

21 Oct, 2019



Interaction-driven symmetry protected exceptional torus with many-body chiral symmetry

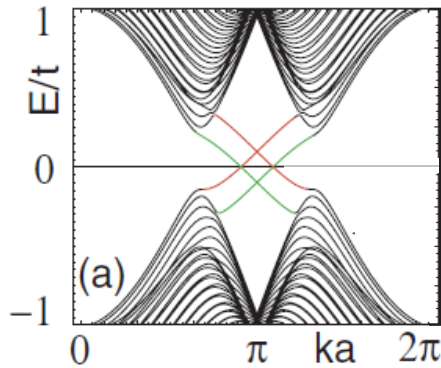
Speaker: Kazuhiro Kimura (Kyoto Univ.)

with T. Yoshida (Univ. of Tsukuba), N. Kawakami (Kyoto Univ.)

Topology has become a ubiquitous issue in condensed matter physics !

Hermitian topological phases

Topological insulator (TI)



- insulating Bulk
- metallic Edge

Protected by local symmetry

C. L. Kane, PRL 95 146802 (2005).

Extension of TIs.

- metallic (gapless) phase
⇒ topological semi-metals
S. Murakami, NJP 9 356 (2007).
- crystalline (non-local) symmetry
⇒ topological crystalline insulators
L. Fu, PRB 106 106802 (2011).

They have been of intense interest !

Non-Hermitian topological phases

The notion of band topology extended to **non-Hermitian** matrix

⇒ New type topological phenomena!

- Open quantum system
Cold atom w/ dissipation
- Classical optics
Photonic crystal w/ gain & loss
- Another aspect of Equilibrium
Lifetime effect

We focus on it in **strongly correlated system.**

Introduction

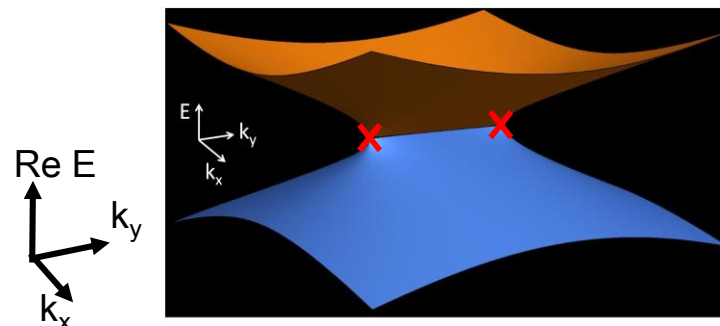
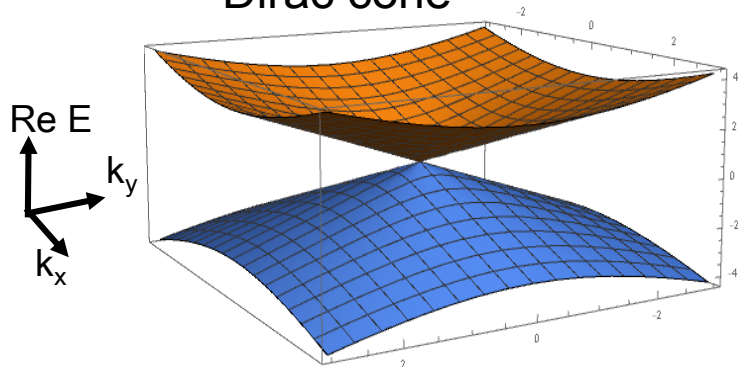
Non-Hermiticity

Non-Hermitian Band theory

$$H(\mathbf{k}) = (k_x + i\kappa_x)\sigma_x + (k_y + i\kappa_y)\sigma_y + (m + i\delta)\sigma_z,$$

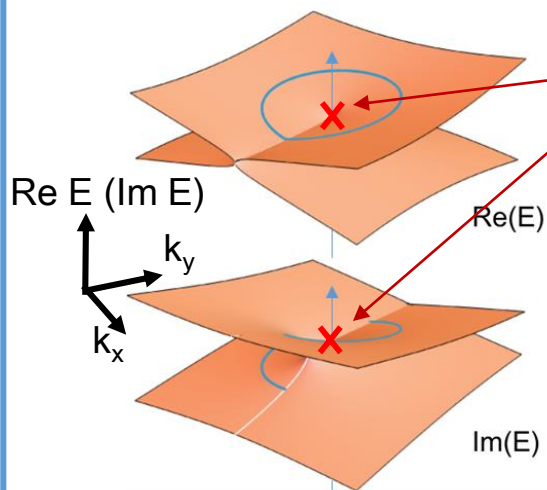
A pair of Exceptional Points

Dirac cone



Topological number on complex energy plane

H. Shen, PRL 120, 146402 (2018)



Exceptional point

Winding number on the complex E plane

$$\nu_{mn}(\Gamma) = -\frac{1}{2\pi} \oint_{\Gamma} \nabla_{\mathbf{k}} \arg [E_m(\mathbf{k}) - E_n(\mathbf{k})] \cdot d\mathbf{k},$$

Topologically stable band degeneracies appear

Non-Hermiticity induced by the lifetime effect !!

V. Kozii and L. Fu arXiv (2017)

Effective Hamiltonian

$$H_{\text{eff}}(\mathbf{k}, \omega) = H_0(\mathbf{k}) + \Sigma(\omega)$$

$$i \text{Im} \Sigma(\omega) = \begin{pmatrix} i\Gamma_1(\omega) & 0 \\ 0 & i\Gamma_2(\omega) \end{pmatrix}$$

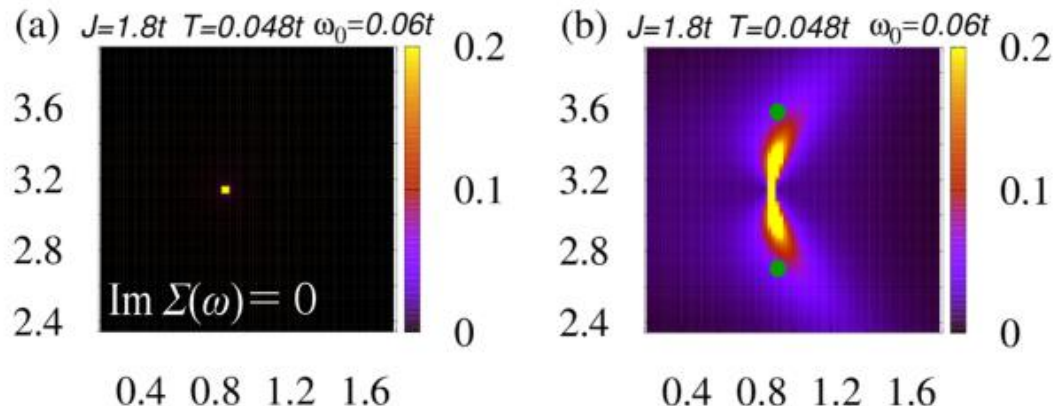
Spectral function

$$A_{\mathbf{k}}(\omega) = -\frac{1}{\pi} \text{Im}[\text{tr} G_{\mathbf{k}}(\omega)]$$

$$G_{\mathbf{k}}(\omega) = [(\omega + i\delta)1 - H_{\text{eff}}(\mathbf{k}, \omega)]^{-1}$$

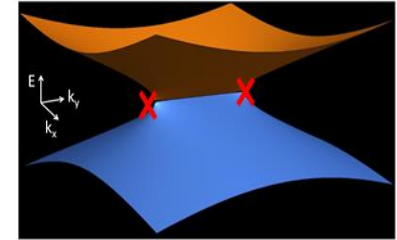
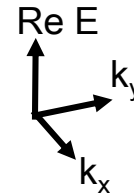
Heavy-Fermions (DMFT)

$A_{\mathbf{k}}(\omega = 0)$



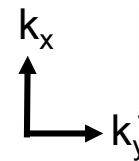
Y. Nagai, (JPS Meeting 2018). T. Yoshida, PRB 98, 035141 (2018)

Dispersion of $H_{\text{eff}}(\mathbf{k}, 0)$



$A_{\mathbf{k}}(\omega = 0)$

open Fermi surface



V. Kozii & L. Fu arXiv (2017)

NH band degeneracy & symmetry protection

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2x2 Non-Hermitian Hamiltonian

$$H_{\text{eff}} = (d_0 + ib_0) \cdot \tau_0 + (\mathbf{d} + i\mathbf{b}) \cdot \boldsymbol{\tau}$$

$$E_{\pm} = d_0 + ib_0 \pm \sqrt{\mathbf{d}^2 - \mathbf{b}^2 + 2i\mathbf{b} \cdot \mathbf{d}}$$

Condition of Band degeneracy

$$\mathbf{d}^2 - \mathbf{b}^2 = 0, \quad \mathbf{b} \cdot \mathbf{d} = 0$$

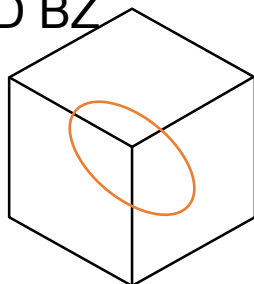
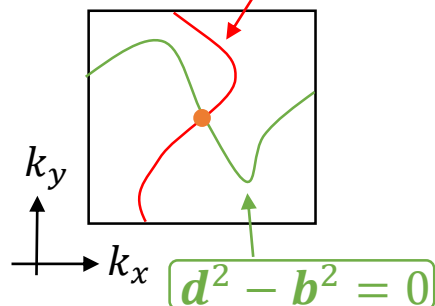
+ $\mathbf{b} \neq 0 \Rightarrow \mathbf{H}$ is defective

Defective manifold & dimension

2D BZ

$$\mathbf{b} \cdot \mathbf{d} = 0$$

3D BZ



Defective region

= system dim - constraint cond.

$$0 = 2 - 2 \Rightarrow \text{point}$$

$$1 = 3 - 2 \Rightarrow \text{ring}$$

Symmetry (Ex. parity & time-reversal)

$$(PT)H_{\text{eff}}(\mathbf{k})(PT)^{-1} = H_{\text{eff}}(\mathbf{k})$$

$$PT = K\sigma_0 \quad K: \text{complex conjugation}$$

R. Okugawa, PRB2019

$\mathbf{b} \cdot \mathbf{d} = 0$ is automatically satisfied

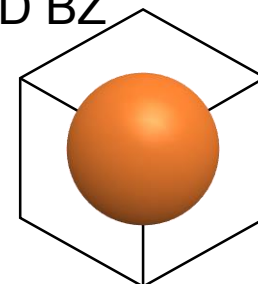
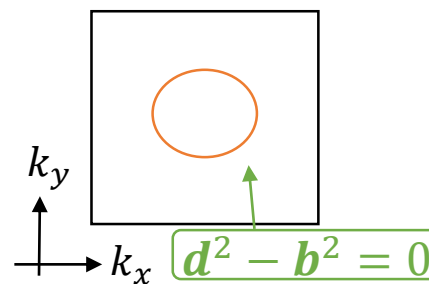
Condition of Band degeneracy

$$\mathbf{d}^2 - \mathbf{b}^2 = 0,$$

Defective manifold

2D BZ

3D BZ

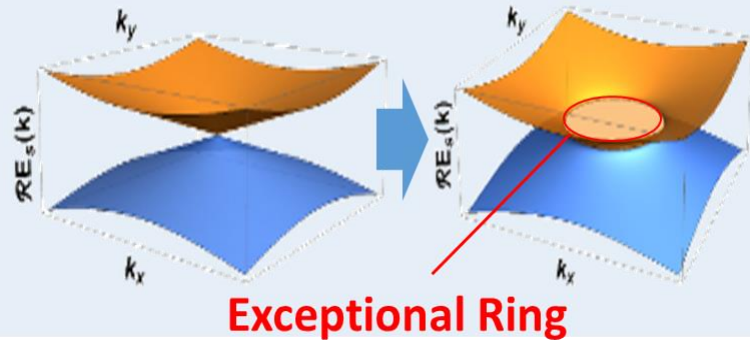


$$1 = 2 - 1 \Rightarrow \text{ring}$$

$$2 = 3 - 1 \Rightarrow \text{surface/torus}$$

Without Symmetry protection

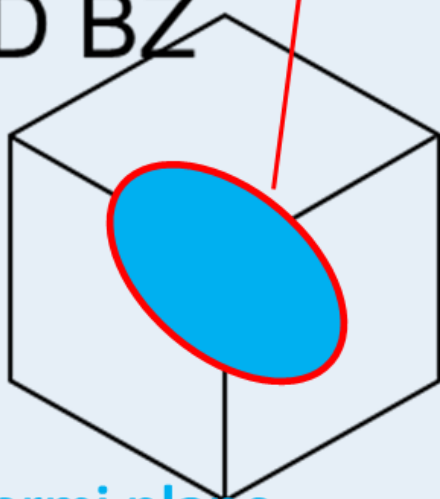
Weyl semi-metal + disorder



Y. Xu, PRL 118, 045701 (2017),
A. A. Zyuzin, PRB 97, 041203(R) (2018)

Exceptional Ring

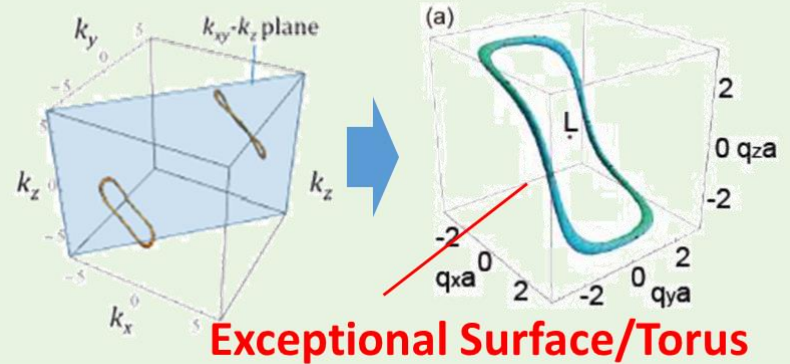
3D BZ



Fermi plane

With Symmetry Protection

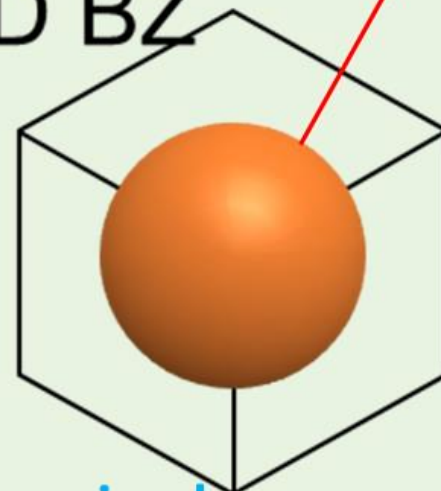
Nodal-line semimetals + gain & loss



R. Okugawa, PRB 99, 041202 (2019)

Exceptional surface

3D BZ



Fermi volume

But almost all studies have been discussed about energy dispersion so far...

Previous study: Symmetry-protected NH band degeneracy

R. Okugawa & T. Yokoyama, PRB 99, 041202 (2019)
J. C. Budich, et. al., PRB 99, 041406 (2019)
T. Yoshida, et. al., PRB 99, 121101 (2019)
K. Kawabata, T. Bessho, & M. Sato, PRL 123, 066405 (2019)
T. Yoshida & Y. Hatugai, PRB 100, 054109 (2019)

Our study

Can the Symmetry-Protected Exceptional Torus emerge in strongly correlated system ?

We focus on nodal-line semimetals
with many-body chiral symmetry.

Effect of many-body chiral symmetry ?

**Emergence of 3D open Fermi surface enclosed by ET !
EP locked on the Fermi level**

How physical properties affected by NH structure ?

NH structure on the Fermi level increase the Spin susceptibility !

The main idea: Many-body chiral symmetry

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Many-body chiral symmetry

$$\hat{U}_\Gamma^\dagger \hat{H}^* \hat{U}_\Gamma = \hat{H}$$

$$\hat{U}_\Gamma^\dagger \hat{c}_{is\sigma}^\dagger \hat{U}_\Gamma = \text{sgn}(s) \hat{c}_{is\sigma}^\dagger$$

$\text{sgn}(s)$ takes 1 & -1 for $s=A$ & $s=B$

Green's function

$$G(\omega + i\delta) = -U_\Gamma^\dagger G^\dagger(-\omega + i\delta) U_\Gamma,$$

chiral matrix $U_\Gamma = \tau_z$

$$G(\mathbf{k}, \omega) = [\omega \mathbb{1} - h(\mathbf{k}) - \Sigma^R(\mathbf{k}, \omega)]^{-1},$$

Effective Hamiltonian

$$H_{\text{eff}}(\omega, \mathbf{k}) = -U_\Gamma^\dagger H_{\text{eff}}^\dagger(-\omega, \mathbf{k}) U_\Gamma$$

$$G^{-1}(\omega + i\delta) = \omega \mathbb{1} - H_{\text{eff}}(\omega, \mathbf{k}).$$

Focus on the Fermi level

Many-body chiral symmetry

$$H_{\text{eff}}(k, 0) = -\tau_z H_{\text{eff}}^\dagger(k, 0) \tau_z$$

T. Yoshida, PRB 99, 121101 (2019)

2×2 Non-Hermitian Hamiltonian

$$H_{\text{eff}}(k, 0) = (d_0 + ib_0) \tau_0 + (\mathbf{d} + i\mathbf{b}) \cdot \boldsymbol{\tau}$$

$$E_\pm = d_0 + ib_0 \pm \sqrt{\mathbf{d}^2 - \mathbf{b}^2 + 2i\mathbf{b} \cdot \mathbf{d}}$$

$$\Rightarrow d_0 = 0, \mathbf{d} = \begin{pmatrix} d_1 \\ d_2 \\ 0 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 0 \\ 0 \\ b_3 \end{pmatrix}$$

$\mathbf{b} \cdot \mathbf{d} = 0$ is satisfied

on the Fermi level !!

**Symmetry-protected EPs
are locked on $\omega = 0$!**

Model & Method

Hubbard model on diamond lattice @ half-filling

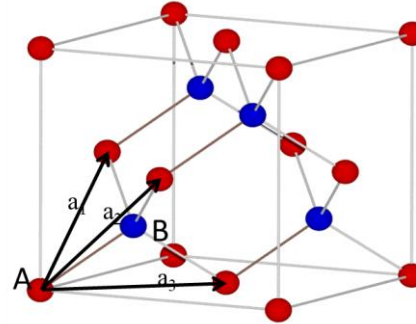
with spatially modulated on-site interaction $U_A \neq U_B$

$$\hat{H} = \sum_{\langle i\alpha, j\alpha' \rangle \sigma} t_{ij} \hat{c}_{i\alpha\sigma}^\dagger \hat{c}_{j\alpha'\sigma} + \sum_{i\alpha} U_\alpha \left(\hat{n}_{i\alpha\uparrow} - \frac{1}{2} \right) \left(\hat{n}_{i\alpha\downarrow} - \frac{1}{2} \right),$$

Noninteracting part

$$h(\mathbf{k}) = \begin{pmatrix} 0 & D_{\mathbf{k}} \\ D_{\mathbf{k}}^* & 0 \end{pmatrix} \otimes \sigma_0 \leftarrow \text{Spin}$$

$$D_{\mathbf{k}} = t_0 + \sum_{j=1,2,3} t_j e^{i\mathbf{k} \cdot \mathbf{a}_j}$$



DMFT

Impurity solver : Iterated perturbation theory (IPT)

$$\Sigma_\alpha^{(2)}(\omega) = U^2 \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dz \rho_\alpha^0(x) \rho_\alpha^0(y) \rho_\alpha^0(z) \frac{f(-x)f(-y)f(z) + f(x)f(y)f(-z)}{\omega - x - y + z + i\delta},$$

DMFT-Random Phase Approximation

$$\chi_{\alpha\beta}^0(\mathbf{q}, i\epsilon_m) = -\frac{T}{N} \sum_{\mathbf{k}, n} \underline{G_{\alpha\beta}(\mathbf{q} + \mathbf{k}, i\omega_n + i\epsilon_m) G_{\beta\alpha}(\mathbf{k}, i\omega_n)},$$

Green's function from DMFT

$$\chi^{\text{RPA}}(\mathbf{q}, i\epsilon_m) := (\mathbf{1} - \chi^0 U)^{-1} \chi^0, \quad U := \text{diag}(U_A, U_B),$$

Result: Interaction driven SPETs

Emergence of Chiral-Symmetry Protected Exceptional Torus

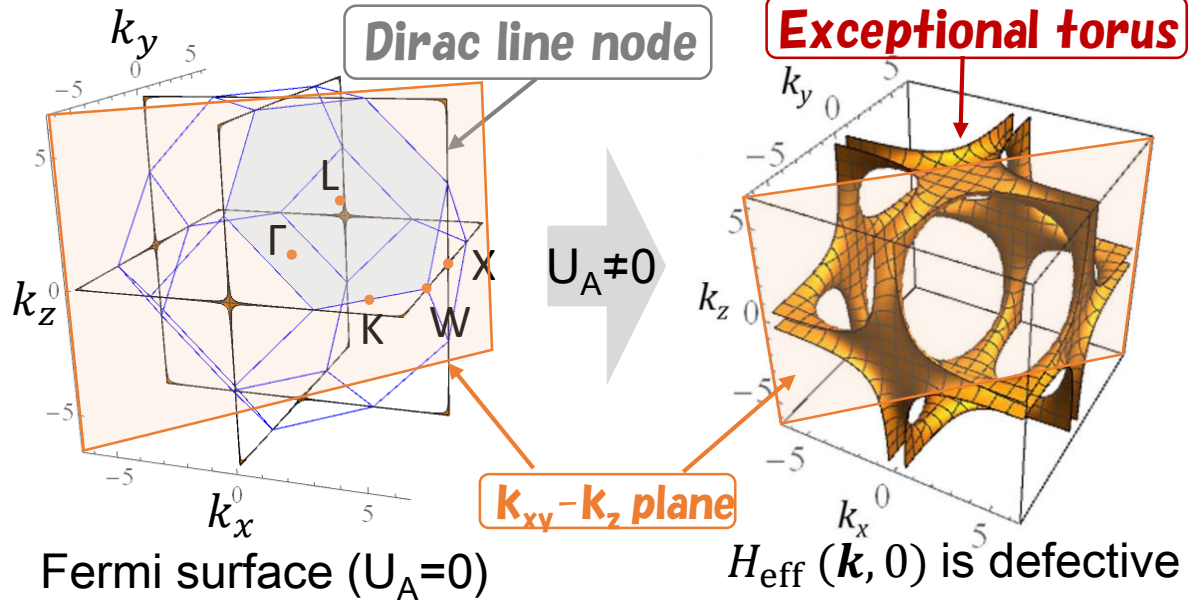
Effective Hamiltonian

$$H_{\text{eff}}(\mathbf{k}, \omega) = H_{\mathbf{k}} + \Sigma^R(\omega)$$

from DMFT-IPT

$$= \begin{pmatrix} 0 & D_{\mathbf{k}} \\ D_{\mathbf{k}}^* & 0 \end{pmatrix} + \begin{pmatrix} \Sigma_A(\omega) & 0 \\ 0 & 0 \end{pmatrix}$$

@T=0.8t, $U_A=8t$, $U_B=0$



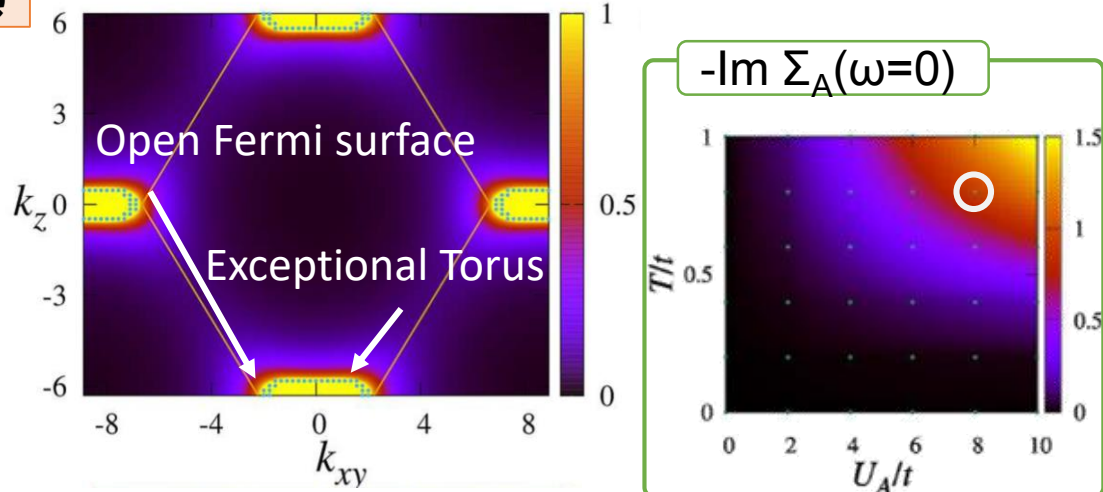
Spectral function on $k_{xy}-k_z$ plane

Spectral function

$$A_{\mathbf{k}}(\omega = 0) = -\frac{1}{\pi} \text{Im}[\text{tr}G_{\mathbf{k}}(0)]$$

$$G_{\mathbf{k}}(\omega) = [(\omega + i\delta)1 - H_{\text{eff}}(\mathbf{k}, \omega)]^{-1}$$

@T=0.8t, $U_A=8t$, $U_B=0$

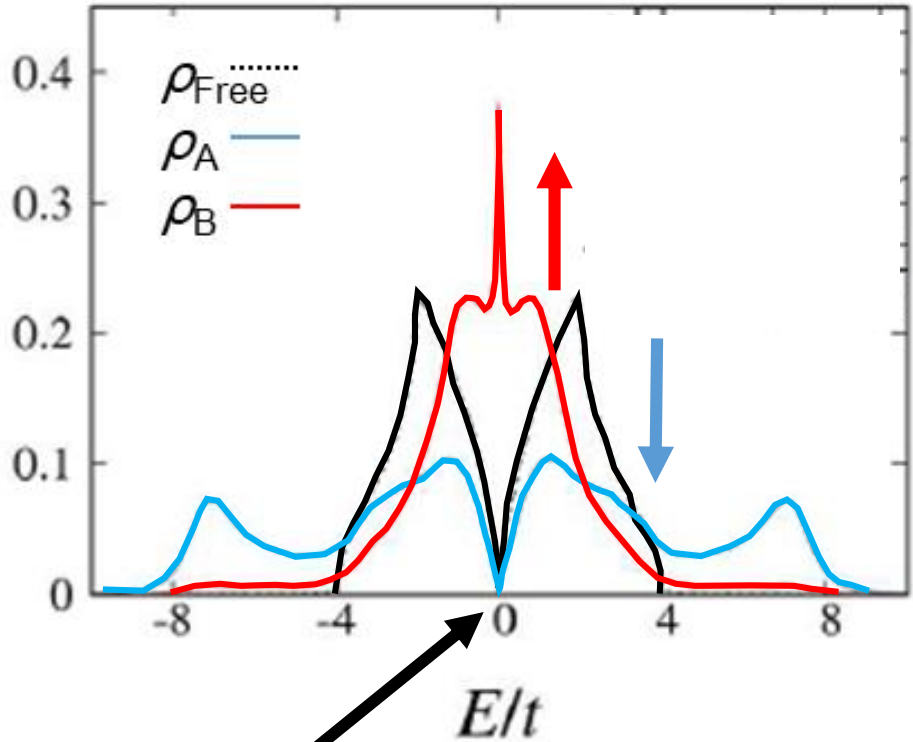


Result: The peak structure of LDOS

Local Density of State

@T=0.8t, $U_A=8t$, $U_B=0$

$$\rho_\alpha(\omega) = -\frac{1}{\pi} \text{Im}[G_k(\omega)]_{\alpha\alpha}$$



Topological semimetal

DOS of A is suppressed (renormalization)

But the DOS of B is enhanced with increasing U_A (& T) !!

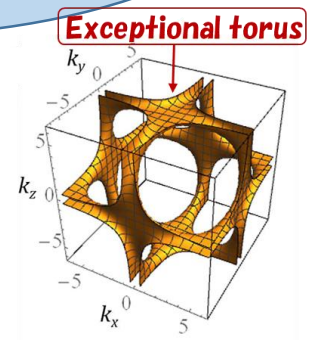
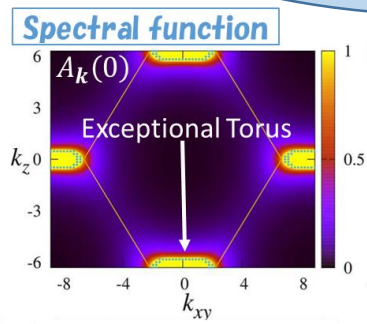
not renormalization factor but lifetime effect of $\Sigma(\omega)$!!

non-Hermitian effect

Strange !? But we recall ETs!!

Peak structure is formed by ET

@ $\omega=0$ locked by Chiral symmetry



Result: Magnetic susceptibilities

Scenario

LDOS of B is enhanced by Chiral-symmetry protected ET

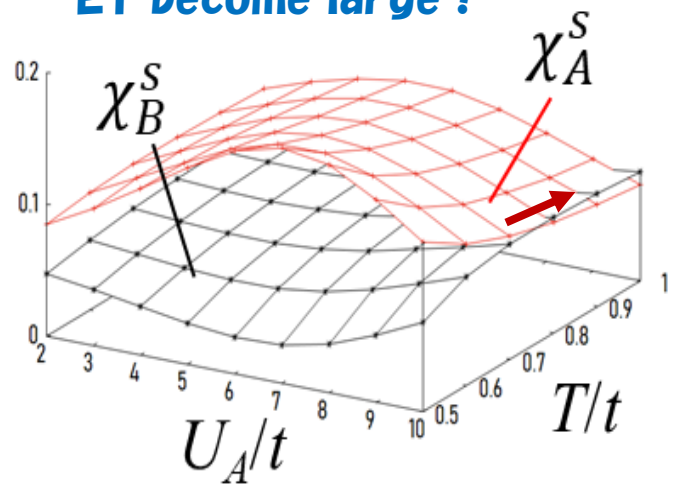
Does the magnetic response for B-sublat. become large?

$$\chi_B = \frac{M_B}{B} \Big|_{B \rightarrow 0}$$

Numerical data form DMFT-RPA

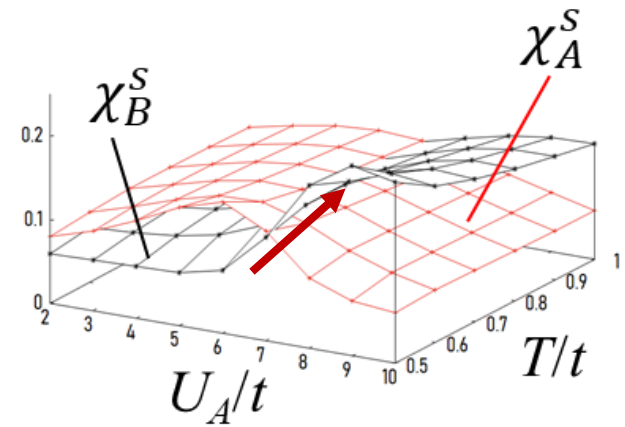
With increasing U_A (& T),

ET become large !



χ_B weakly enhanced !

Another parm. $U_B = U_A/2$



χ_B enhanced by forming of ET!

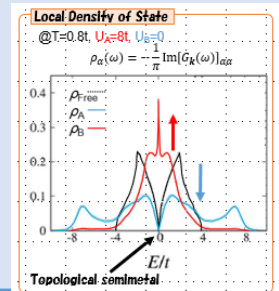
Magnetic susceptibility $\chi_B^S > \chi_A^S$ although Interaction is opposite $U_B < U_A$.

Interaction-driven Chiral-Symmetry Protected Exceptional Torus

- The low-energy excitations are locked on the Fermi level.
“Fermi volume” enclosed by ET

- The unusual peak structure of LDOS induced by ET with increasing T & U_A .

This is not usual renormalization effect but the lifetime effect !!



- How is the physical properties affected by ET ?

The magnetic response for B-sublat. becomes large.