Melting a Valence Plaquette Solid:
Emergent Fracton and algebra quantum liquid

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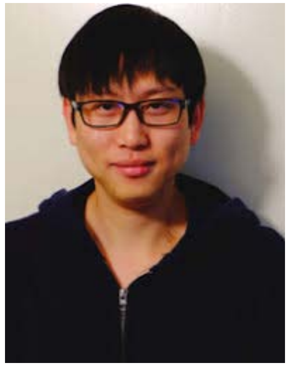
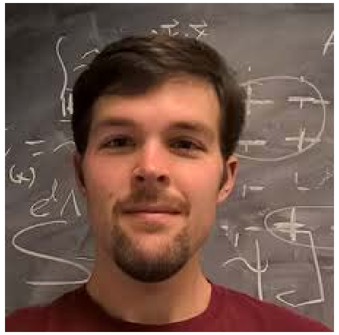
arXiv:1908.08540



Zhen Bi (MIT)



Mike Pretko (Boulder)

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Diverse phenomenology:

✓ Ordered phase:
Magnetic order (Neel) ,
Crystalline order (VBS;VPS)

✓ Exotic quantum phases:
Z_2, U(1) Spin liquid….

✓ Exotic quantum critical points between different order

VBS→ Neel transition
Breaks different symmetry;
DQCP forbidden by GL paradigm

Deconfined Quantum Critical Points

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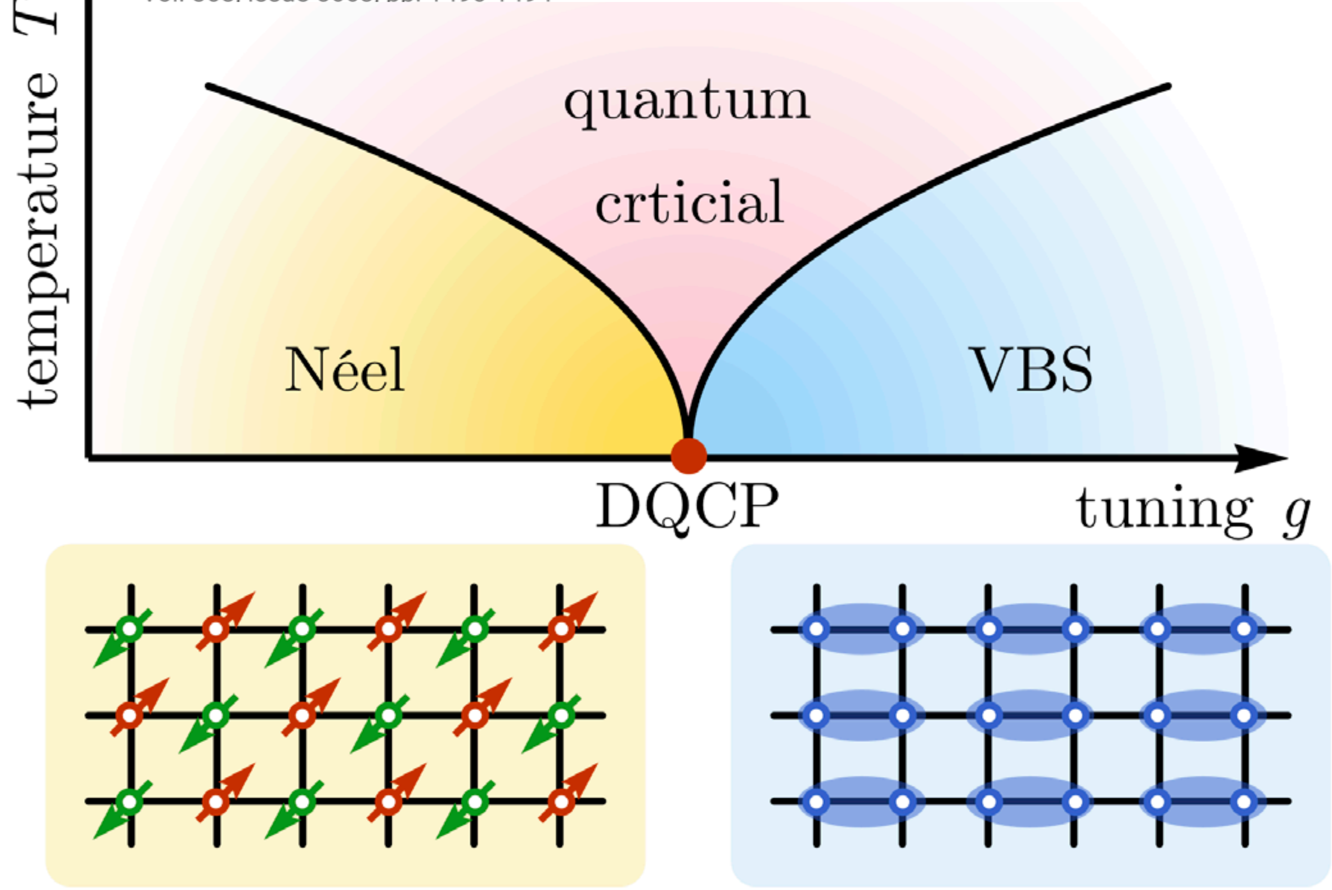
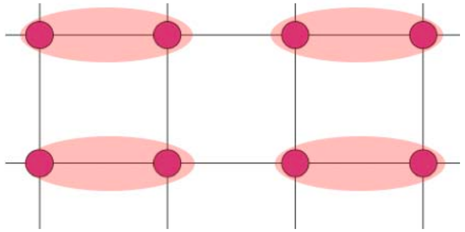
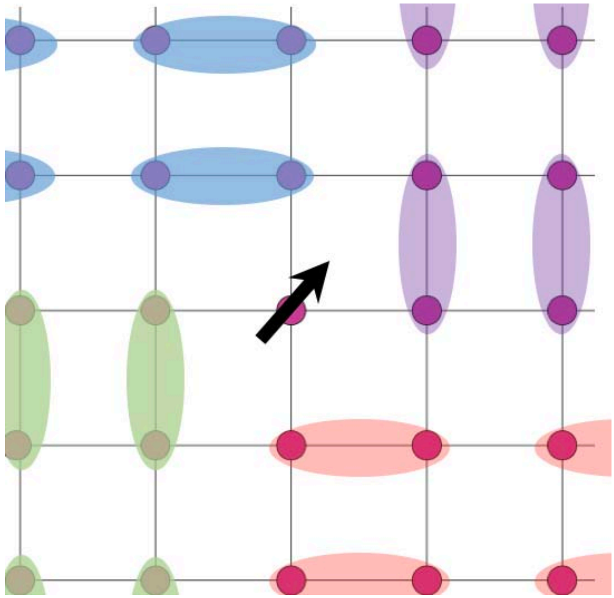
Science 05 Mar 2004:
Vol. 303, Issue 5663, pp. 1490-1494





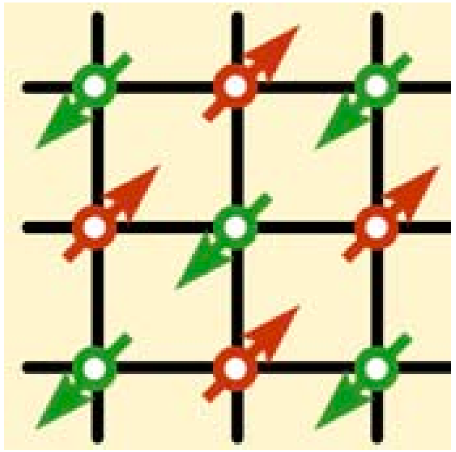
Valence Bond Solid

✓ Paramagnetic crystal
✓ Breaks translation and C_4



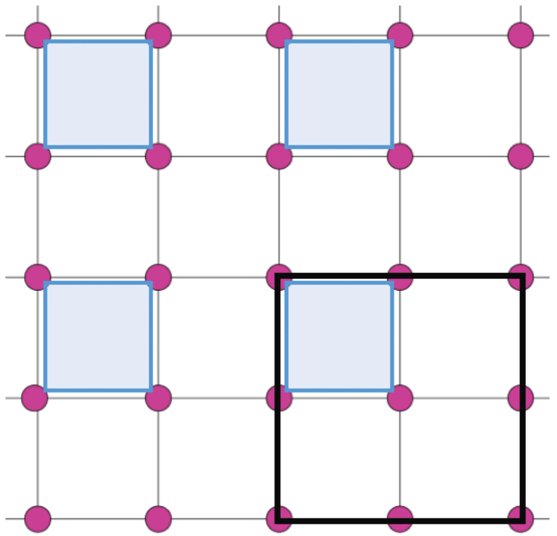
Condense VBS defect
= Condense spinon

NCCP1 theory



Defect of the VBS order:
Carry a **spinon** ! *(Levin-2004)*

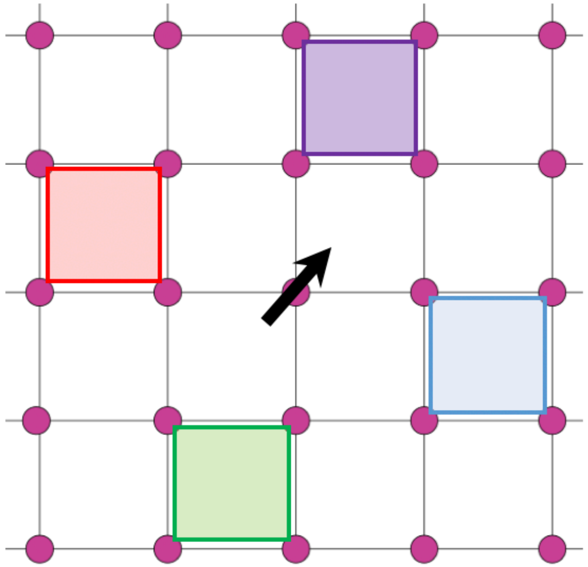
Restore $\mathbf{C_4}$
+ Break **spin rotation**
= Magnetic order



Another candidate for Paramagnetic Crystal

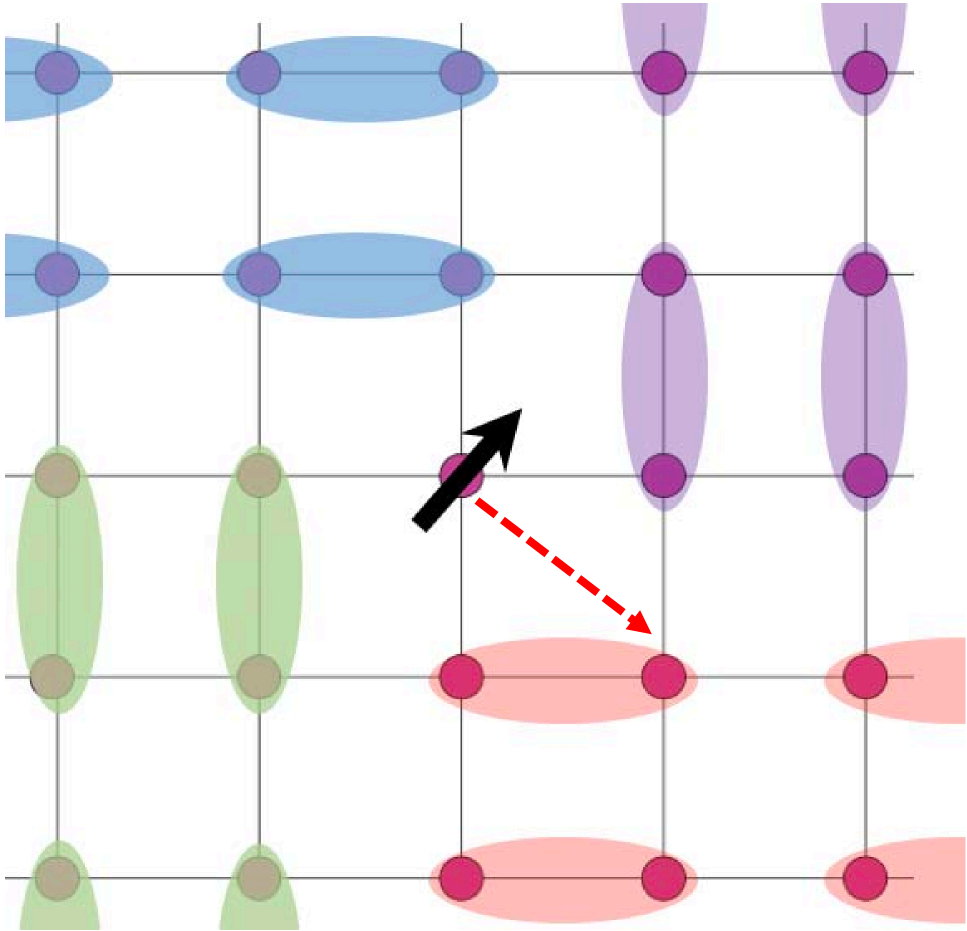
Valence Plaquette Solid (**$SrCu_2(BO_3)_2$** *Nature Physics 13, 962–966*)

Breaks T_x, T_y , C_4



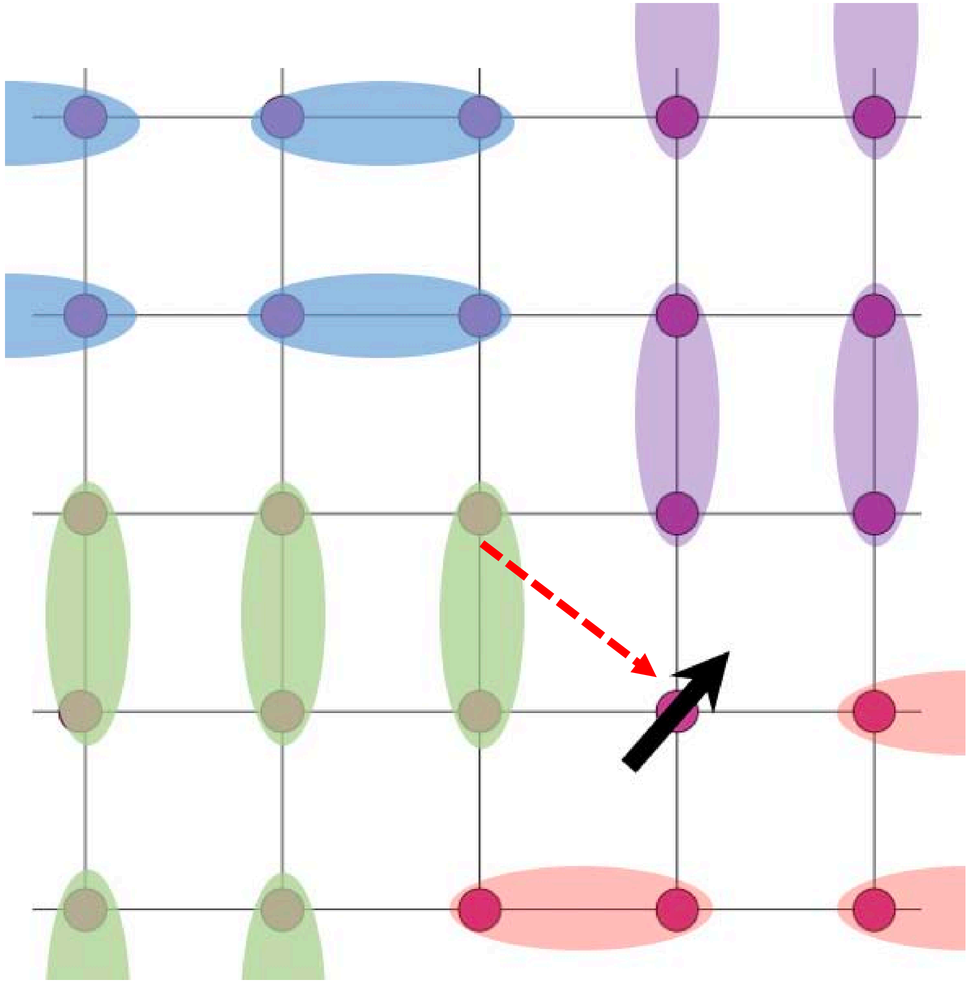
Defect of the VPS order: Carry a **spinon** !

Can we condense the `defect+spinon' → Magnetic order ?



When spinon is energetically deconfined,

It can fluctuate in the dimer background!



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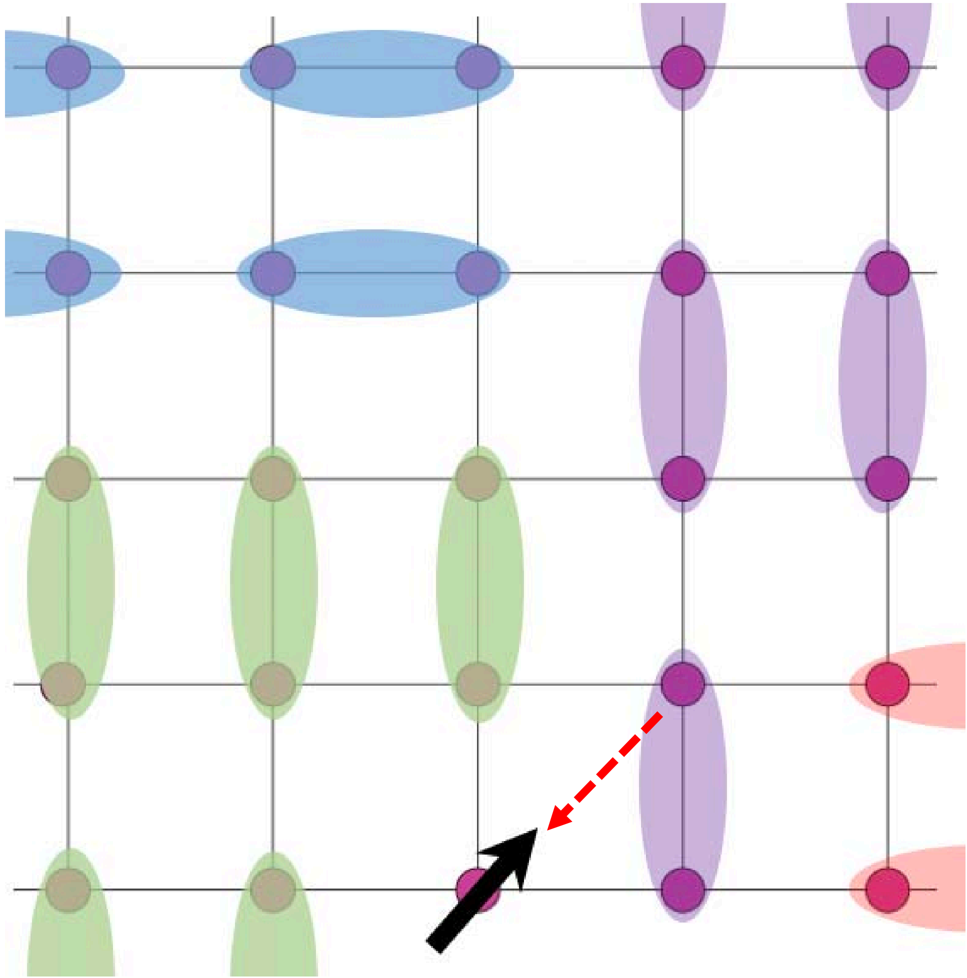

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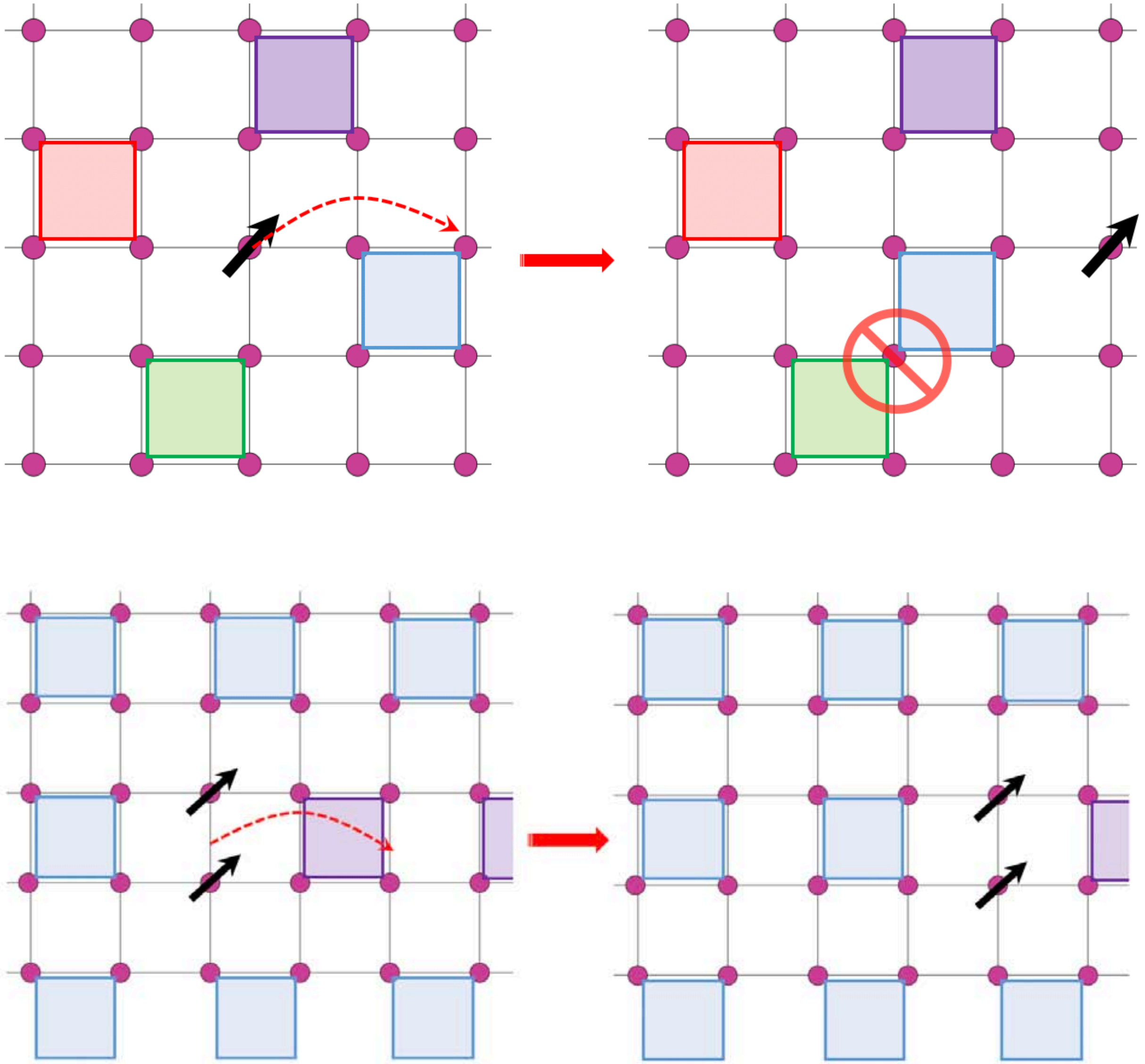


Spinon **cannot** fluctuate and hop in the Plaquette background?

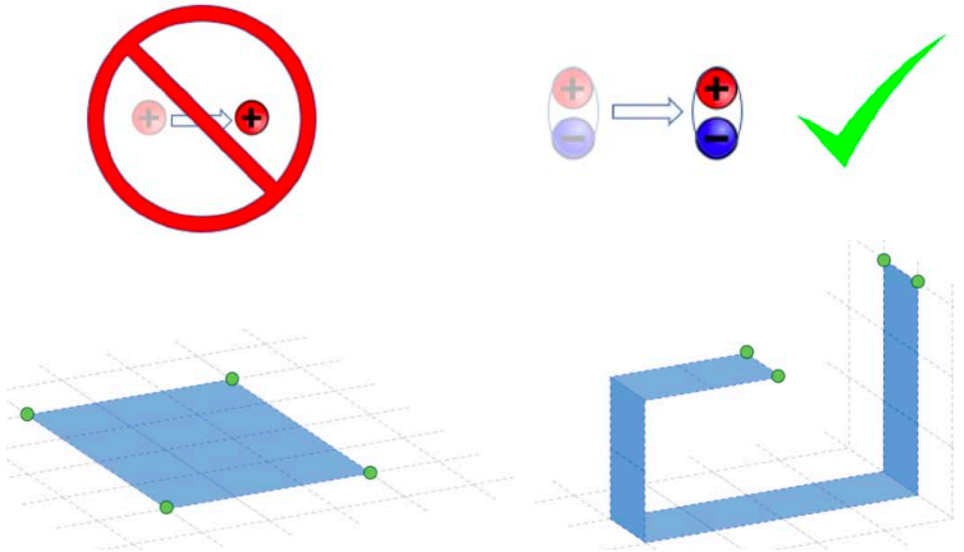
Spinon is a **Fracton**?

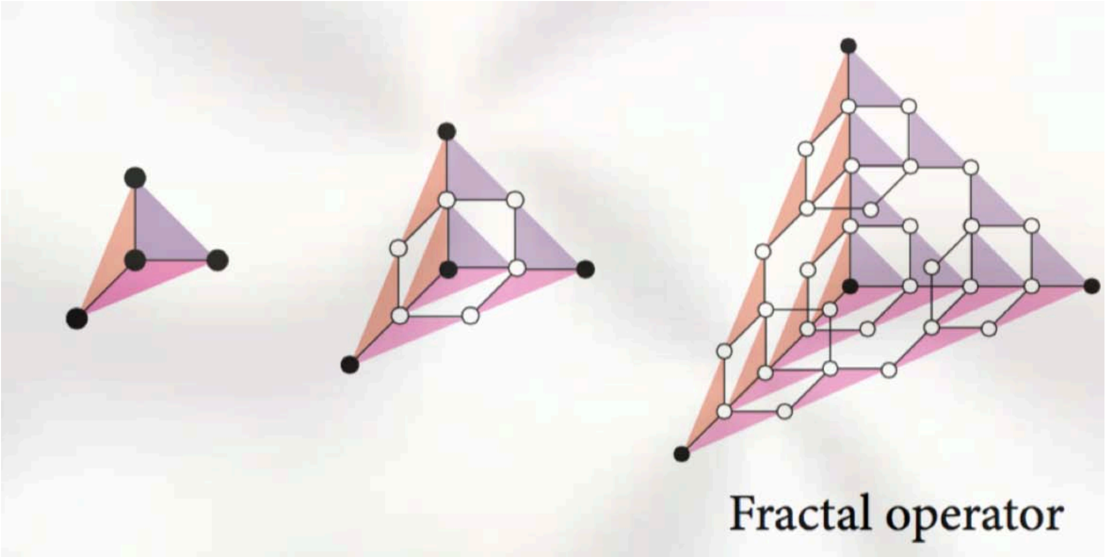
A pair of spinon (as a dipole) **only** fluctuate along the transverse stripe!

Fracton → QP with restricted mobility *(Vijay-2016, Pretko-2017)*

✓Deconfined but immobile particle

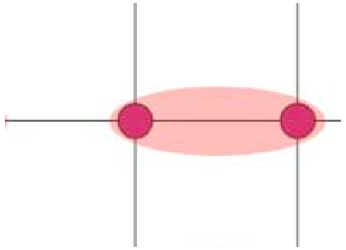
✓ Energy barrier -> Particle moved with restricted direction
-> **Subsystem conservation law**





Warm up

Mapping: VB ←→ U(1) gauge theory



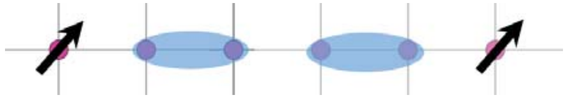
VB coverage = ***E*** field

$$E_i(\boldsymbol{r}) = (-1)^{i_{\boldsymbol{r}}} D_i(\boldsymbol{r})$$

Gauss Law → charge conservation

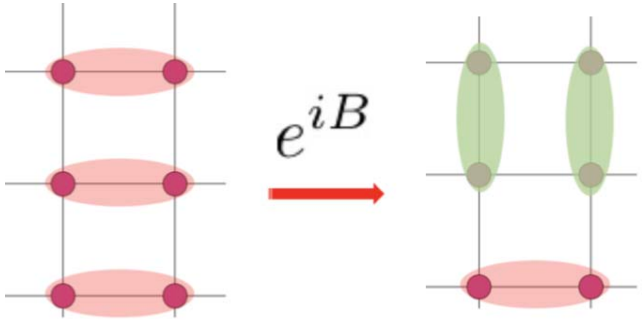
$$\partial_i E_i(\boldsymbol{r}) = (-1)^{i_{\boldsymbol{r}}} (1 - q(\boldsymbol{r}))$$

No dimer adjacent to site → spinon
1 dimer adjacent to site → no spinon

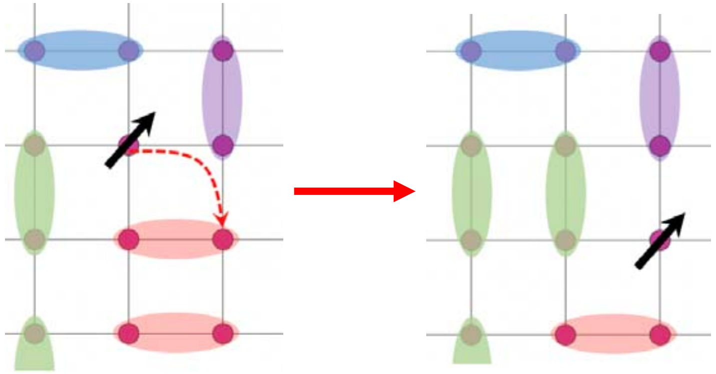


$$[A_i(\boldsymbol{x}), E_j(\boldsymbol{y})] = \frac{i}{2\pi} \delta_{ij} \delta_{\boldsymbol{xy}}$$

Create/annihilate VB



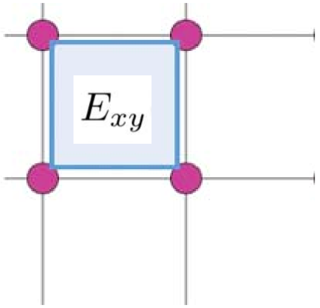
VB fluctuation
= ***B*** field



$$= z_a^\dagger e^{\int_a^b A_i dx_i} z_b$$

Mapping

VP ←→ Higher-rank gauge theory *(Xu-2004)*



VP coverage
= $\boldsymbol{E_{xy}}$ field

$$E_{xy}(\boldsymbol{r}) = (-1)^{i_{\boldsymbol{r}}} P(\boldsymbol{r})$$

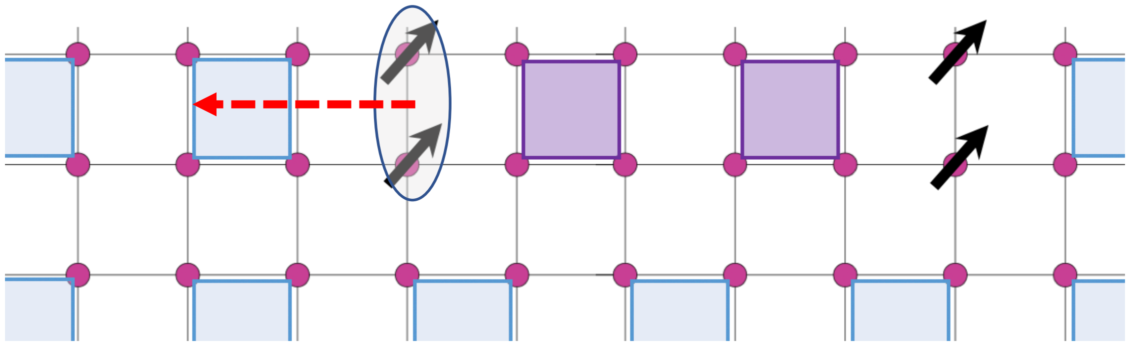
`Gauss Law '→ Charge Conservation

$$\partial_x \partial_y E_{xy}(\boldsymbol{r}) = (-1)^{i_{\boldsymbol{r}}} (1 - q(\boldsymbol{r}))$$

Charge conservation on each row !

$$\int dx_i \ \rho = 0$$

Defect configurations

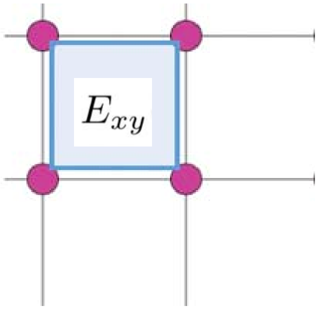


Spinon inside VPS defect appear in quartet

Conserved in each row/column!

Mapping

VP ←→ Higher-rank U(1) gauge theory



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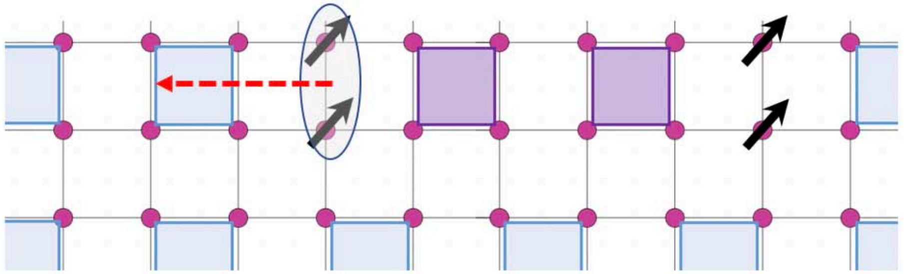
Charge conservation on each row !

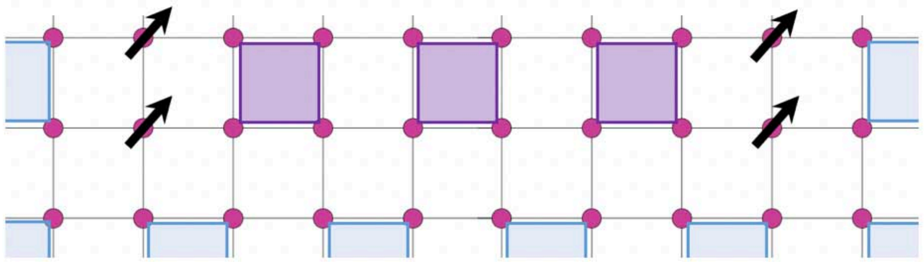
$$\int dx_i \;\; \rho = 0$$

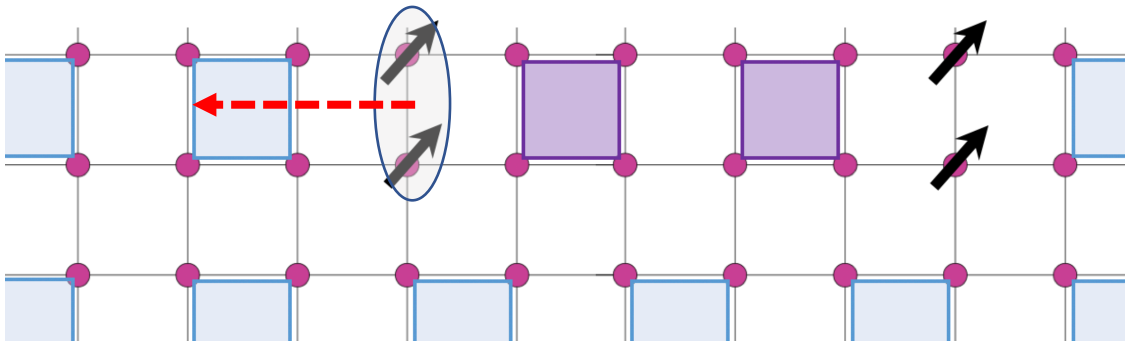
$$A_{xy} \to A_{xy} + \partial_x \partial_y \alpha,$$

Create/annihilate VB



$$z_r^\dagger z_{r+e_x} e^{i \int dy A_{xy}} z_{r+y} z_{r+e_x+y}^\dagger$$





$$\boldsymbol{S} = \frac{1}{2} z^\dagger \boldsymbol{\sigma} z$$

$$z_a^\dagger \sim e^{i\theta_a}$$

$$\mathcal{H} = u \sum_{\boldsymbol{r}} \Big(\sum_{a=1,2} \hat{n}_a - 1 \Big)^2 + t \sum_{a=1,2} \cos(\partial_x \partial_y \theta_a + A_{xy})$$

A pair of spinon (as a dipole) can hop along the transverse stripe!

SWA: Guassian theory

$$\mathcal{L} = \frac{K}{2} \sum_{a=1,2} (\partial_t \theta_a)^2 - \frac{K}{2} \sum_{a=1,2} (\partial_x \partial_y \theta_a + A_{xy})^2$$

$$K = \sqrt{t/u}$$

→ Luttinger Parameter

Large K:

SWA is valid (instanton irrelevant)

Large K, liquid phase *(`exciton liquid' Paramekanti-Balents-Fisher; `algebra bond liquid' Xu-Fisher)*

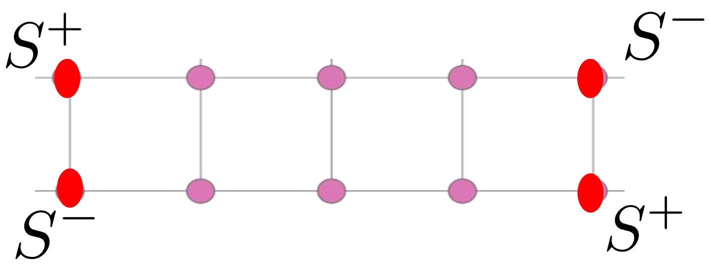
$$\mathcal{L} = \frac{K}{2}\sum_{a=1,2}(\partial_t\theta_a)^2 - \frac{K}{2}\sum_{a=1,2}(\partial_x\partial_y\theta_a + A_{xy})^2$$

✓ Dipole: $\partial_i\theta$ fluctuate in **1d**
✓ Dipole condense?
→ no long-range order due to Mermin–Wagner theorem

→ Quasi-long-range order between dipoles!

$$\langle S^+(r)S^-(r+e_y)S^+(r+x)S^-(r+e_y+x)\rangle$$
$$= \frac{1}{(x)^{1/(K\pi^2)}}$$

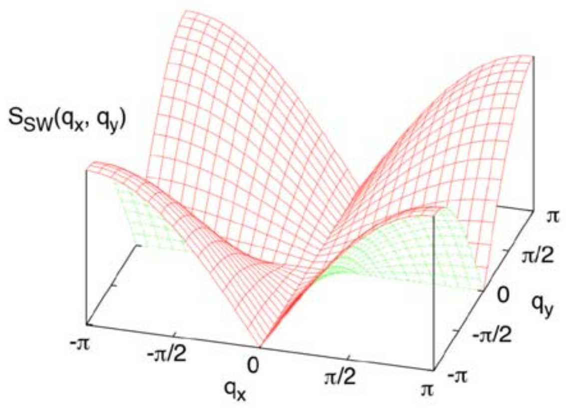
Power-law correlation



Algebra quantum liquid phase:
✓ Gapless, but no symmetry breaking or LRO
✓ power-law correlation between dipoles on the same stripe
✓ Stable phase for large K (small K, defect proliferate → stripe/ plaquette order)

Signature of algebra quantum liquid phase *(Paramekanti-2002, Xu-2006)*

Structure factor



$$S^{zz}(k) \sim |\sin(k_x)\sin(k_y)|$$

✓ zero-energy branch line on k_x ,k_y axis

✓ `Bose Fermi surface'?

Specific heat

$$C_v \sim T\ln(1/T)$$

✓ Compared to T^3

✓ Akin to marginal non-Fermi liquid, due to gapless banch line

Disorder?

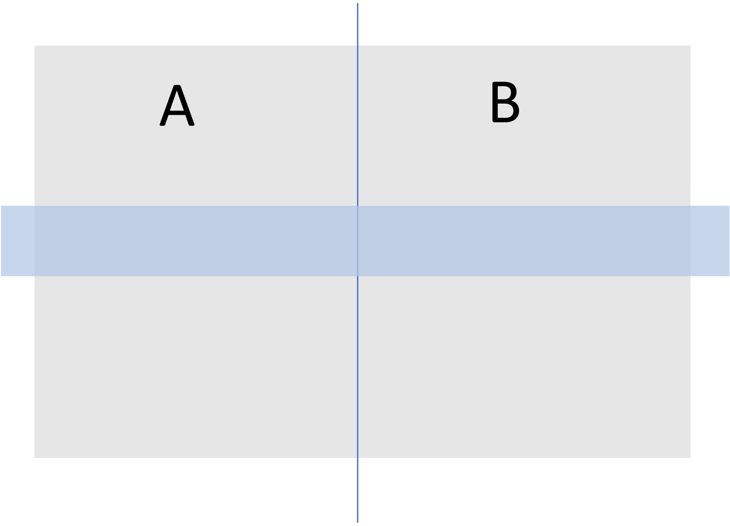
✓ Fragile to disorder quasi-1d behavior?

Entanglement entropy

$$L\ln(L)$$ *(Bonesteel-2013)*

✓ Similar to 2d FS

✓ violation of the area law



Outlook

✓ *Melting Plaquette order on square lattice*

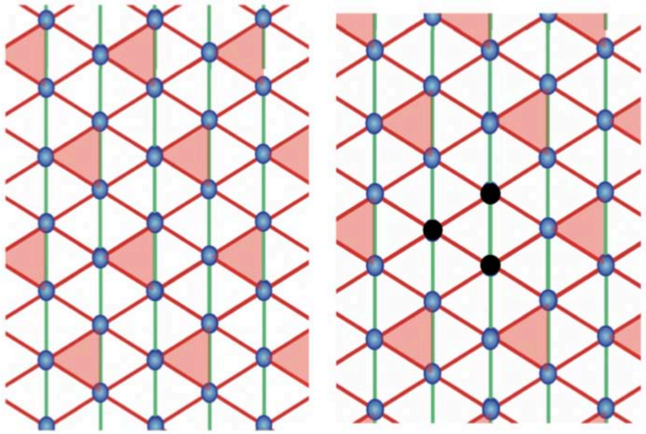
Defect carries spinon, but immobile, cannot condense
Dipole is 1d sub-dimensional→ condensation leads to `Algebra liquid phase', no LRO

✓ *What about 3D plaquette order?*

Map to a compact tensor gauge theory, pure gauge theory is confined.
DQCP ?

✓ *SU(3) plaquette order on triangle lattice?*

Defect carries a spinon →conserved on a fractal



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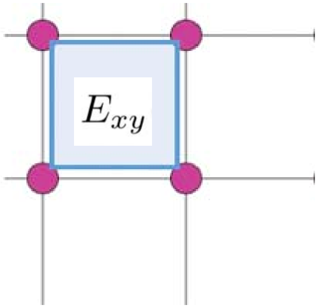

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$$\int dx_i \;\; \rho = 0$$

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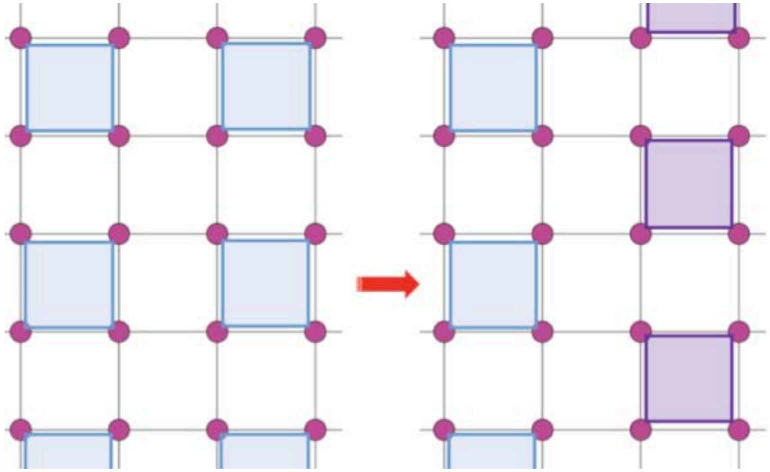
Create/annihilate VB

$$\phi^x(y_i) = \int dx (-1)^x A_{xy}(y_i),$$
$$\phi^y(x_i) = \int dy (-1)^y A_{xy}(x_i)$$

No local B flux!
Only global flux!



VP pattern can't change locally! Can only shift globally!