

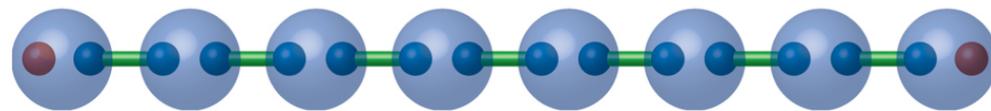
Majorana in Iron Superconductors & Pairing of Spin-Helical Electrons

Liang Fu



Majorana Fermion

Majorana
zero mode:



Kitaev (2000)

Two spatially separated Majoranas γ_1, γ_{2N} constitute a fermionic qubit

At **T=0**, zero-energy Majorana qubits are protected by spatial separation and the gap

“The condition is satisfied ... by proximity of a 3-dimensional **p-wave superconductor**”

“Physical realization ... is a difficult task because electron spectra are usually degenerate with respect to **spin**...

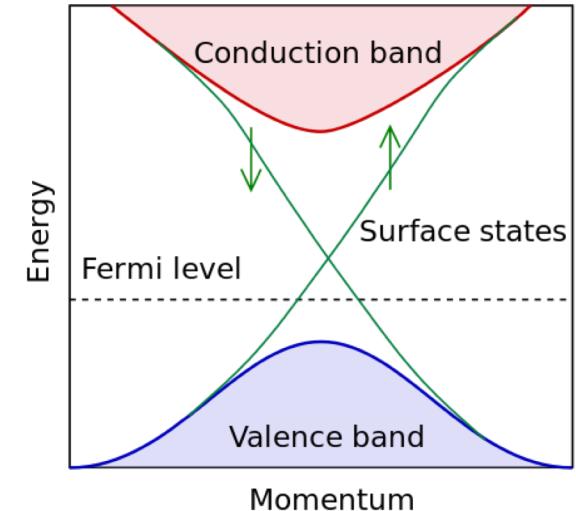
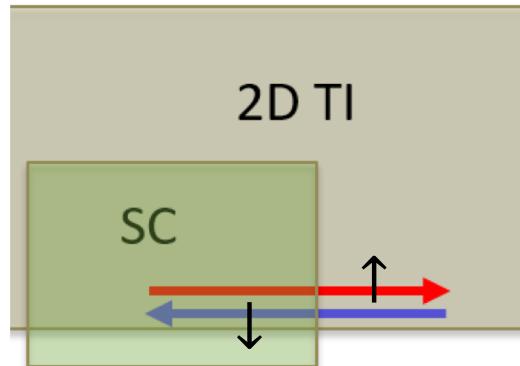
Two difficulties: absence of spin degeneracy, presence of p-wave SC

Creating Majorana from Spin-Helical Electrons

s-wave SC + spin-helical (instead of spin-polarized) electrons

Surface of TI:

- spin degeneracy removed
- opposite spins at k and $-k$; T-symmetry maintained



Under π rotation $x \rightarrow -x$

- spinless: $c_k^+ c_{-k}^+ \rightarrow -c_k^+ c_{-k}^+$ p-wave
- spin-helical: $|\uparrow\rangle \rightarrow |\downarrow\rangle, |\downarrow\rangle \rightarrow -|\uparrow\rangle$ s-wave!
 $\Rightarrow c_{k\uparrow}^+ c_{-k\downarrow}^+ \rightarrow c_{k\uparrow}^+ c_{-k\downarrow}^+$

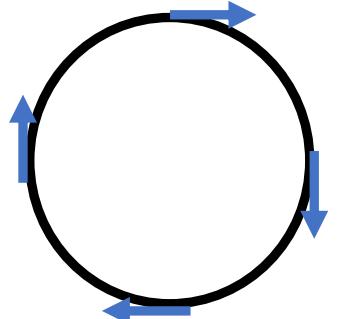
(2π rotation on spin-1/2 gives -1)

LF & Kane (2008)

Superconductivity in Topological Insulator

$$H_0 = \psi^\dagger (-iv\vec{\sigma} \cdot \nabla - \mu)\psi. \quad \Delta\psi_\uparrow^\dagger\psi_\downarrow^\dagger + h.c.$$

p-wave dispersion + s-wave pairing

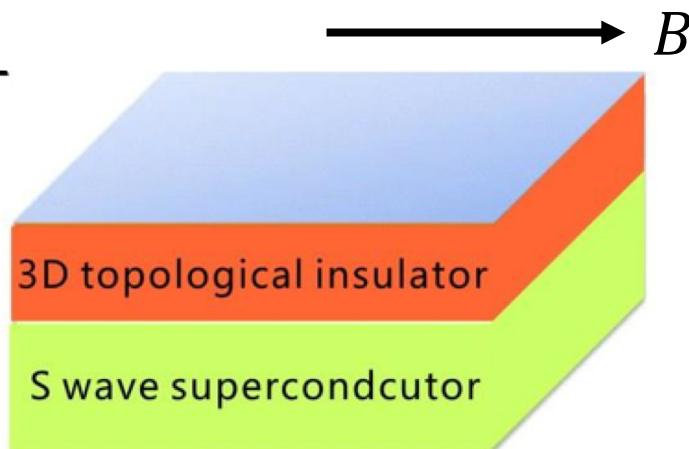


$$c_{\mathbf{k}} = (\psi_{\uparrow\mathbf{k}} + e^{i\theta_{\mathbf{k}}}\psi_{\downarrow\mathbf{k}})/\sqrt{2}$$

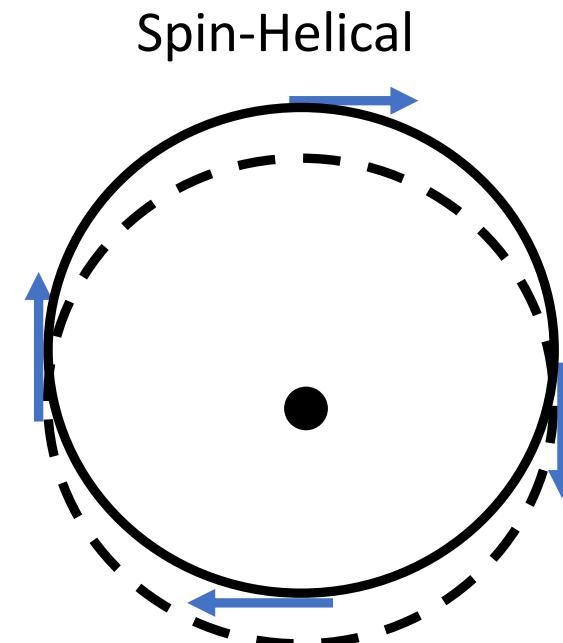
$$H_{\text{eff}} = \sum_{\mathbf{k}} v c_{\mathbf{k}}^+ (|\mathbf{k}| - k_F) c_{\mathbf{k}} + \Delta (e^{i\theta_{\mathbf{k}}} c_{\mathbf{k}}^+ c_{-\mathbf{k}}^+ + h.c)$$

resembles s-wave dispersion + p-wave pairing

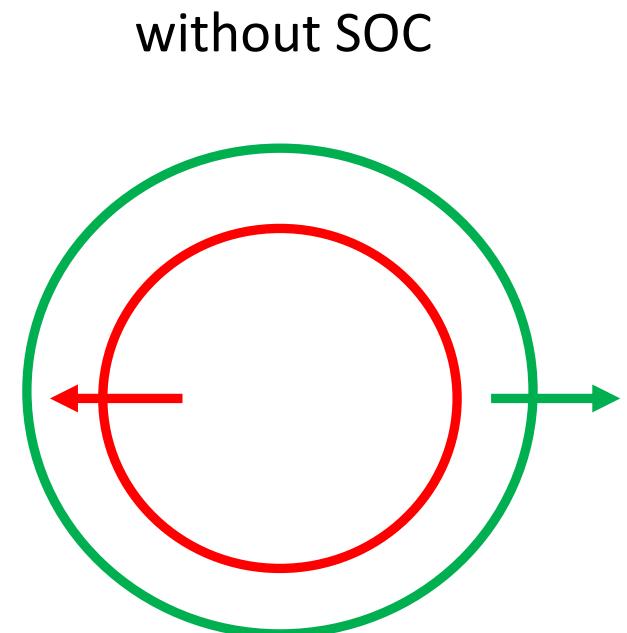
Proximitized Topological Insulator under In-Plane B Field



Noah Yuan & LF, PRB (2018)



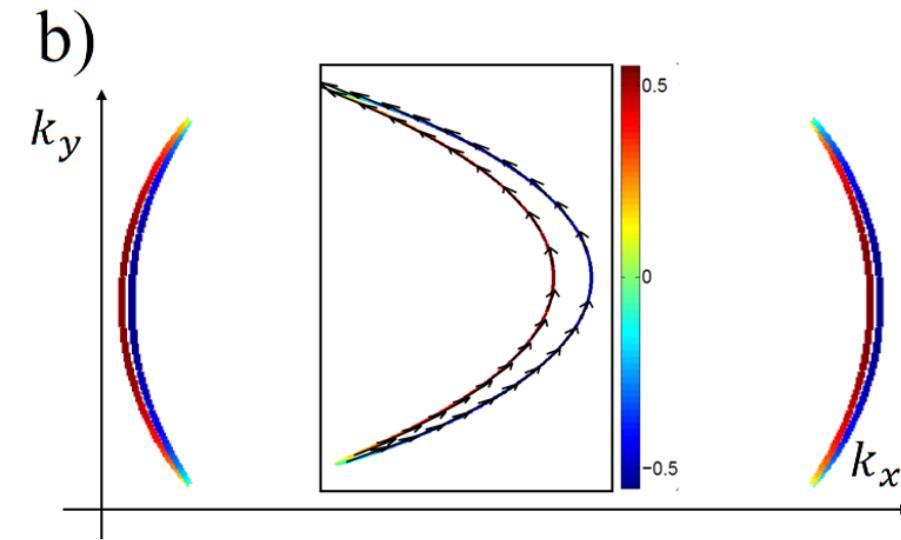
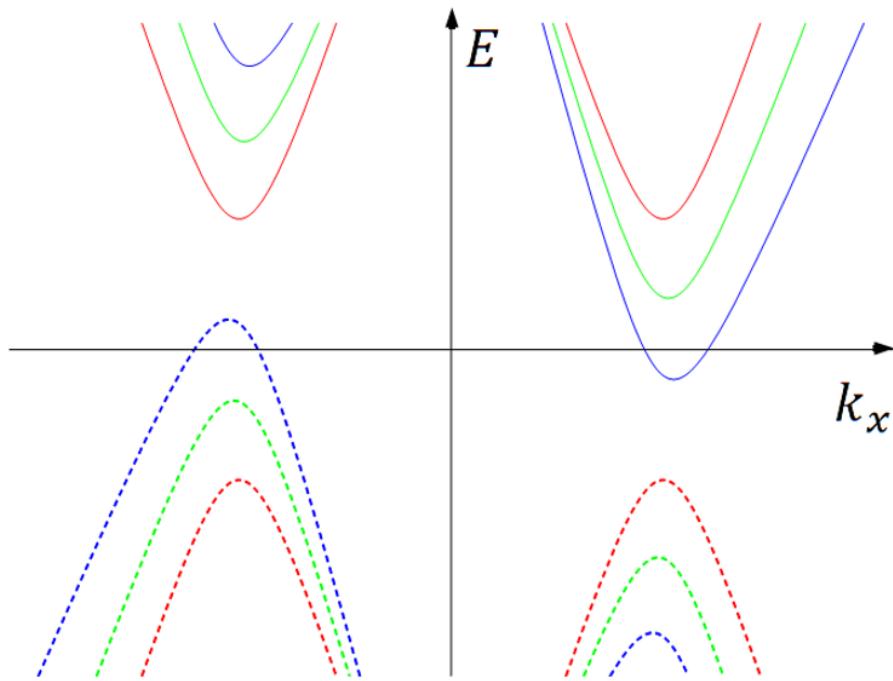
Fermi surface shift in k space



Spin splitting

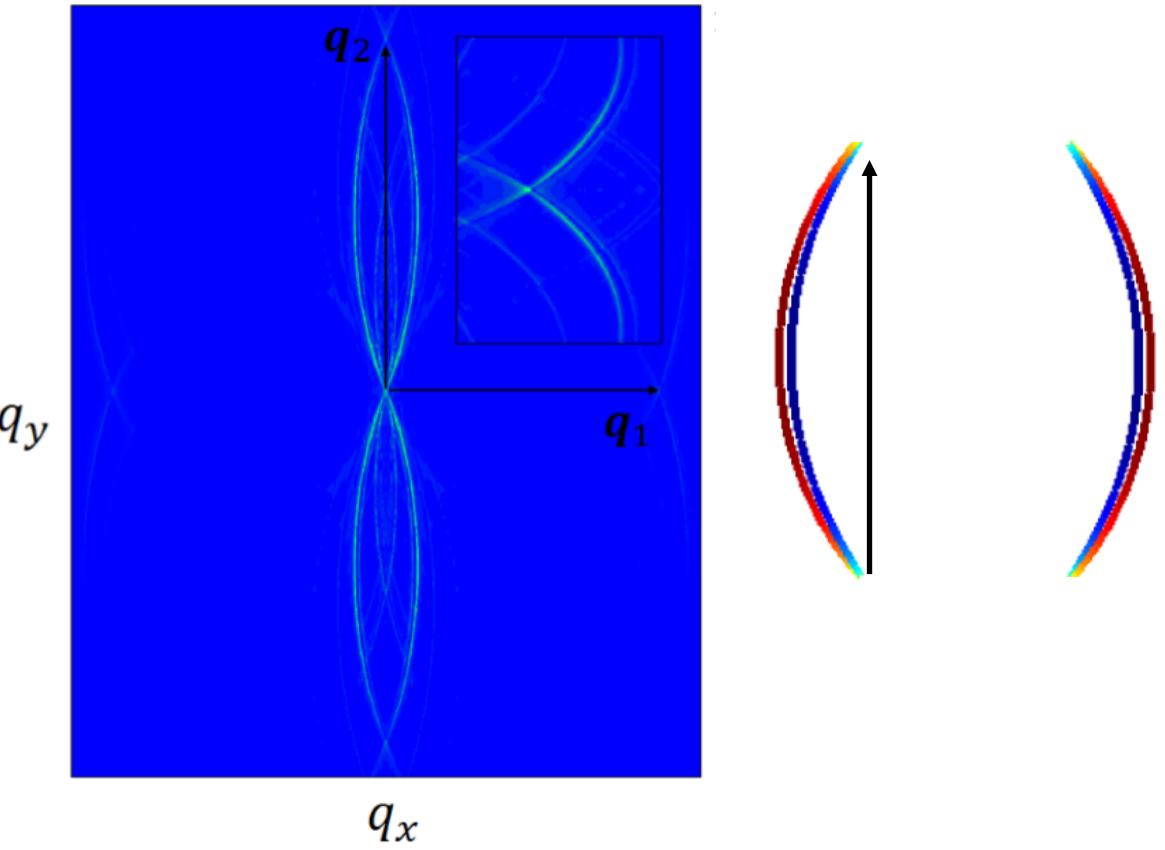
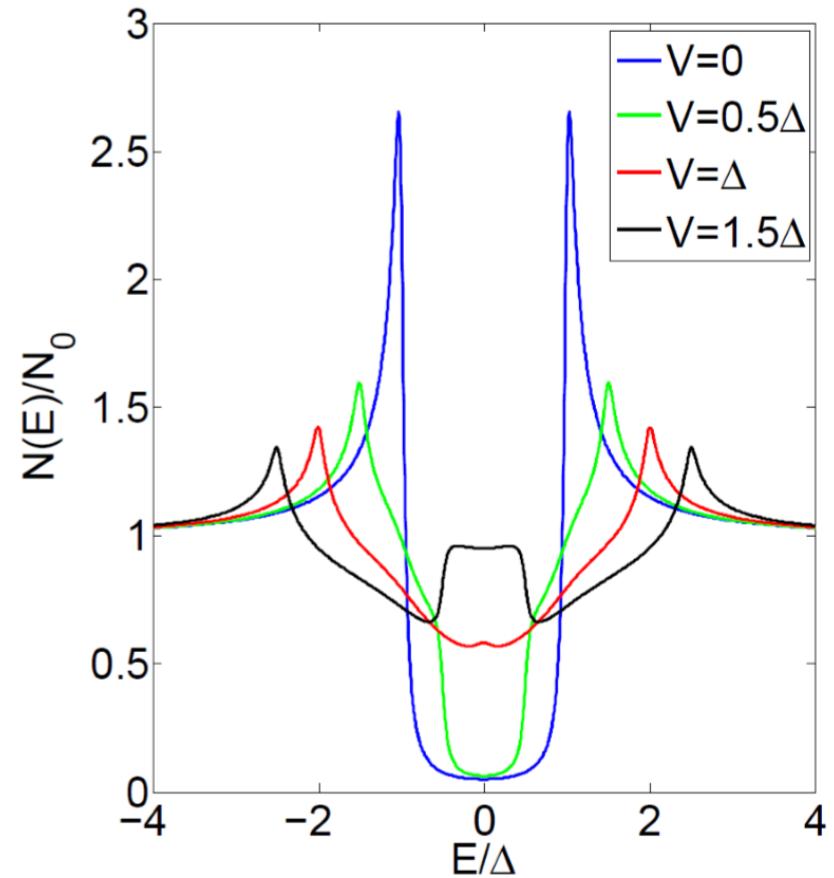
- In-plane Zeeman field couples to spin-helical electrons as vector potential
- At finite $B < B_p$, the external conventional SC maintains $Q=0$ pairing

Partial Fermi Surface in Superconductor: FFLO Gap Structure at Q=0



Depairing of helical electrons is strongly direction dependent.
At $B > B_0$, Fermi surface is partially gapped and partially gapless.
Gap structure is identical to FFLO SC at finite Q and ordinary SC with finite supercurrent

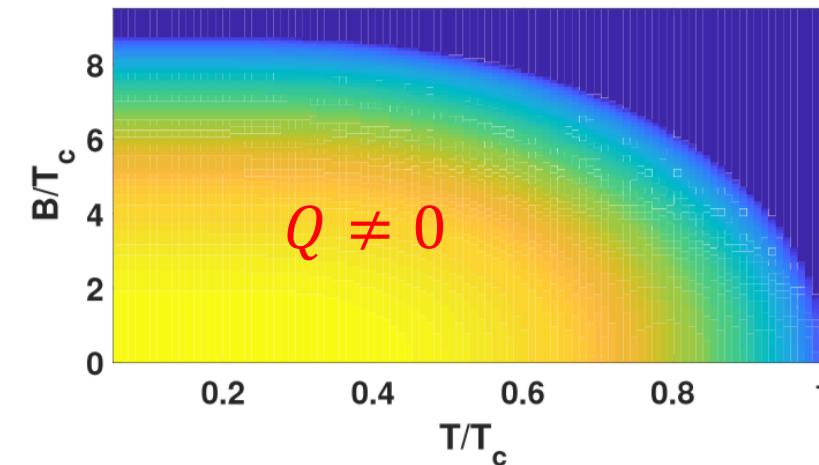
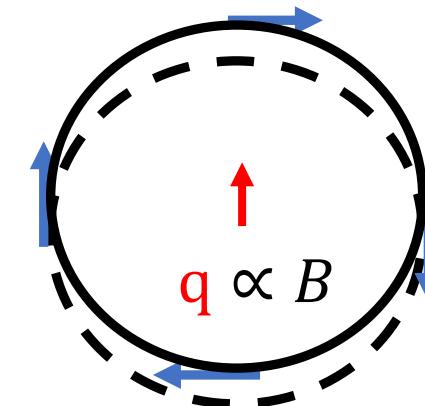
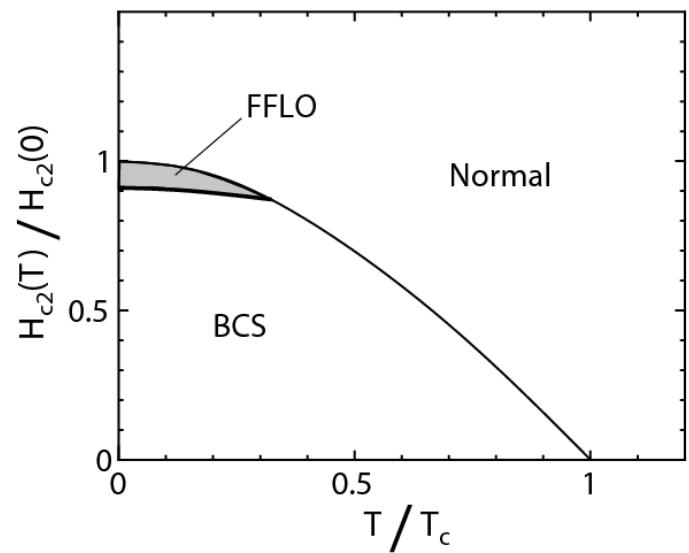
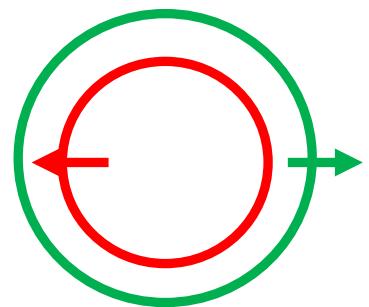
Quasiparticle DOS and Interference



Coherence peak moves to higher energy under B !
In-gap states appear abruptly at $B > B_0$ (Lifshitz transition)

Noah Yuan & LF, PRB (2018) + in progress

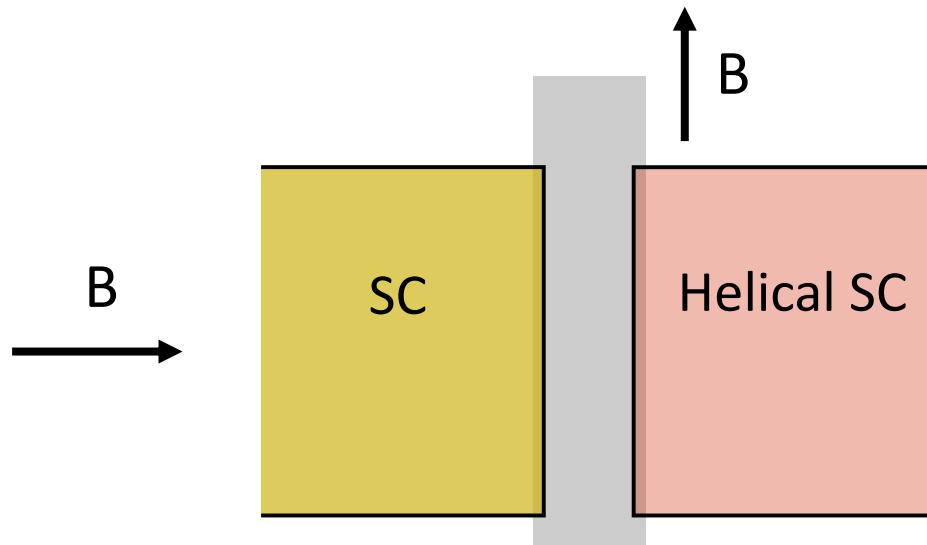
Intrinsic Superconductivity of Helical Electrons under In-Plane B



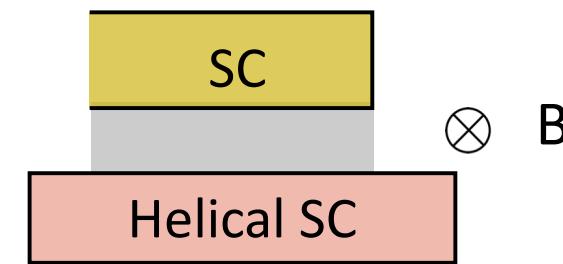
Finite momentum pairing **immediately** appears



Detecting Finite-Momentum Pairing

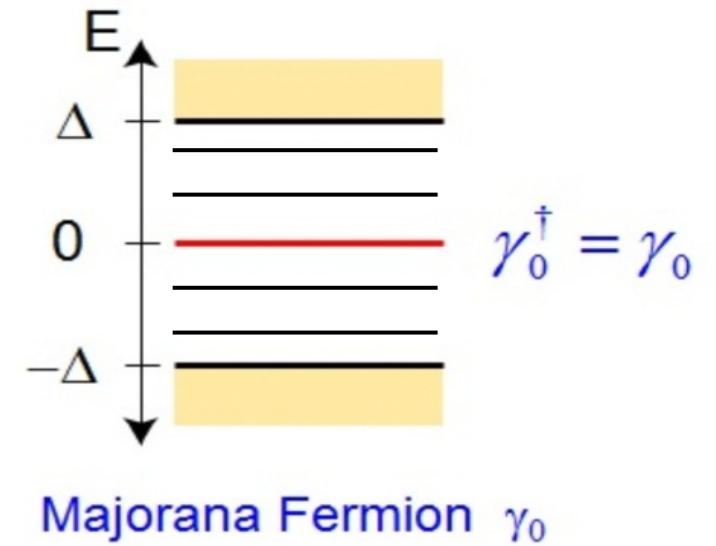
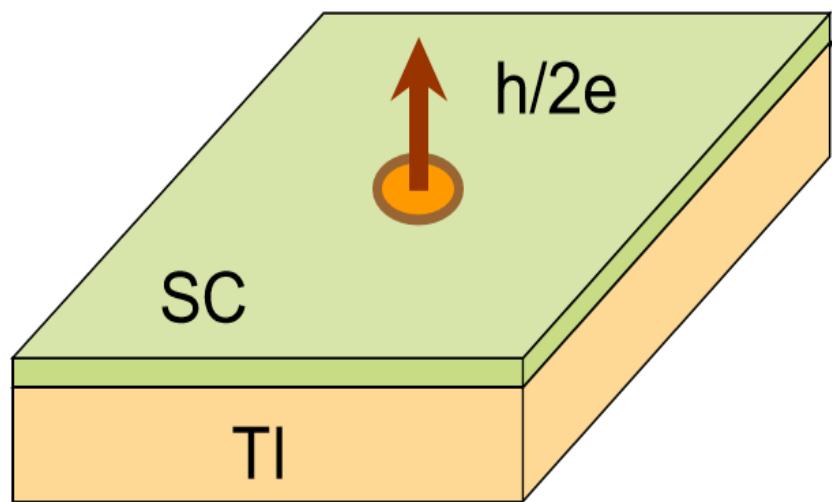


Cooper momentum and Josephson current depends on B and its orientation



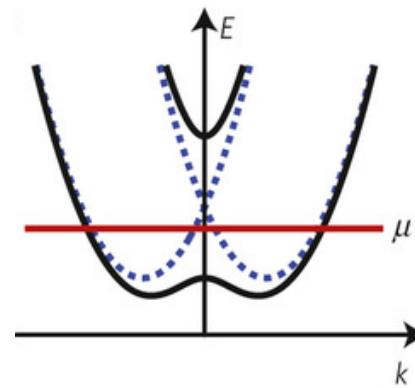
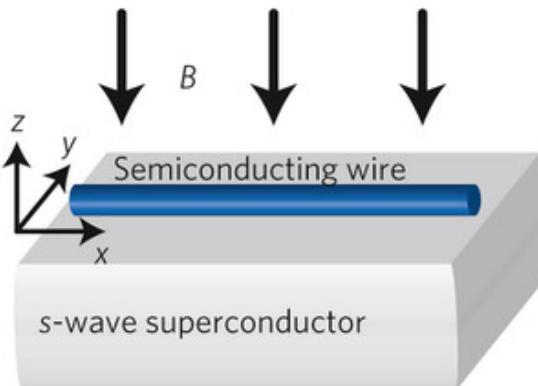
Fraunhofer pattern has an anomalous period

Majorana in Superconducting Topological Insulator



Minigap Δ^2/E_F for $E_F \gg \Delta$

Majorana Wire



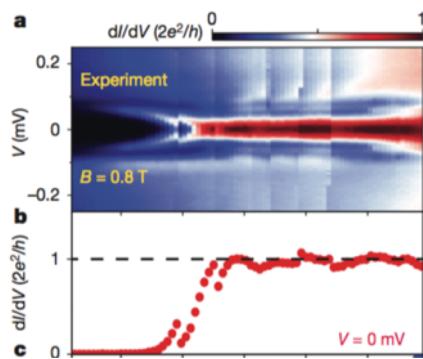
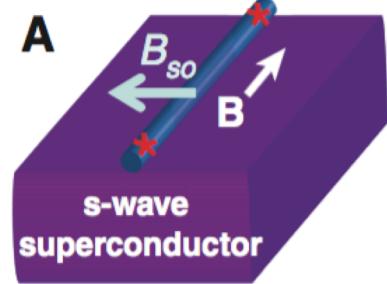
Lutchyn, Sau & Das Sarma;
Oreg, von Oppen & Refael;
Potter & Lee ... (since 2010)

Spin-orbit-coupled nanowire under a magnetic field hosts spin-helical electrons when Fermi level is tuned into the Zeeman gap.

Platform for Majorana modes

- #1: Intrinsic chiral p -wave (Order parameter) 2D $p+ip$ TSC or 1D-Kitaev chain
- #2: Proximity effect (Single Dirac Fermi surface + full SC gap)

Nanowire / SC

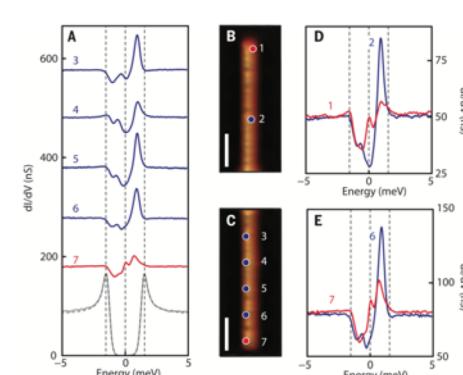
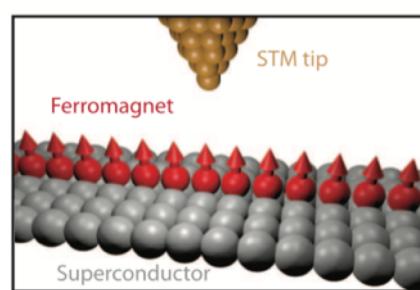


Delft, CPH and others

Common properties

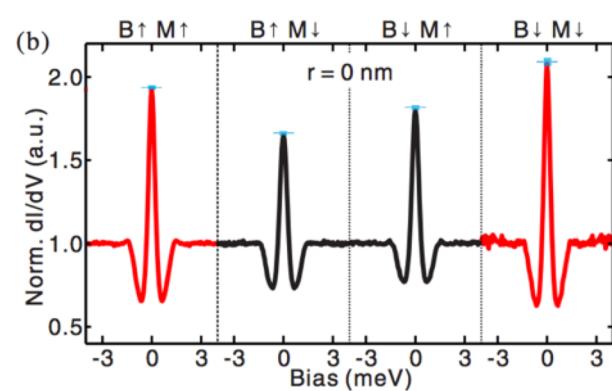
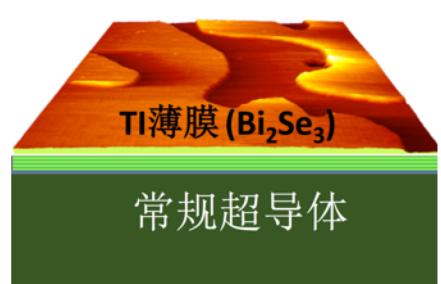
1. Heterostructure

Atomic Chain / SC



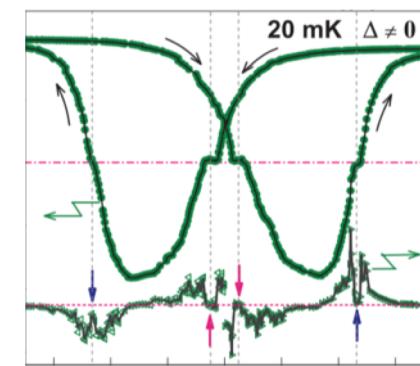
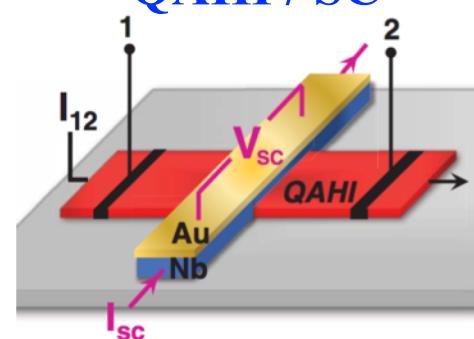
Princeton and others

TI / SC



Shanghai Jiao-Tong

QAH / SC



UCLA and others

2. Ultra-low temperature

3. Small topological gap

Majorana in Iron Superconductors

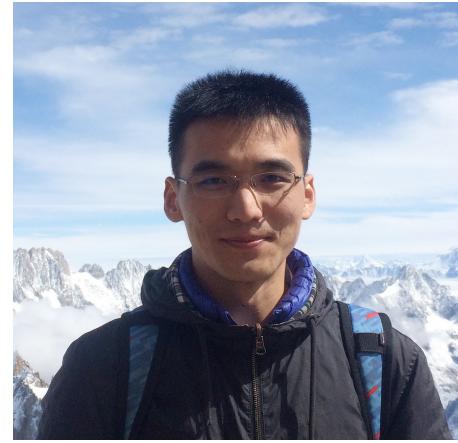
Institute of Physics, Beijing



Hong Ding



Hong-Jun Gao



Lingyuan Kong

Brookhaven



Genda Gu

MIT

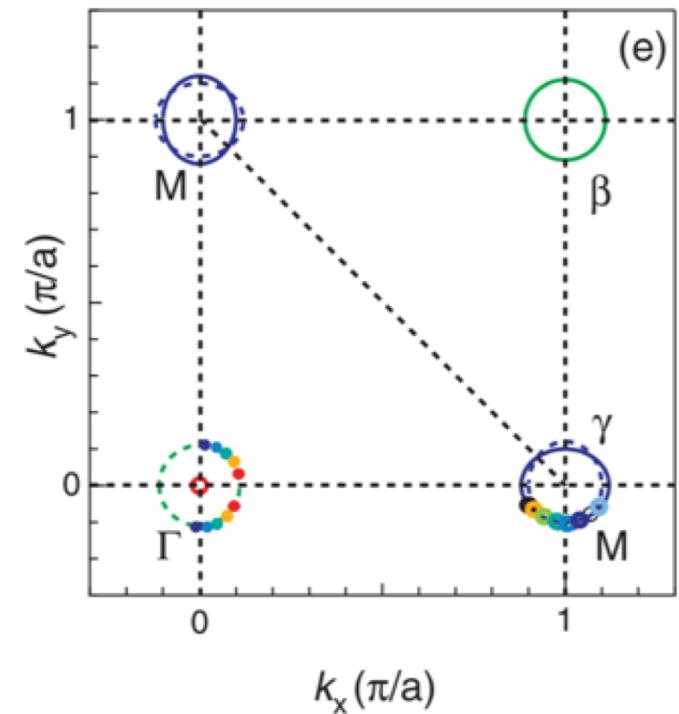
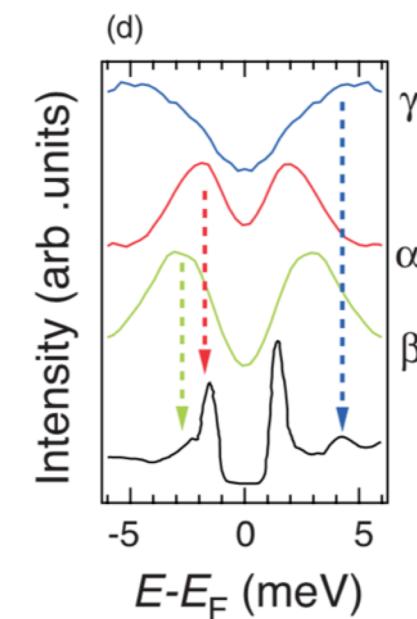
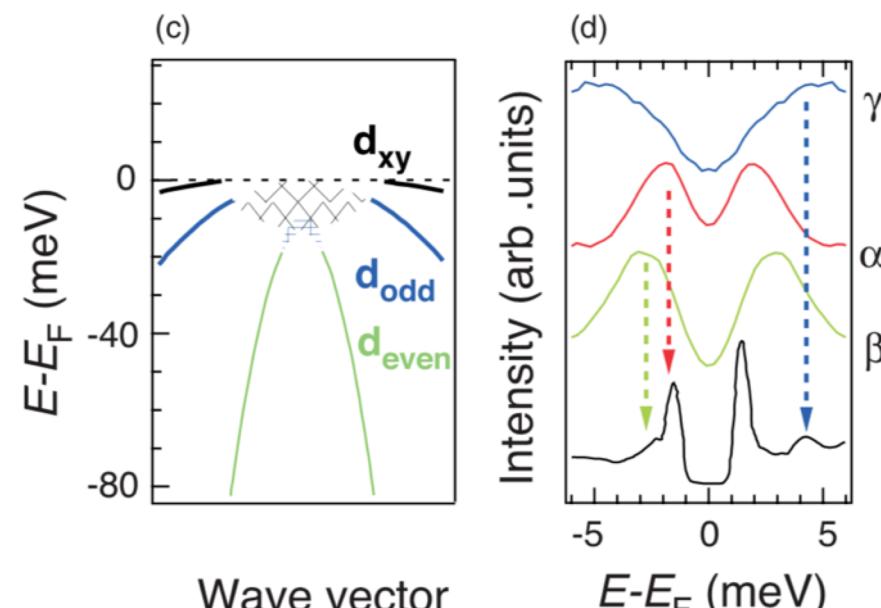
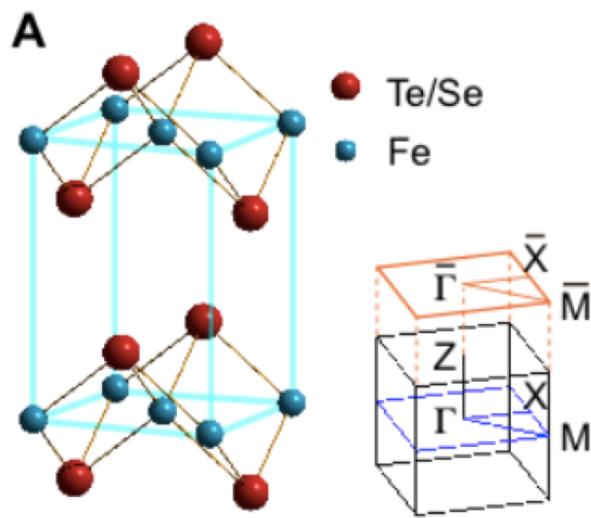


Michal Papaj

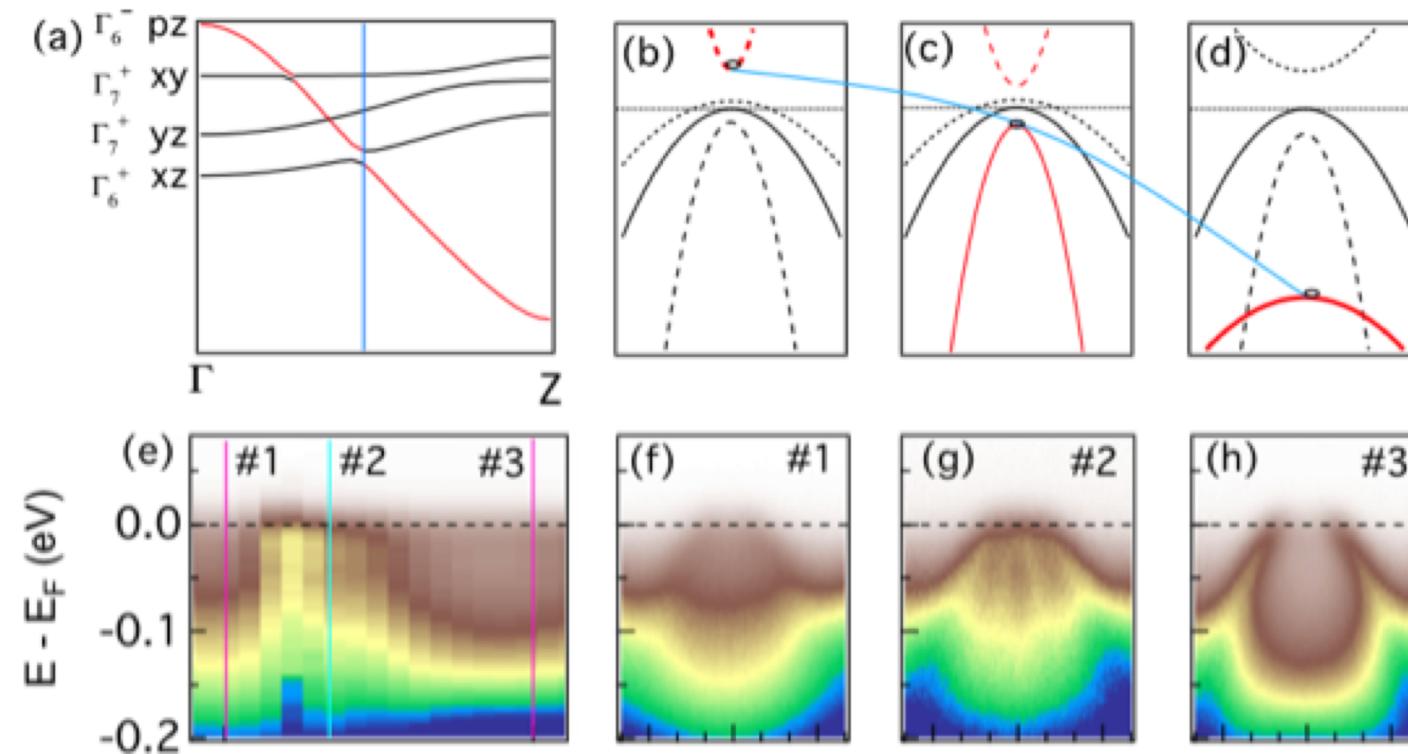
+ Peng Fan, Dongfei Wang, Shiyu Zhu, Hui Chen, Lu Cao,
Wenya Liu, Yujie Sun, Yuyang Zhang
Yuqing Xing, Fazhi Yang.

+ John Schneeloch
Ruidan Zhong

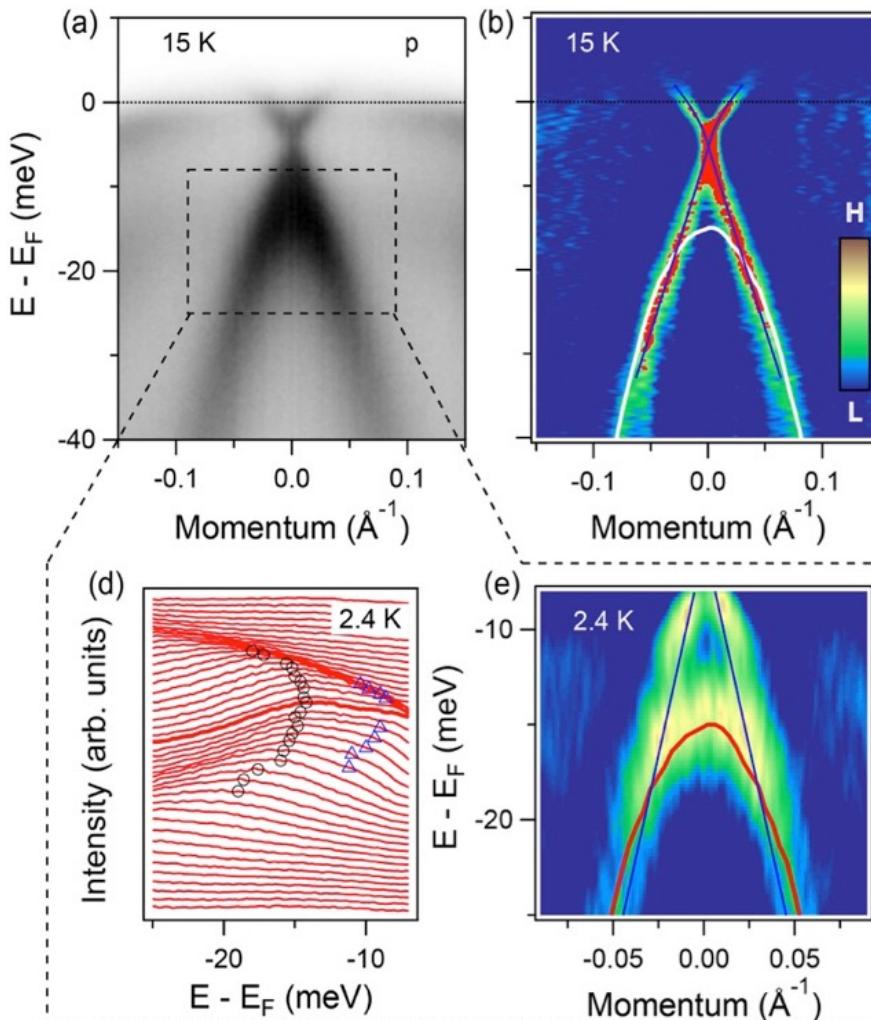
Fe(Te, Se): small E_F with large correlations



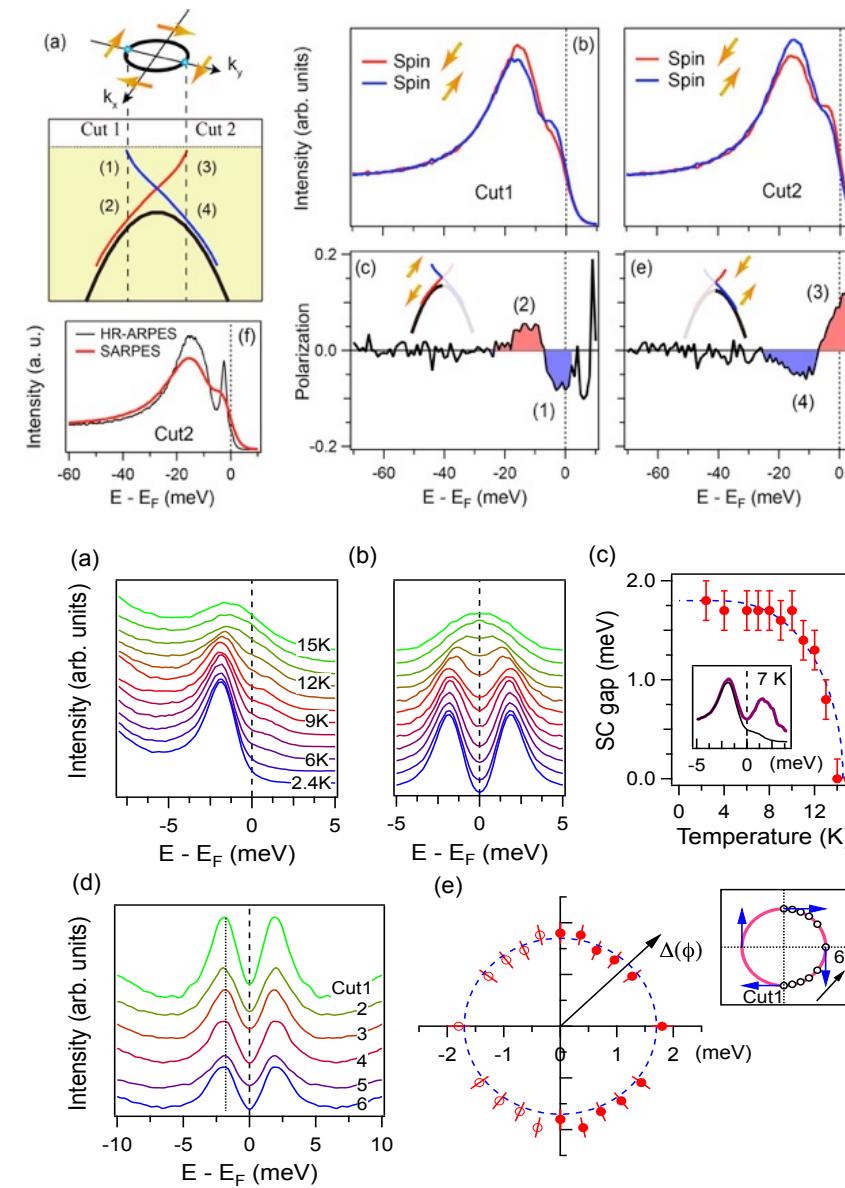
Band Inversion by Te/Se Composition



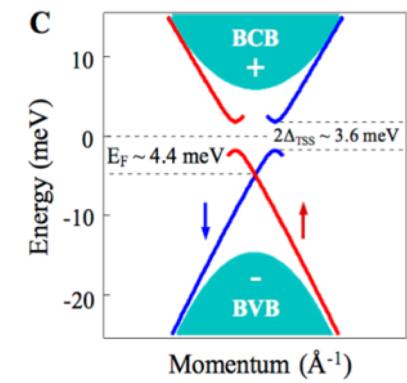
Topological Surface States on FeTe_{0.55}Se_{0.45}



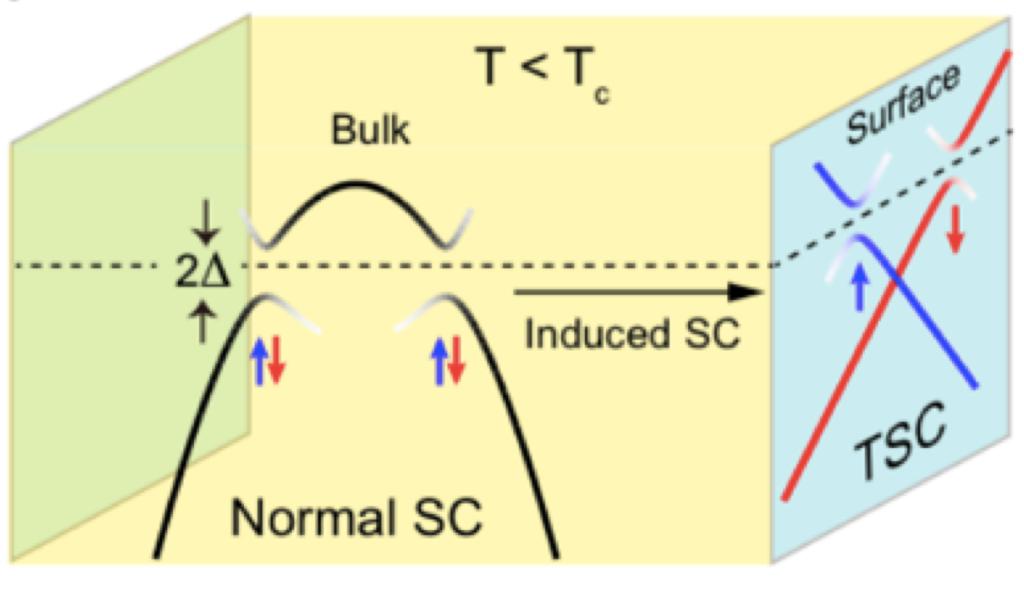
P. Zhang et al., Science 360, 182 (2018)
(IOP + ISSP + Brookhaven)



- #1. Linear dispersion of surface states.
- #2. Spin-momentums locking.
- #3. s-wave SC gap on the surface states.
- Small Fermi energy comparable to SC gap



Fe(Te,Se): Nature's Gift



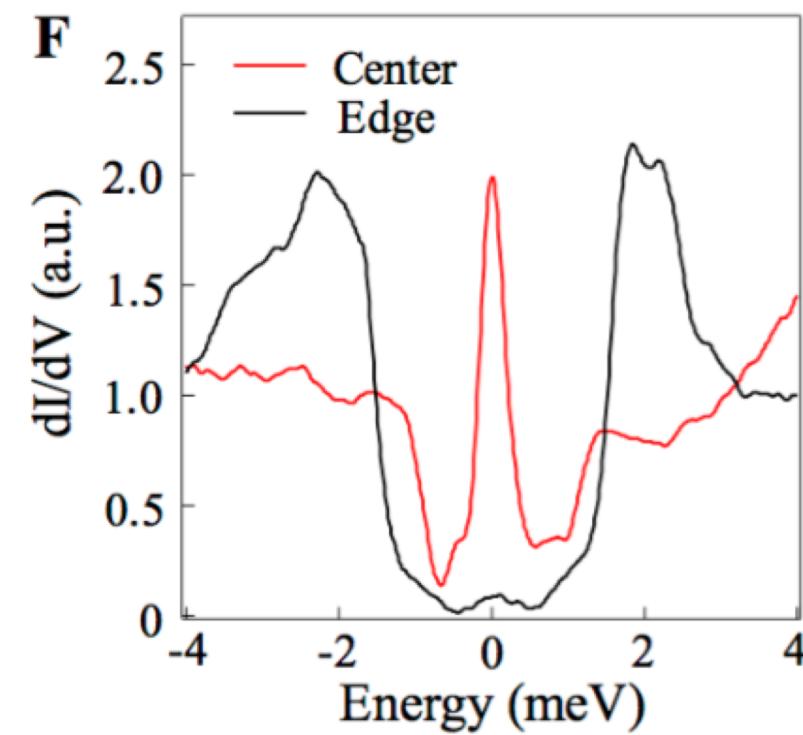
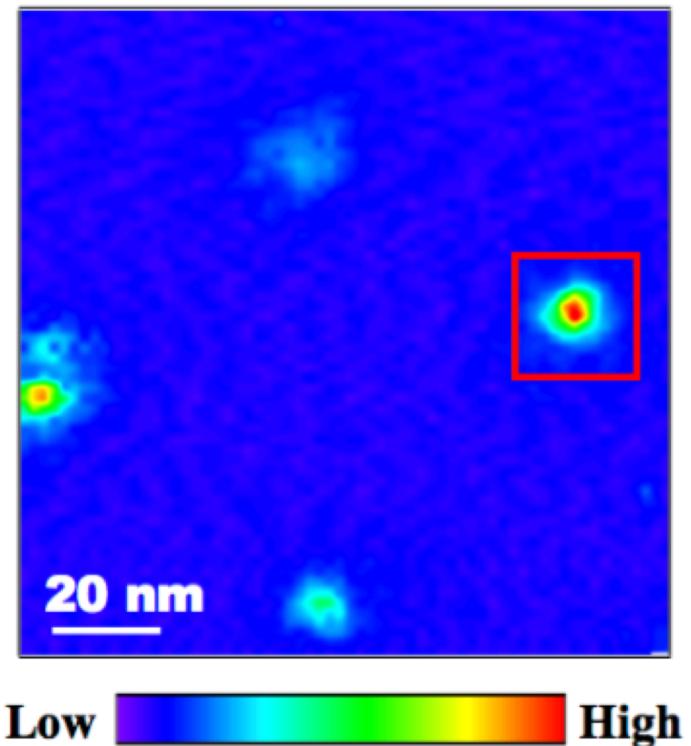
S-wave + spin-helical

High- T_c SC

Small E_F , Large gap

Single Material

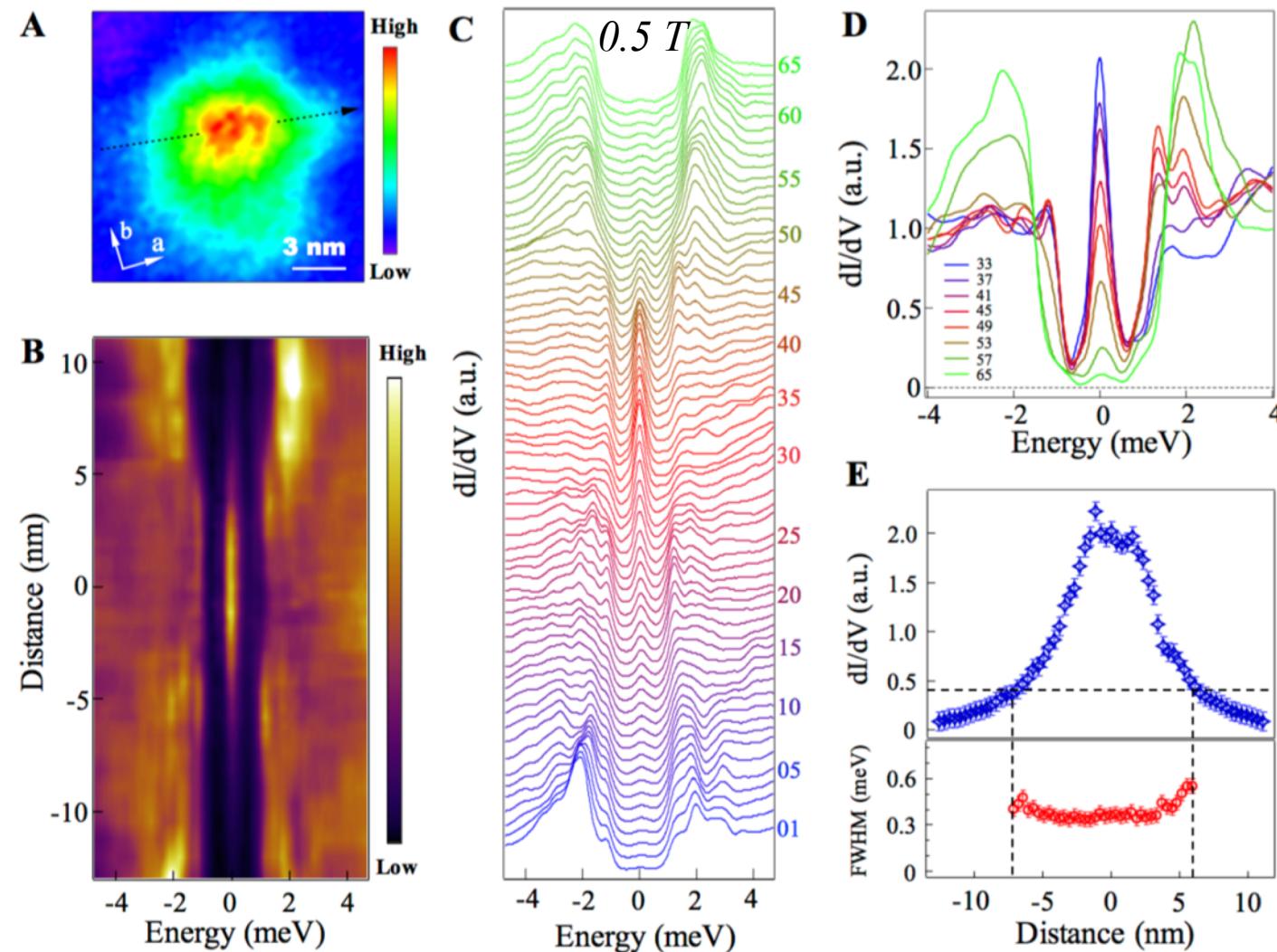
Zero-bias conductance Peak



Science 362, 333 (2018)

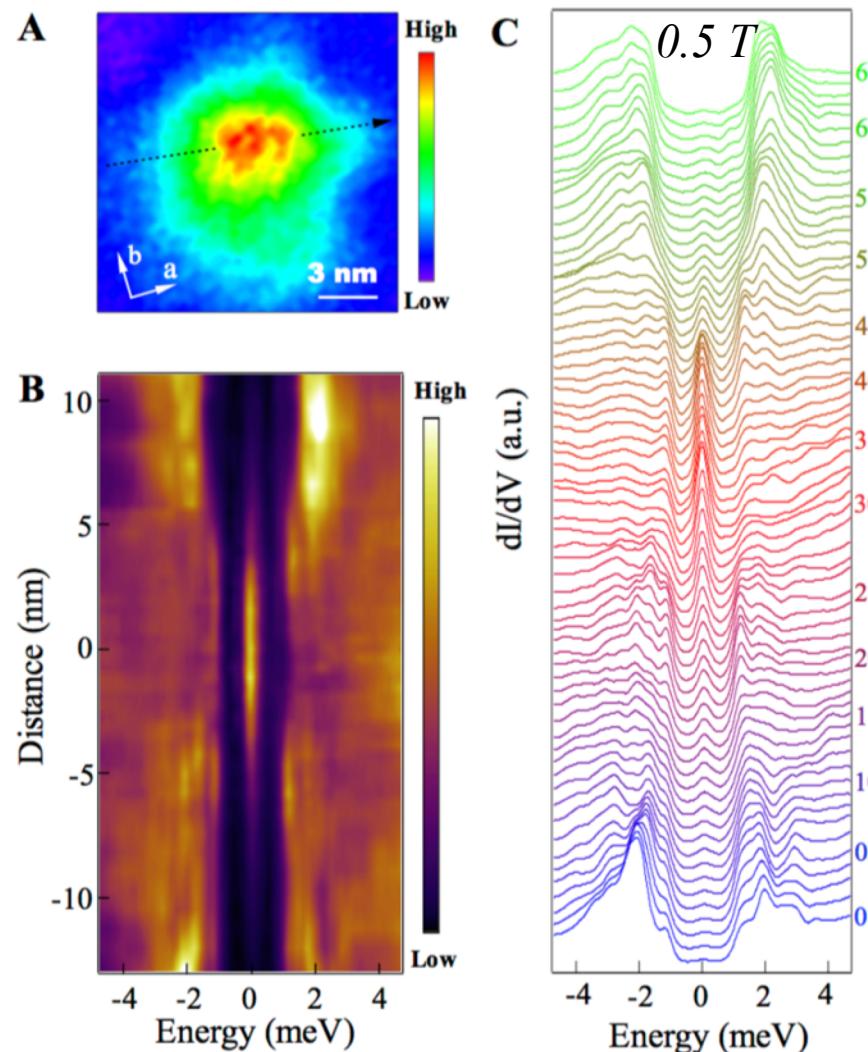
Spatial Line Profile

➤ Majorana: Non-split spatial dispersion (Pure)

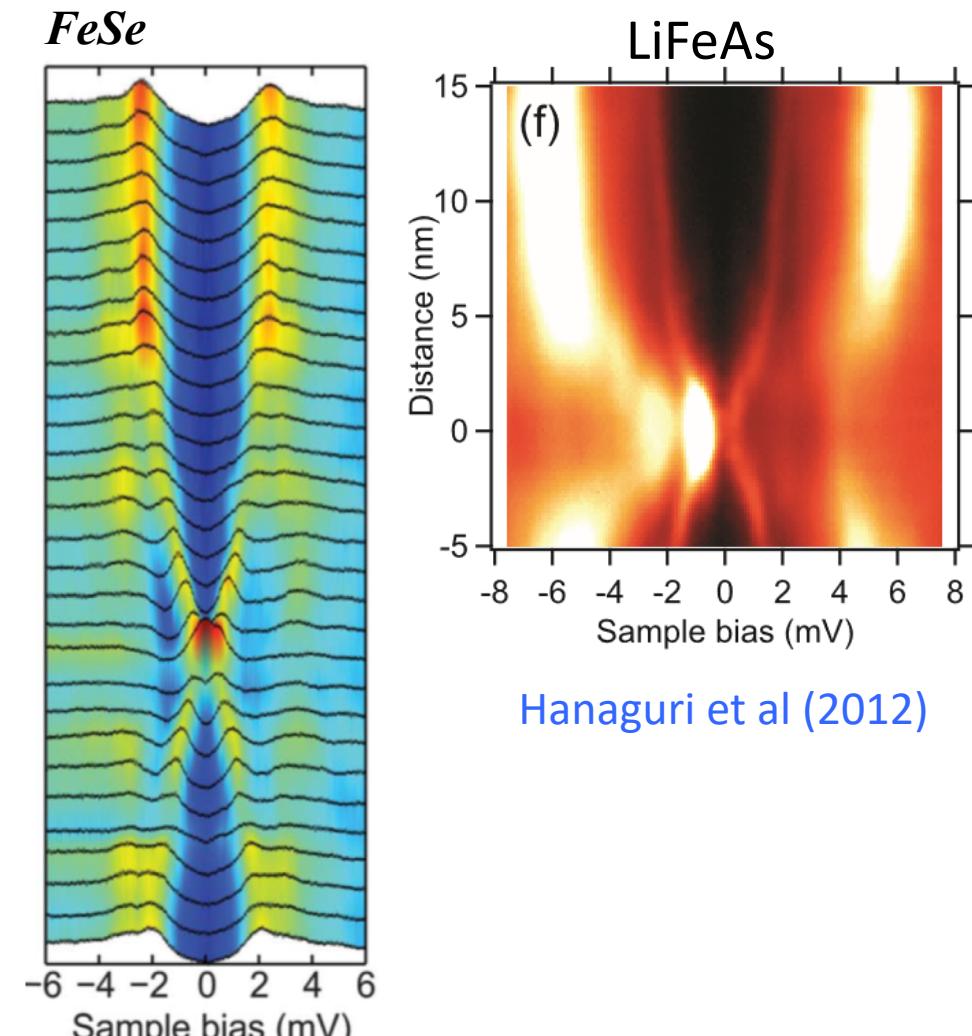


Spatial Line Profile

➤ Majorana: Non-split spatial dispersion (Pure)



➤ CdGM split



Science 332, 1410

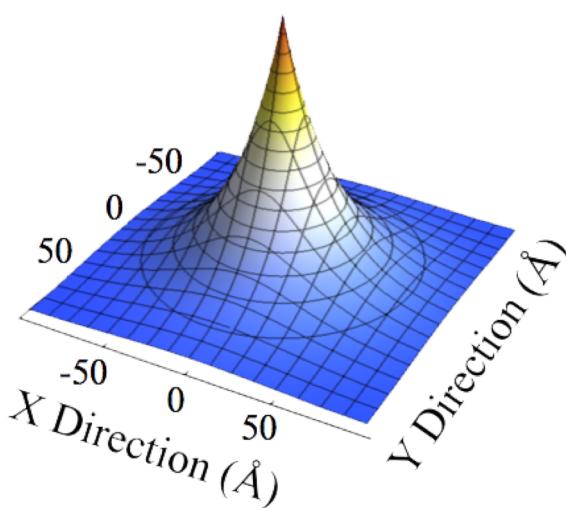
Majorana Bound State Wavefunction

Majorana Wavefunction Calculation

$$|u|^2 = |f(r)|^2 + |g(r)|^2$$

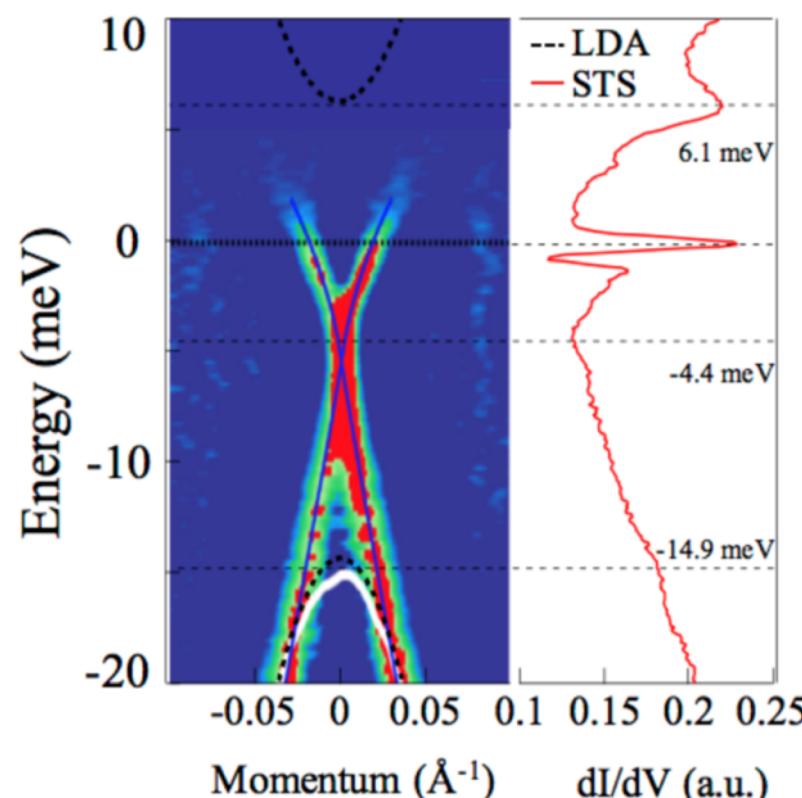
$$f(r) = J_0\left(\frac{E_F r}{v_F}\right) \exp\left[-\int^r \frac{\Delta(r')}{v_F} dr'\right] (i+1)$$

$$g(r) = J_1\left(\frac{E_F r}{v_F}\right) \exp\left[-\int^r \frac{\Delta(r')}{v_F} dr'\right] (i-1)$$

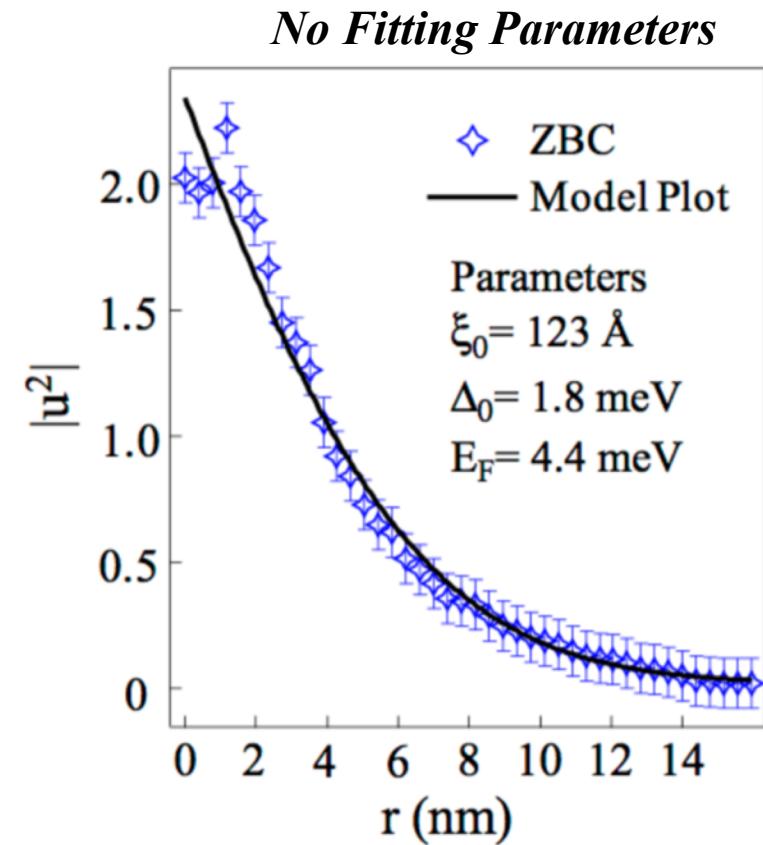


Fu & Kane PRL 100, 096407 (2008)
Wang & Fu PRL 119, 187003 (2017)
Hughes et.al., PRB (2011)

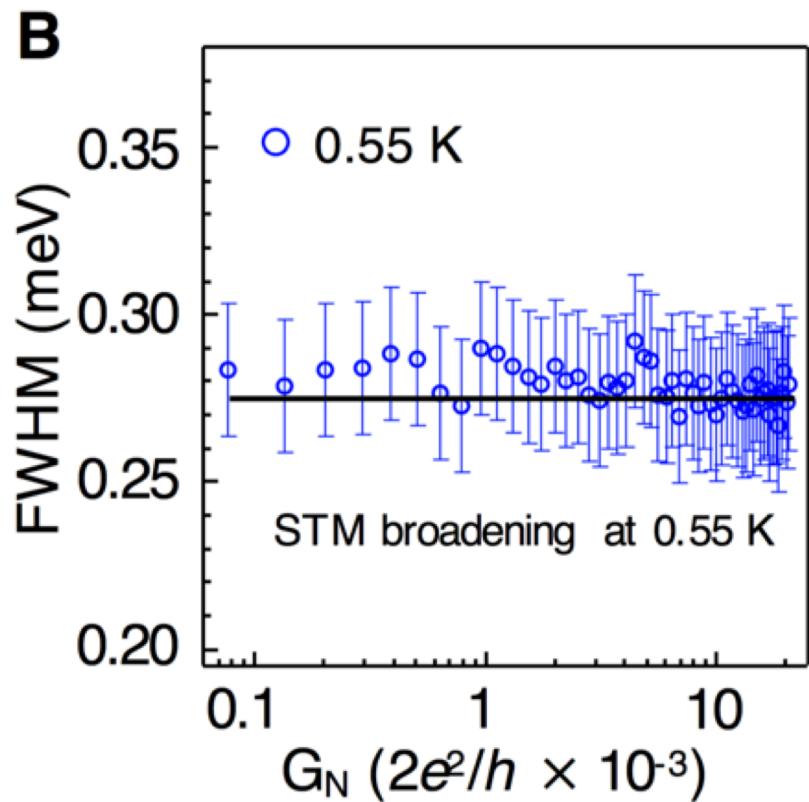
Weak tunneling regime: $\Gamma \ll k_B T$



$$\frac{dI}{dV} \propto |u|^2$$



Linewidth



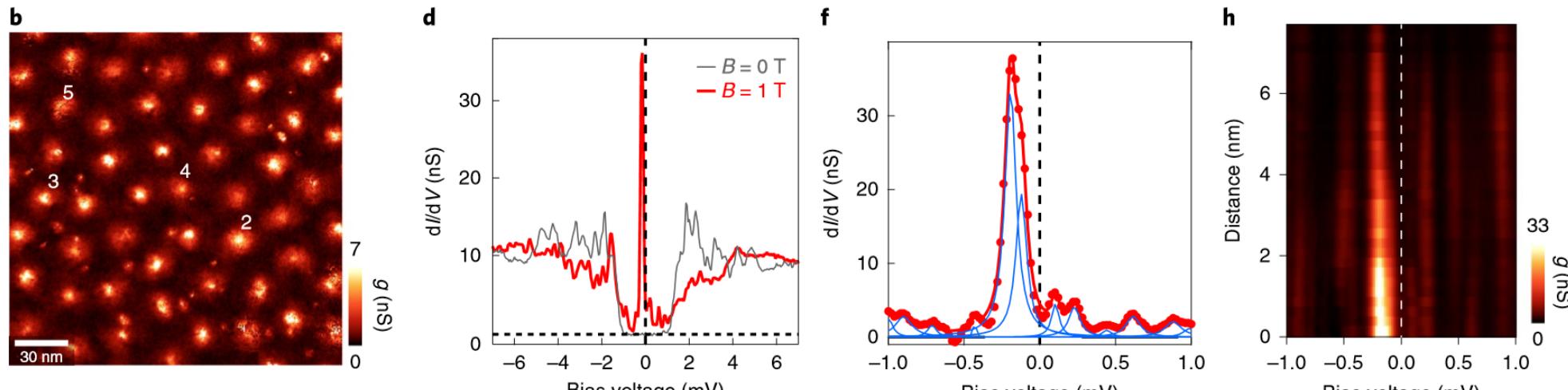
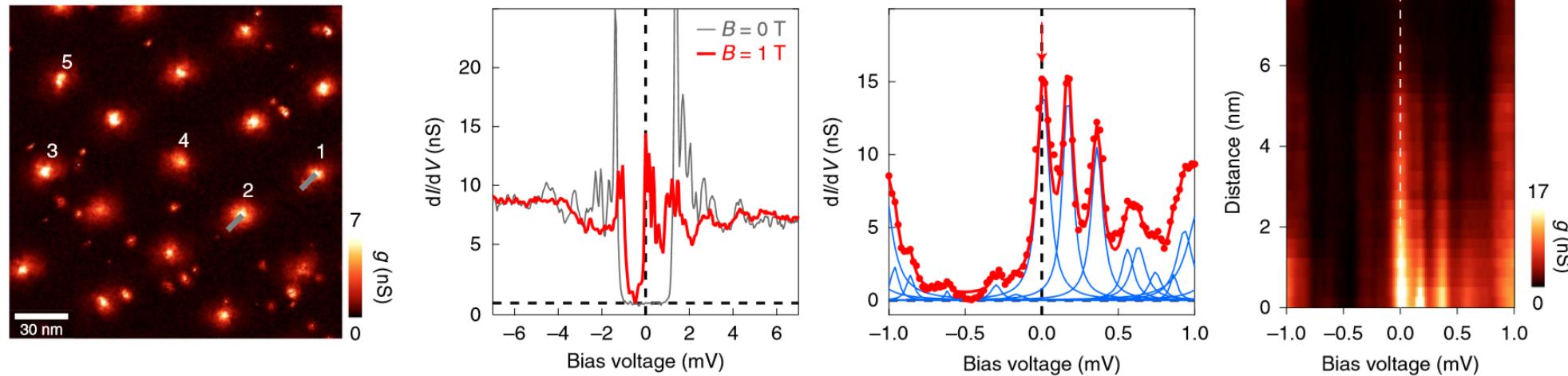
$$\begin{aligned} \text{SysWidth} &= \sqrt{(0.23^2)_{STM} + (0.16^2)_{Tem}} \\ &= 0.28 \text{ meV} \end{aligned}$$

- FWHM of ZBP *approaches resolution limit* in good cases.
- In bad cases extra broadening related to quasiparticle poisoning

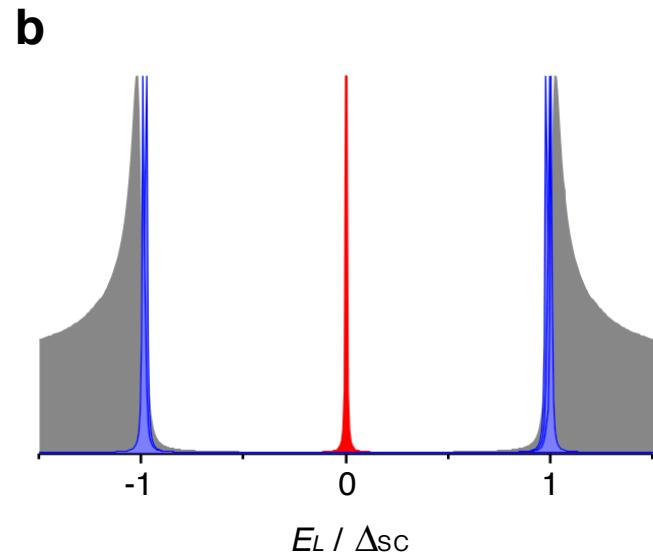
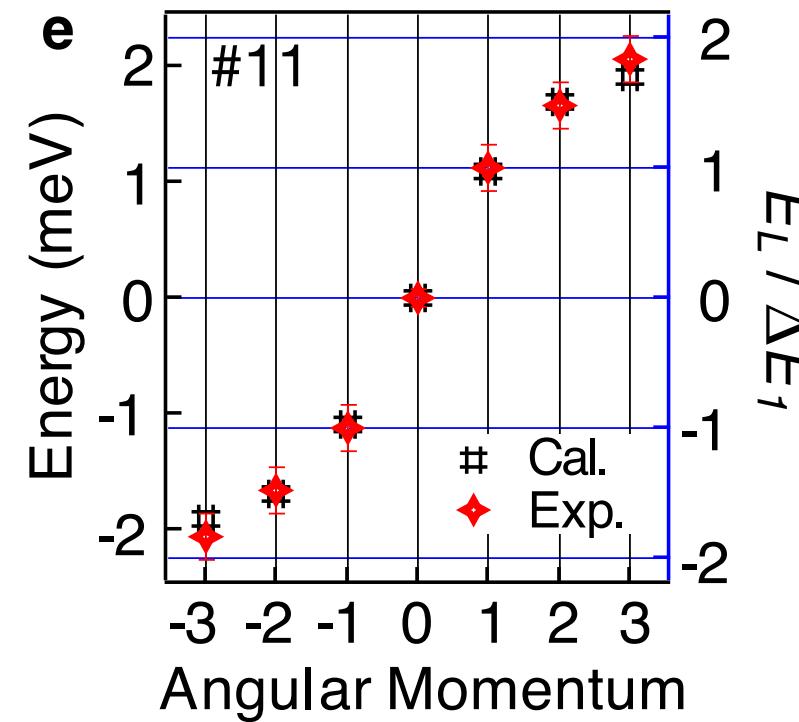
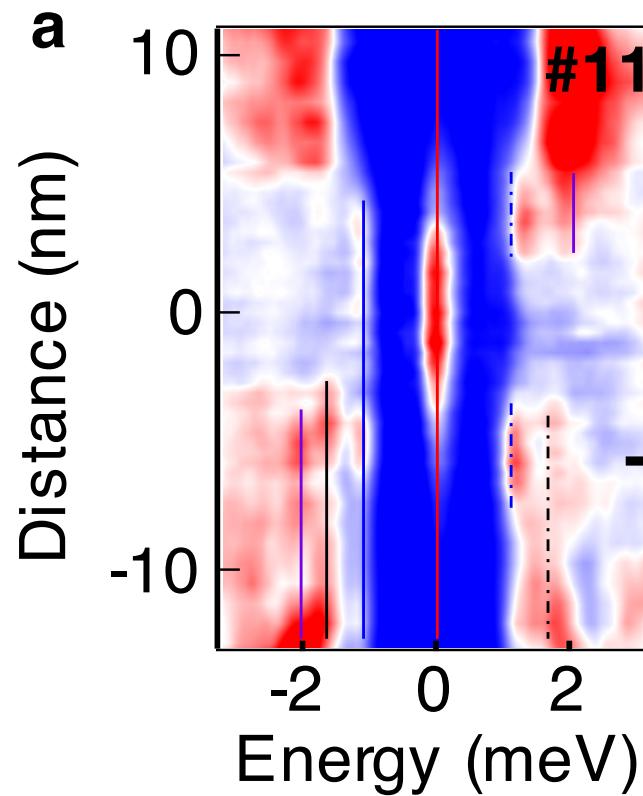
Topological and Ordinary Vortex

$T_{\text{eff}} \approx 85 \text{ mK}$, ultrahigh energy resolution of $\sim 20 \mu\text{eV}$. [Hanaguri et.al., Nat. Mat \(2019\)](#)

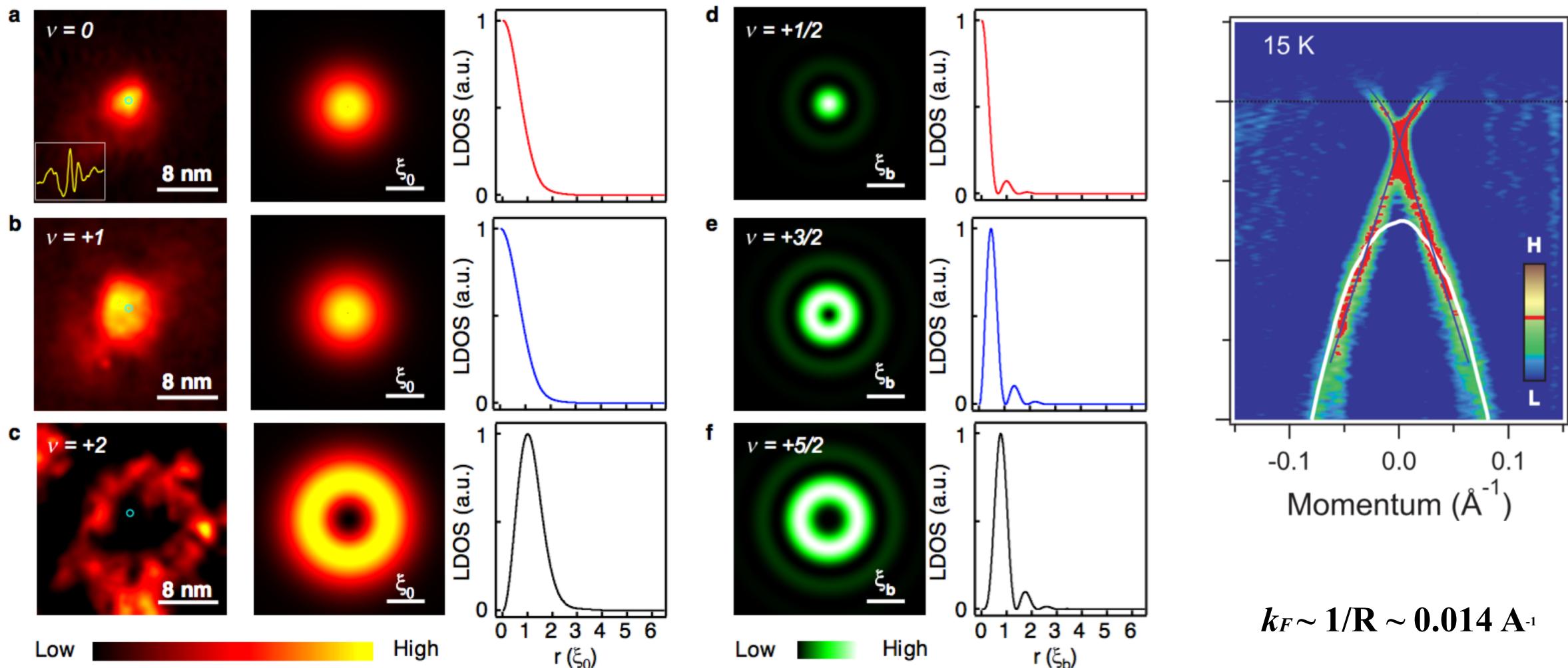
“80% and 40% of the vortices host the ZVBS (peak energy $|E| < 20 \mu\text{eV}$) at $B = 1$ and 3 T, respectively”



To approach the ‘sweet spot’: zero-doping limit

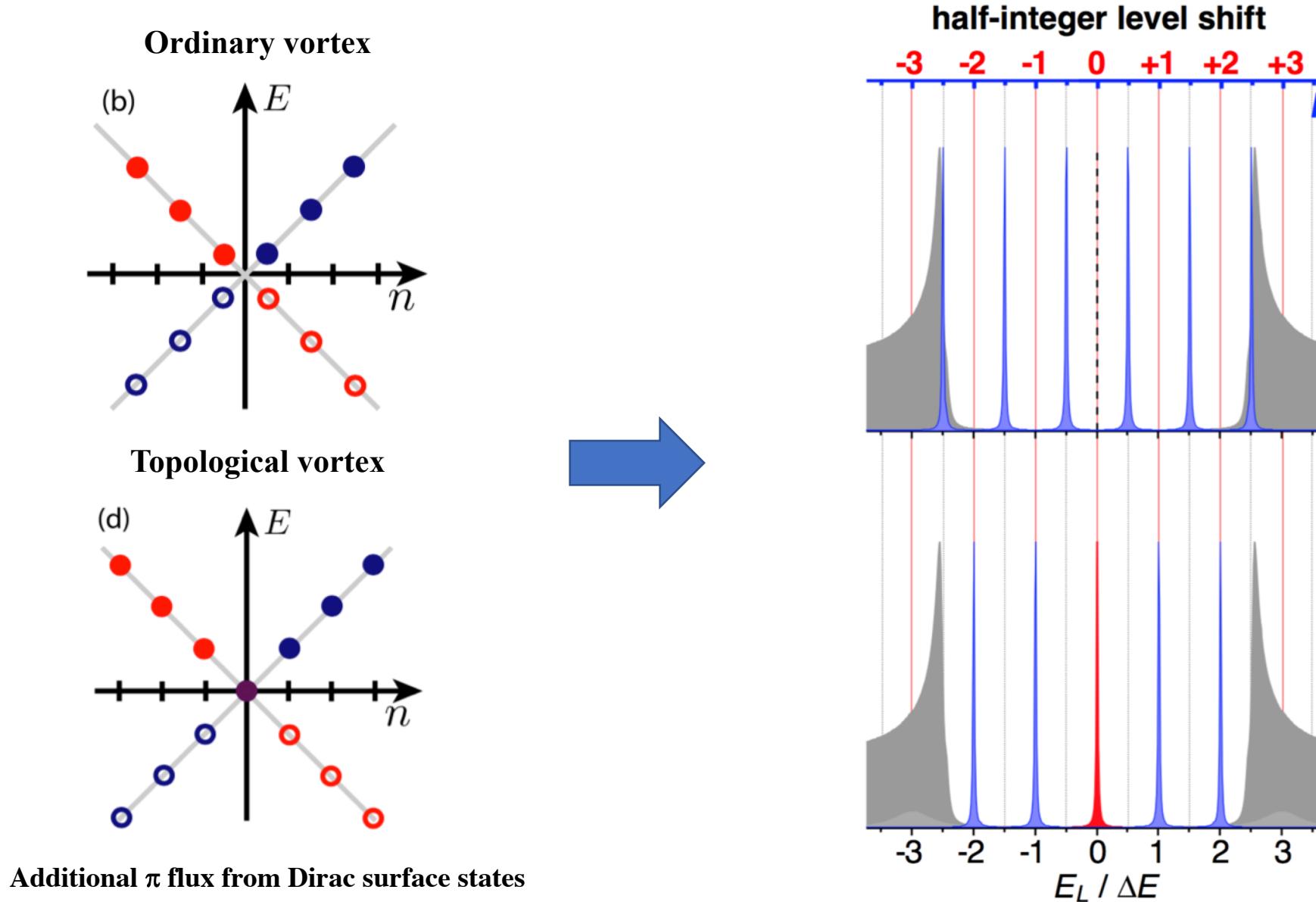


Spatial pattern of topological vortices

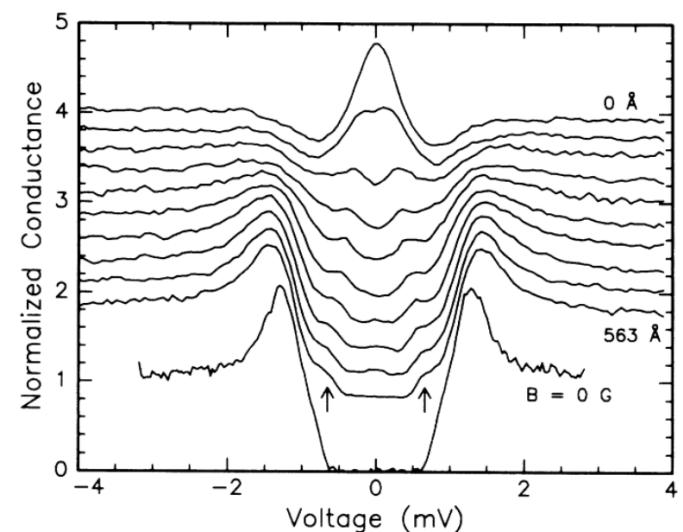


Ultra-small k_F of TSS enables direct observation of spatial pattern of vortex bound states

Half-integer level shift by Dirac surface state



Quantum-limit vortex bound states

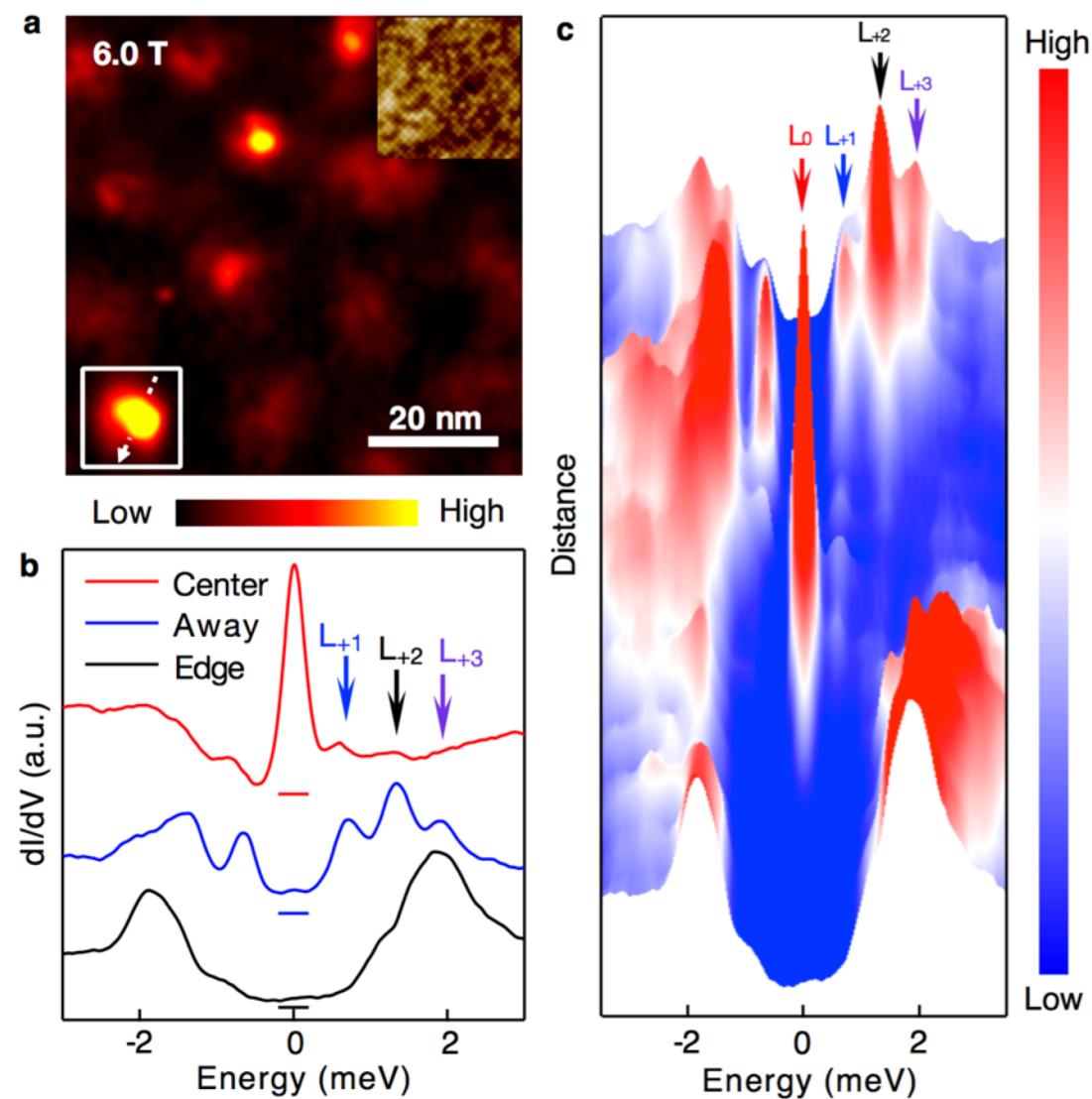


NbSe₂: Hess et al (1990)

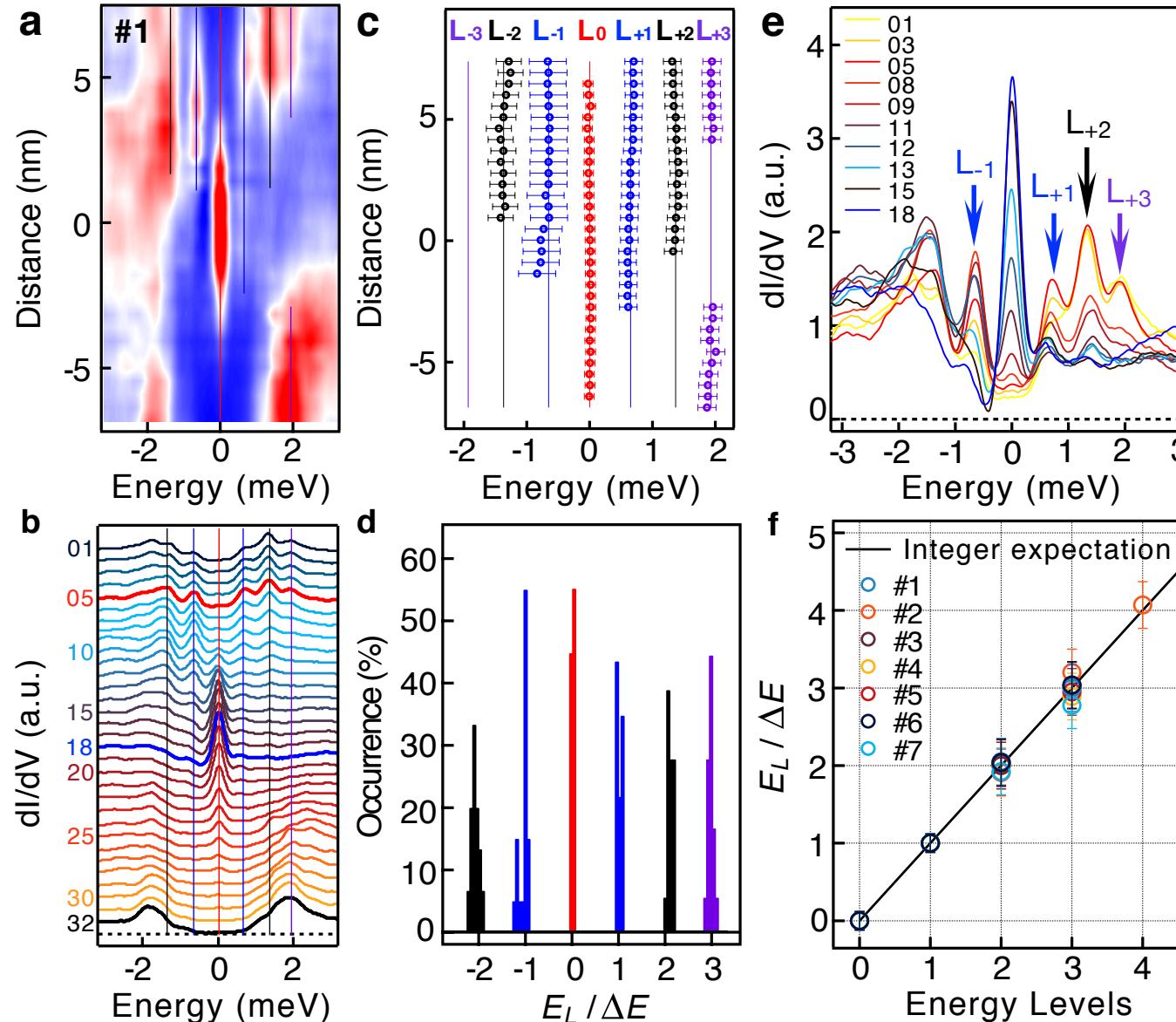
Parameters on Fe(Te, Se)

$T_{\text{exp}} = 0.4 \text{ K}$	$\Delta \text{ (meV)}$	$E_F \text{ (meV)}$	$T_{\text{QL}} \text{ (K)}$
TSS	1.8	4.4	5.9
Γ (bulk)	2.5	$5 \sim 30$	$1.2 \sim 7.25$
M(bulk)	4.2	$15 \sim 40$	$1.5 \sim 4.1$

Fe(Te,Se) satisfies quantum limit in our experiments



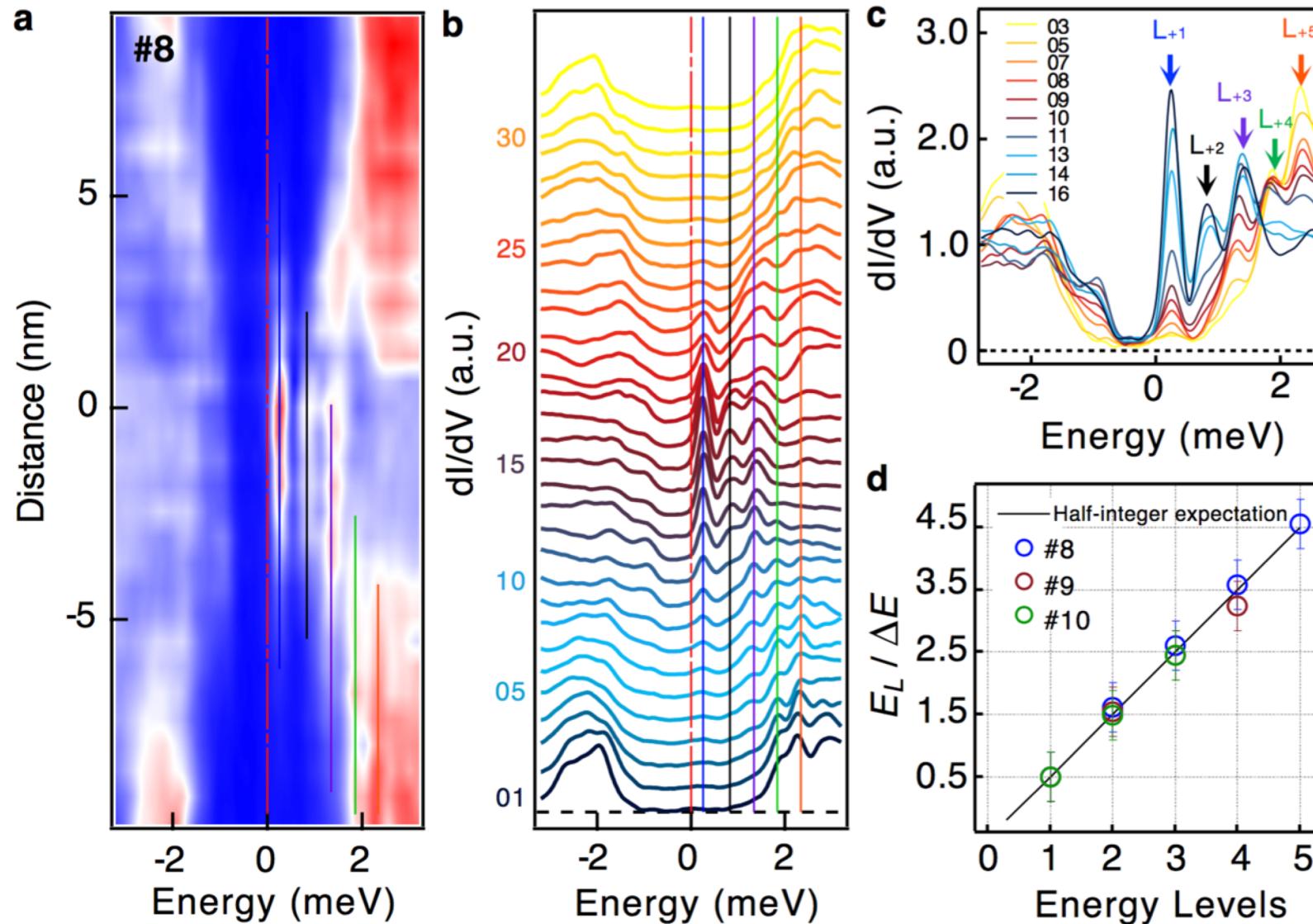
Topological vortices and integer quantized CdGM states



#1_6.0 T		
Sample#1_STM#2		
	E _L (meV)	E _L / ΔE
E ₀	0	0
E ₁	0.65	1.00
E ₂	1.37	2.11
E ₃	1.93	2.97

MBS is the 0th level of the integer-quantized CdGM states

Ordinary vortices: half-odd-integer quantized CdGM states



b

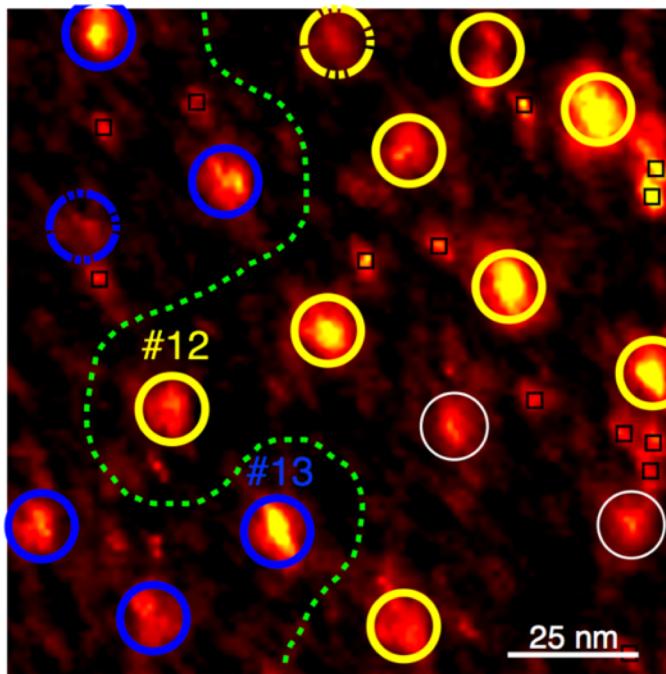
#8_2.0 T

Sample#3_STM#2

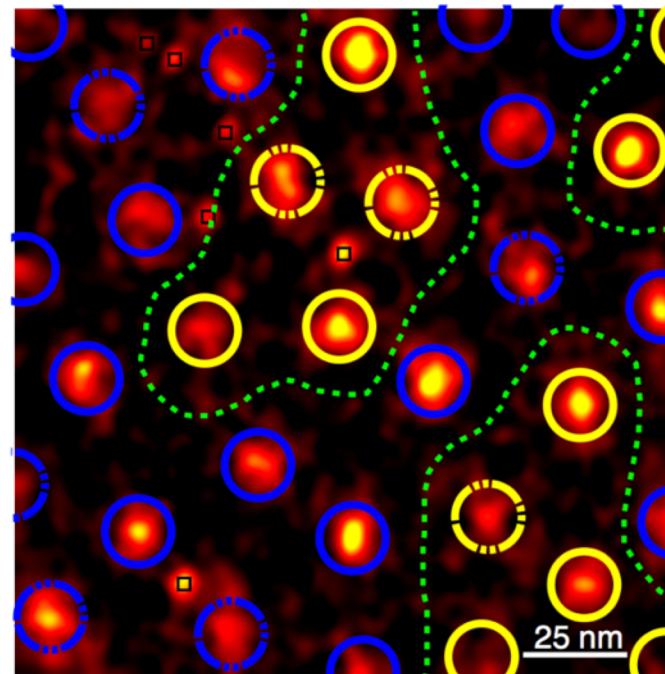
	E _L (meV)	E _L / ΔE
E ₁	0.26	0.50
E ₂	0.83	1.60
E ₃	1.34	2.58
E ₄	1.84	3.54
E ₅	2.34	4.5

Vortex statistics

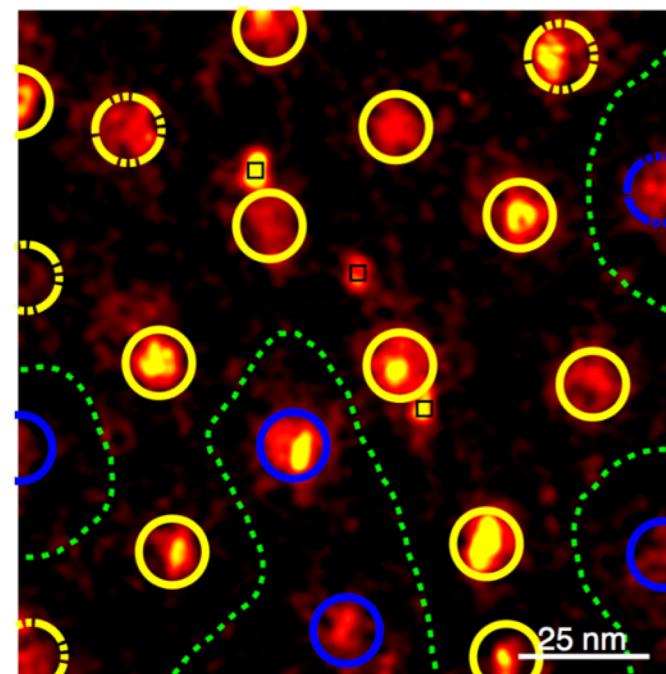
a Region#1 (126 nm × 126 nm)



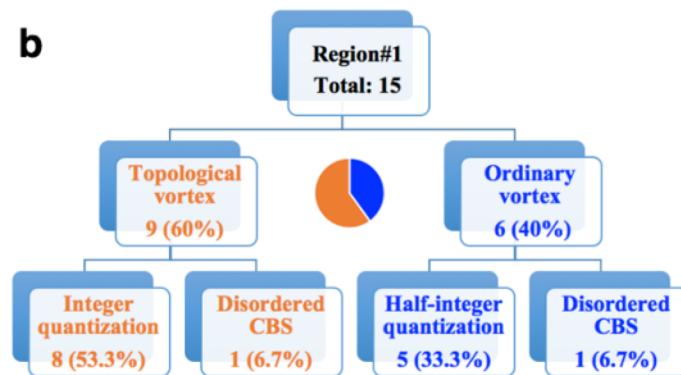
c Region#2 (157 nm × 157 nm)



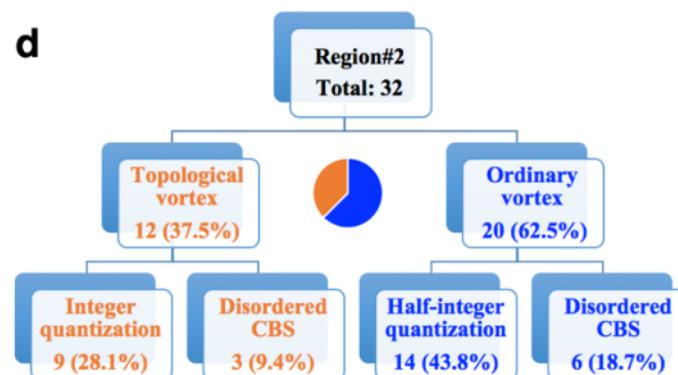
e Region#3 (126 nm × 126 nm)



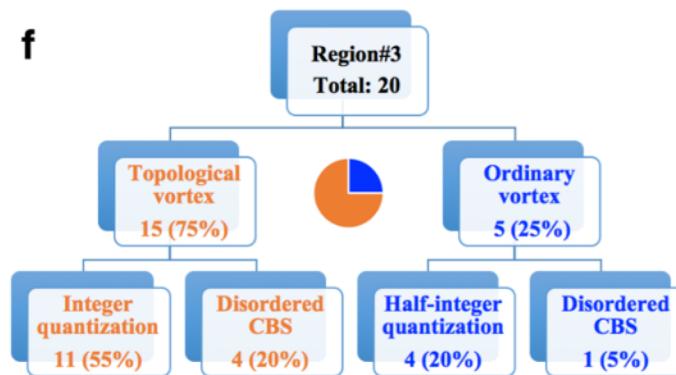
b



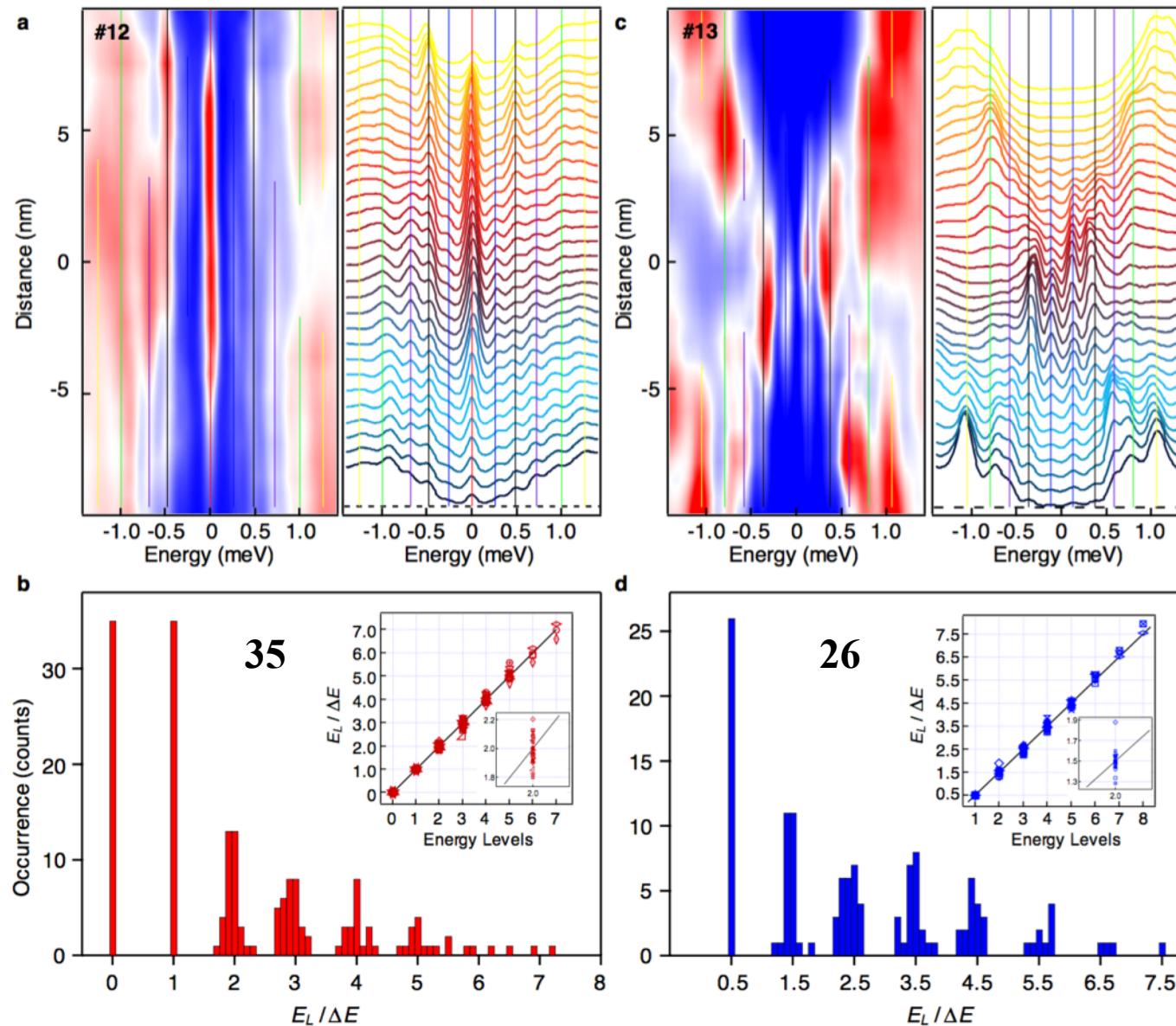
d



f

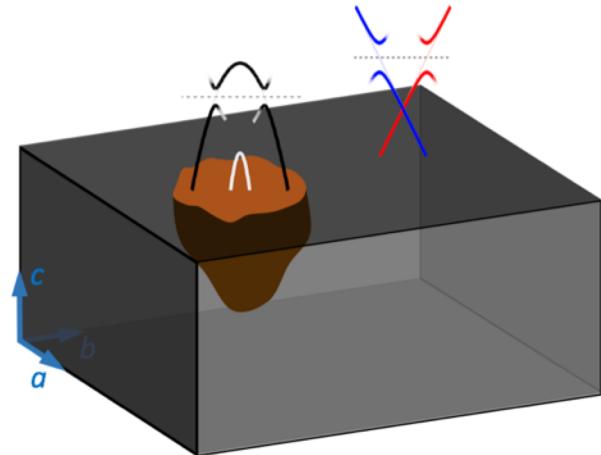


Vortex statistics

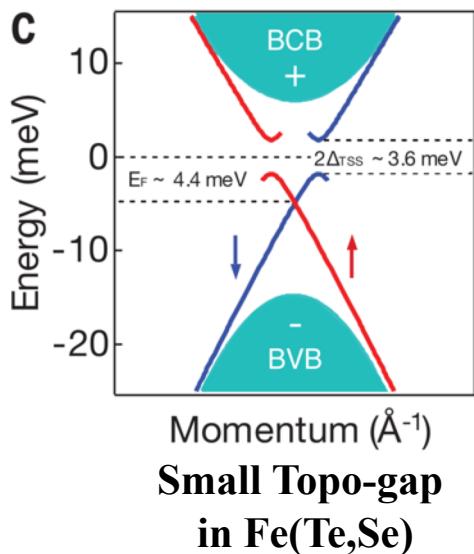


The inhomogeneity of Telluride/Selenide alloy

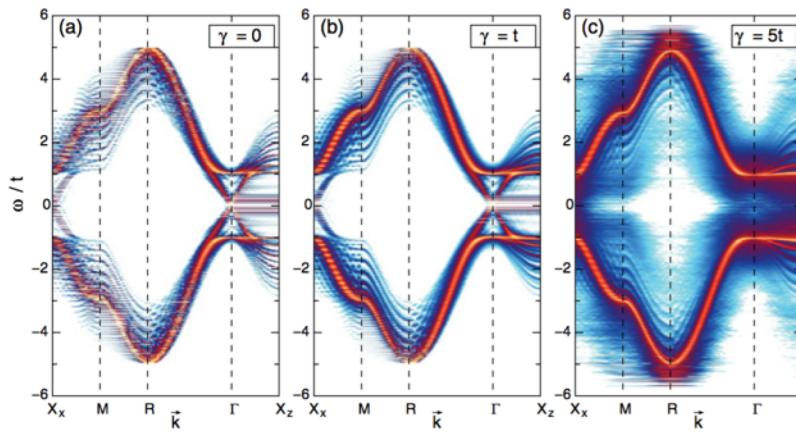
➤ Strong inhomogeneity (break down strong-TI)



➤ Normal insulating state: easy to conquer 20 meV



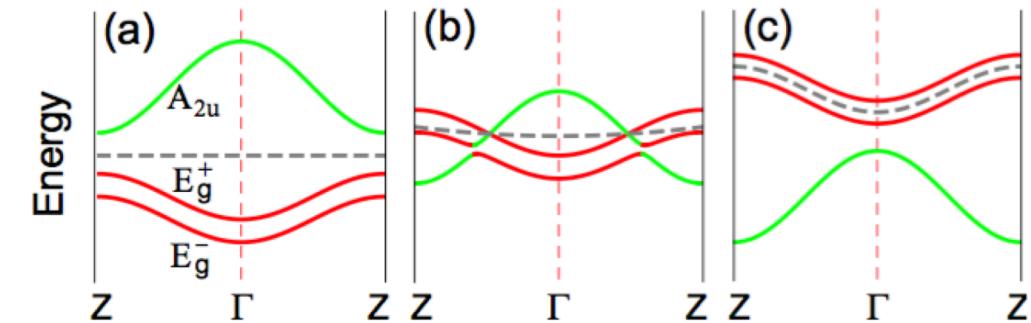
Small Topo-gap
in Fe(Te,Se)



G. Schubert et al.,
PRB 85, 201105(R) (2012)

- No TSS, Ordinary vortex, No MZM
- With TSS, Topo. vortex, MZM

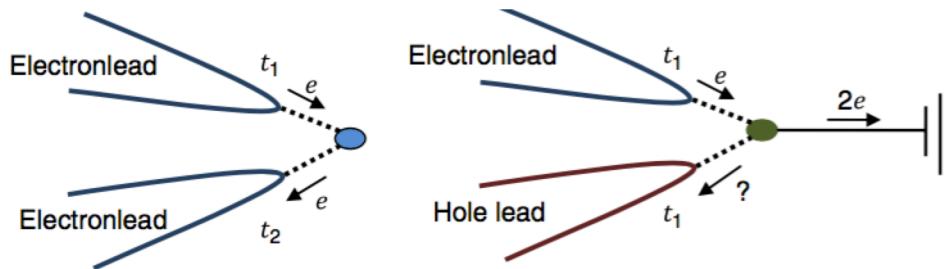
➤ Weak Topological insulator state



S.-S. Qin, L.-H. Hu et al. arXiv:1901.03120v1

Quantized Majorana Conductance

- Majorana modes induced resonant Andreev reflection

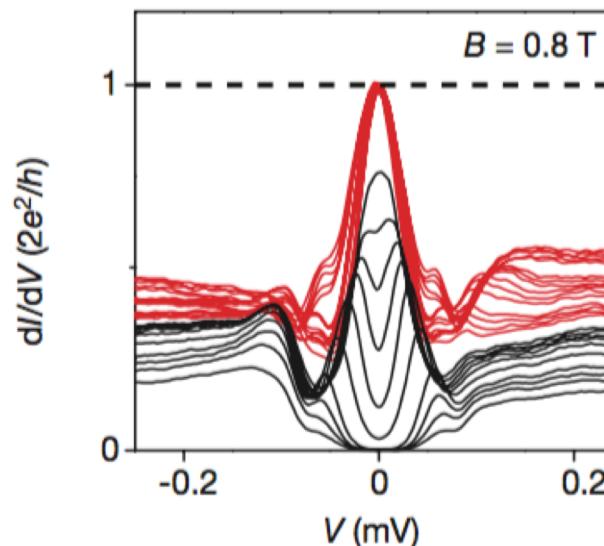
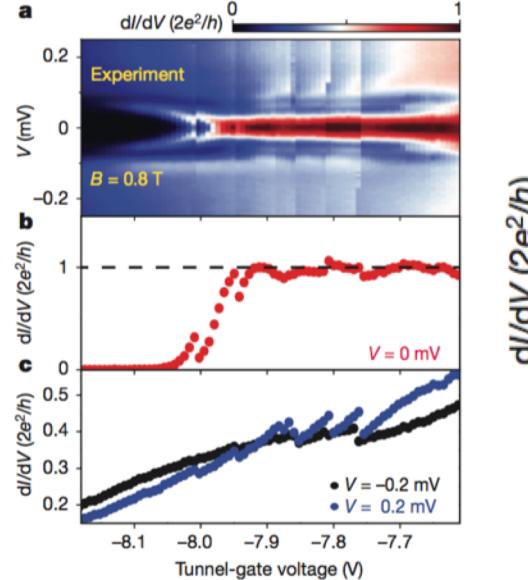
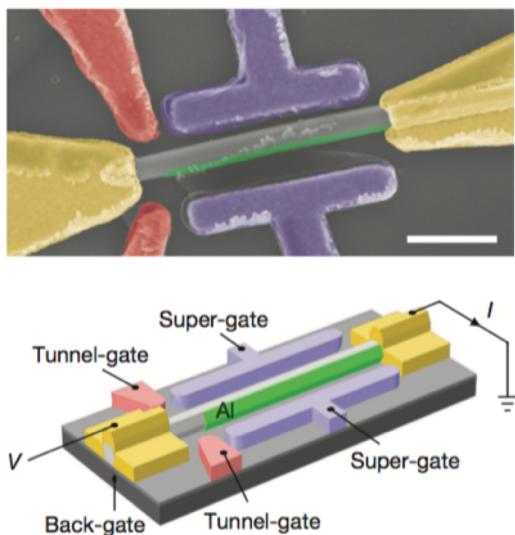


Law, Lee and Ng. PRL (2009)

$$\frac{dI}{dV} = \frac{2e^2}{h} \frac{4\Gamma^2}{(eV)^2 + 4\Gamma^2}$$

On resonance ($V=0$), universal conductance ($2e^2/h$) regardless of tunnel barrier (Γ) at low temperature $kT < \Gamma$

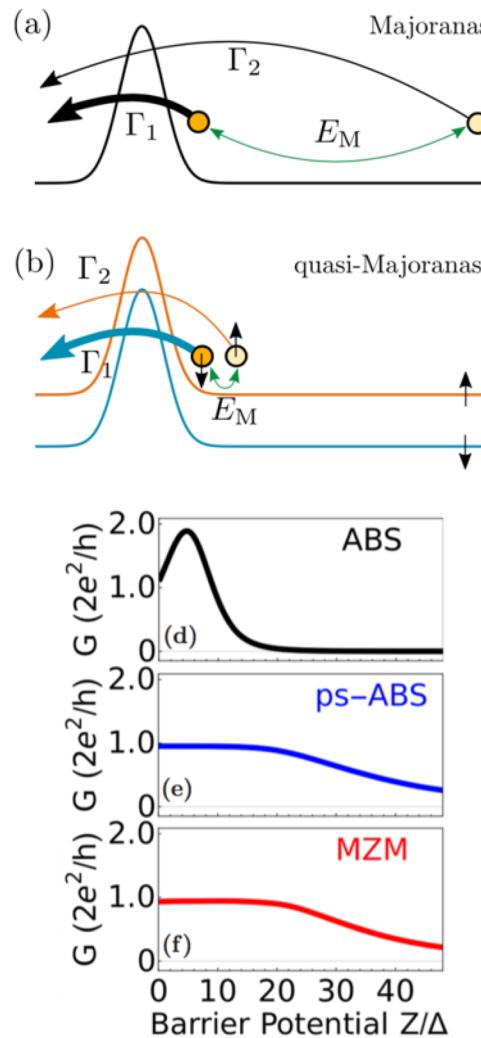
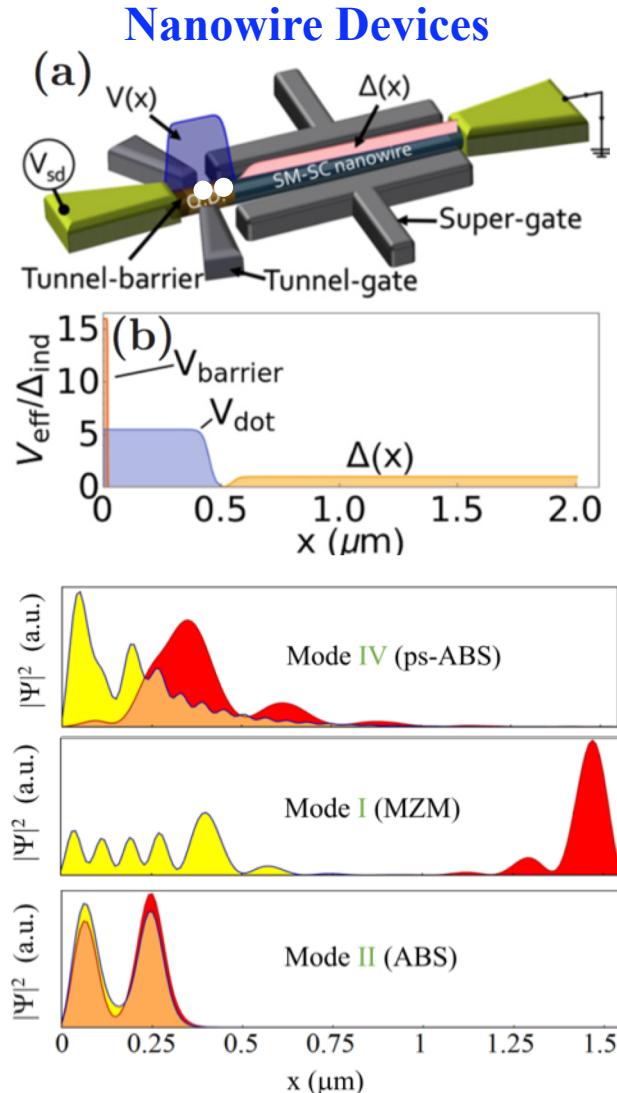
- Majorana conductance plateau observed in nanowires



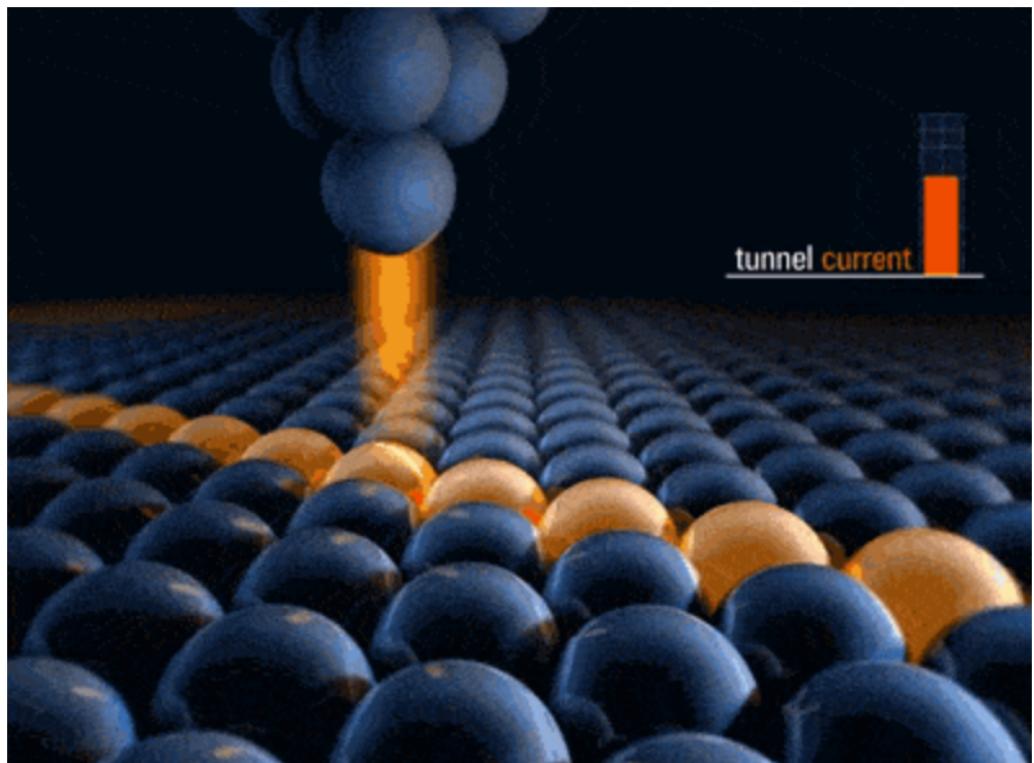
H. Zhang et al.
Nature 556, 74 (2018)

Partially separated Andreev bound state

Smooth confinement potential at the end of the nanowire raise the possibility to exist a pair of spatially-overlapping MZMs localized at the same end of the nanowire arXiv:1806.02801 Phys. Rev. B 98,155314 (2018); Phys. Rev. B 97,165302 (2018)

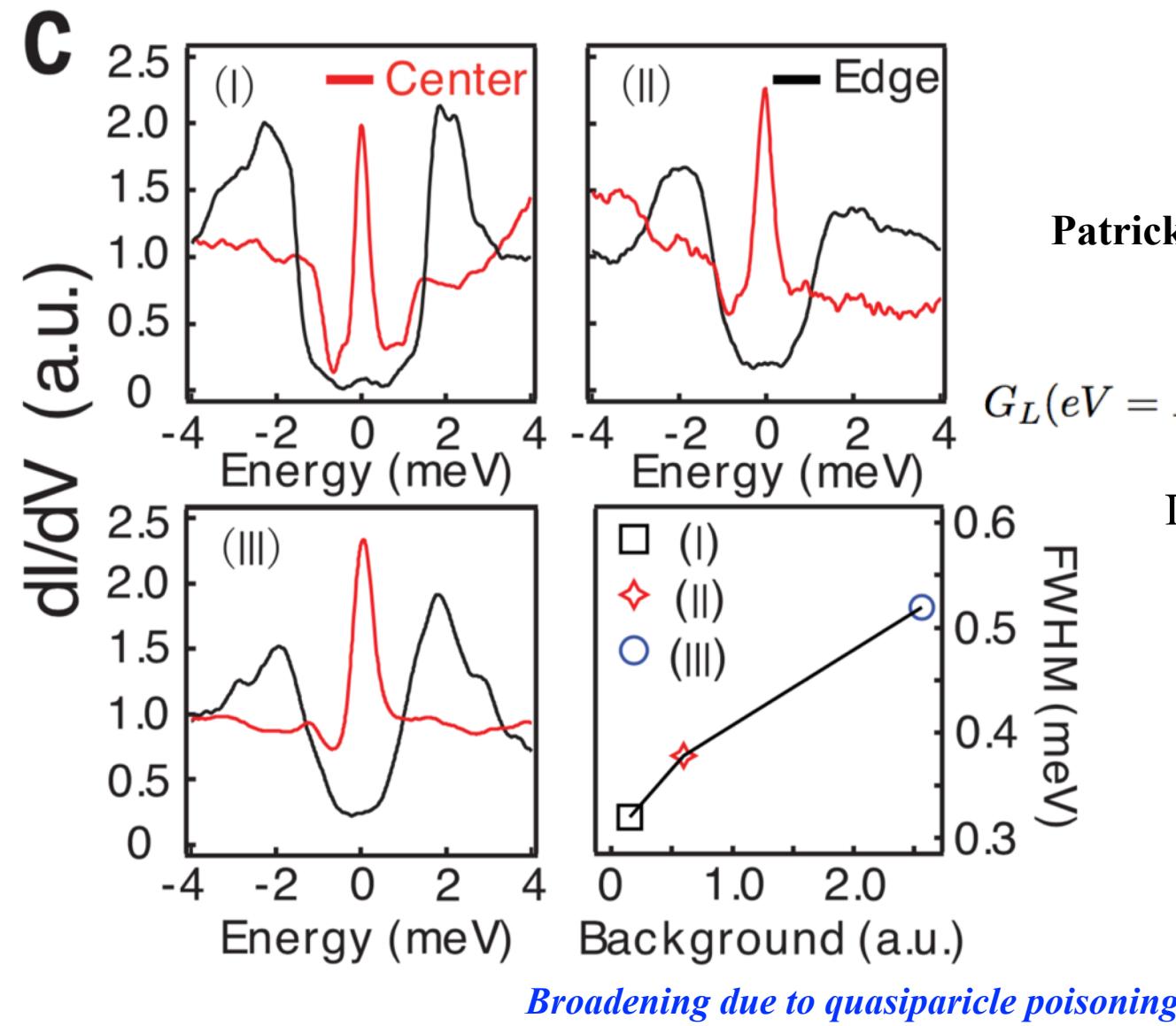
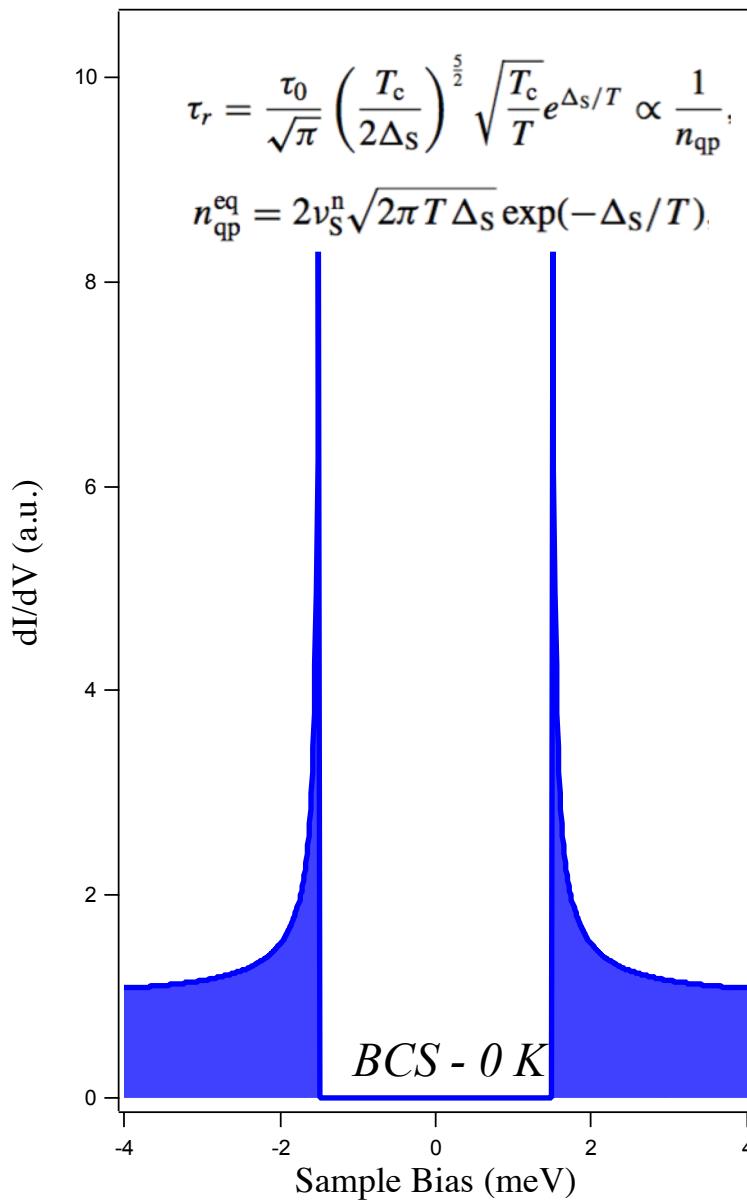


STM: vacuum tunneling



Sharp confinement tunnel-barrier remove the possibility of partially separated Andreev bound state.

Quasiparticle poisoning

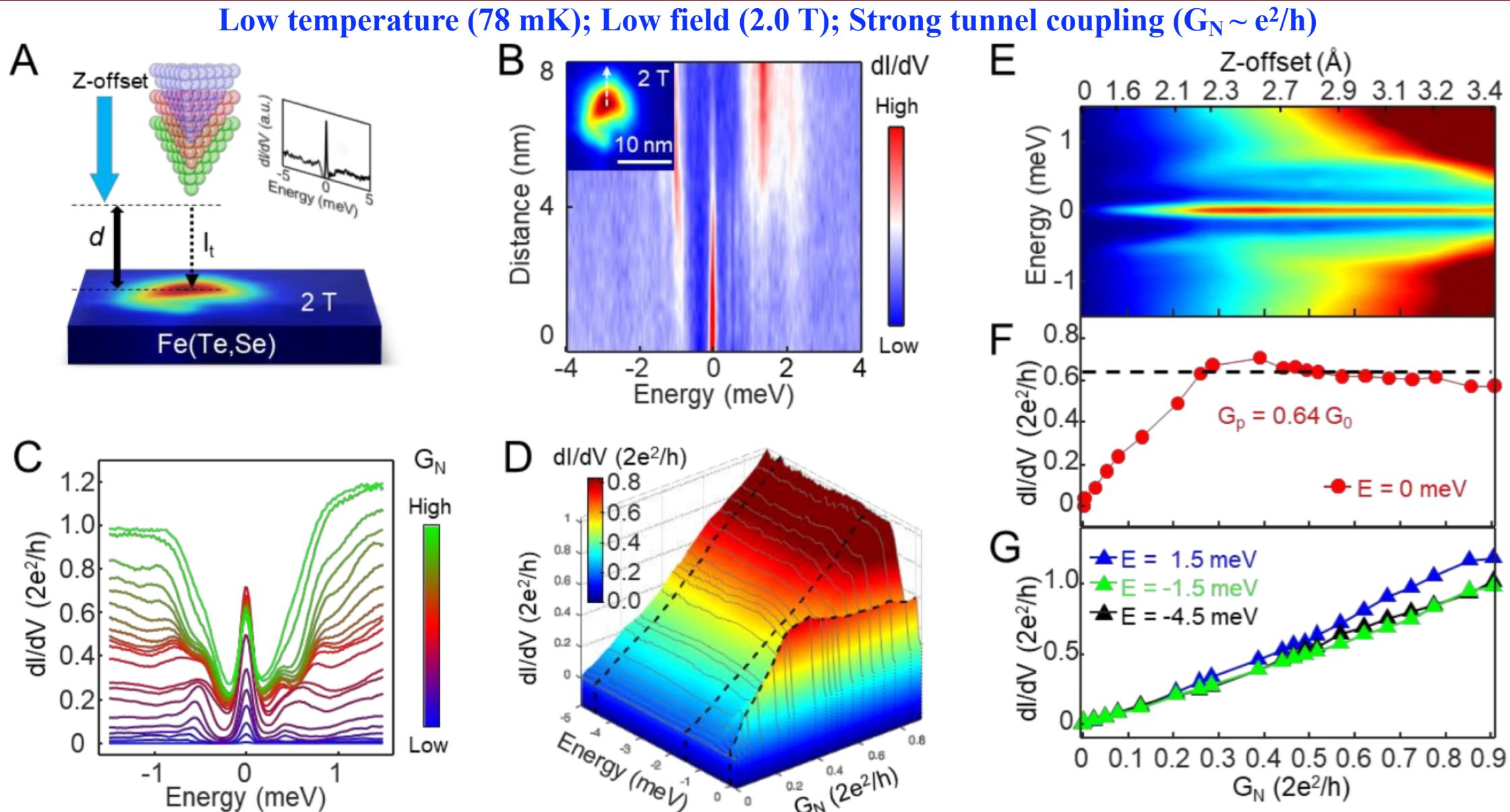


Patrick Lee, PRB (2014)

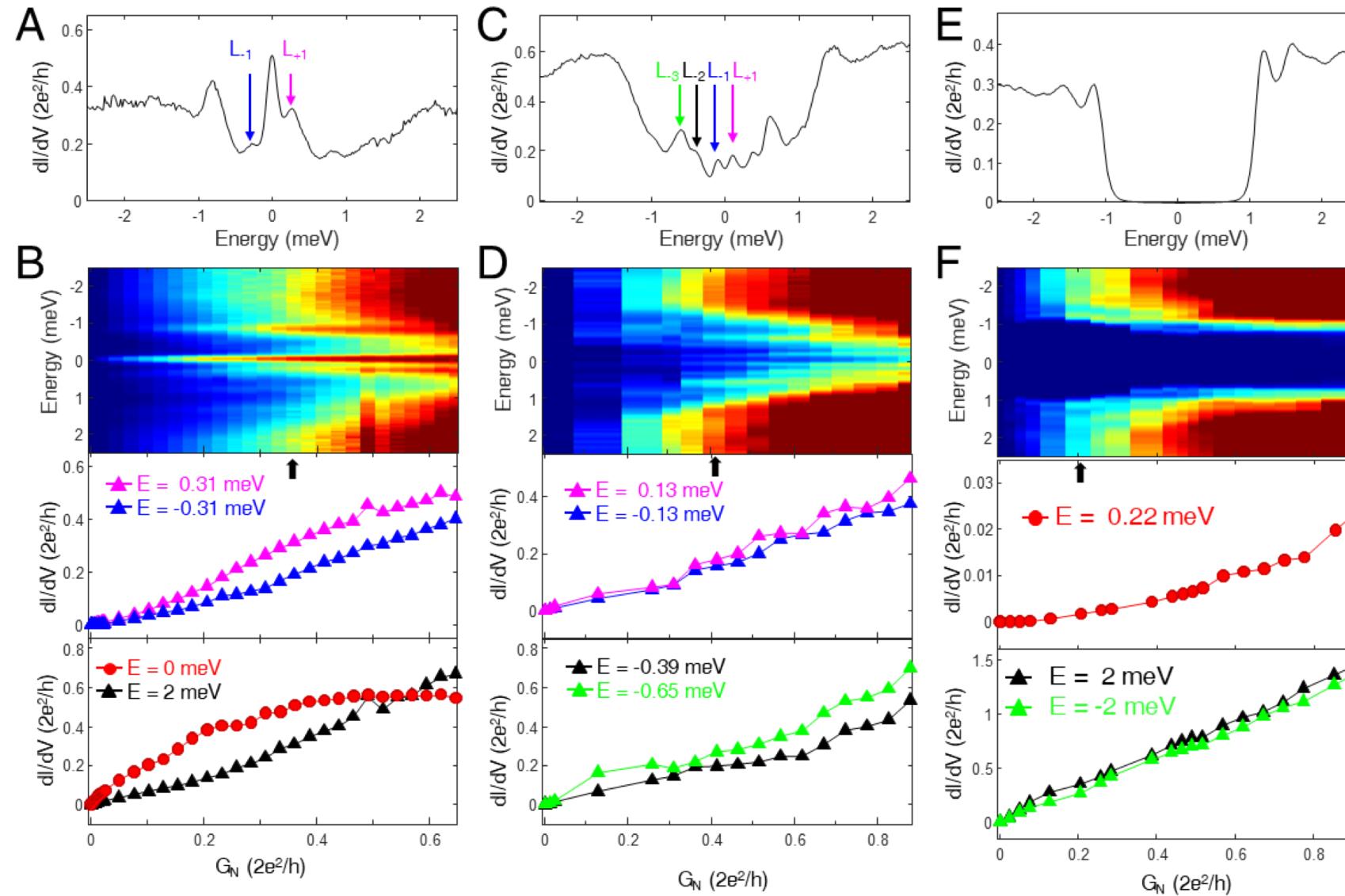
$$G_L(eV = E) = \frac{2e^2}{h} \frac{\gamma_L^t \Gamma_L}{E^2 + \Gamma_L^2}$$

$$\Gamma_L = \gamma_L^t + \gamma_L^p$$

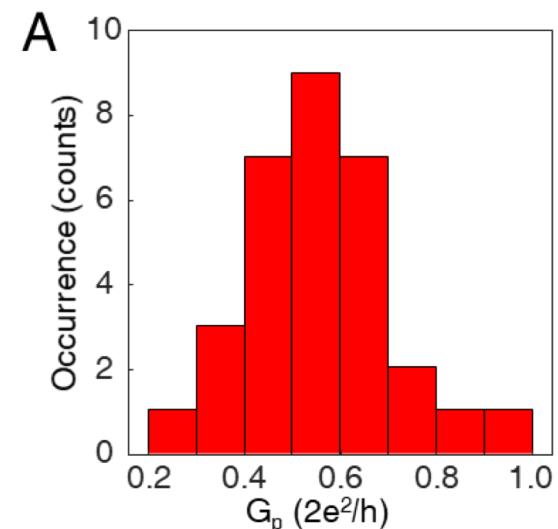
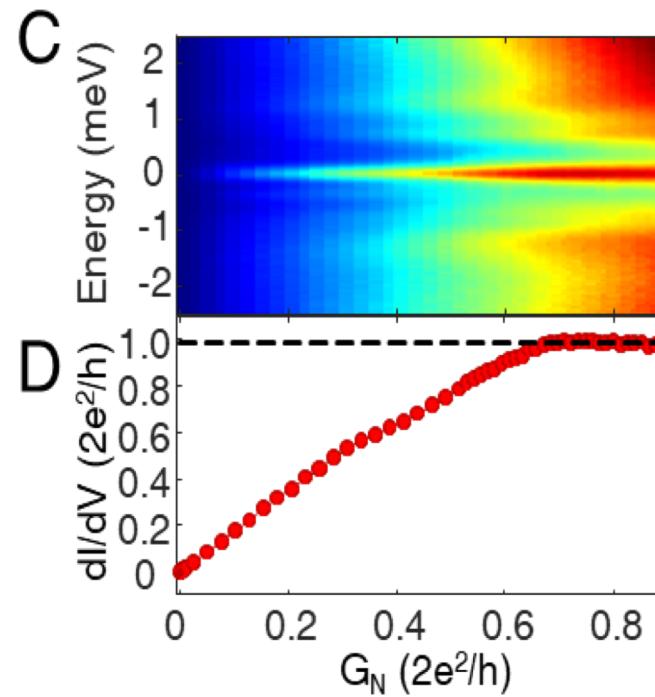
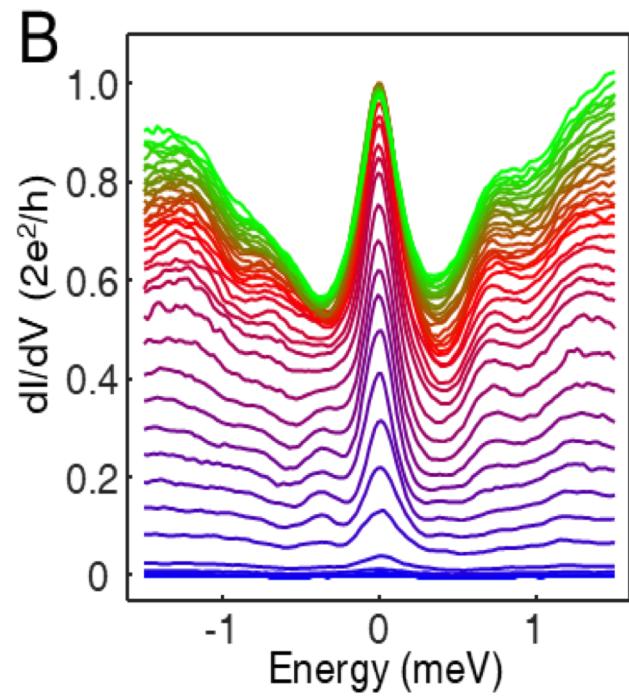
Zero-bias conductance plateau observed on $\text{FeTe}_{0.55}\text{Se}_{0.45}$



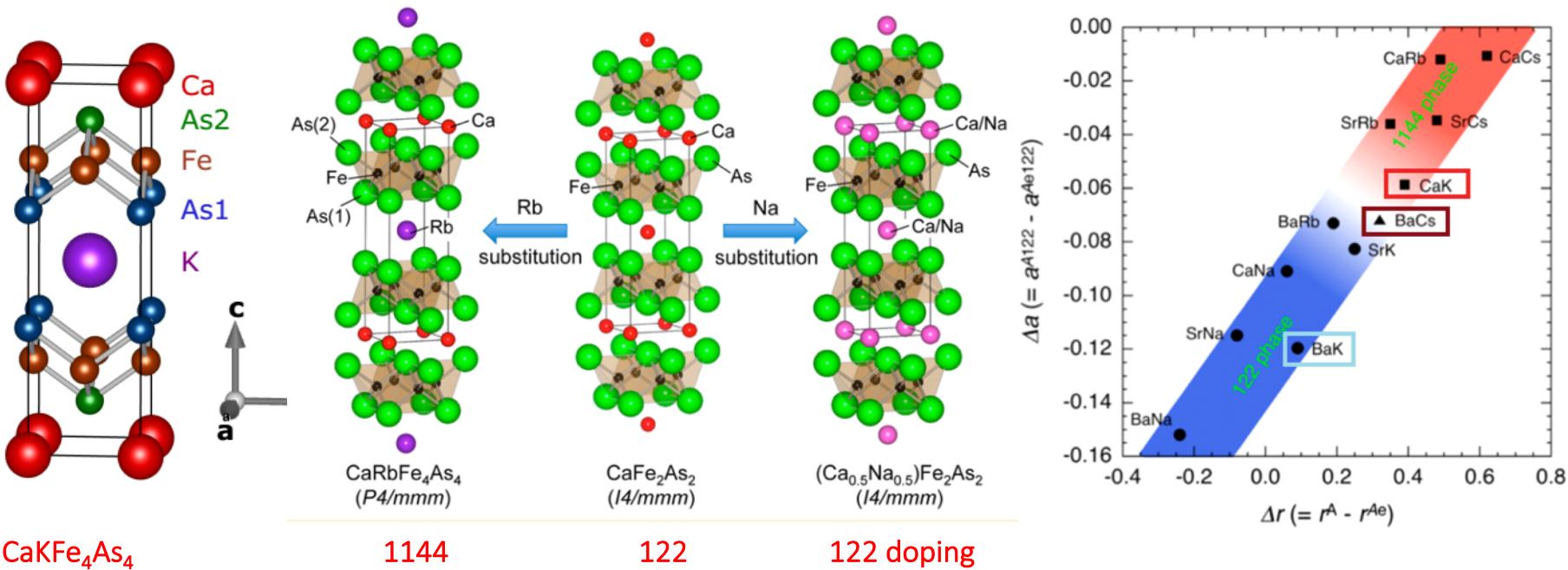
In stark contrasts with CBS and continues states



Distribution of the plateau value



CaKFe₄As₄ (T_c = 33K): double Fe-As layers, self-doping



CaKFe₄As₄

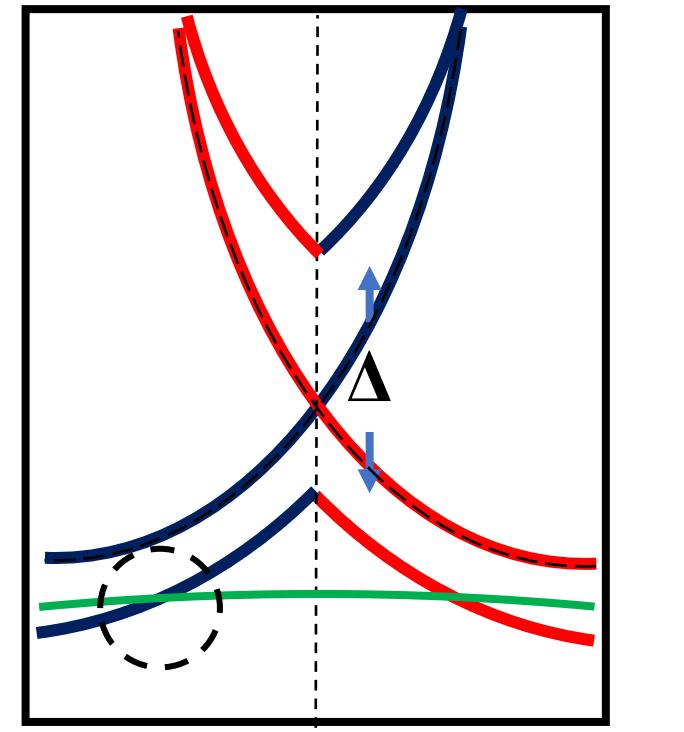
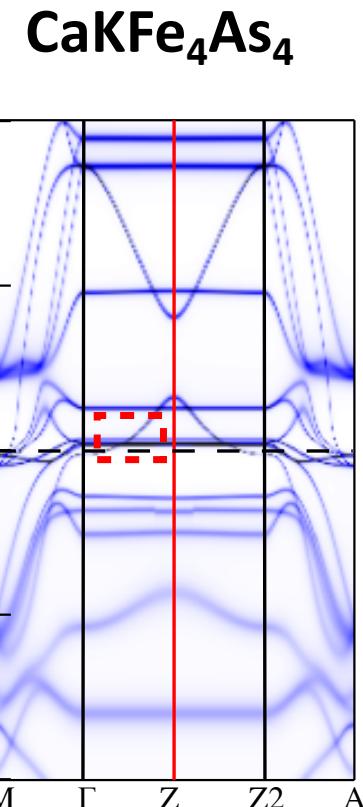
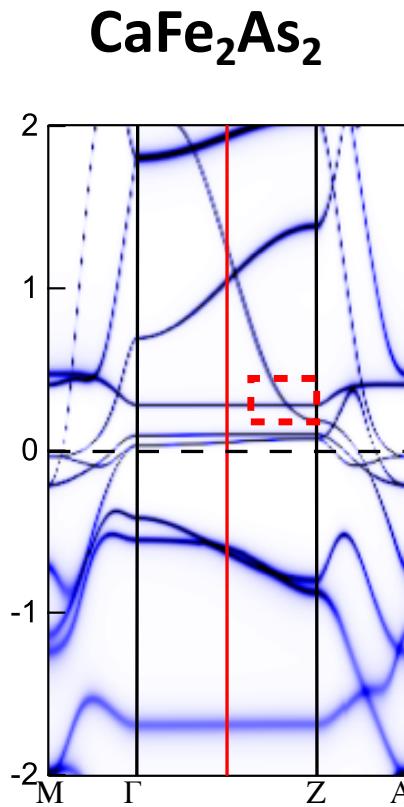
1144

122

122 doping

compound	space group ^a	<i>a</i> (Å)	<i>c</i> (Å)	T _c (K)
CaKFe ₄ As ₄	<i>P</i>	3.866(1)	12.817(5)	33.1
CaRbFe ₄ As ₄	<i>P</i>	3.8757(9)	13.104(3)	35.0
CaCsFe ₄ As ₄	<i>P</i>	3.891 (1)	13.414(2)	31.6
SrRbFe ₄ As ₄	<i>P</i>	3.897(1)	13.417(5)	35.1
SrCsFe ₄ As ₄	<i>P</i>	3.910(1)	13.729(3)	36.8
BaCsFe ₄ As ₄ or (Ba,Cs)Fe ₂ As ₂	<i>P</i> or <i>I</i>	3.927(2)	14.134(6)	26
CaFe ₂ As ₂	<i>I</i>	3.900(1)	11.62(1)	NS ^c
SrFe ₂ As ₂	<i>I</i>	3.9266(5)	12.370(2)	NS ^c
BaFe ₂ As ₂	<i>I</i>	3.9612(3)	13.006(1)	NS ^c
NaFe ₂ As ₂	<i>I</i>	3.8090(5)	12.441(3)	25
KFe ₂ As ₂	<i>I</i>	3.8414(2)	13.837(1)	3.8
RbFe ₂ As ₂	<i>I</i>	3.888(2)	14.534(7)	2.6
CsFe ₂ As ₂	<i>I</i>	3.8894(2)	15.066(5)	1.8

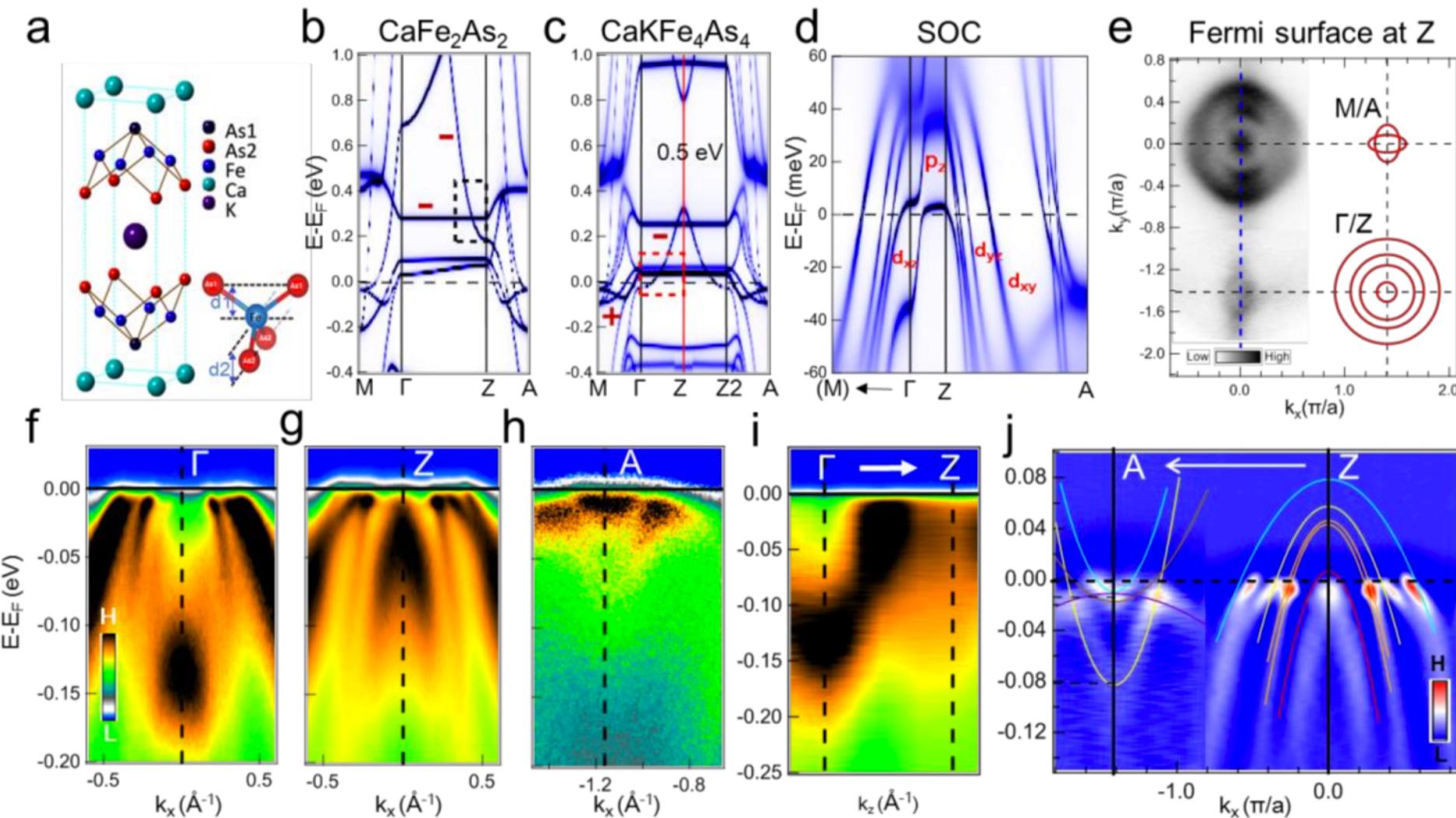
CaKFe₄As₄: band inversion induced by symmetry breaking



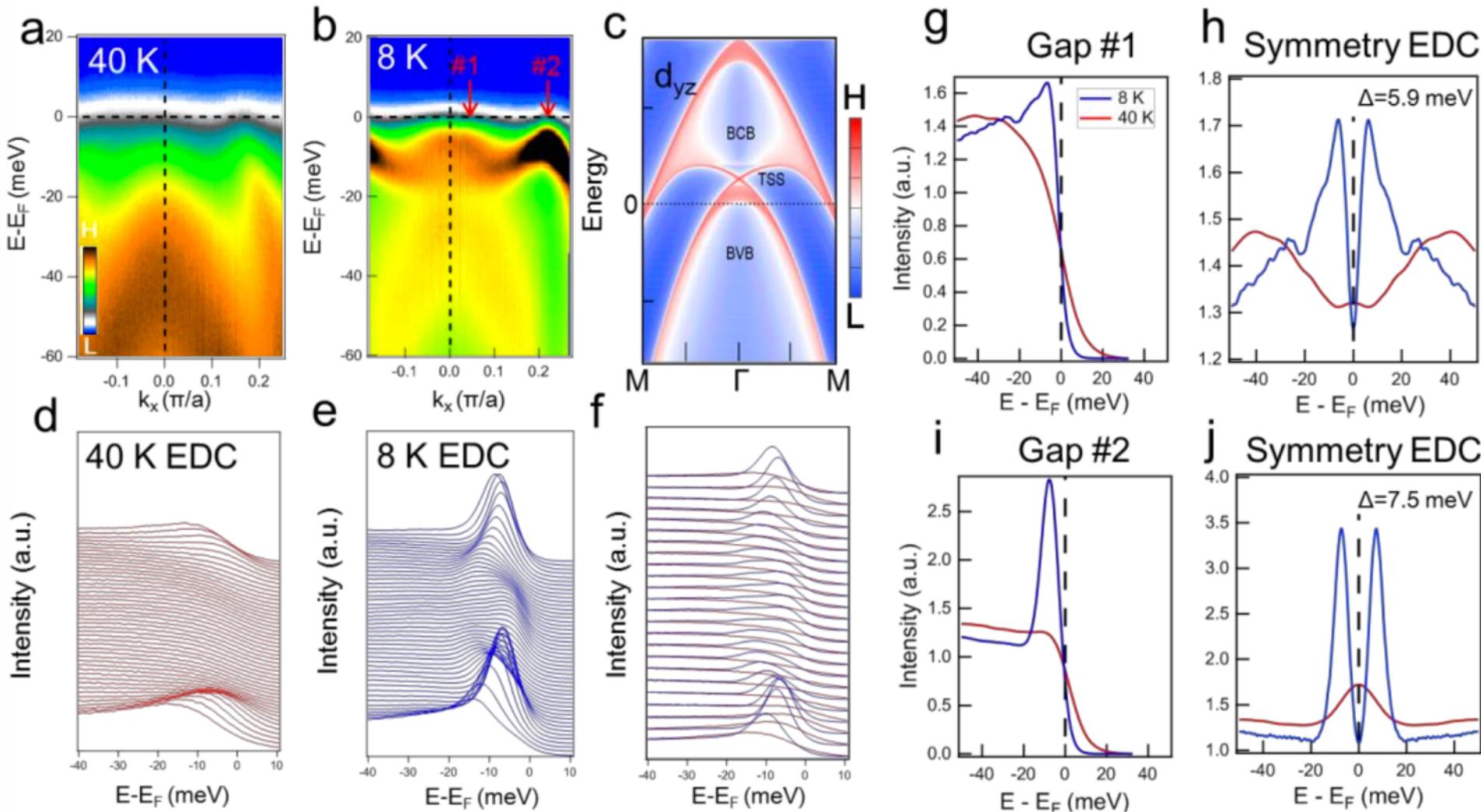
Γ

Z2

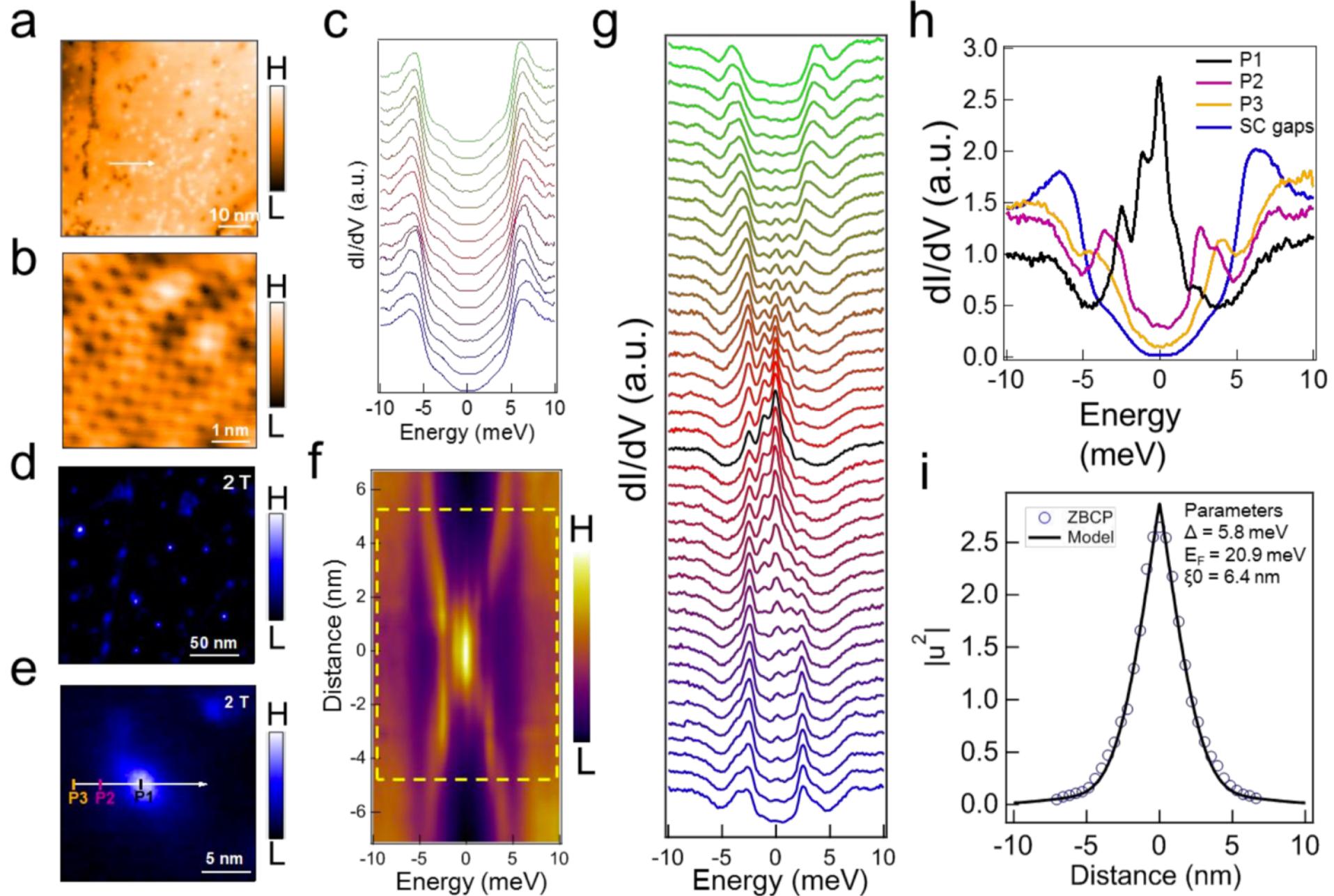
Comparison between ARPES and DMFT caculation



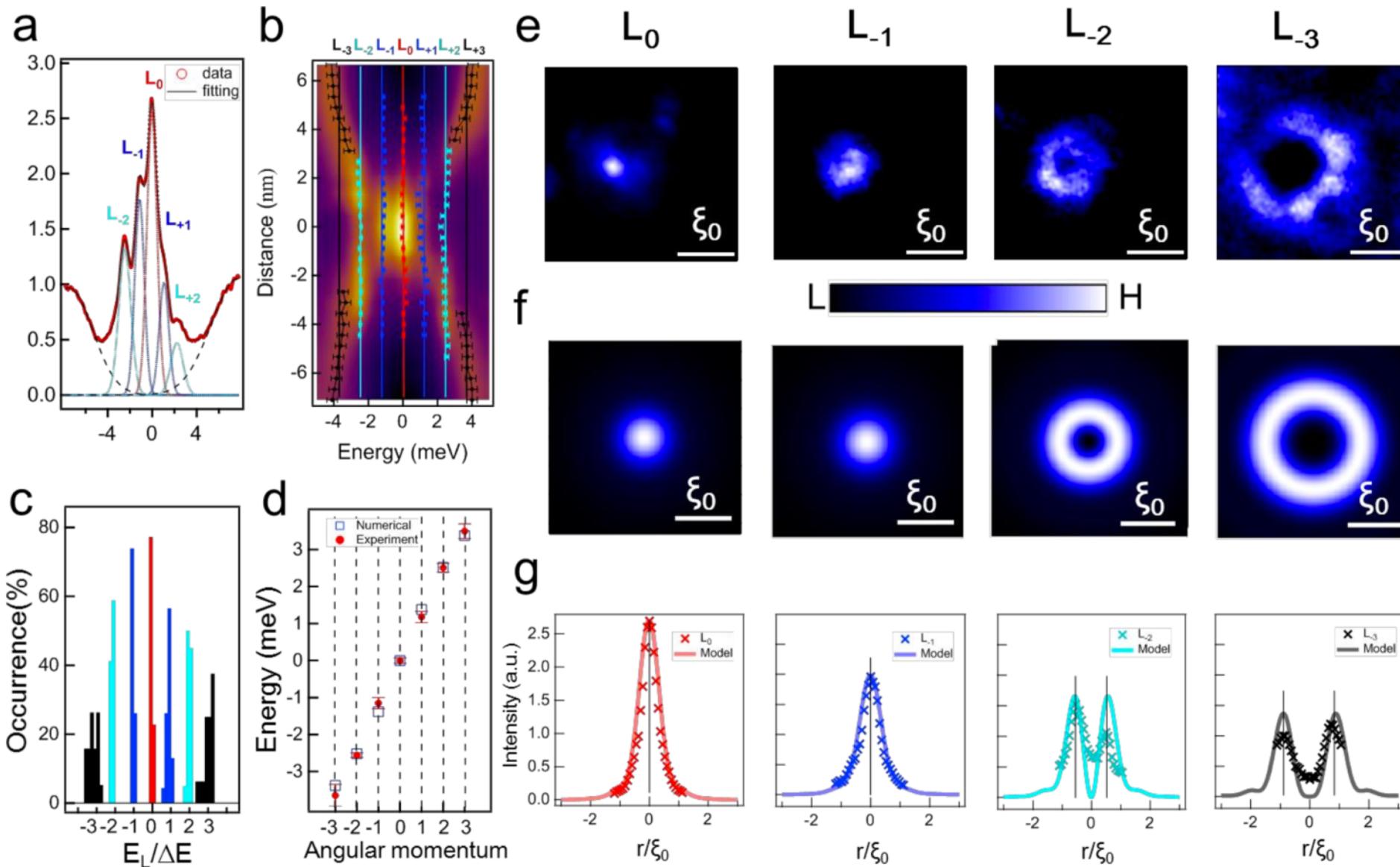
ARPES measurements of superconducting gap



STM observation of MZM and CBSs



Integer level spacing and “ring-pattern” of CBSs



An amazing new field for Majorana

Experiments:

- Zhang et al. Science 360, 182 (2018)**
Wang et al. Science 362, 333 (2018)
Machida et al. Nat. Mater. (2019)
Kong et al. arXiv: 1901.02293 (2019)
Zhu et al. arXiv: 1904.06124 (2019)
Liu et al. Phys. Rev. X 8, 041056 (2018)
Zhao et al. Phys. Rev. B.97.224504 (2018)
Liu et al. arXiv: 1807.07259 (2018)
Chen et al. Sci. Adv. 4, eaat1084 (2018)
Zhang et al. Nat. Phys. 15, 41 (2019)
Peng et al. arXiv: 1903.05968 (2019)
Rameau et al, Phys. Rev. B 99. 205117 (2019)
Gray et al, arXiv: 1902.10723 (2019)
Wang et al, arXiv: 1903.00515 (2019)
Chen et al. Chin.Phys.Lett. 36, 057403 (2019)
Liu et al, arXiv: 1907.00904 (2019)
Chen et al. arXiv: 1905.05735 (2019)
Yuan, Xue (WS2) et al, Nat. Phys. (2019)
.....

Theory:

- Berthod et al. Phys. Rev. B.98.144519 (2018)
Jiang et al. Phys. Rev. X 9. 011033 (2019)
Liu et al. arXiv: 1901.06083 (2019)
Chiu et al. arXiv: 1904.13374 (2019)
Qin et al. arXiv: 1901.03120 (2019)
Qin et al. arXiv: 1901.04932 (2019)
Konig et al. Phys. Rev. Lett.122.207001 (2019)
Zhang et al. Phys. Rev. Lett.122.187001 (2019)
November et al. arXiv: 1905.09792 (2019)
Zhang et al. arXiv: 1905.10647 (2019)
Wu et al. arXiv: 1905.10648 (2019)
Hu et al. arXiv: 1906.01754 (2019)
Kawakami et al. arXiv: 1906.09286 (2019)
Ghazaryan et al. arXiv: 1907.02077 (2019)
.....

Journal Club for Condensed Matter Physics
<https://www.condmatjclub.org>

JCCM_January_2018_01

Iron-based superconductors went topological ?

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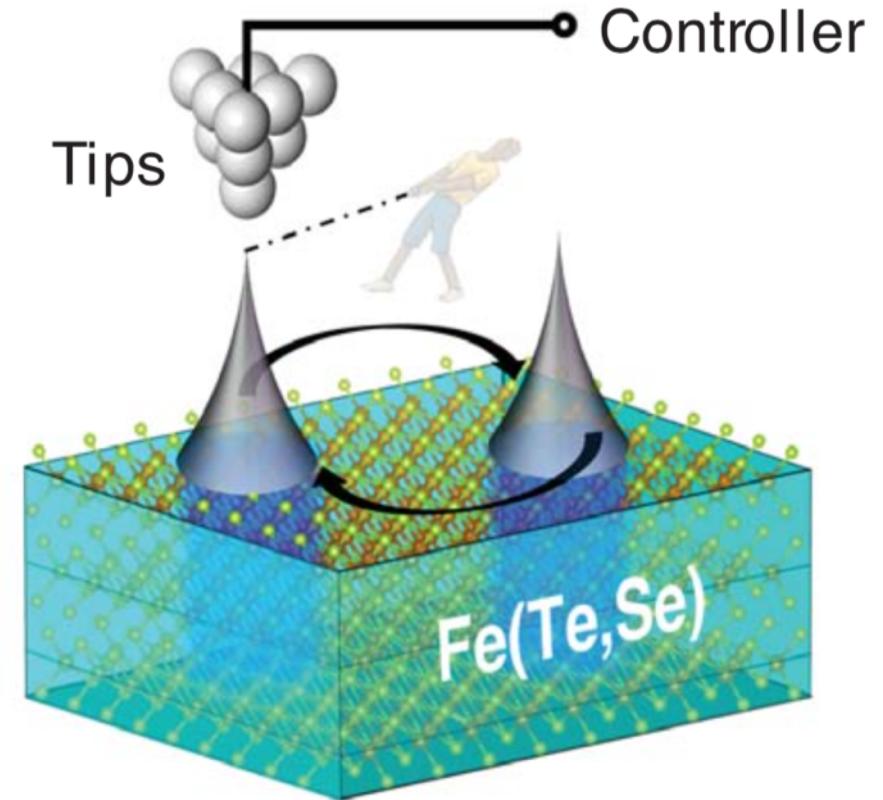
JCCM_December_2018_03

Spontaneous vortex formation and Majorana zero mode in iron based superconductor.

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<https://www.condmatjclub.org>

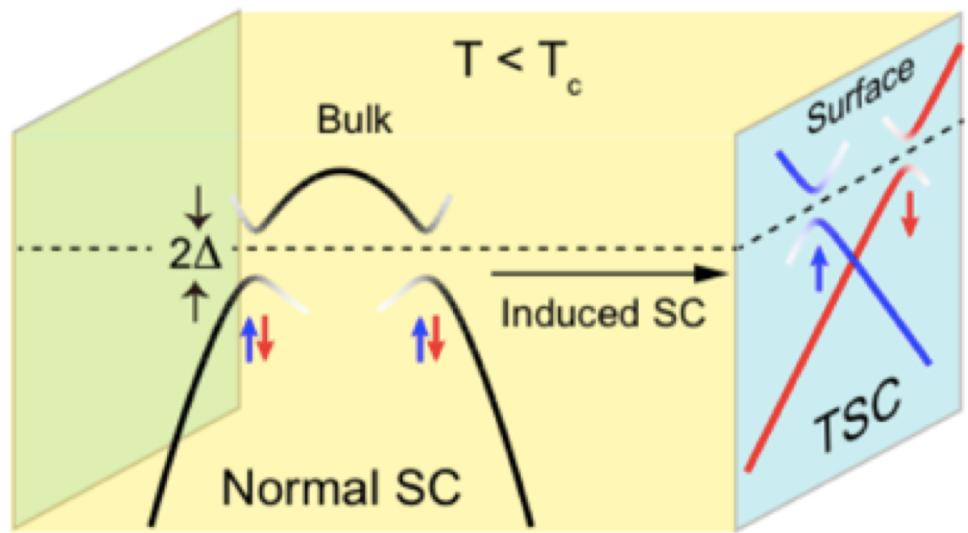
JCCM_June_2019_02

Trails of Mobile Majoranas in an Iron Chalcogenide?



Exchanging MBSs

Iron Home for Majorana: Nature's Gift



S-wave + spin-helical

High- T_c SC

Small E_F , Large gap

Single Material

