researches. XII. The susceptibility of solid oxygen in two forms.

(*Translated from: Verslag van de Gewone Vergadering der Wis- en Natuurkundige Afdeeling der Kon. Akad. van Wetenschappen te Amsterdam*, 28 **Februari 1914**, p. 1004—1011.)

EDUARD IJDO — PRINTER — LEIDEN.

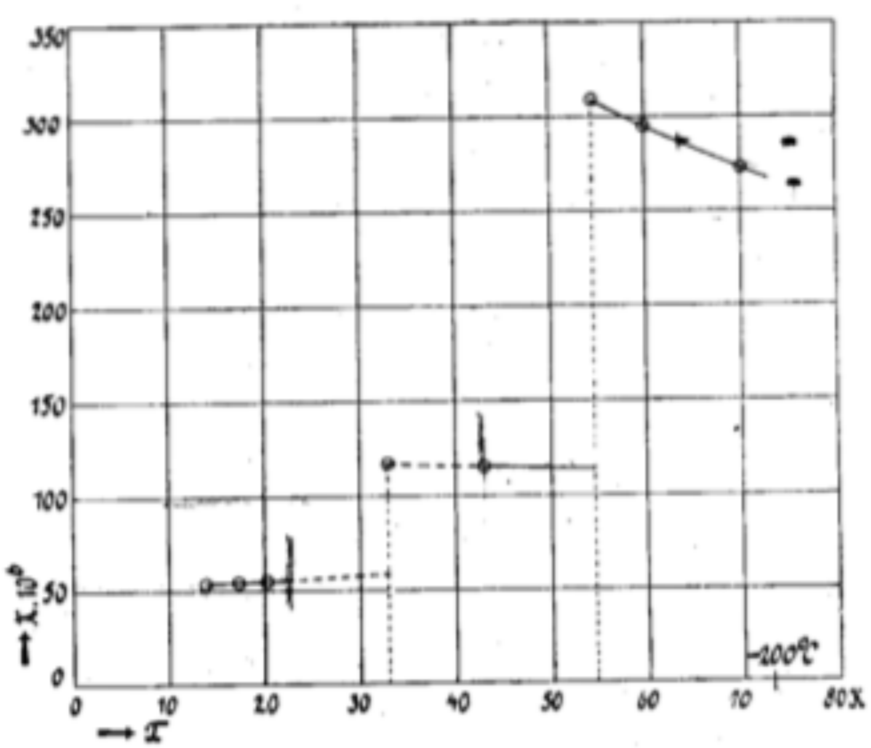
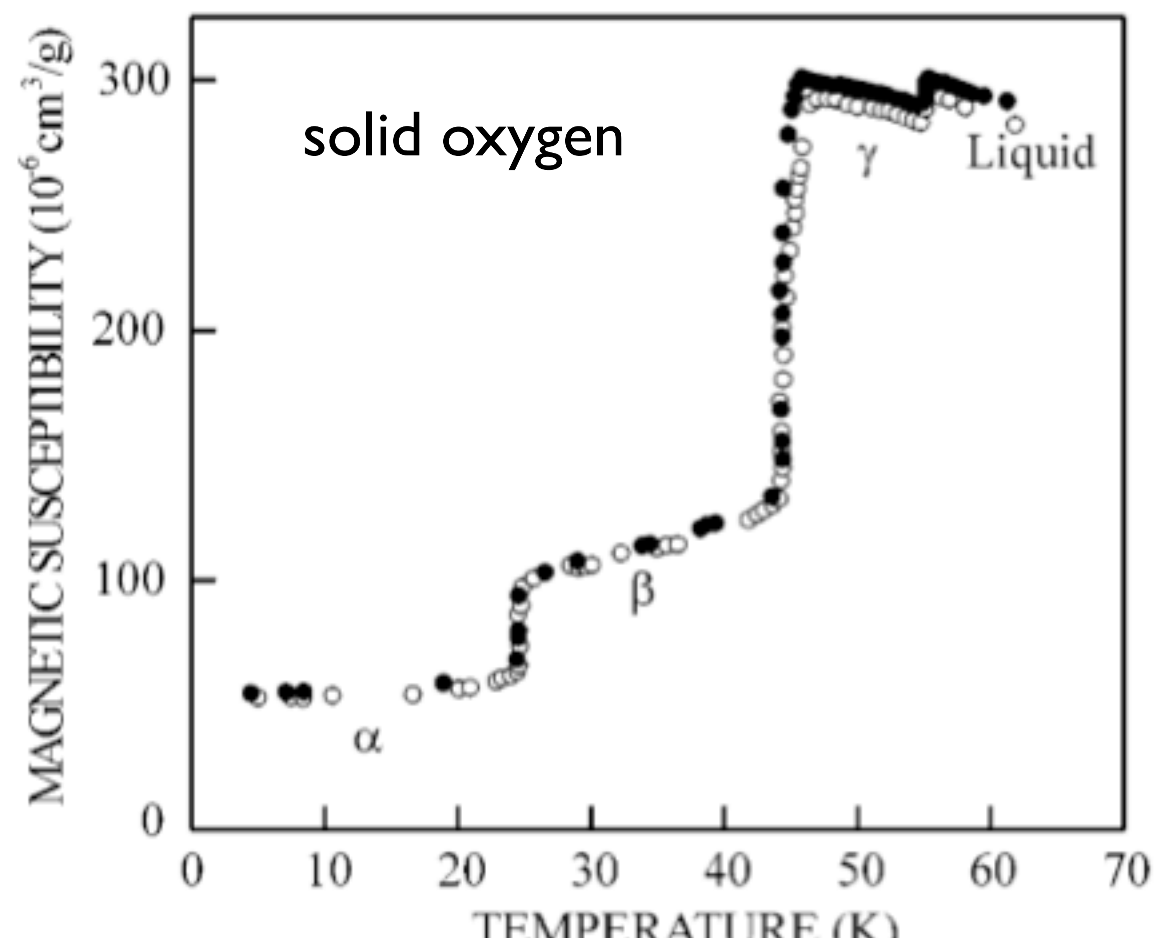


Fig. 2.



Fermions in antiferromagnets

Regarding statistics of spin excitations:

"The experimental facts available suggest that the magnons are submitted to the Fermi statistics; namely, when T << T_{CW} the susceptibility tends to a constant limit, which is of the order of const/T_{CW} ([5]) [for T > T_{CW}, χ=const/(T + T_{CW})]. Evidently we have here to deal with the Pauli paramagnetism which can be directly obtained from the Fermi distribution. Therefore, we shall assume the Fermi statistics for the magnons."

Debye temp < Curie temp

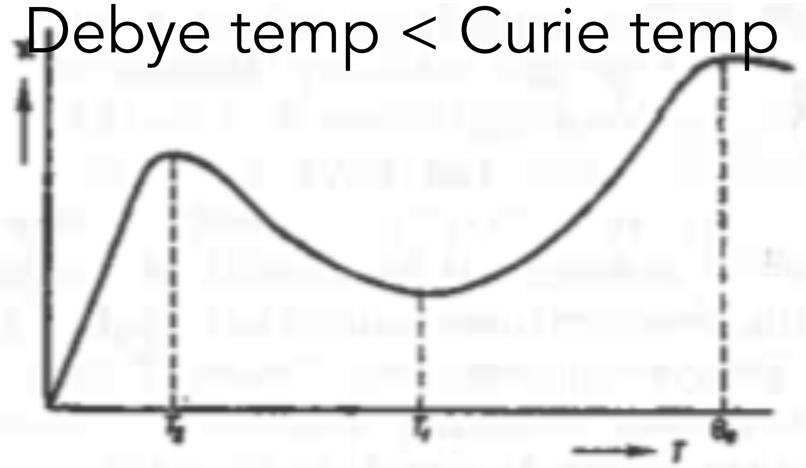


Fig. 1. Characteristic curve of thermal conductivity, when $\theta_D < \theta_k$ (assuming $\varkappa \sim 1/T$, when $T \gg \theta_D$)

Debye temp > Curie temp

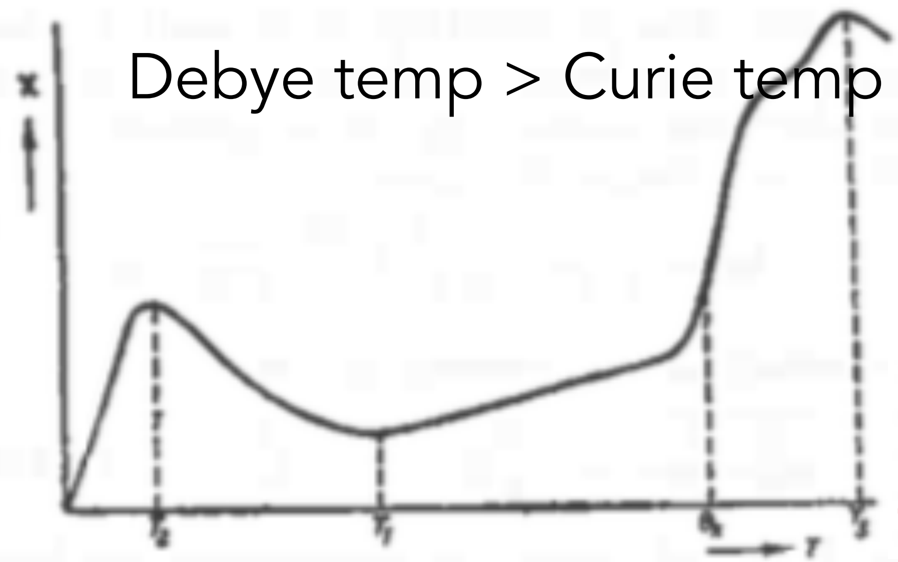


Fig. 2. Characteristic curve of the thermal conductivity, when $\theta_D > \theta_k$

And a hint of more exotic physics:

We are intended also to publish a paper in which will be considered the dependence of the thermal conductivity in paramagnetic dielectrics with a non-Fermi distribution of magnons. A discussion of the experimental data will also be given there.

thermal conductivity of spinons + phonons

The first magnon was fermion

Modern analysis

Transport properties of a spinon Fermi surface coupled to a U(1) gauge field

Cody P. Nave and Patrick A. Lee
Phys. Rev. B **76**, 235124 – Published 21 December 2007

Outline

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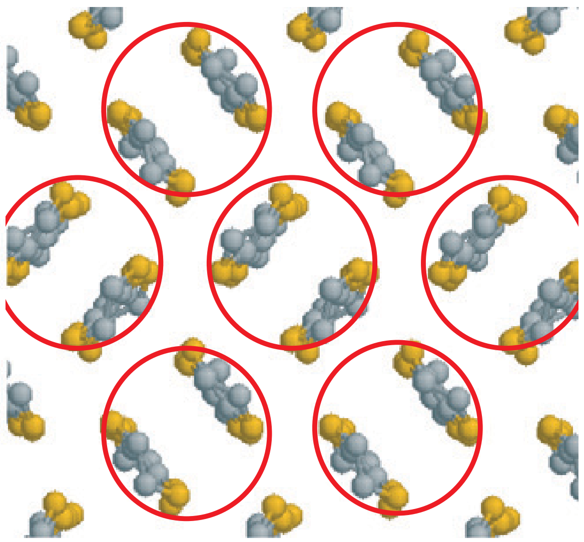
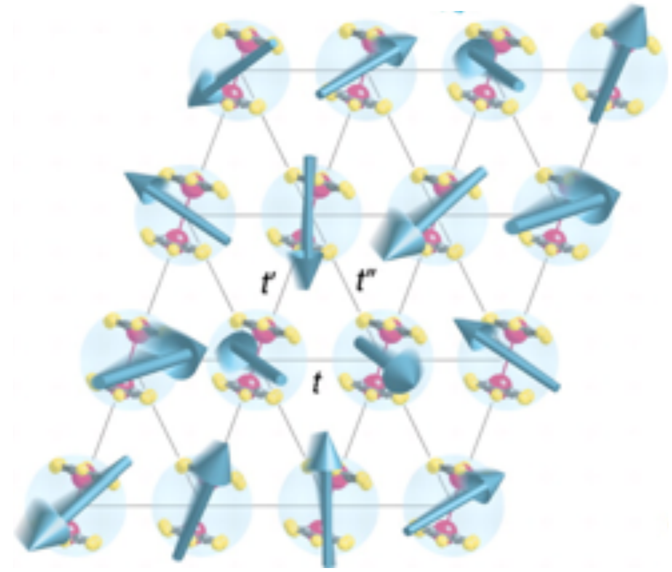
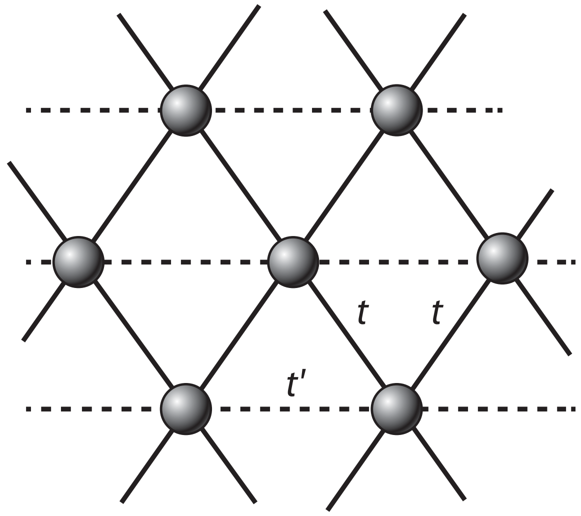
Triangular organics: low energy signatures



κ-$(ET)_2X$



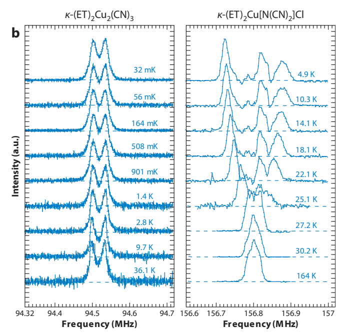
β′-$Pd(dmit)_2$

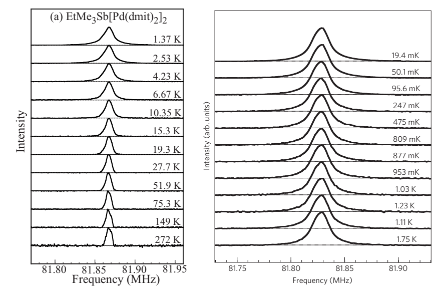


- Molecular materials which behave as effective triangular lattice S=1/2 antiferromagnets with J ~ 250K
  - significant charge fluctuations

Experimental Evidence

- NMR

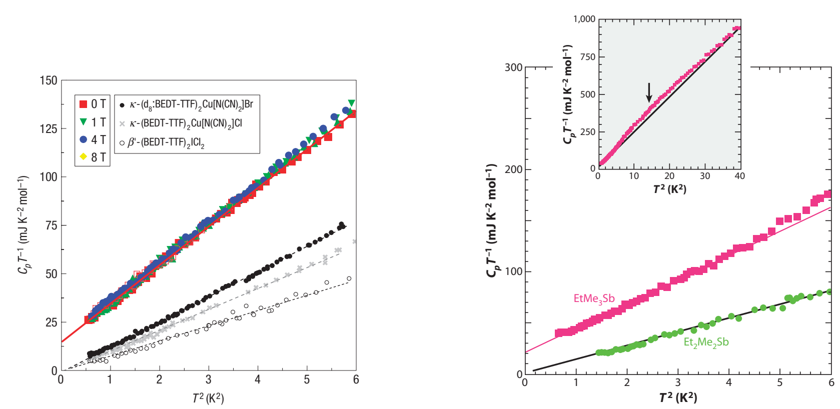




Y. Shimizu *et al*, 2003   T. Itou *et al*, 2008,2010

no magnetic order

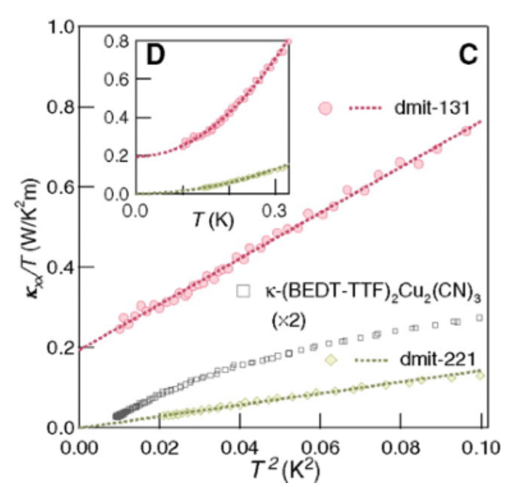
- Specific heat



S. Yamashita *et al*, 2008

Sommerfeld law

- Thermal conductivity

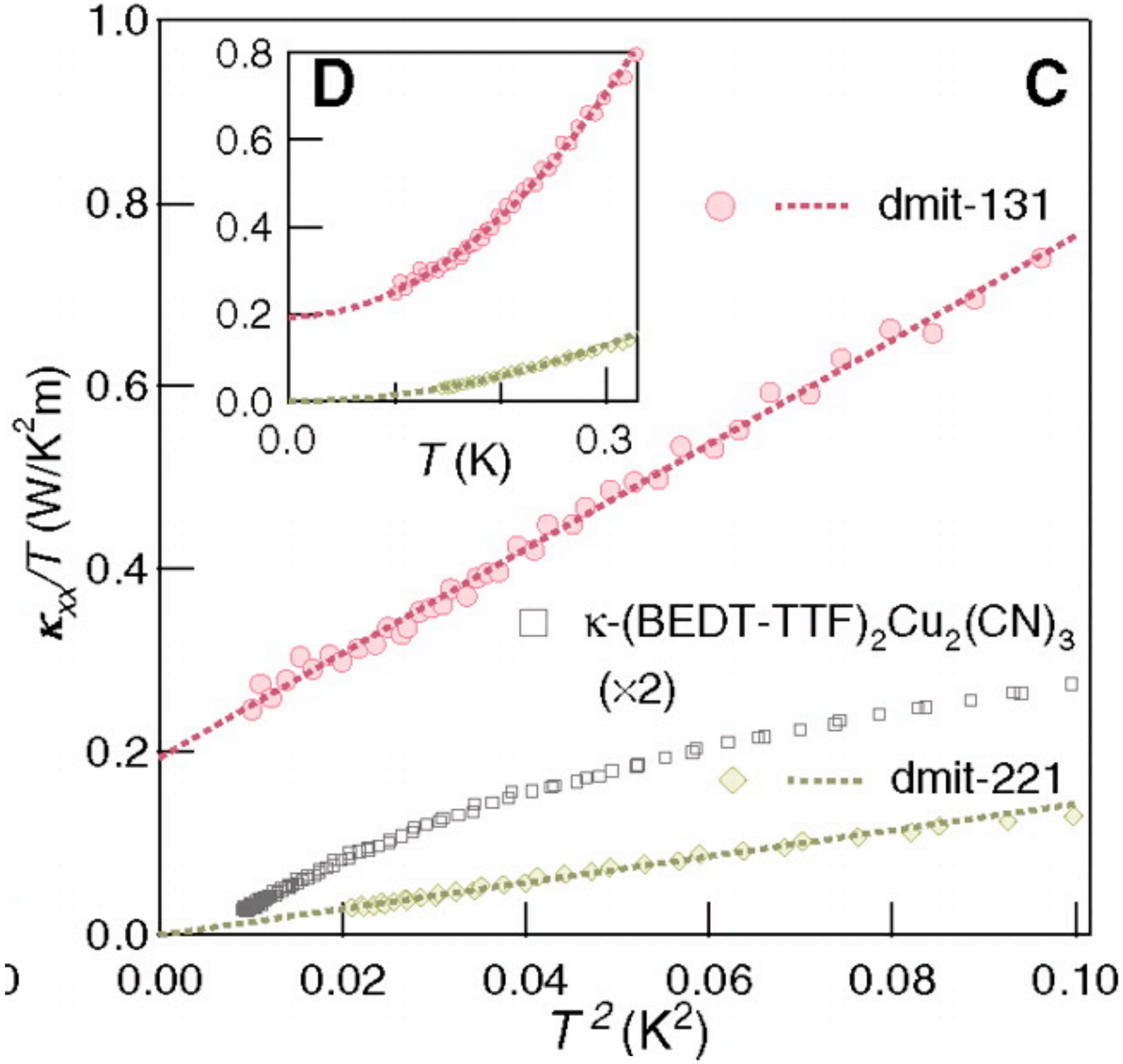


M. Yamashita *et al*, 2010

itinerant fermions?

Organic Mott insulators: Spin liquid with spinon Fermi surface?

THERMAL TRANSPORT



EtMe$_3$Sb[Pd(dmit)$_2$]$_2$ (dmit-131)

Spin liquid?

M. Yamashita et al, Science 2010

Et$_2$Me$_2$Sb[Pd(dmit)$_2$]$_2$ (dmit-221)

Non-magnetic charge-ordered

theory: O. Motrunich 2005,
S.-S. Lee and P. A. Lee 2005

Highly Mobile Gapless Excitations in a Two-Dimensional Candidate Quantum Spin Liquid 4 JUNE 2010 VOL 328 SCIENCE

Minoru Yamashita,[1*] Norihito Nakata,[1] Yoshinori Senshu,[1] Masaki Nagata,[1]
Hiroshi M. Yamamoto,[2,3] Reizo Kato,[2] Takasada Shibauchi,[1] Yuji Matsuda[1*]

electrical insulator,
but metal-like thermal conductor

Experimental controversy

Absence of magnetic thermal conductivity in the quantum spin li
$EtMe_3Sb[Pd(dmit)_2]_2$ – revisited

J. M. Ni,[1] B. L. Pan,[1] Y. Y. Huang,[1] J. Y. Zeng,[1] Y. J. Yu,[1] E. J. Cheng,[1] L. S. Wang,[1] R.
[1] *State Key Laboratory of Surface Physics, Department of Physics,*
and Laboratory of Advanced Materials, Fudan University, Shanghai 200438, (
[2] *RIKEN, Condensed Molecular Materials Laboratory, Wako 351-0198, Jap*
[3] *Collaborative Innovation Center of Advanced Microstructures, Nanjing 210093, China*
(Dated: April 25, 2019)

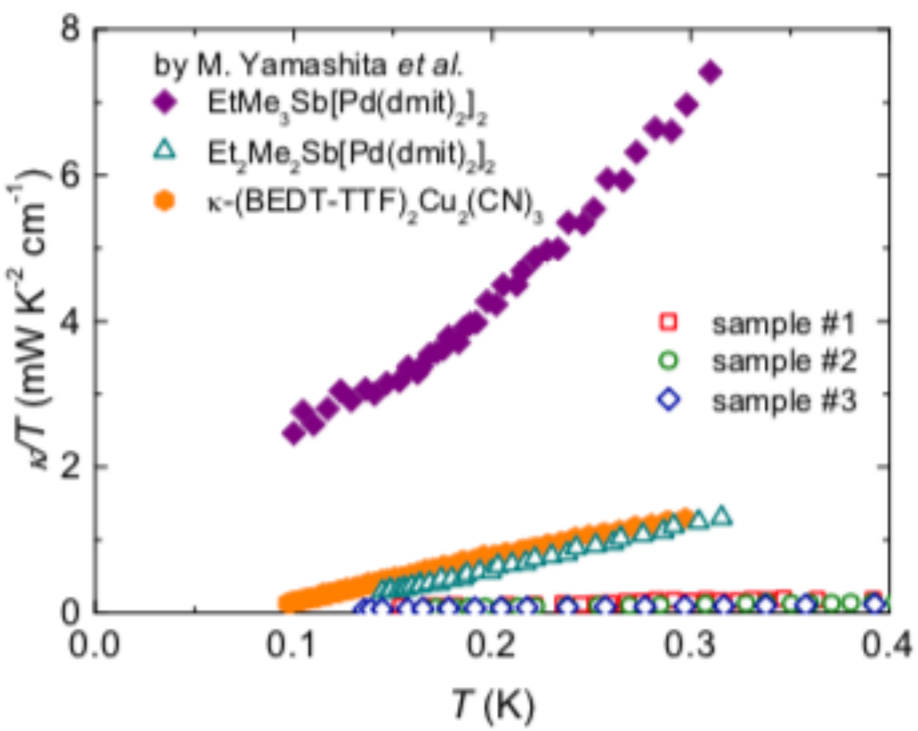
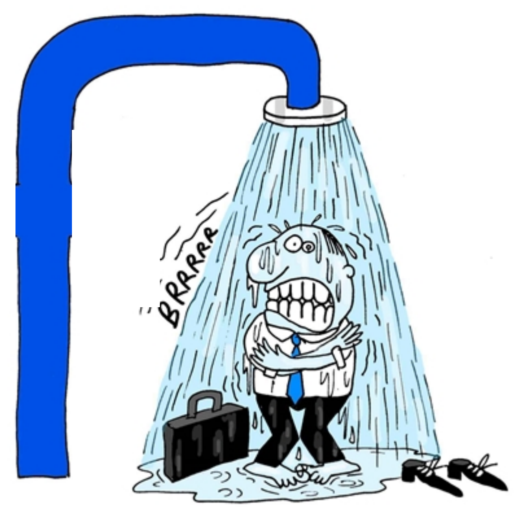


FIG. 4. Comparison of our thermal conductivity data with the data of $EtMe_3Sb[Pd(dmit)_2]_2$ and $Et_2Me_2Sb[Pd(dmit)_2]_2$ in Ref. [12] and κ-$(BEDT\text{-}TTF)_2Cu_2(CN)_3$ in Ref. [9]. The absolute value of our data are much smaller than that in Ref. [12], even 10 times smaller than the nonmagnetic reference compound $Et_2Me_2Sb[Pd(dmit)_2]_2$, indicating phonons being strongly scattered in our samples.



Thermal conductivity of the quantum spin liquid candidate $EtMe_3Sb[Pd(dmit)_2]_2$: No evidence of mobile gapless excitations

P. Bourgeois-Hope,[1] F. Laliberté,[1] E. Lefrançois,[1] G. Grissonnanche,[1]
S. René de Cotret,[1] R. Gordon,[1] R. Kato,[2] L. Taillefer,[1,3,*] and N. Doiron-Leyraud[1,†]
[1] *Institut Quantique, Département de physique & RQMP, Université de Sherbrooke, Sherbrooke, Québec, Canada*
[2] *RIKEN, Wako-shi, Saitama 351-0198, Japan*
[3] *Canadian Institute for Advanced Research, Toronto, Ontario, Canada*
(Dated: April 26, 2019)

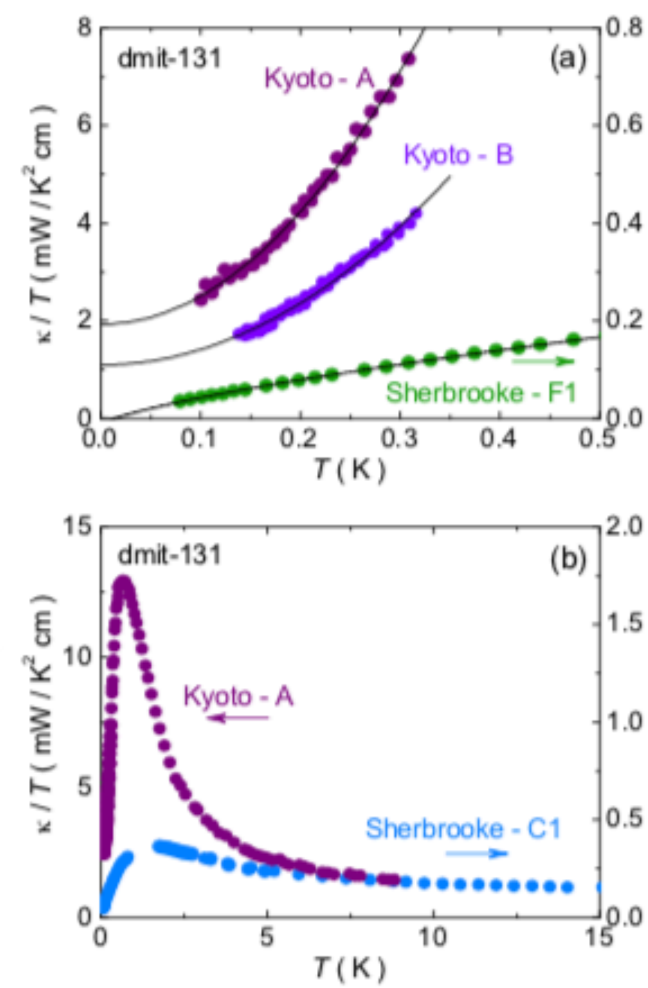


FIG. 4. (a) κ/T vs T for our sample F1 (green, right scale) and for samples A (purple) and B (violet) of ref. [10] (left scale). Note that the left and right vertical scales differ by a factor 10. The three samples were grown in identical conditions, using the same starting material. For our data, the black line is the free power law fit below 0.55 K shown in Fig. 6(b). For the Kyoto data, the lines are quadratic fits reproduced from Ref. [10]. (b) κ/T vs T up to 15 K for our sample C1 (blue, right scale) and for sample A of ref. [10] (purple, left scale). Note that the left and right vertical scales differ by a factor 7.5.

Recent developments

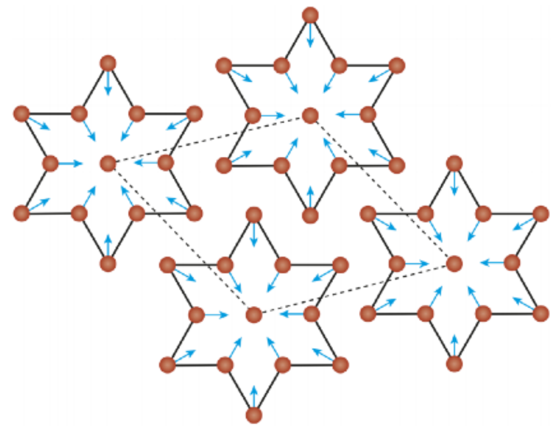


Fig. 1. In the cluster Mott phase of 1T-TaS_2, Ta atoms (red dots) belonging to a star of David move toward the Ta atom at the center. Thirteen Ta atoms form a unit cell, and these unit cells form a triangular lattice. The directions and lengths of the arrows are schematic.

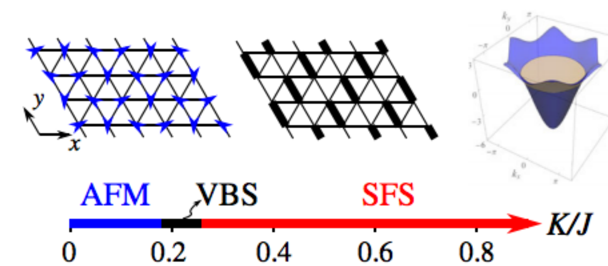
1T-TaS_2 as a quantum spin liquid

K. T. Law[a] and Patrick A. Lee[b,1]

PHYSICAL REVIEW LETTERS **121**, 046401 (2018)

Spinon Fermi Surface in a Cluster Mott Insulator Model on a Triangular Lattice and Possible Application to 1*T*-TaS_2

Wen-Yu He,[1] Xiao Yan Xu,[1,*] Gang Chen,[2,3] K. T. Law,[1,‡] and Patrick A. Lee[4,†]



Emergence of a field-driven U(1) spin liquid in the Kitaev honeycomb model

Ciarán Hickey & Simon Trebst

Spinon Fermi surface in
AFM Kitaev in [111] magnetic field

Nature Communications **10**, Article number: 530 (2019) | Download Citation

Field-induced quantum spin liquid in the Kitaev-Heisenberg model and its relation to α-$RuCl_3$

Yi-Fan Jiang, Thomas P. Devereaux, Hong-Chen Jiang

(Submitted on 26 Jan 2019)

Field-induced QCD_3-Chern-Simons quantum criticalities in Kitaev materials

Liujun Zou[1,2] and Yin-Chen He[3]

Magnetic field induced intermediate quantum spin-liquid with a spinon Fermi surface

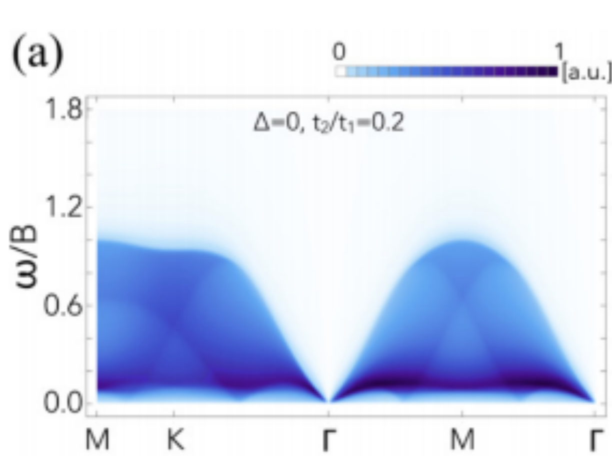
Niravkumar D. Patel[1] and Nandini Trivedi[1]

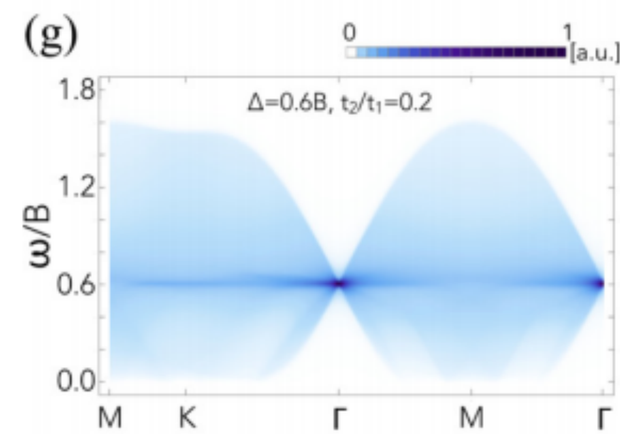
YbMgGaO$_4$ in applied magnetic field

PHYSICAL REVIEW B **96**, 075105 (2017)

Detecting spin fractionalization in a spinon Fermi surface spin liquid

Yao-Dong Li[1] and Gang Chen[1,2,*]





Article | OPEN | Published: 08 October 2018

Fractionalized excitations in the partially magnetized spin liquid candidate YbMgGaO$_4$

Yao Shen, Yao-Dong Li, H. C. Walker, P. Steffens, M. Boehm, Xiaowen Zhang, Shoudong Shen, Hongliang Wo, Gang Chen & Jun Zhao

Nature Communications **9**, Article number: 4138 (2018) | Download Citation

What should it be for a spinon FS state?

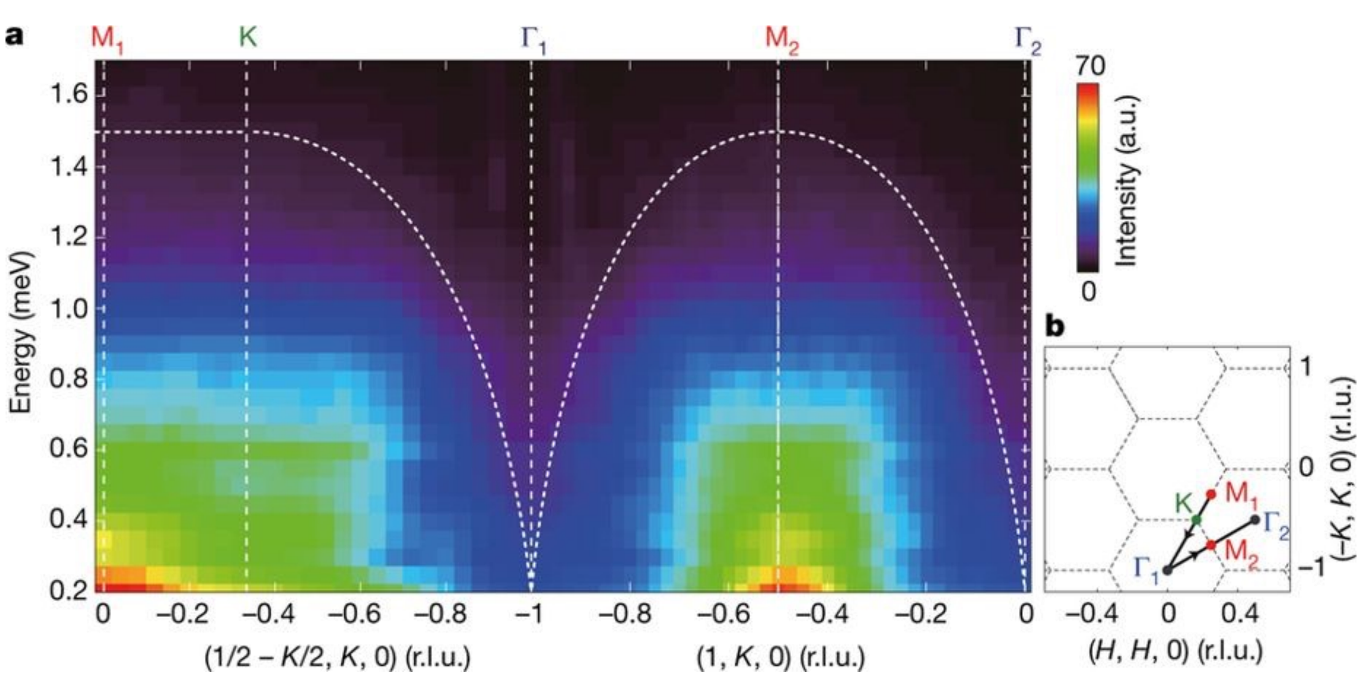
Letter | Published: 05 December 2016

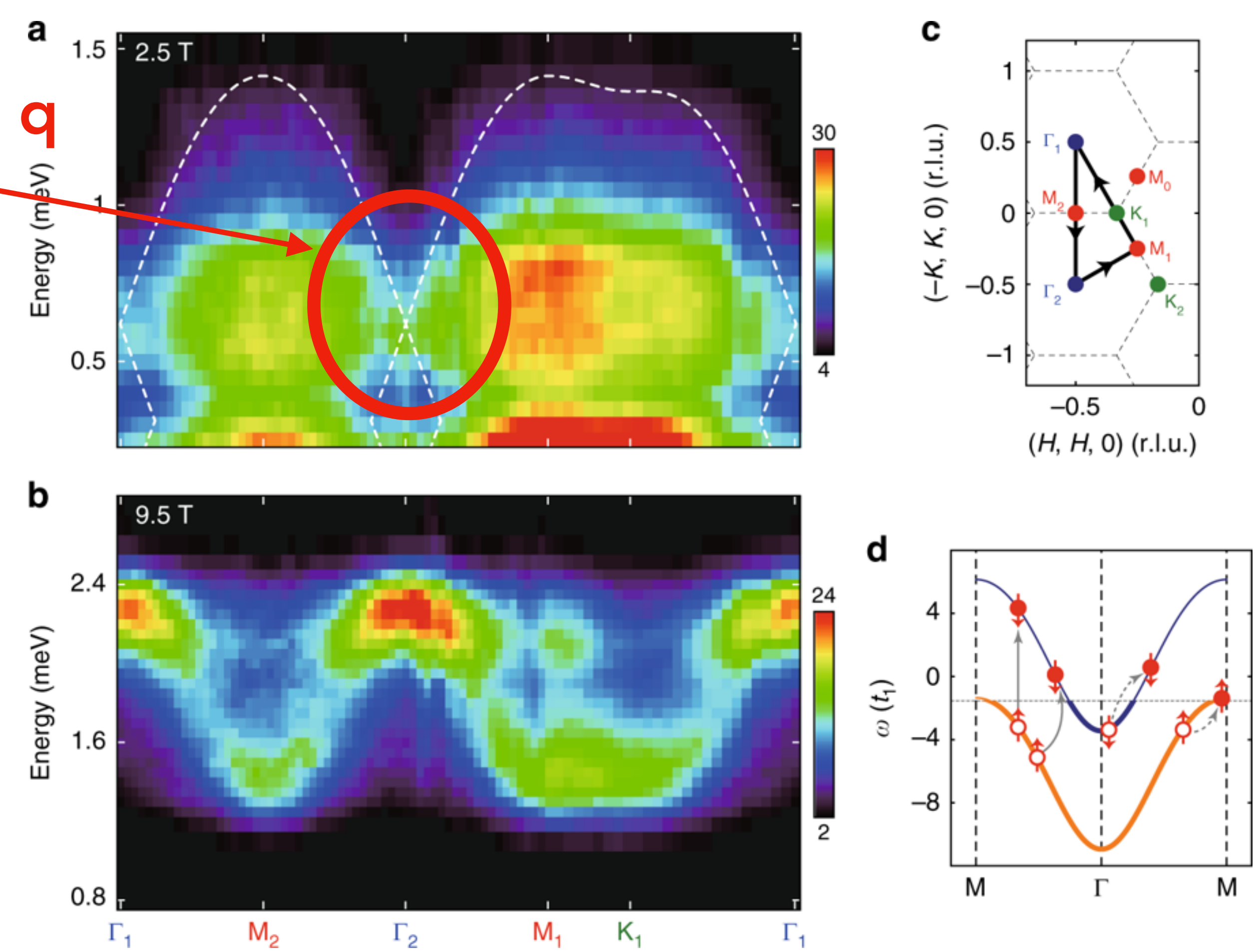
Evidence for a spinon Fermi surface in a triangular-lattice quantum-spin-liquid candidate

Yao Shen, Yao-Dong Li, Hongliang Wo, Yuesheng Li, Shoudong Shen, Bingying Pan, Qisi Wang, H. C. Walker, P. Steffens, M. Boehm, Yiqing Hao, D. L. Quintero-Castro, L. W. Harriger, M. D. Frontzek, Lijie Hao, Siqin Meng, Qingming Zhang, Gang Chen & Jun Zhao

Nature **540**, 559–562 (22 December 2016) | Download Citation

Small momentum **q** structure





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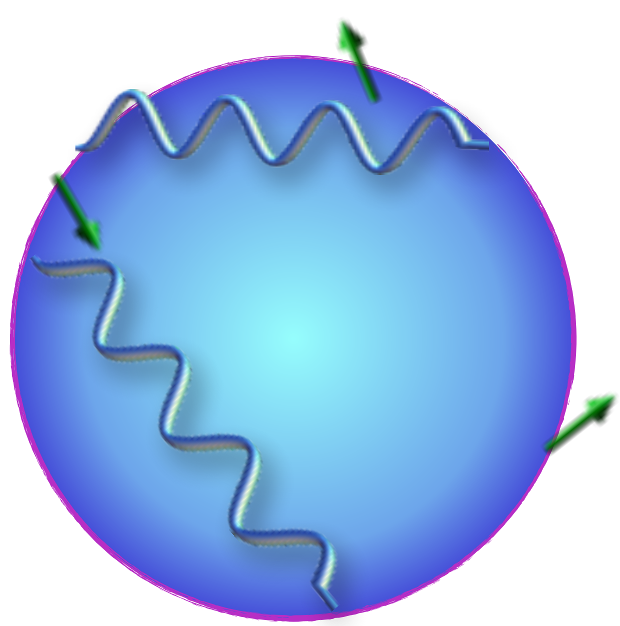
Spinon Fermi surface

$$|\Psi\rangle = \prod_i \hat{n}_i(2-\hat{n}_i) \prod_{k<k_F} c^\dagger_{k\uparrow} c^\dagger_{k\downarrow}|0\rangle$$



- The most gapless/highly entangled QSL state
- Like a "metal" of neutral fermions w/ a U(1) gauge field
- Prototype "non-Fermi liquid" state of great theoretical interest

U(1) gauge theory: main technical steps

1) $S^a_{\mathbf{r}} = \frac{1}{2}\psi^\dagger_{\mathbf{r},\alpha}\sigma^a_{\alpha\beta}\psi_{\mathbf{r},\beta}$ provided $\sum_\alpha \psi^\dagger_{\mathbf{r},\alpha}\psi_{\mathbf{r},\alpha} = 1$ enforced by **A_0**

2) $\vec{S}_{\mathbf{r}} \cdot \vec{S}_{\mathbf{r}'} \to |\Delta_{\mathbf{r}\mathbf{r}'}| e^{iA_{\mathbf{r}\mathbf{r}'}} \psi^\dagger_{\mathbf{r}\alpha}\psi_{\mathbf{r}'\alpha}$ → $A_{\mathbf{r}\mathbf{r}'} \to (\mathbf{r}-\mathbf{r}')\cdot \mathbf{A}\left(\frac{\mathbf{r}+\mathbf{r}'}{2}\right)$

3) $S_\psi = \int d\tau d^2\mathbf{r}\, \psi^\dagger_\alpha \left(\left[\partial_\tau - iA_0 - \mu - \frac{(\boldsymbol{\nabla}_{\mathbf{r}} - i\mathbf{A})^2}{2m}\right]\delta_{\alpha\beta} - \omega_B \sigma^3_{\alpha\beta}\right)\psi_\beta.$

Spinon

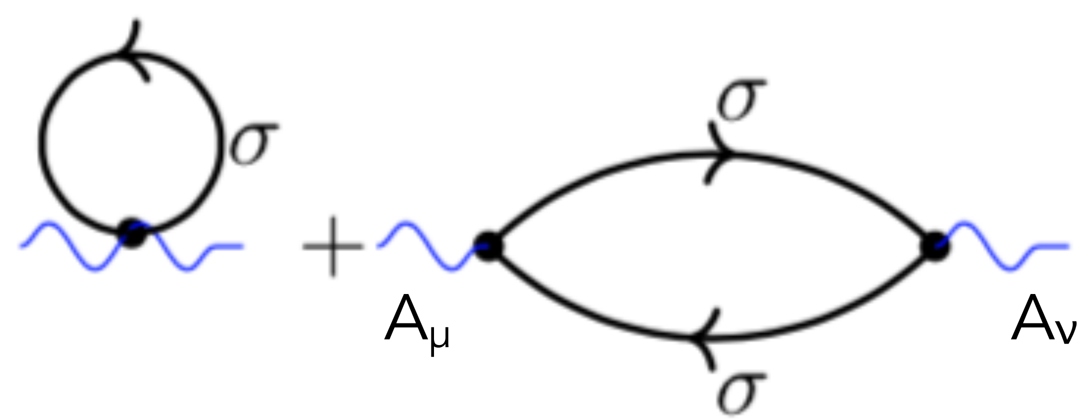
External magnetic field

Gauge field

Gapless fermions and gauge fields in dielectrics
L. B. Ioffe and A. I. Larkin
Phys. Rev. B **39**, 8988 – Published 1 May 1989

Gauge theory of the normal state of high-T_c superconductors
Patrick A. Lee and Naoto Nagaosa
Phys. Rev. B **46**, 5621 – Published 1 September 1992

4) Gauge dynamics generated by spinons

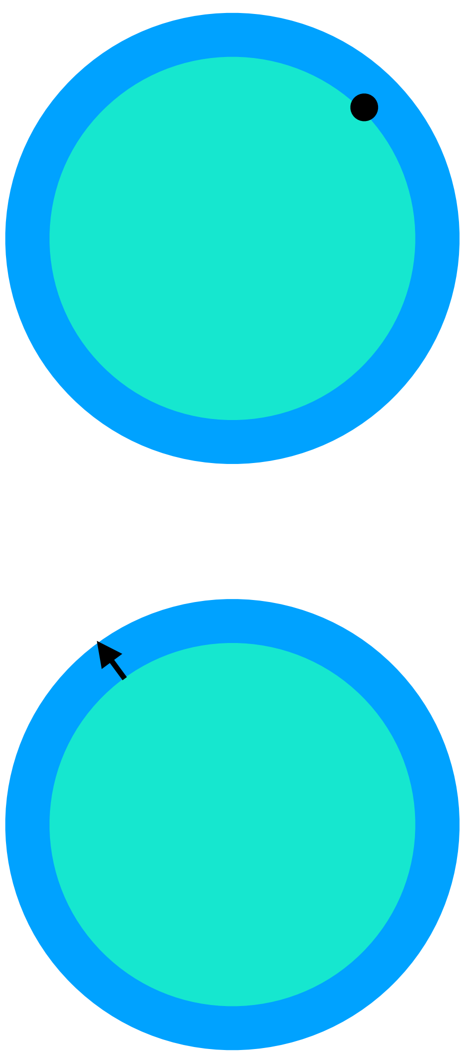


Spinon particle-hole continuum

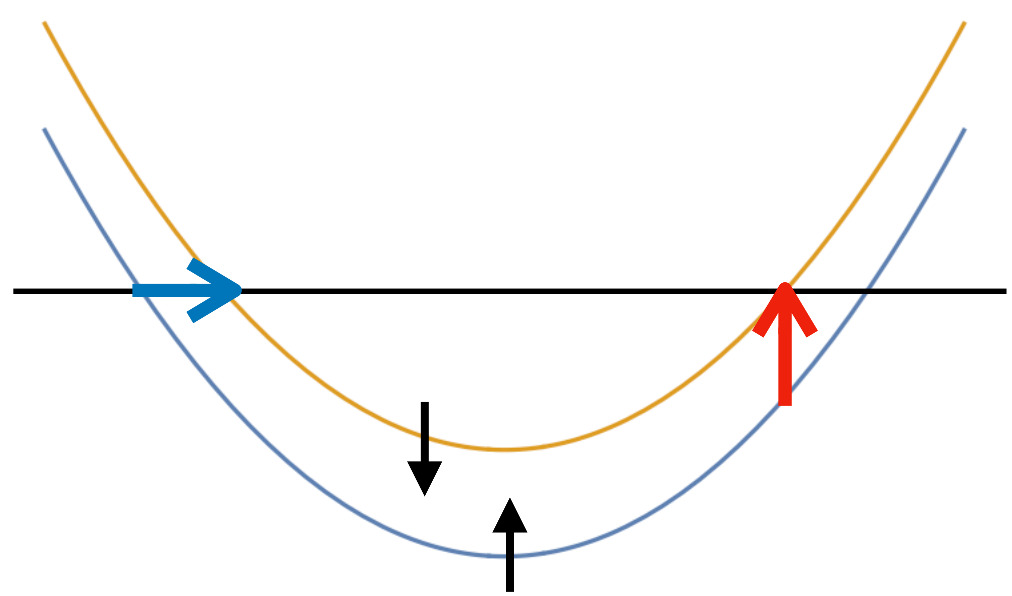
+ Zeeman field

$$\chi_{\pm}(\boldsymbol{q},\omega) = i\int_0^{\infty} dt \langle [S^{\dagger}_{\mathbf{q}}(t), S^{-}_{-\mathbf{q}}(0)]\rangle e^{i\omega t}$$

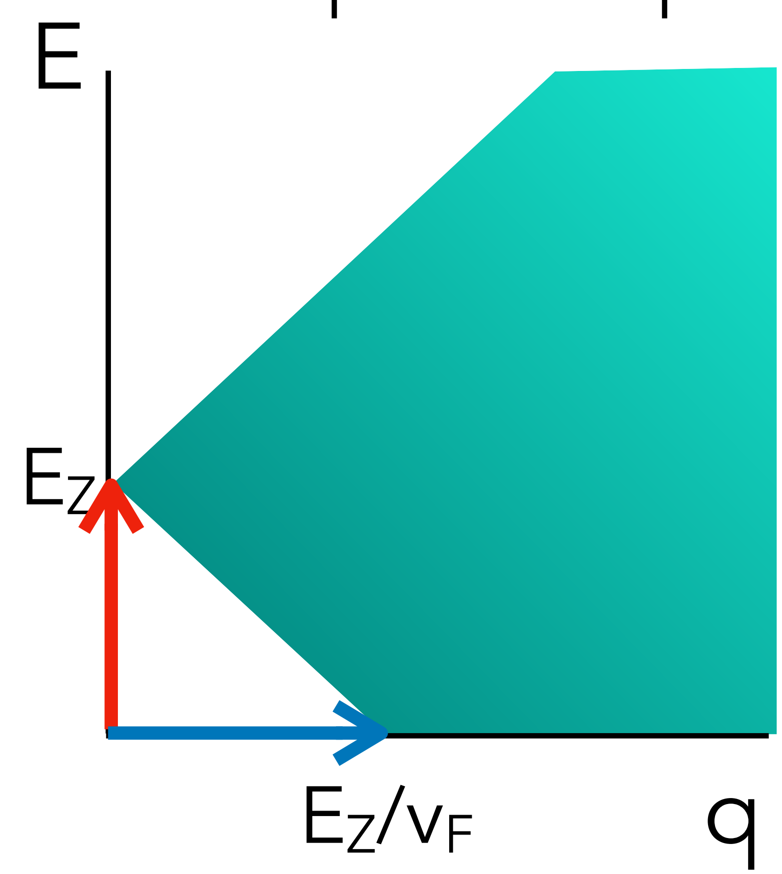
Transverse spin susceptibility



q=0 costs Zeeman energy

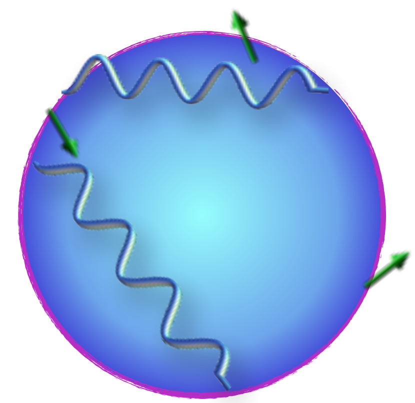


zero energy when $v_F q$ = Zeeman



U(1) gauge theory

Claim: spin susceptibility at small **q** is strongly affected by gauge fluctuations **in the presence of external magnetic field**



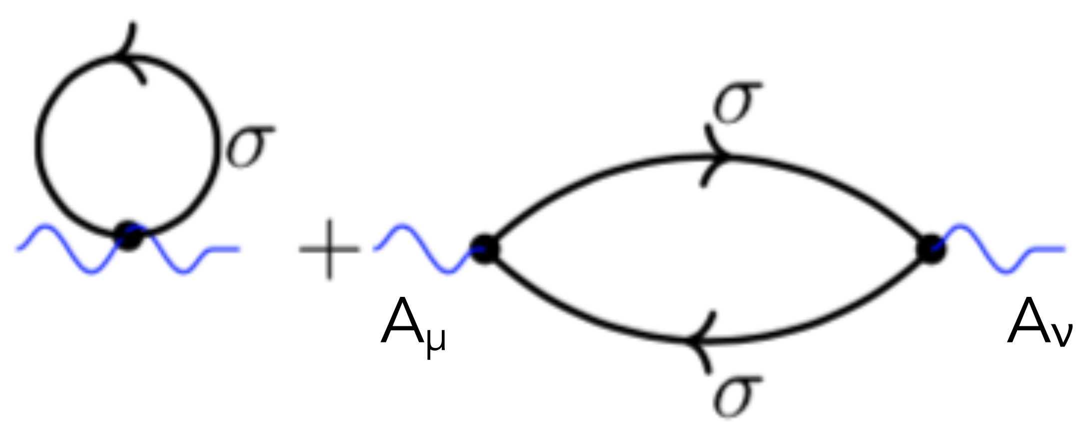
magnetic field

$$S_\psi = \int d\tau d^2\mathbf{r}\psi^\dagger_\alpha \Big([\partial_\tau - iA_0 - \mu - \frac{(\mathbf{\nabla}_\mathbf{r} - i\mathbf{A})^2}{2m}]\delta_{\alpha\beta} - \omega_B \sigma^3_{\alpha\beta}\Big)\psi_\beta.$$

Gauge field

Spinon

Gauge dynamics generated by spinons



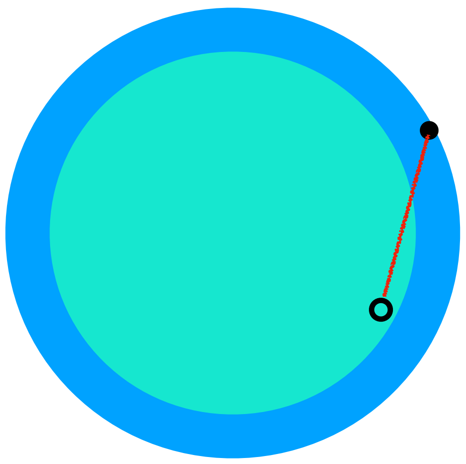
Interactions

- Longitudinal

$$A_0\psi^\dagger\psi$$

screened Coulomb interaction

boring!?

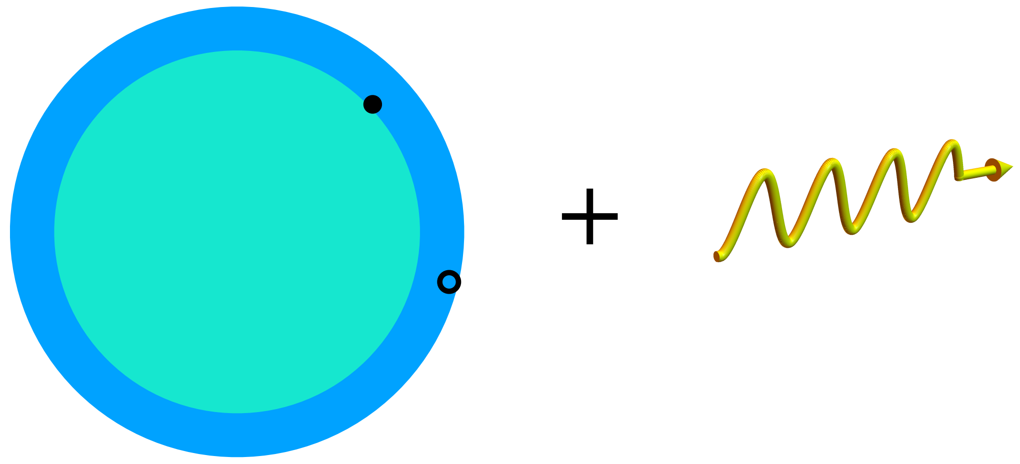


- Transverse

$$i\boldsymbol{A}\cdot(\psi^\dagger\boldsymbol{\nabla}\psi-\boldsymbol{\nabla}\psi^\dagger\psi)$$

coupling to dynamical photons

Source of non-analytical behaviors $\omega^{2/3}$, etc



Equivalent formulation of U(1) gauge theory

Finite spinon density of states on the Fermi surface leads to the screening of longitudinal gauge field

$$S_{A_0} \approx \frac{-m}{4\pi} \int \frac{d\omega_n d^2\mathbf{q}}{(2\pi)^3} |A_0(\mathbf{q}, \omega_n)|^2$$

Integrate out $\quad iA_0 \psi^\dagger_\alpha \psi_\alpha$

$$S_\psi = \int d\tau d^2\mathbf{r} \psi^\dagger_\alpha \Big([\partial_\tau - \mu - \frac{(\boldsymbol{\nabla}_\mathbf{r} - i\mathbf{A})^2}{2m}] \delta_{\alpha\beta} - \omega_B \sigma^3_{\alpha\beta} \Big) \psi_\beta,$$

$$S_A = \frac{1}{2} \int \frac{d\omega_n d^2\mathbf{q}}{(2\pi)^3} A_\mu(\mathbf{q}, \omega_n) \Big(\gamma \frac{|\omega_n|}{q} + \chi q^2 \Big) A_\nu(-\mathbf{q}, -\omega_n),$$

$$S_u = \int d\tau d^2\mathbf{r}\, u\, \psi^\dagger_\uparrow(\mathbf{r}, \tau) \psi_\uparrow(\mathbf{r}, \tau) \psi^\dagger_\downarrow(\mathbf{r}, \tau) \psi_\downarrow(\mathbf{r}, \tau).$$

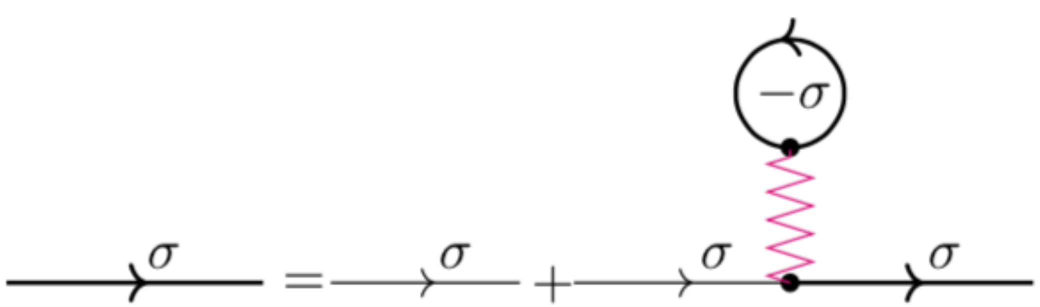
Short-ranged repulsion

Spinon self energy

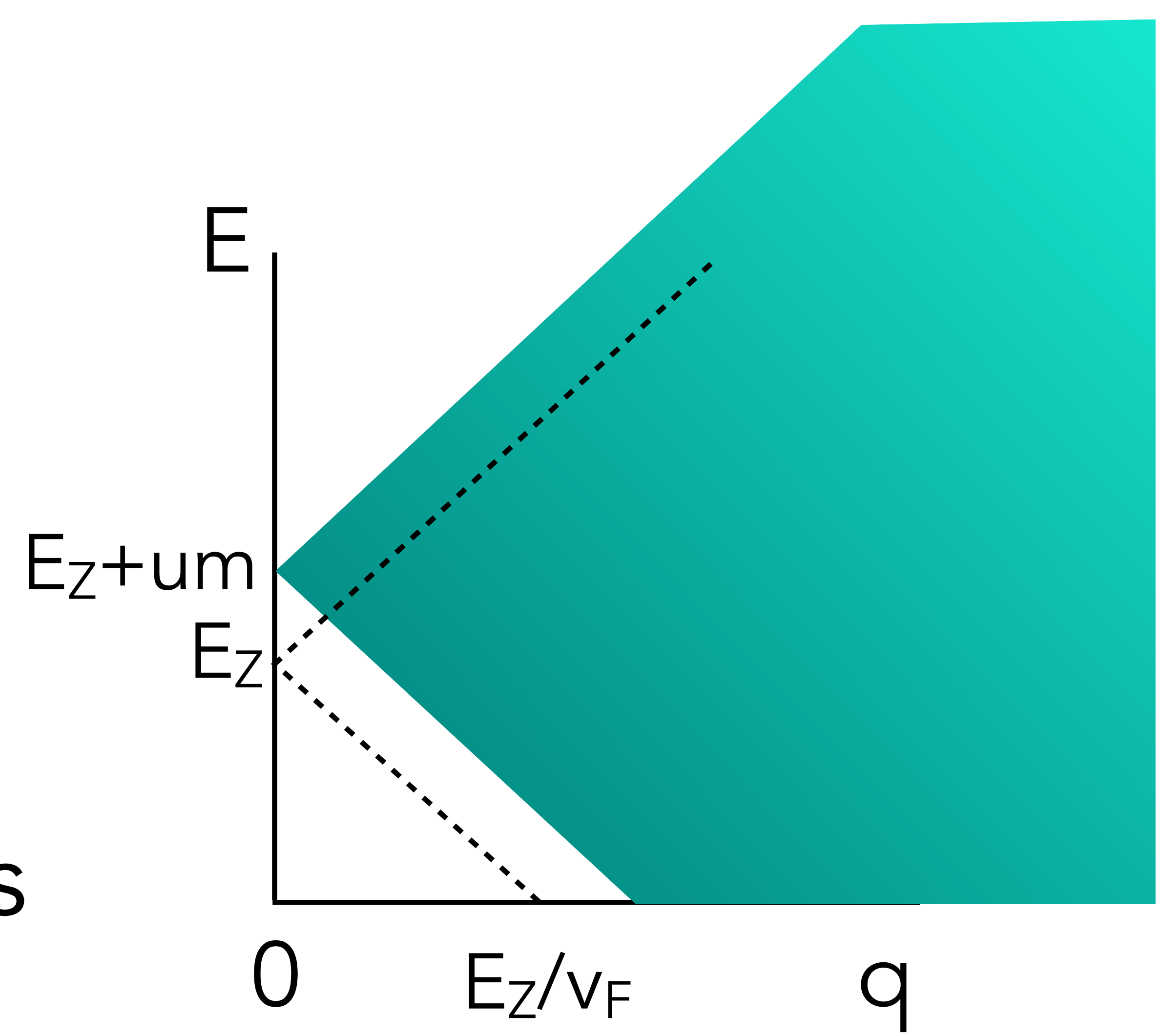
- Longitudinal $a_0\psi^\dagger\psi \rightarrow u\psi^\dagger_\uparrow\psi_\uparrow\psi^\dagger_\downarrow\psi_\downarrow$

$$= -um\left(\psi^\dagger_\uparrow\psi_\uparrow - \psi^\dagger_\downarrow\psi_\downarrow\right)$$

mean field shift



Spinon continuum shifts **up** in energy by **uM**



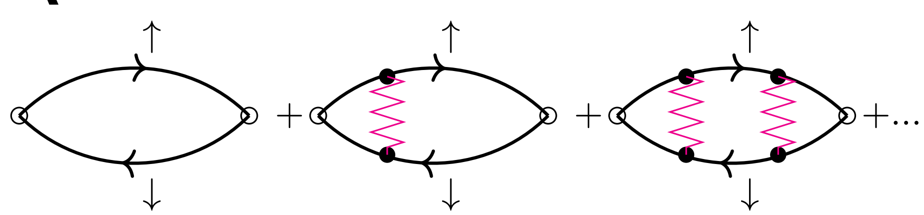
$$E_Z = 2\omega_B$$

Silin spin wave

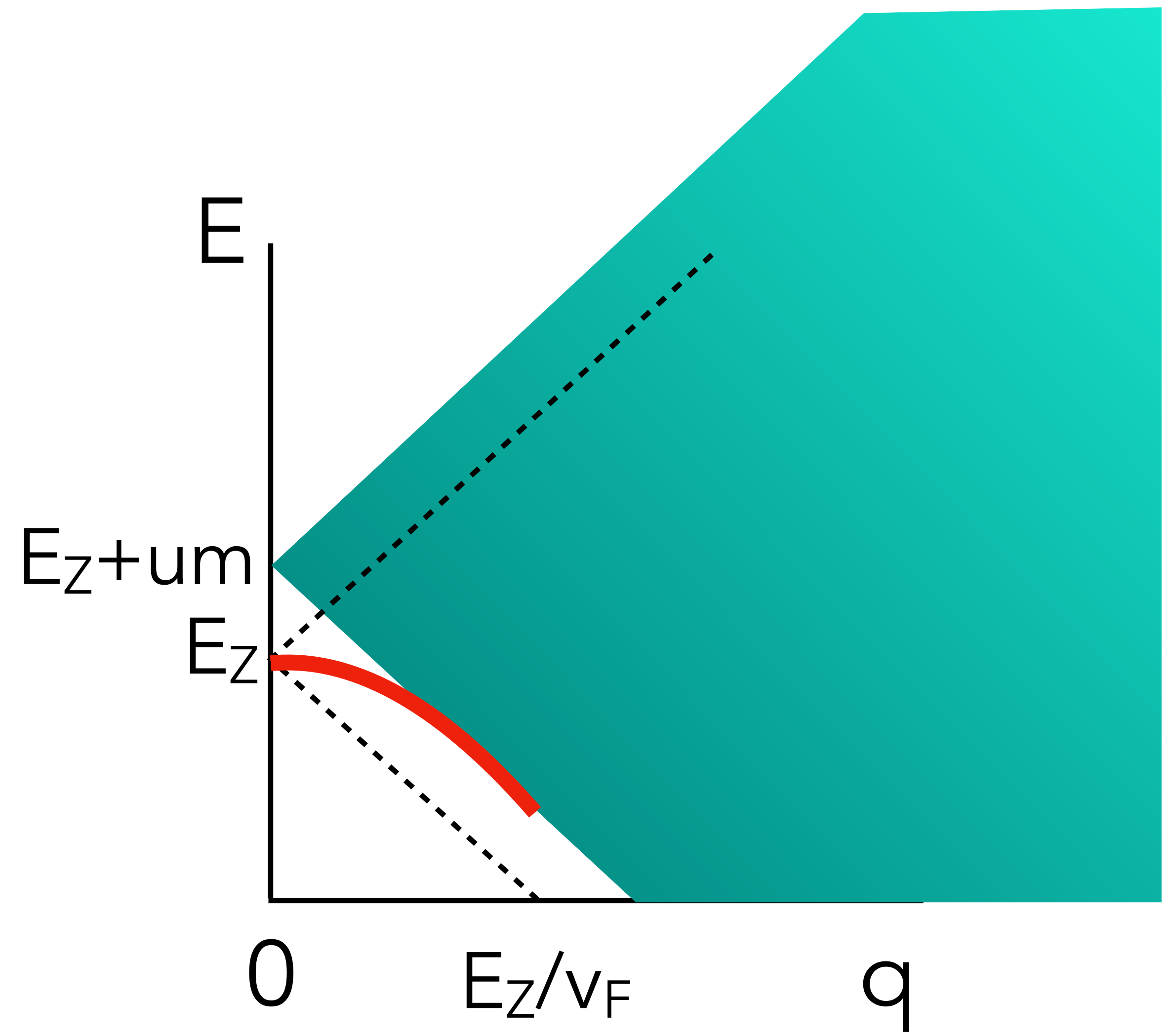
<u>Larmor theorem</u>: **q**=0 excitation *must* be at E_Z

$$\frac{dS^{+}_{\text{total}}}{dt} = -ihS^{+}_{\text{total}}$$

RPA



$$\chi(\boldsymbol{q}, i\omega_n) = \frac{\chi^0(\boldsymbol{q}, i\omega_n)}{1 + u\chi^0(\boldsymbol{q}, i\omega_n)}$$



pole: collective mode

$$\omega = E_Z + um - \sqrt{u^2m^2 + v_F^2 q^2}$$

"Silin spin wave"

[1]V. P. Silin, Zh. Eksperim. i Teor. Fiz. 35, 1243 (1958) [Sov. Phys. JETP 8, 870 (1959)].
[2]S. Schultz and G. Dunifer, Phys. Rev. Letters 18, 283 (1967). Conduction electron spin resonance
[3]P. M. Platzman and P. A. Wolff, Phys. Rev. Letters 18, 280 (1967). Kinetic equation for Landau Fermi liquid

STATISTICAL PHYSICS

Part 2

Theory of the Condensed State

by

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Volume 9 of *Course of Theoretical Physics*

Translated from the Russian by
J. B. SYKES and M. J. KEARSLEY

§3 magnetic field. In calculating the magnetic susceptibility, we must therefore write the quasi-particle energy change operator as

$$\delta\hat{\varepsilon} = -\beta\boldsymbol{\sigma}.\mathbf{H} + \mathrm{tr}' \int \hat{f}\, \delta\hat{n}'\, d\tau'. \tag{3.1}$$

The change in the distribution function is given in terms of $\delta\hat{\varepsilon}$ by $\delta\hat{n} = (\partial n/\partial\varepsilon)\,\delta\hat{\varepsilon}$;† we thus have

$$\delta\hat{\varepsilon}(\mathbf{p}) = -\beta\boldsymbol{\sigma}.\mathbf{H} + \mathrm{tr}' \int \hat{f}(\mathbf{p}, \mathbf{p}')\,(dn'/d\varepsilon')\,\delta\hat{\varepsilon}(\mathbf{p}')\, d\tau'. \tag{3.2}$$

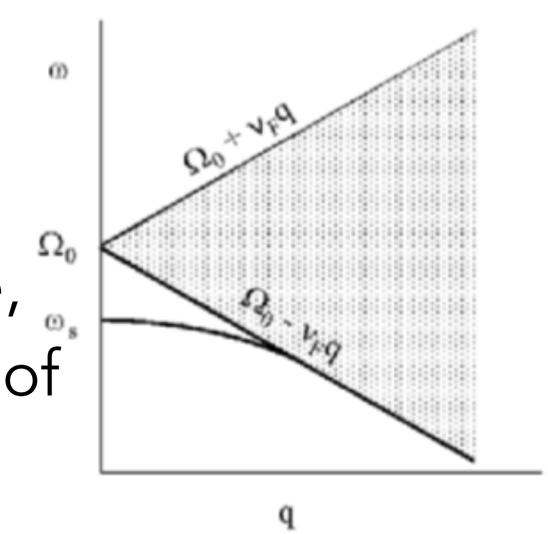
The ultimate opinion

$$\chi = \frac{\beta^2 p_F m^*}{\pi^2\hbar^3(1+\bar{G})} = \frac{3\gamma\beta^2}{\pi^2(1+\bar{G})}, \tag{3.5}$$

where γ is the coefficient in the linear specific heat law (1.15). The expression $\chi = 3\gamma\beta^2/\pi^2$ gives the susceptibility of a denegerate Fermi gas of particles with magnetic moment β; see Part 1, (59.5). The factor $1/(1+\bar{G})$ represents the difference between a Fermi liquid and a Fermi gas.†

† For He^3, $\bar{G} \approx -2/3$.

§5 Spin waves of another kind can be propagated in a Fermi liquid when a magnetic field is present (V. P. Silin 1958). Here we shall consider only vibrations with $\mathbf{k} = 0$, in which $\delta\hat{n}$ is independent of the coordinates.



\+ R. M. White, Quantum theory of Magnetism

The first frequency ω_{00} corresponds to vibrations with $\boldsymbol{\mu}$ = constant; then $\boldsymbol{\rho} = \boldsymbol{\mu}(1+\bar{G})$, and (5.6) becomes

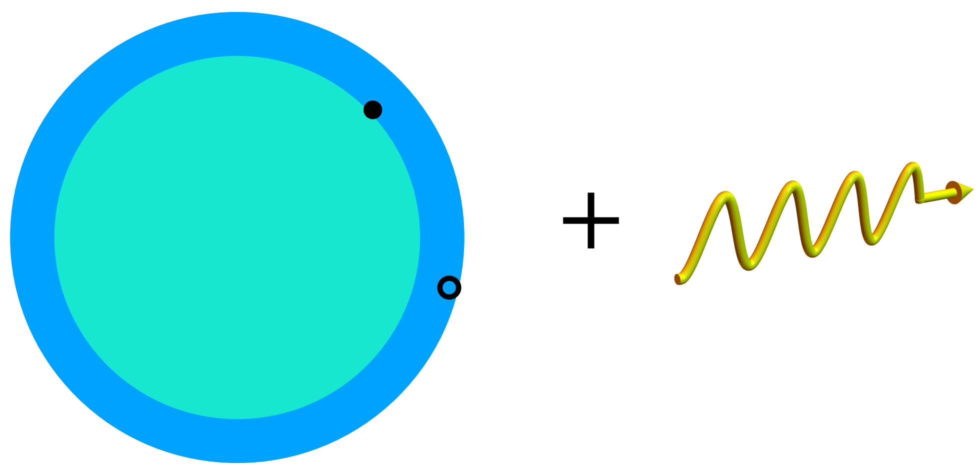
$$i\omega_{00}\boldsymbol{\mu} = (2\beta/\hbar)\,\mathbf{H}\times\boldsymbol{\mu};$$

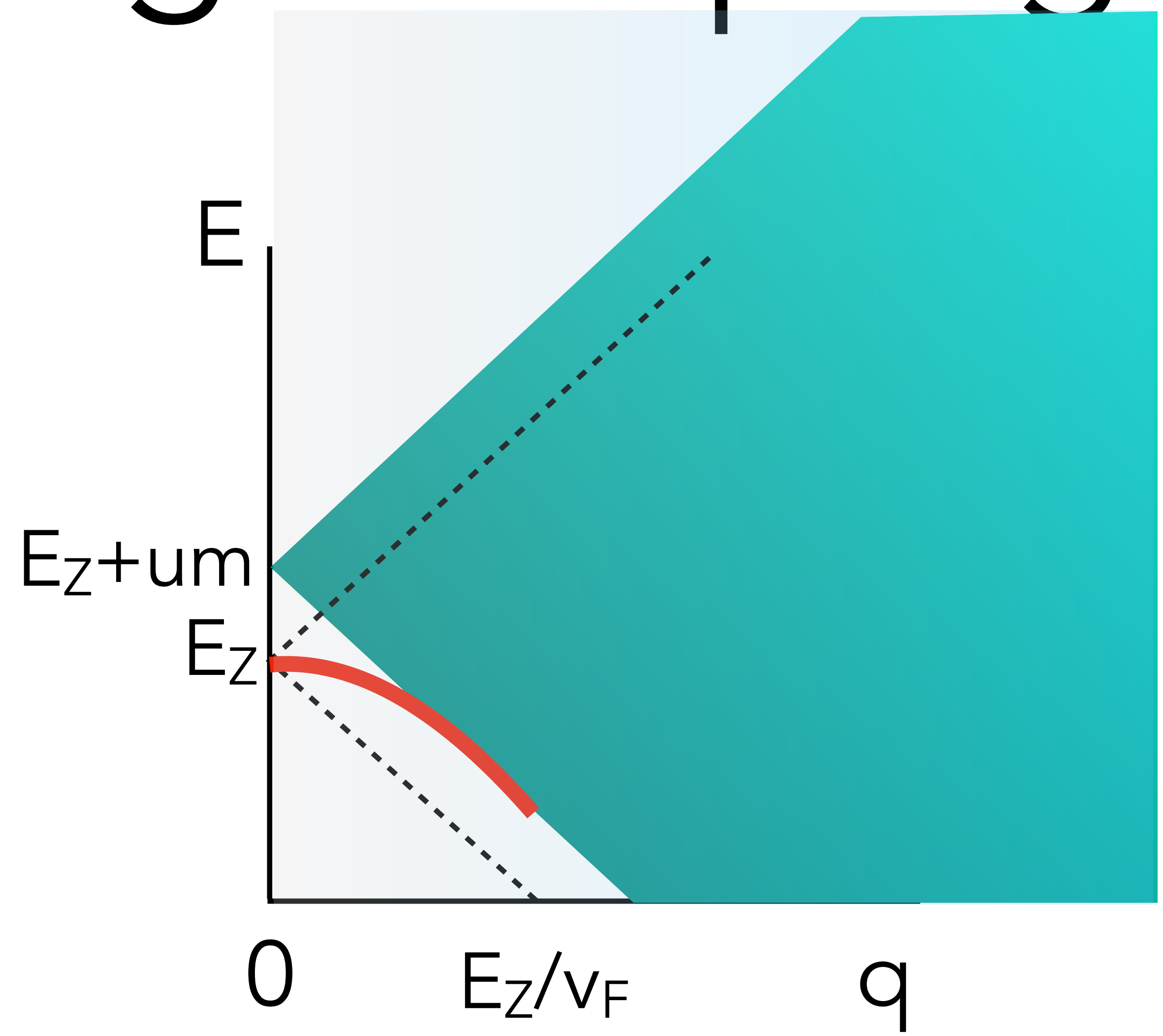
the vibrations are transverse to the field ($\boldsymbol{\mu} \perp \mathbf{H}$). Writing the equation in components in the plane perpendicular to $\mathbf{H}$ and taking the determinant, we find the frequency

$$\omega_{00} = 2\beta H/\hbar. \tag{5.8}$$

Here β is the magnetic moment of a particle (actual) in the liquid. Thus ω_{00} is independent of the specific properties of the liquid. The values of all the other frequencies ω_{lm}, however, depend on the specific form of the function $G(\vartheta)$.

Transverse gauge coupling





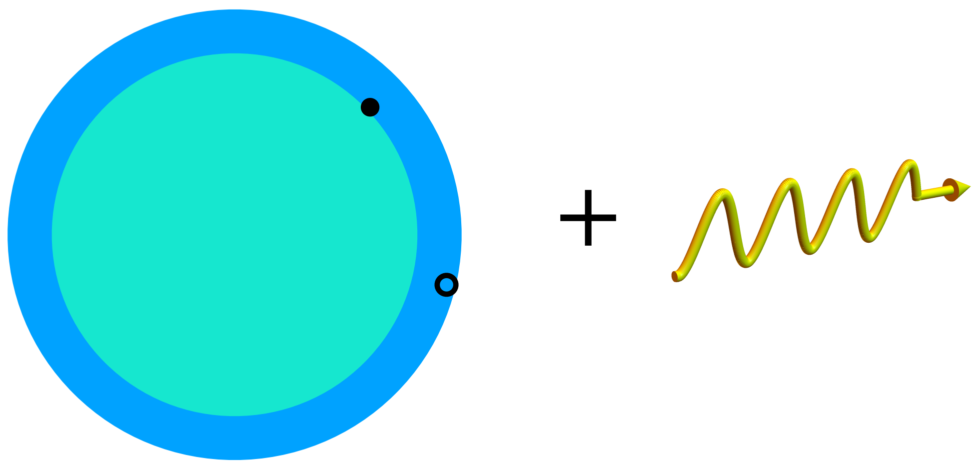
Simple picture:

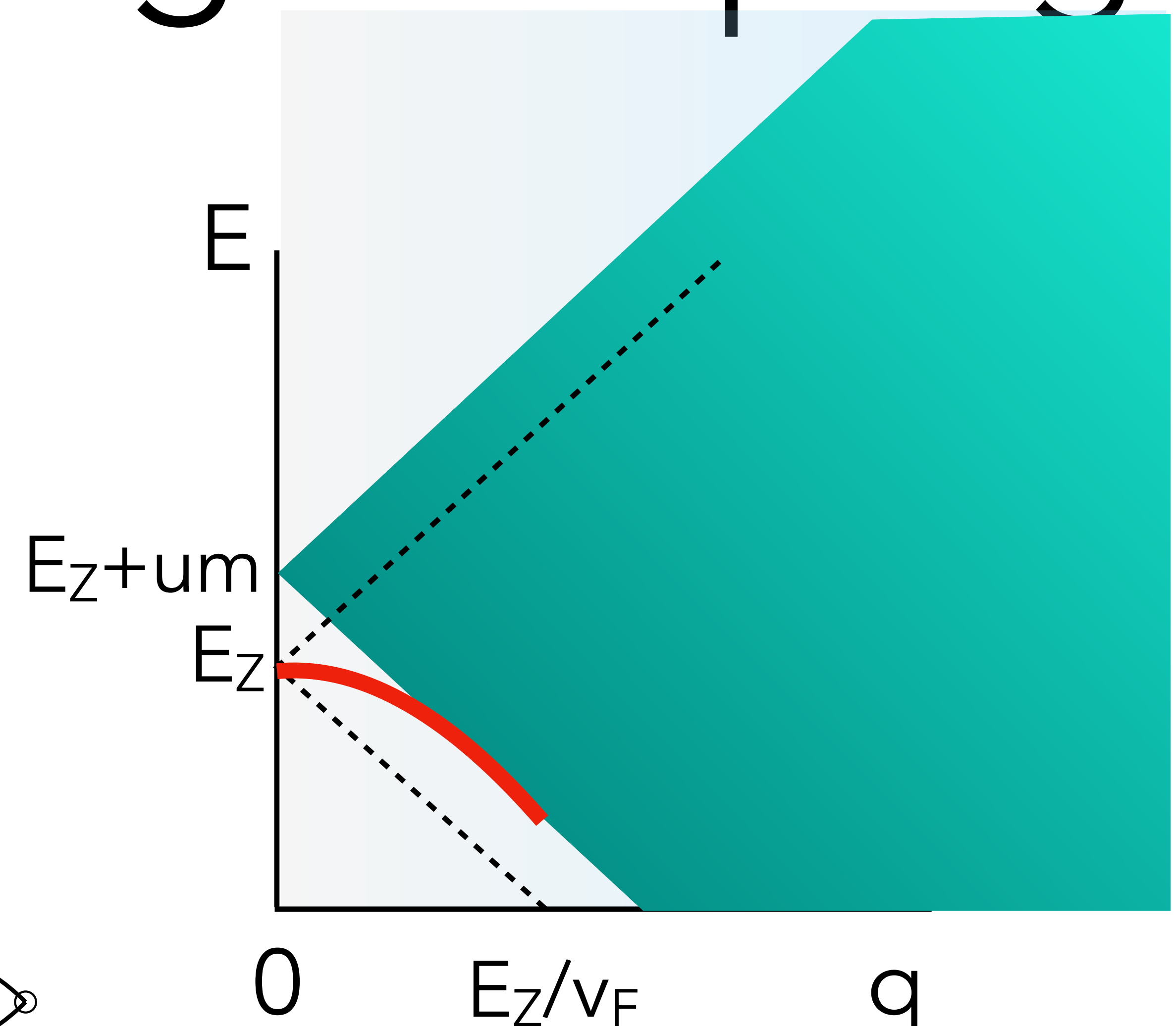
3-particle process:

$$E = E_{\mathrm{p/h}}(q-k) + \underbrace{E_{\mathrm{photon}}(k)}_{\sim c\, k^3}$$

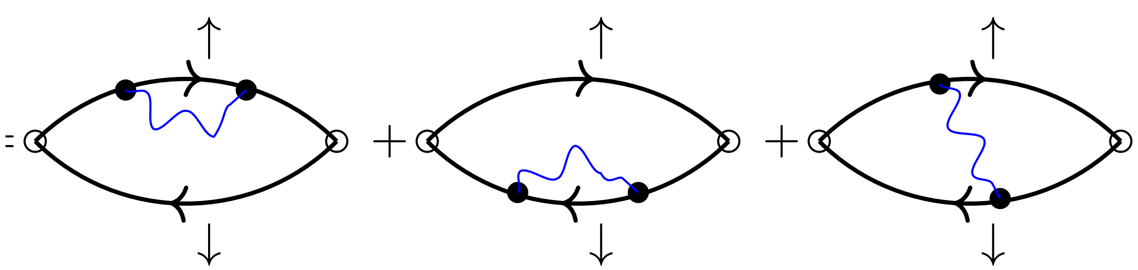
Does this smear out all the Fermi liquid structure?

Transverse gauge coupling





Actual calculation:

$\chi^1 =$ 

c.f. Y. B. Kim, A. Furusaki, X.-G. Wen, and P. A. Lee, Phys Rev. B **50**, 17917 (1994).

V. A. Zyuzin, P. Sharma, and D. L. Maslov, Phys. Rev. B **98**, 115139 (2018).

$$\chi = \frac{\chi^0 + \chi^1}{1 + u(\chi^0 + \chi^1)}$$

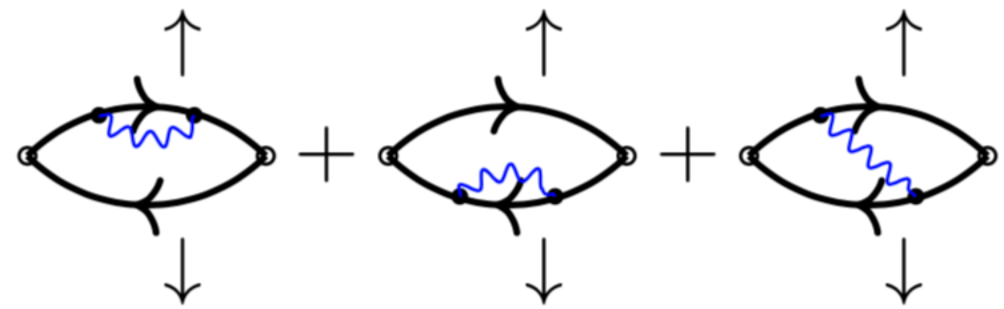
weight at all $\mathbf{q} \neq 0$

but weak enough to preserve structure

$$\chi_1''(\mathbf{q}, \omega) \propto \omega^{7/3} v_F^2 q^2 / [(v_F k_F)^{1/3} (uM)^4]$$

Few details

$$\chi^1_{\pm}(\boldsymbol{q}, i\omega_n) =$$



The diagrams

$$\chi_1^1(q,\omega_n) = \int (dk)(dp) G_\uparrow^2(\boldsymbol{k},k_n) G_\uparrow(\boldsymbol{k}+\boldsymbol{p},k_n+p_n) G_\downarrow(\boldsymbol{k}+\boldsymbol{q},k_n+\omega_n) \frac{2k_\mu+p_\mu}{2m}\frac{2k_\nu+p_\nu}{2m} D_{\mu\nu}(\boldsymbol{p},p_n),$$

Self-energy

$$\chi_2^1(q,\omega_n) = \int (dk)(dp) G_\uparrow(\boldsymbol{k},k_n) G_\downarrow^2(\boldsymbol{k}+\boldsymbol{q},k_n+\omega_n) G_\downarrow(\boldsymbol{k}+\boldsymbol{p}+\boldsymbol{q},k_n+p_n+\omega_n)$$
$$\times \frac{2k_\mu+2q_\mu+p_\mu}{2m}\frac{2k_\nu+2q_\nu+p_\nu}{2m} D_{\mu\nu}(\boldsymbol{p},p_n),$$

$$\chi_3^1(q,\omega_n) = \int (dk)(dp) G_\uparrow(\boldsymbol{k},k_n) G_\uparrow(\boldsymbol{k}+\boldsymbol{p},k_n+p_n) G_\downarrow(\boldsymbol{k}+\boldsymbol{q},k_n+\omega_n) G_\downarrow(\boldsymbol{k}+\boldsymbol{p}+\boldsymbol{q},k_n+p_n+\omega_n)$$
$$\times \frac{2k_\mu+p_\mu}{2m}\frac{2k_\nu+2q_\nu+p_\nu}{2m} D_{\mu\nu}(\boldsymbol{p},p_n),$$

Vertex

$$D_{\mu\nu}(\boldsymbol{p},p_n) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) D(\boldsymbol{p},p_n),$$

The gauge propagator

$$D(\boldsymbol{p},p_n) = \frac{1}{\gamma|p_n|/p + \chi p^2}.$$

The results:

$$\operatorname{Im}\chi^1(\boldsymbol{q},\omega) \approx -\frac{\sqrt{3}\gamma^{1/3}k_F}{56\pi^2\chi^{4/3}}\frac{q^2\omega^{7/3}}{(\omega-2\tilde{\omega}_B)^4}, \qquad 2\tilde{\omega}_B = 2\omega_B + uM$$

$$\operatorname{Re}\chi^1(\boldsymbol{q},\omega) \simeq \frac{k_F^2 q^2(\omega+2\tilde{\omega}_B)}{16\pi^2\chi(2\tilde{\omega}_B-\omega)^3}\ln\left(\frac{\Lambda\chi}{\gamma}\right)$$

Dispersion of the **spinon spin** wave

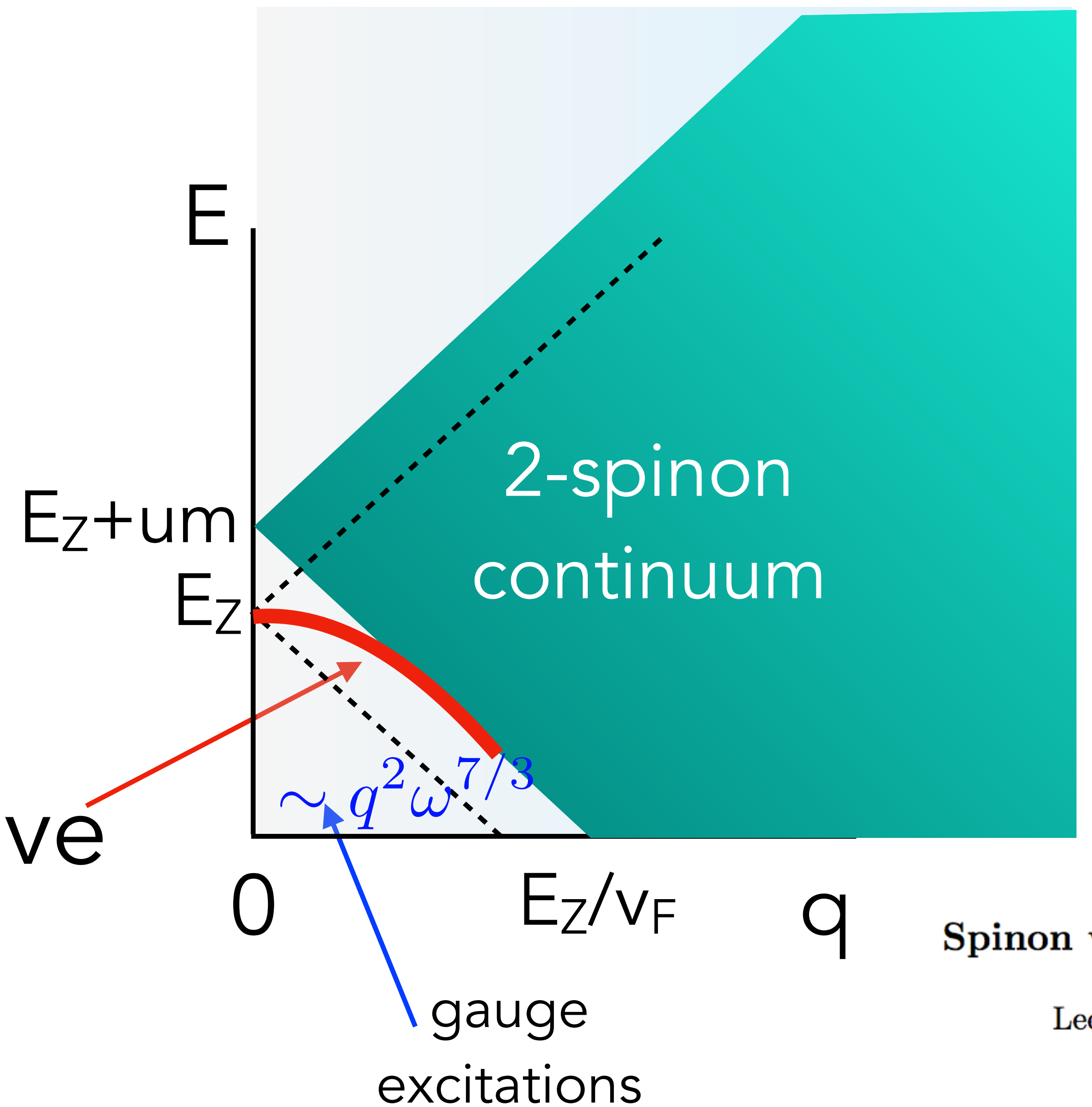
$$\omega_{\mathbf{q}} = 2\omega_B - \frac{\mathbf{q}^2}{2m^*} \qquad \frac{1}{m^*} = \frac{v_F^2}{2uM} - \frac{u(2\omega_B+uM)\ln\left(\frac{\Lambda\chi}{\gamma}\right)}{4\pi^2\chi(2uM)^3}$$

Summary

Spinon waves in magnetized spin liquids

Leon Balents[1] and Oleg A. Starykh[2] arXiv:1904.02117

Distinct signature of spinons, interactions, and gauge fields



spinon spin wave

$$\frac{E_Z}{v_F} \sim \sqrt{mE_Z}\sqrt{\frac{E_Z}{E_F}}$$

Spinon waves in magnetized spin liquids

Leon Balents[1] and Oleg A. Starykh[2]

arXiv:1904.02117

Outline

- QSL, spinon Fermi surface
  - Some history
  - Candidate materials
- Spinon continuum in magnetic field
- Spin-orbit interactions
  - Weak spin-orbit: ESR linewidth due to gauge fluctuations
  - Strong spin-orbit: spinon resonance
- Conclusions

Symmetry-breaking spin-orbit terms

1. Total spin is not conserved
2. No Larmor theorem
3. Weak SOC: collective mode shifts, acquires **finite lifetime**

PRL 114, 156803 (2015) PHYSICAL REVIEW LETTERS week ending 17 APRIL 2015

Intrinsic Damping of Collective Spin Modes in a Two-Dimensional Fermi Liquid with Spin-Orbit Coupling

Saurabh Maiti[1,2] and Dmitrii L. Maslov[1]

Fermi liquid;
Details depend on
specific model

$$\mathcal{H}_k = \left(\frac{k^2}{2m} - \mu\right)\sigma_0 + \lambda\vec{\sigma}\cdot\vec{f}(\vec{k})$$

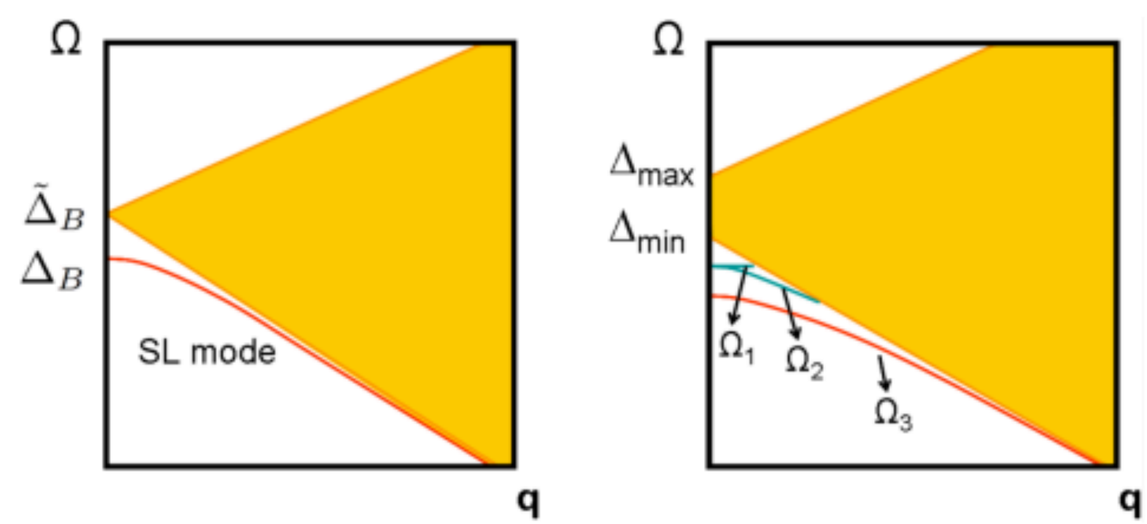
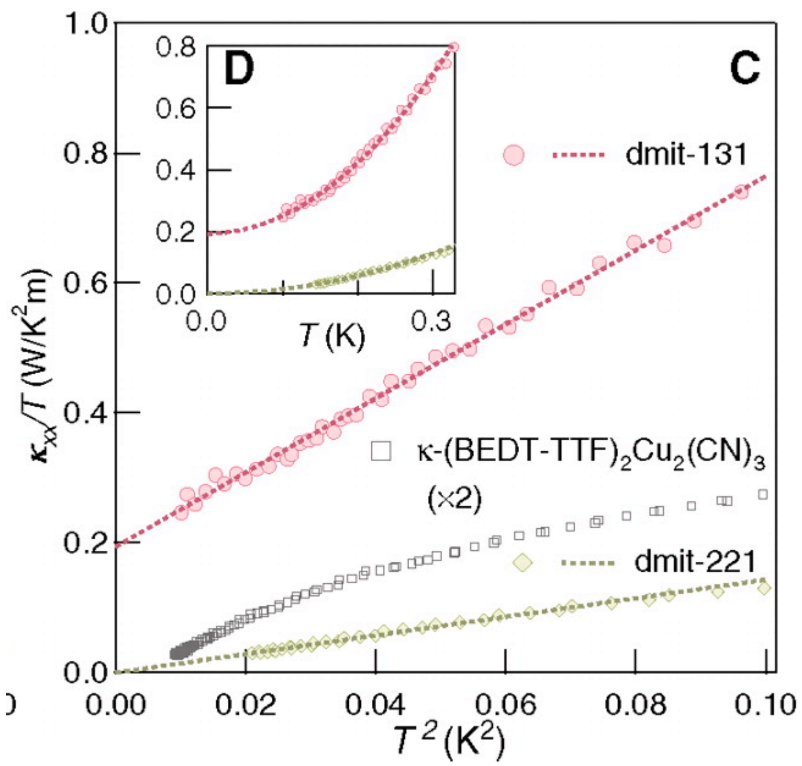


FIG. 1 (color online). Left: The Silin-Leggett mode in a partially spin-polarized FL. Right: The chiral-spin modes in a FL with Rashba spin-orbit coupling. The shaded regions denote the particle-hole continua, Δ_B is the Larmor frequency, $\tilde{\Delta}_B$ is the quasiparticle Zeeman energy, and $\Delta_{\min(\max)}$ is the lower (upper) boundary of the continuum at $q = 0$.

Organic Mott insulators: Spin liquid with spinon Fermi surface?



$EtMe_3Sb[Pd(dmit)_2]_2$ (dmit-131)

Spin liquid?

M. Yamashita et al, Science 2010

$Et_2Me_2Sb[Pd(dmit)_2]_2$ (dmit-221)

Non-magnetic charge-ordered

Spin-orbit interaction is present in closely related materials

PHYSICAL REVIEW B **68**, 024512 (2003)

Dzialoshinskii-Moriya interaction in the organic superconductor κ-(BEDT-TTF)$_2$Cu[N(CN)$_2$]Cl

Dylan F. Smith,* Stewart M. De Soto,† and Charles P. Slichter
Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA

John A. Schlueter, Aravinda M. Kini, and Roxanne G. Daugherty
Materials Science Division, Argonne National Laboratory, Argonne, Illinois 60439, USA
(Received 11 February 2003; published 23 July 2003)

The authors report ^{13}C NMR and magnetization measurements on the magnetic state of oriented single crystals of the organic superconductor κ-(BEDT-TTF)$_2$Cu[N(CN)$_2$]Cl. To understand these data a spin Hamiltonian based on the *Pnma* symmetry of the crystal is developed. When interpreted in the context of this Hamiltonian, the measurements provide a detailed picture of the spin ordering. It is found that the Dzialoshinskii-Moriya (DM) interaction is largely responsible for the details of the ordering above the spin-flop field. Of particular note, the interplane correlations are determined by the intraplane DM interactions and the direction of the applied field.

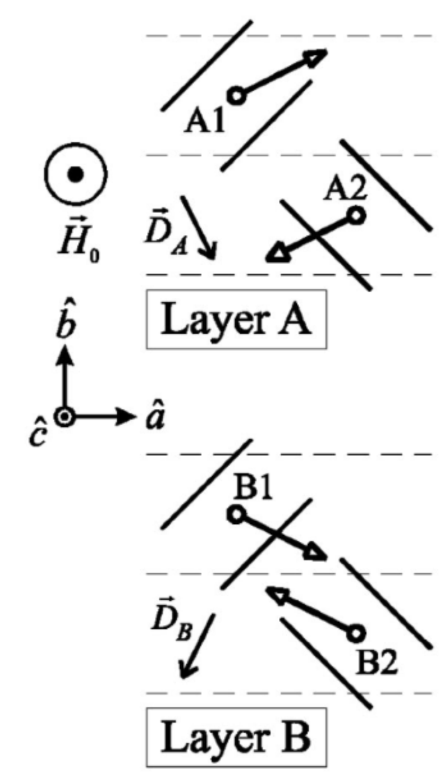
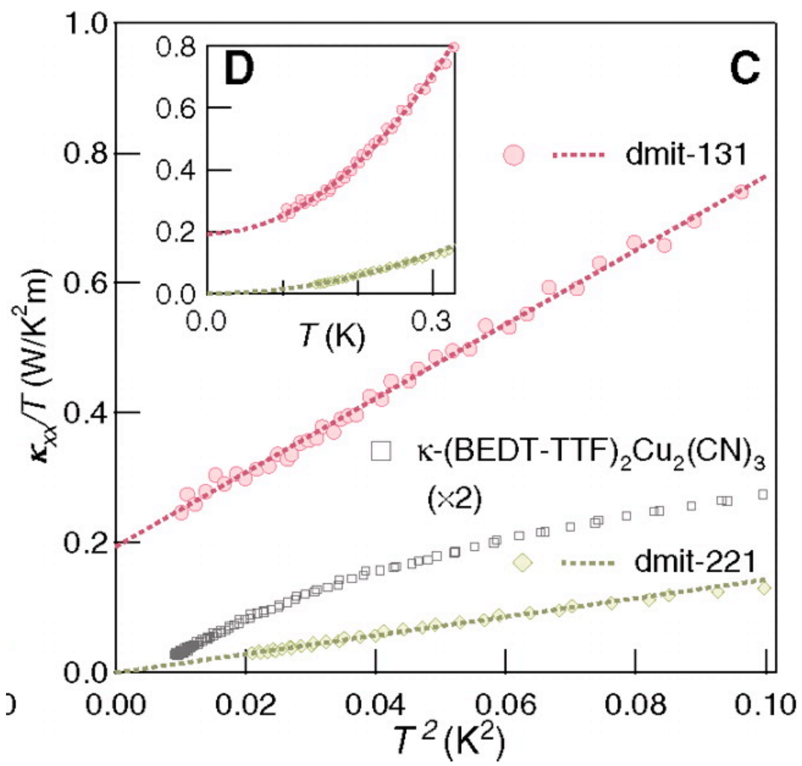


FIG. 6. Depiction of DM vectors and electron spin ordering

Organic Mott insulators: Spin liquid with spinon Fermi surface?



EtMe$_3$Sb[Pd(dmit)$_2$]$_2$ (dmit-131)

Spin liquid?

M. Yamashita et al, Science 2010

Et$_2$Me$_2$Sb[Pd(dmit)$_2$]$_2$ (dmit-221)

Non-magnetic charge-ordered

Spin-orbit interaction is present in closely related materials

PHYSICAL REVIEW B **95**, 060404(R) (2017)

Importance of spin-orbit coupling in layered organic salts

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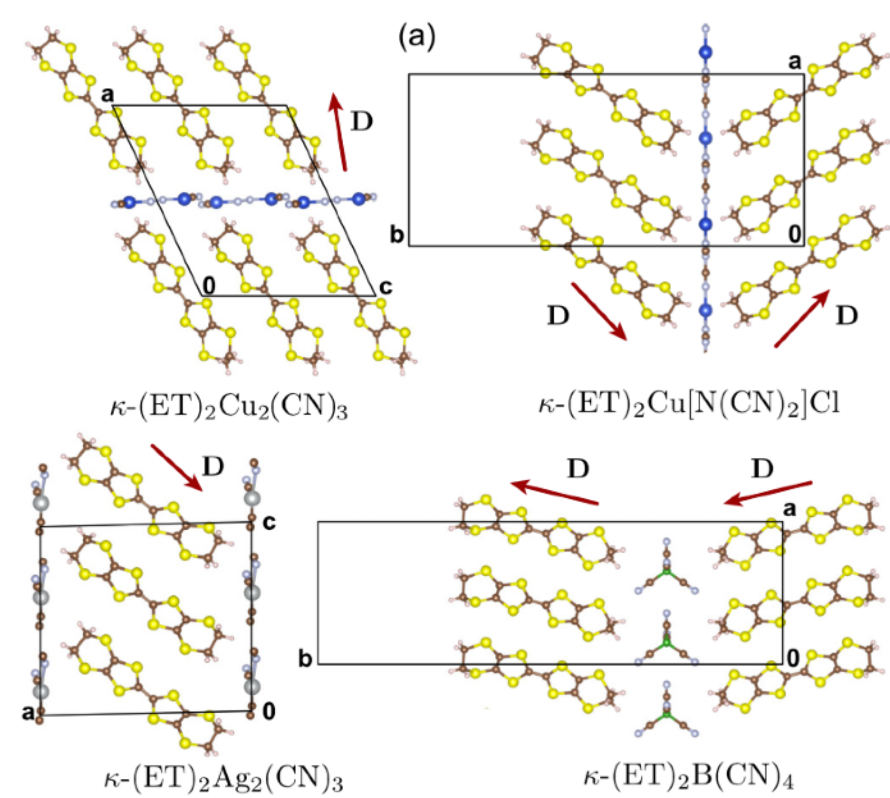
(Received 2 November 2016; revised manuscript received 4 January 2017; published 7 February 2017)

We investigate the spin-orbit coupling (SOC) effects in α- and κ-phase BEDT-TTF and BEDT-TSF organic salts. Contrary to the assumption that SOC in organics is negligible due to light C, S, and H atoms, we show the relevance of such an interaction in a few representative cases. In the weakly correlated regime, SOC manifests primarily in the opening of energy gaps at degenerate band touching points. This effect becomes especially important for Dirac semimetals such as α-(ET)$_2$I$_3$. Furthermore, in the magnetic insulating phase, SOC results in additional anisotropic exchange interactions, which provide a compelling explanation for the puzzling field-induced behavior of the quantum spin-liquid candidate κ-(ET)$_2$Cu$_2$(CN)$_3$. We conclude by discussing the importance of SOC for the description of low-energy properties in organics.



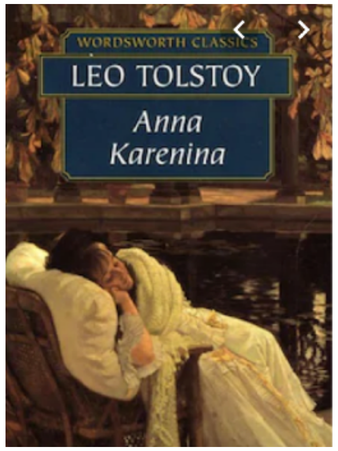
General analysis of spin-orbital effects in QSL is not possible.

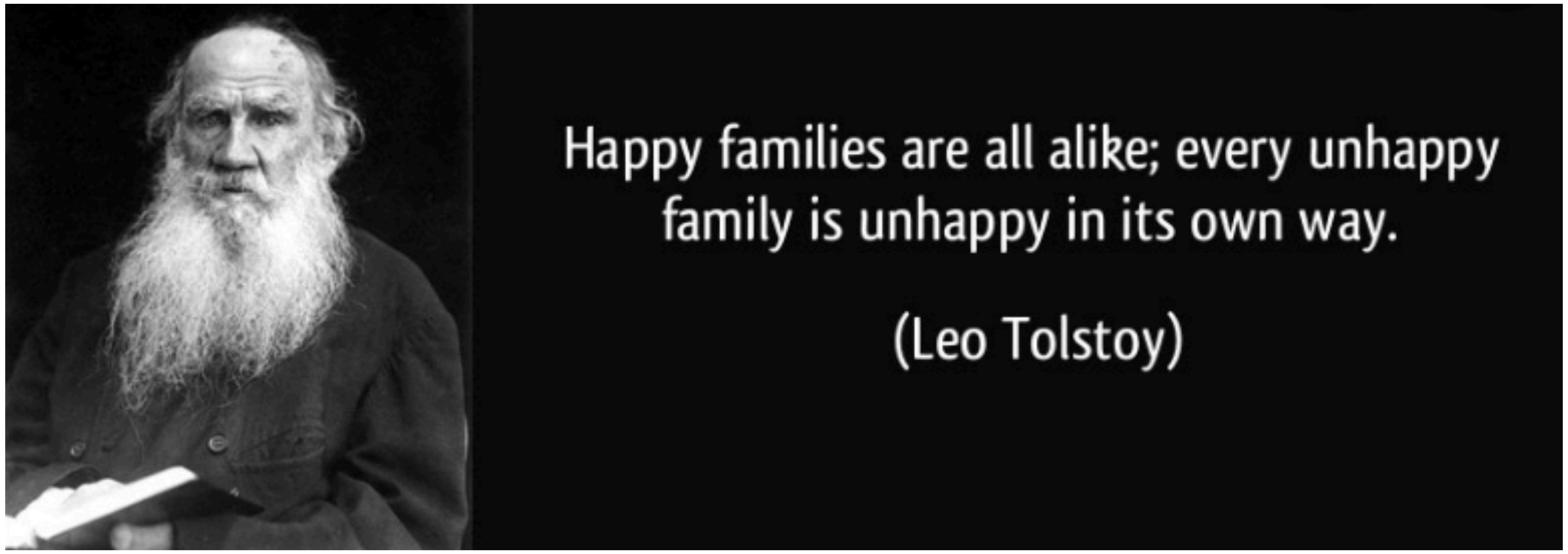
Today: Case study of ESR line width for spinons with Rashba spin-orbit anisotropy

Spinon waves in magnetized spin liquids

Leon Balents[1] and Oleg A. Starykh[2] arXiv:1904.02117

Isotropic magnets are all alike, every anisotropic magnet is anisotropic in its own way.





Linewidth *at (relatively)* high T

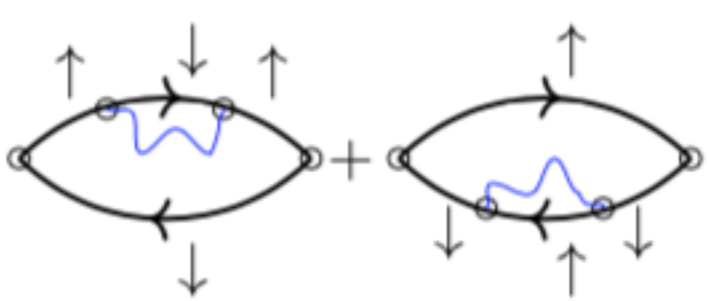
- Spinon band structure determines *line shape* of absorption.
- Interactions determine h,T-dependent *line width* !

Ideal U(1) spin liquid

$$L_{u(1)} = \psi^\dagger_\alpha \Big(\partial_t - i{\color{red}A_0} + \epsilon(\nabla - i{\color{red}\vec{A}})\Big)\psi_\alpha$$

Rashba-like perturbation due to spin orbit interaction

$$\delta L_R = {\color{blue}\alpha_R}\psi^\dagger_\alpha \Big((p_x + {\color{red}A_x})\sigma^y - (p_y + {\color{red}A_y})\sigma^x\Big)\psi_\alpha$$



Spin-orbit mediates interaction between gauge fluctuations and spins

Mori-Kawasaki formalism (q=0)

Oshikawa, Affleck 2002

Retarded spin GF $$G^R_{S^+S^-}(\omega) \propto 1/(\omega - h - \Sigma(\omega))$$

Line width $$\eta(\omega = h) = \mathrm{Im}\Sigma(\omega = h) = -\frac{\mathrm{Im}\{G^R_{\mathcal{A}\mathcal{A}^\dagger}(\omega)\}}{2\langle S^z \rangle}$$

Perturbation is encoded in the **composite** operator
(depends on polarization of microwave radiation!)

$$\mathcal{A} = [\delta H_R, S^+] = -2i\alpha_R \sum_{p,q} \psi^\dagger_{p+q}\sigma^z\psi_p(A_{x,q} - iA_{y,q})$$

$$\eta(h) \sim \alpha_R^2 \int d\epsilon[1 + n_B(\epsilon) + n_B(h-\epsilon)]\mathrm{Im}G^R_{S^z_q S^z_{-q}}(\epsilon)\mathrm{Im}G^R_{A^-_q A^+_q}(h-\epsilon)$$

Gauge field propagator $$\mathrm{Im}G^R_{A^-_q A^+_q}(\nu) = \frac{\gamma q \nu}{\gamma^2\nu^2 + \chi^2 q^6}$$

Landau damping,
$\nu \sim q^3$

`Particle-hole' spinon continuum $$\mathrm{Im}G^R_{S^z_q S^z_{-q}}(\epsilon) = \frac{m}{2\pi}\frac{\epsilon}{\sqrt{v^2q^2 - \epsilon^2}}\Theta(vq - |\epsilon|)$$

Linewidth for U(1) QSL + Rashba

Spinon waves in magnetized spin liquids

Leon Balents[1] and Oleg A. Starykh[2] [arXiv:1904.02117]

T = 0,
h>> T

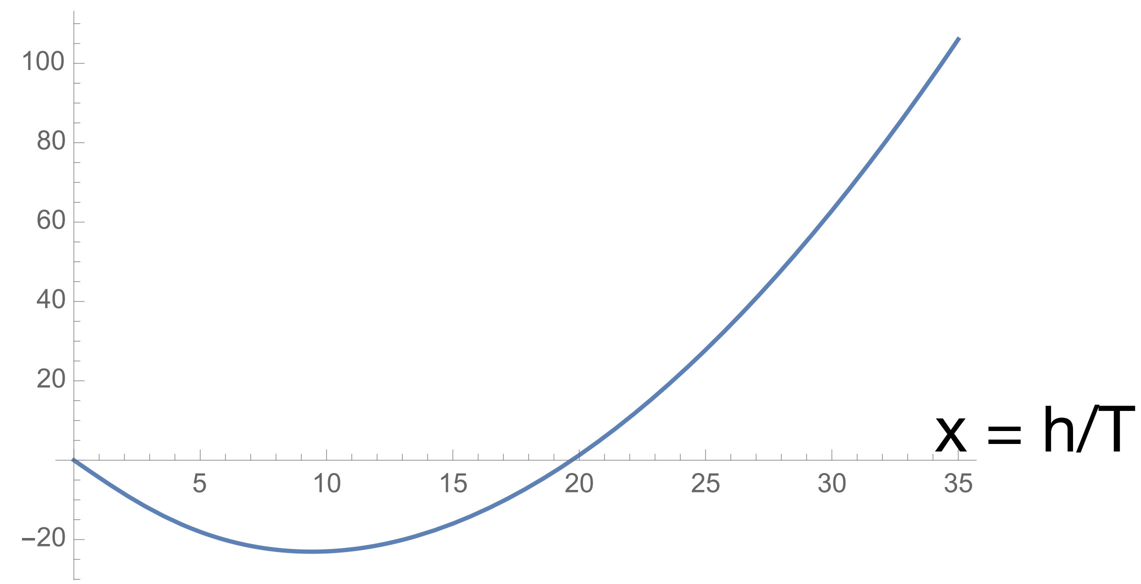
$$\eta \sim \alpha_R^2 \omega^{5/3}/h \sim h^{2/3}, h > 0$$

$$\eta \sim \omega^{2/3}, h = 0$$

T > 0,
h<< T

$$\eta = \frac{1}{2\chi_{\rm u} h}\Big(\frac{mT}{8\pi\chi} + \tilde{c}_0 T^{5/3} f(\frac{h}{T})\Big) \sim \frac{T}{h} + T^{2/3}$$

Scaling function f(x)



$$f(x) \to -4.4x \text{ for } x \ll 1; f(x) \to 0.75x^{5/3} \text{ for } x \gg 1$$

Conclusions

- Spinon Fermi surface hypothesis has testable experimental consequences

✓ Magnetized spin liquid: Collective transverse spin mode with downward dispersion

✓ Gauge fluctuations of QSL produce intrinsic line width, scales as $q^2\omega^{7/3}$

✓ Spin-orbit interaction distorts this picture, causes broadening, non-trivial ESR line width, broad absorption band, etc.

Spinon **liquid** is strongly interacting, *qualitatively* different from spinon **gas**.