Thermal Hall effect in correlated materials

3rd EPiQS-TMS alliance workshop on
Topological Phenomena in Quantum Materials
(TPQM2019)
KITP, Santa Barbara, October 22, 2019

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

PHYSICS
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Giant thermal Hall conductivity in the pseudogap phase of cuprate superconductors

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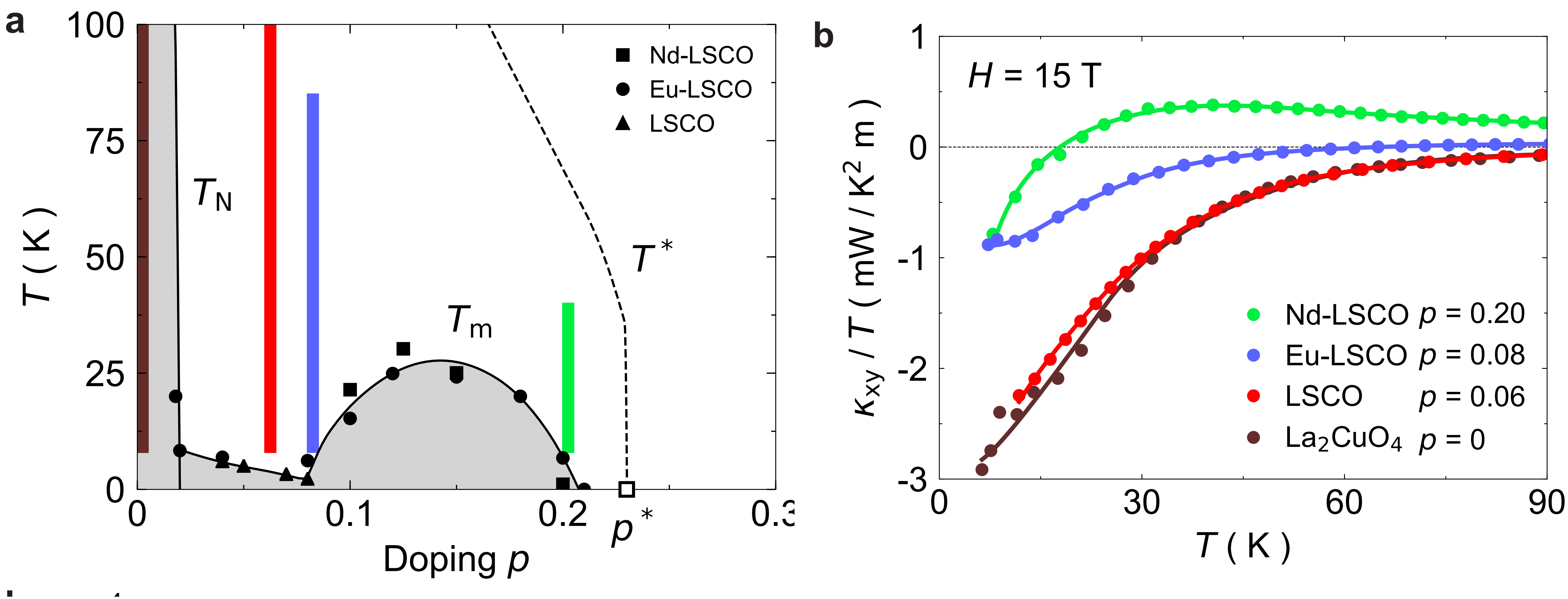

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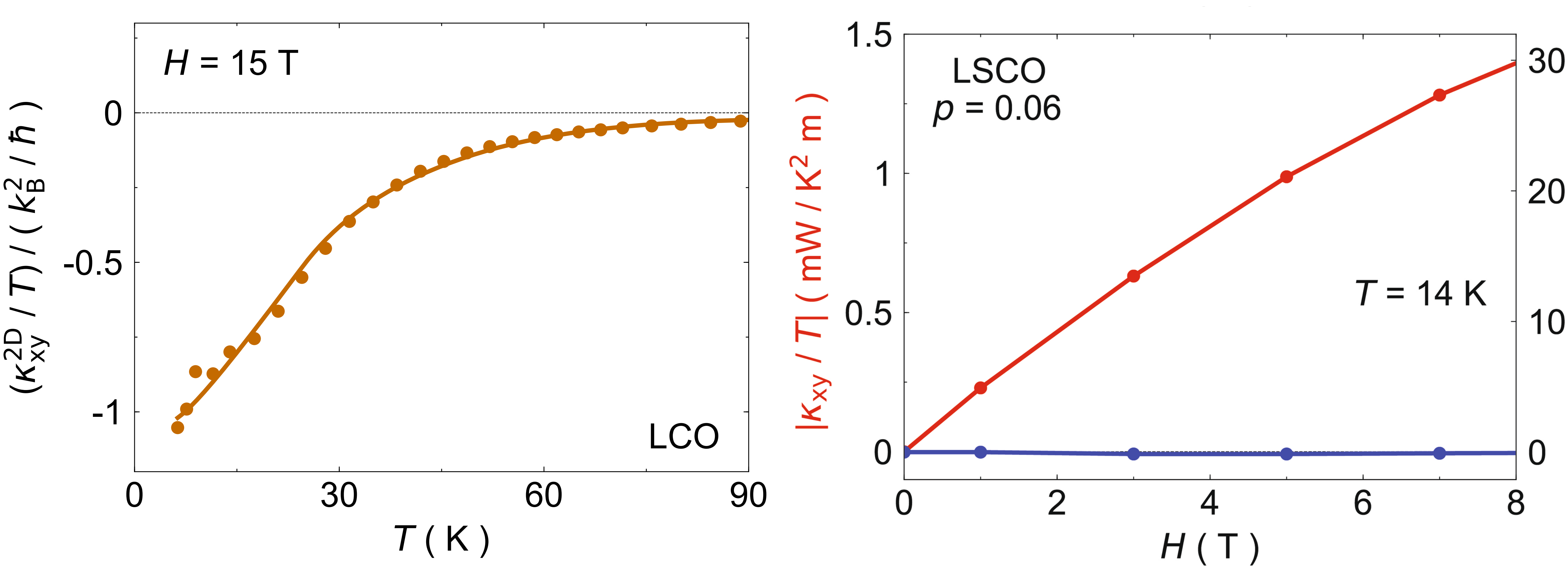
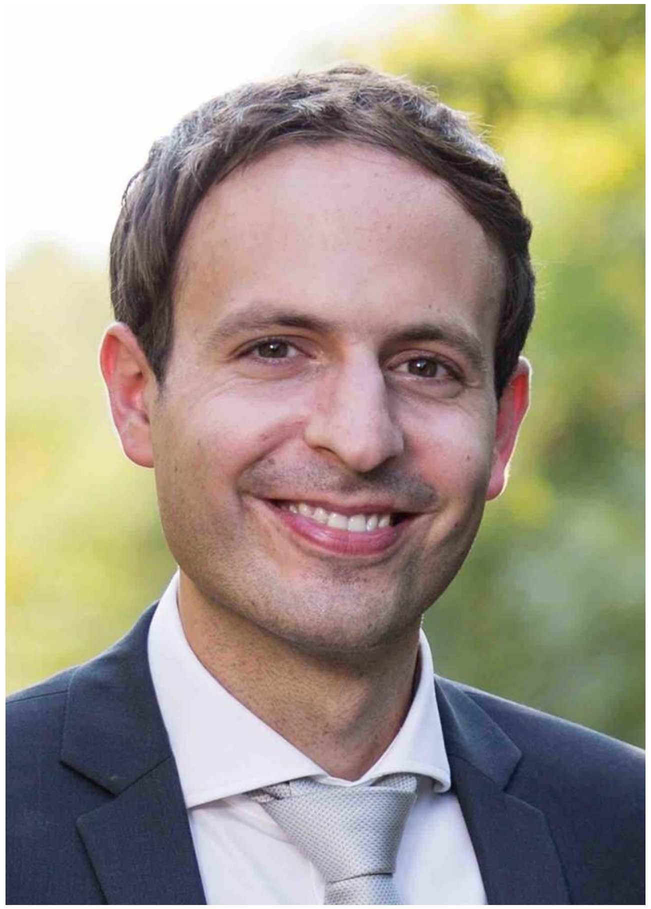


Enhanced thermal Hall effect in the square-lattice Néel state

arXiv:1903.01992, Nature Physics (2019)

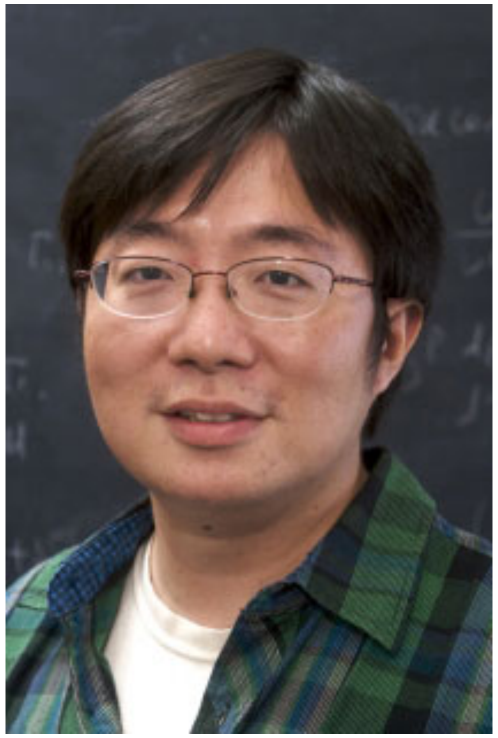
Rhine Samajdar



Mathias Scheurer

Shubhayu Chatterjee

Haoyu Guo



Cenke Xu

Enhanced thermal Hall effect in the square-lattice Néel state

arXiv:1903.01992, Nature Physics (2019)

- The ground state of the square lattice antiferromagnet is a conventional Neel state.
- In a sufficiently large orbital magnetic field, there is a quantum transition to a "chiral spin liquid" co-existing with conventional Neel order.
- Proximity to this quantum transition can enhance the thermal Hall effect at non-zero temperatures, even though the ground state is conventional.

1. Insulator

A. Thermal Hall conductivity across the Neel/Neel+CSL quantum transition

B. Quantum criticality and non-Abelian dualities

2. Pseudogap at non-zero doping

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The Néel state of square lattice antiferromagnets described by

$$H_1 = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \ldots$$

has zero transverse thermal conductivity $\kappa_{xy}/T = 0$.

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In the presence of a magnetic field

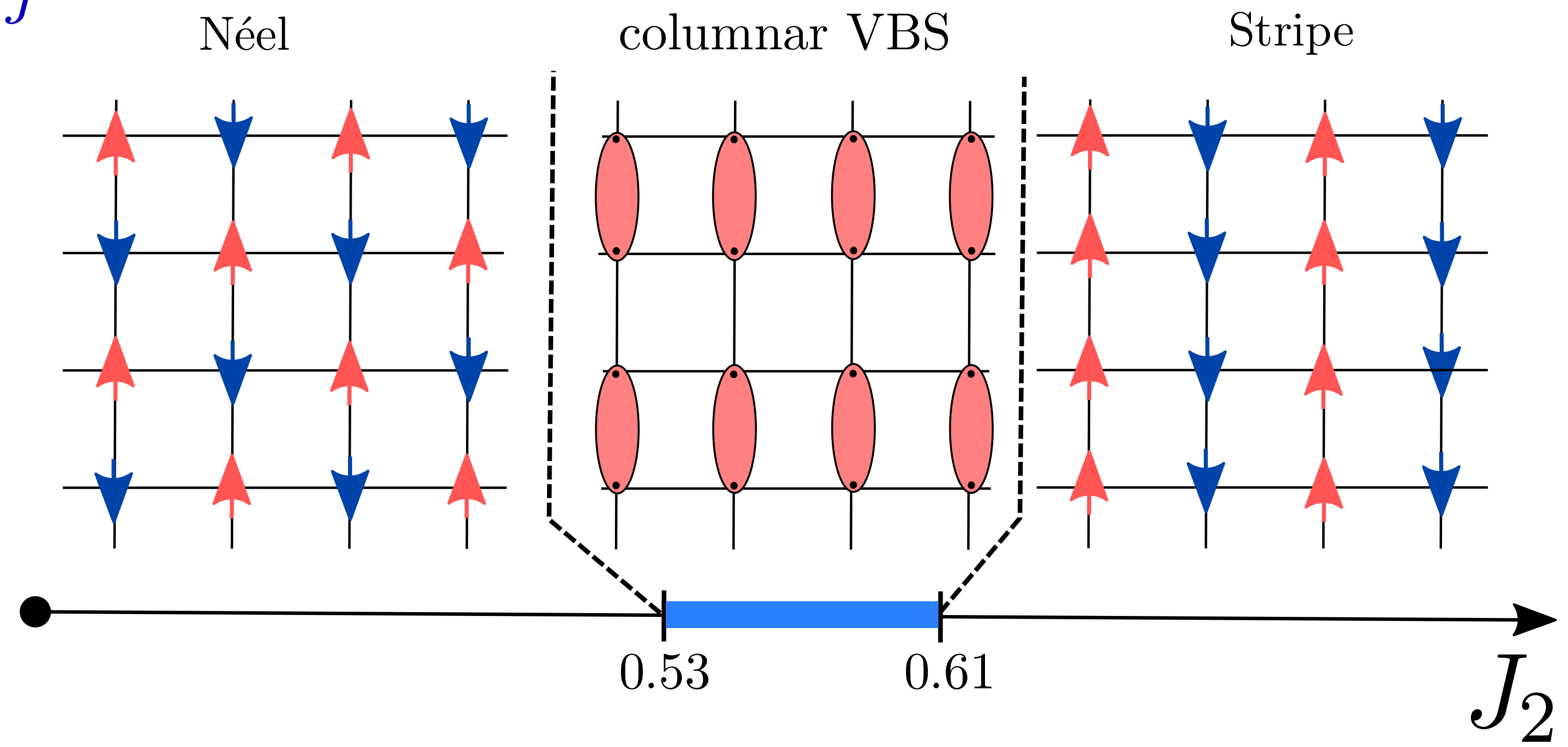
$$H_B = J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot \left(\boldsymbol{S}_j \times \boldsymbol{S}_k \right) - \sum_i \boldsymbol{B}_Z \cdot \boldsymbol{S}_i \,.$$

The *orbital coupling* $J_\chi \propto \boldsymbol{B}_\perp$ induces a non-zero Berry curvature in the spin-wave dispersion, which induces a non-zero κ_{xy}/T.

However, this Berry curvature is small at long wavelengths, and consequently the thermal Hall conductivity is very small $|\kappa_{xy}/T| \ll k_B^2/\hbar$, and vanishes rapidly as $T \to 0$.

$$H_1 = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Nearest ($J_1 = 1$) and next-nearest (J_2) neighbor interactions



$U(1)$-symmetric infinite projected entangled-pair states study of the spin-1/2 square J_1-J_2 Heisenberg model

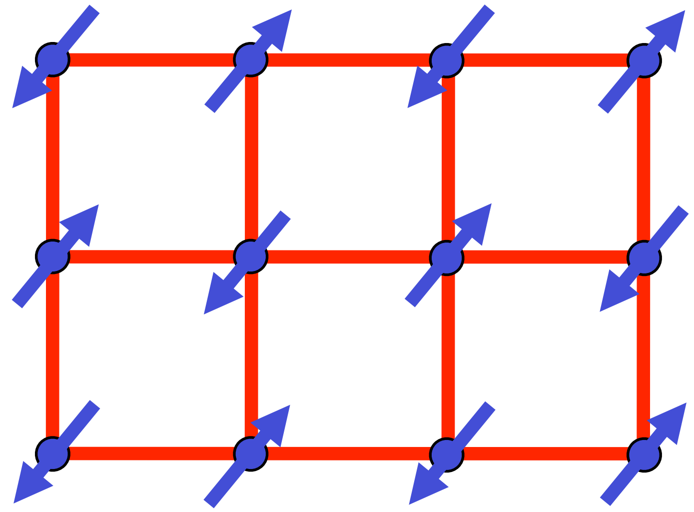
PHYSICAL REVIEW B **97**, 174408 (2018)

R. Haghshenas and D. N. Sheng

By studying the finite-D scaling of the magnetically order parameter, we find a Néel phase for $J_2/J_1 < 0.53$. For $0.53 < J_2/J_1 < 0.61$, a nonmagnetic columnar valence bond solid (VBS) state is established as observed by the pattern of local bond energy. The divergent behavior of correlation length $\xi \sim D^{1.2}$ and vanishing order parameters are consistent with a deconfined Néel-to-VBS transition at $J_2^{c1}/J_1 = 0.530(5)$, where estimated critical anomalous exponents are $\eta_s \sim 0.6$ and $\eta_d \sim 1.9$ for spin and dimer correlations, respectively.

Quantum criticality in a frustrated square lattice antiferromagnet

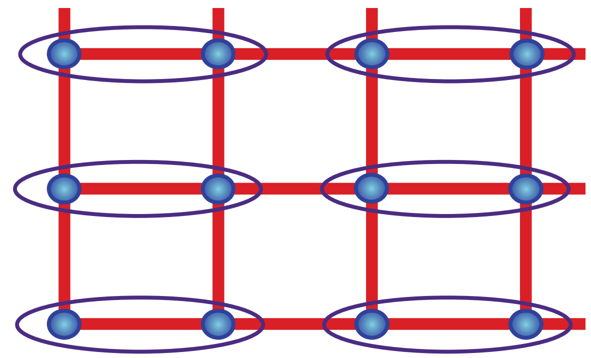
N. Read and S. Sachdev, PRL **62**, 1694 (1989)



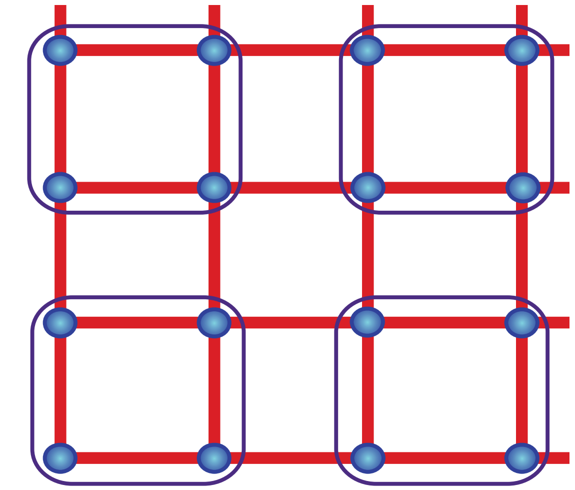
$\langle z_\alpha \rangle \neq 0$

Néel state

$\vec{N} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$



or



$\langle z_\alpha \rangle = 0$

Valence bond solid (VBS) state, V_x, V_y

with a nearly gapless, emergent "photon"

s_c

s

Critical $\mathbb{CP}^1$ theory for photons and deconfined spinons:

$$\mathcal{S}_z = \int d^2 r d\tau \left[|(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2}(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2 \right]$$

O.I. Motrunich and A. Vishwanath, *Phys. Rev. B* **70**, 075104 (2004).

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

A non-Abelian duality

Critical U(1) gauge (a_μ) theory of $N_b = 2$ relativistic bosons
is dual to
SU(2) gauge (A_μ) theory of $N_f = 2$ Dirac fermions.

$$\mathcal{S}_z = \int d^2 r d\tau \left[|(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2}(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2 \right]$$

$$\mathcal{S}_f = \int d^2 r d\tau \sum_{\alpha=1}^{2} \left[\overline{f}_\alpha \gamma^\mu (\partial_\mu - i A_\mu) f_\alpha \right]$$

Chong Wang, A. Nahum, M. A. Metlitski, Cenke Xu, and T. Senthil, PRX **7**, 031051 (2017)

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$$\mathcal{S}_f = \int d^2 r d\tau \sum_{\alpha=1}^{2} \left[\overline{f}_\alpha \gamma^\mu (\partial_\mu - i A_\mu) f_\alpha \right]$$

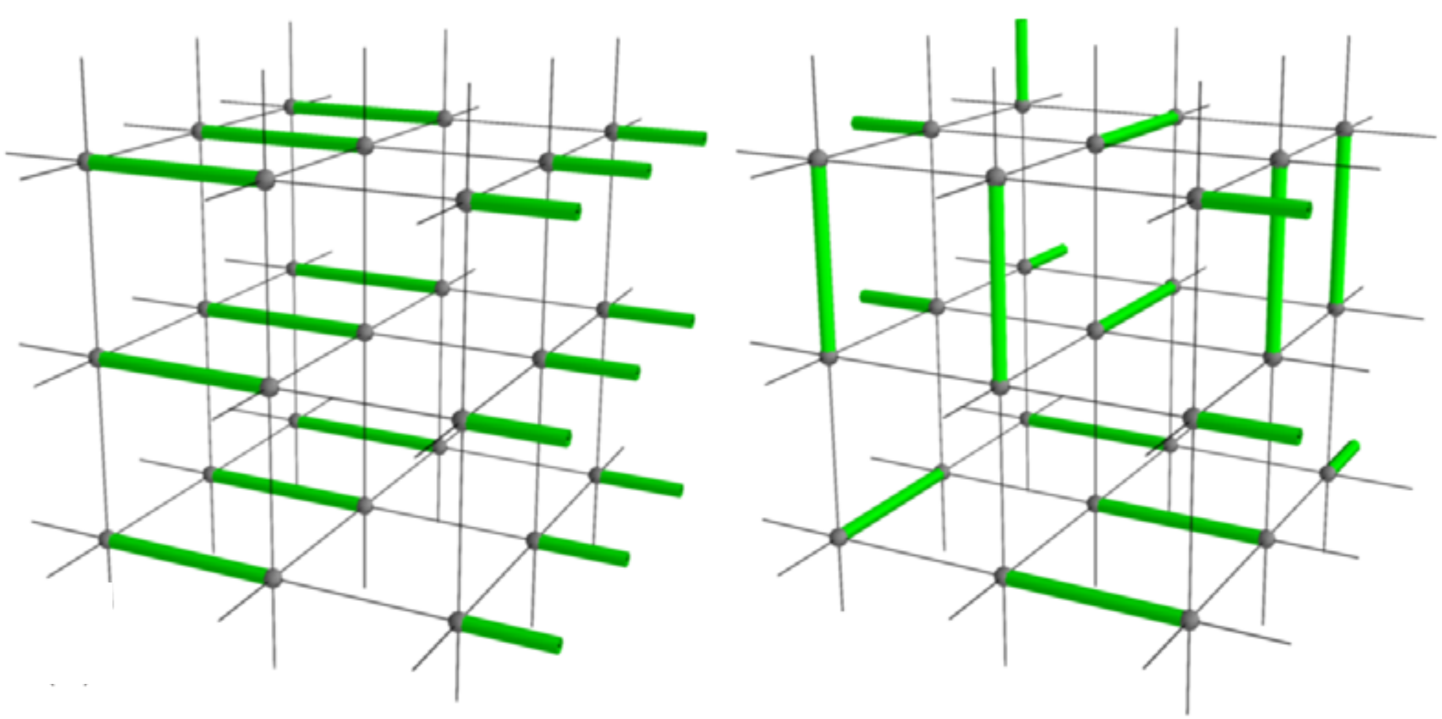
The fermion theory has a SO(5) global flavor symmetry, and the gauge-invariant fermion bilinears form a SO(5) vector which transforms as the Néel and VBS order parameters!

$$(N_x, N_y, N_z, V_x, V_y)$$

Akihiro Tanaka and Xiao Hu, PRL. **95**, 036402 (2005).
T. Senthil and M.P.A. Fisher, PRB **74**, 064405 (2006)
Chong Wang, A. Nahum, M.A. Metlitski, Cenke Xu, and T. Senthil, PRX **7**, 031051 (2017)

Emergent SO(5) Symmetry at the Columnar Ordering Transition in the Classical Cubic Dimer Model

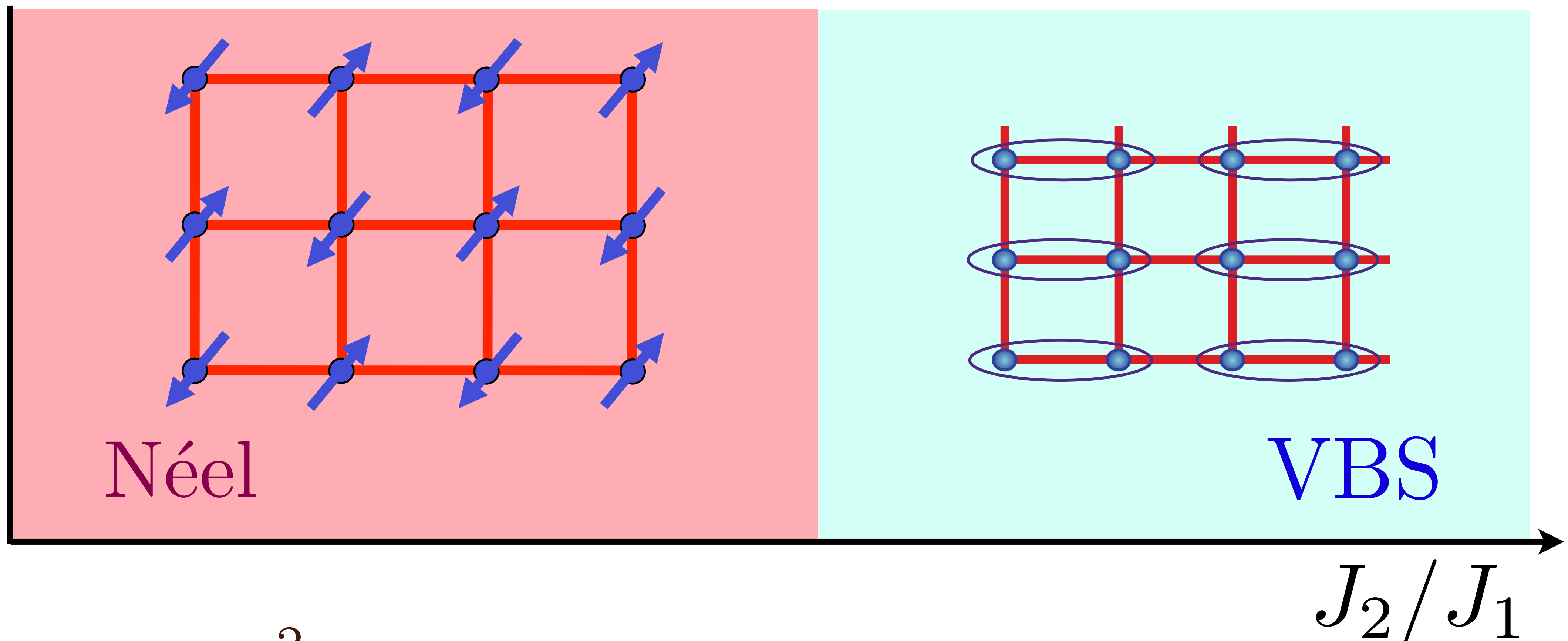
"Studying linear system sizes up to L=96, we find that this symmetry applies with an excellent precision, consistently improving with system size over this range. It is remarkable that SO(5) emerges in a system as basic as the cubic dimer model, with only simple discrete degrees of freedom. Our results are important evidence for the generality of the SO(5) symmetry that has been proposed for the noncompact CP^1 field theory. We describe an interpretation for these results in terms of a consistent hypothesis for the renormalization-group flow structure, allowing for the possibility that SO(5) may ultimately be a near-symmetry rather than exact."



G.J. Sreejith, Stephen Powell, and Adam Nahum
PRL **122**, 080601 (2019)

Stephen Powell and John T. Chalker,
PRB **80**, 134413 (2009)

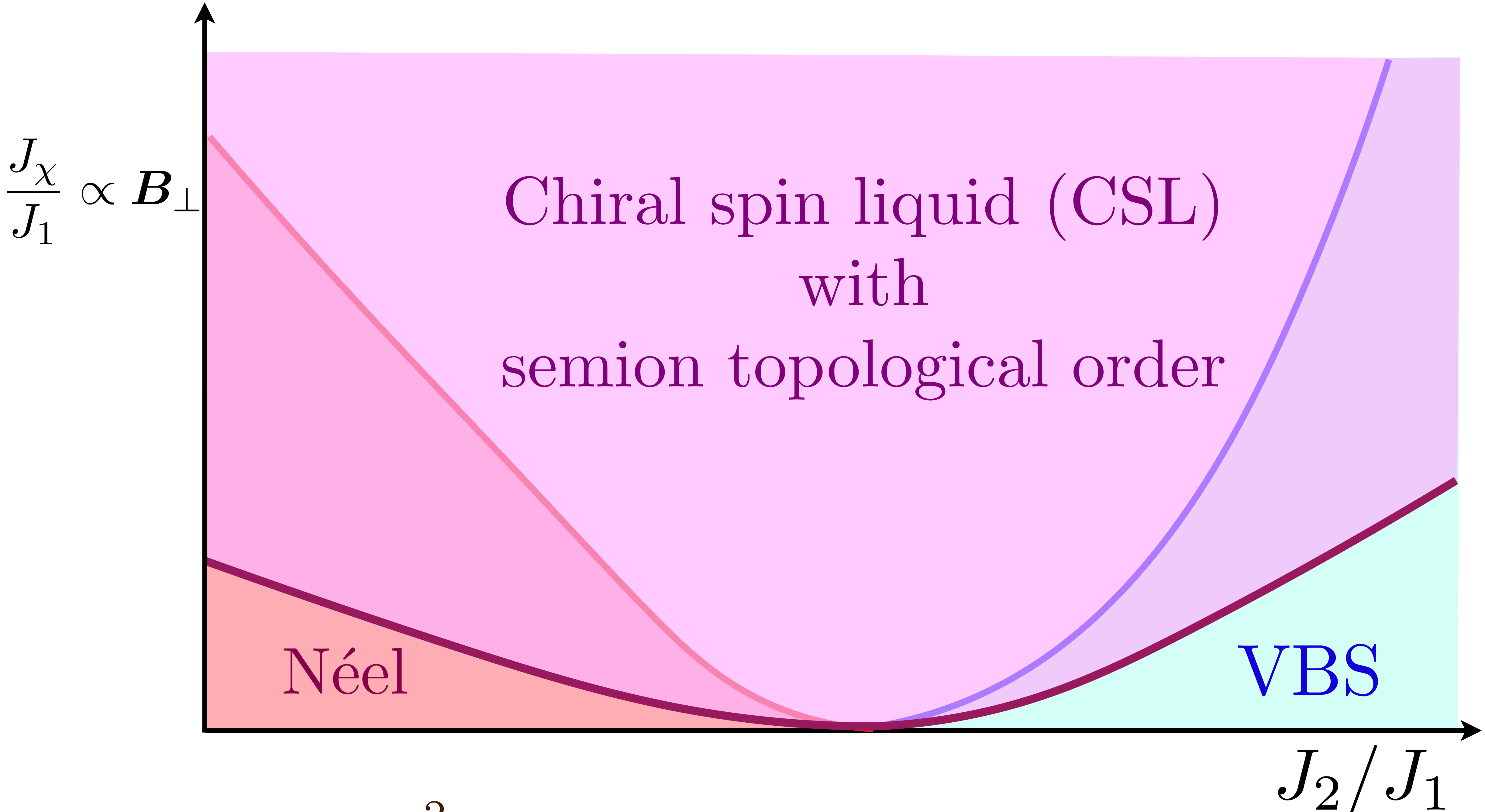
$$H = \sum_{\text{n.n.}} J_1 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \sum_{\text{n.n.n.}} J_2 \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$



$$\mathcal{S}_f = \int d^2 r d\tau \sum_{\alpha=1}^{2} \left[\overline{f}_\alpha \gamma^\mu (\partial_\mu - i A_\mu) f_\alpha \right]$$

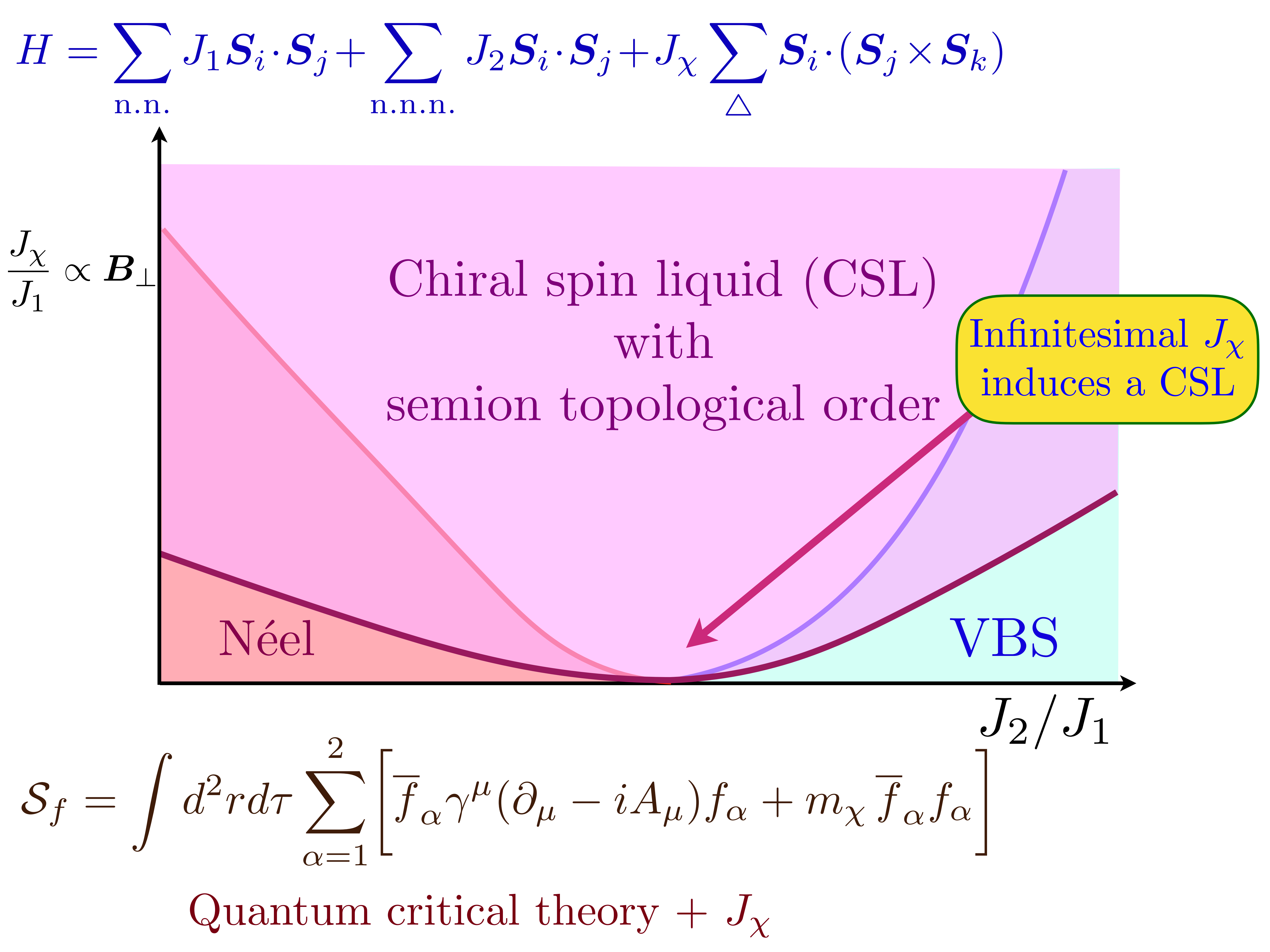
Quantum critical theory

$$H=\sum_{\text{n.n.}} J_1 \boldsymbol{S}_i\cdot\boldsymbol{S}_j+\sum_{\text{n.n.n.}} J_2 \boldsymbol{S}_i\cdot\boldsymbol{S}_j+J_\chi\sum_{\triangle}\boldsymbol{S}_i\cdot(\boldsymbol{S}_j\times\boldsymbol{S}_k)$$

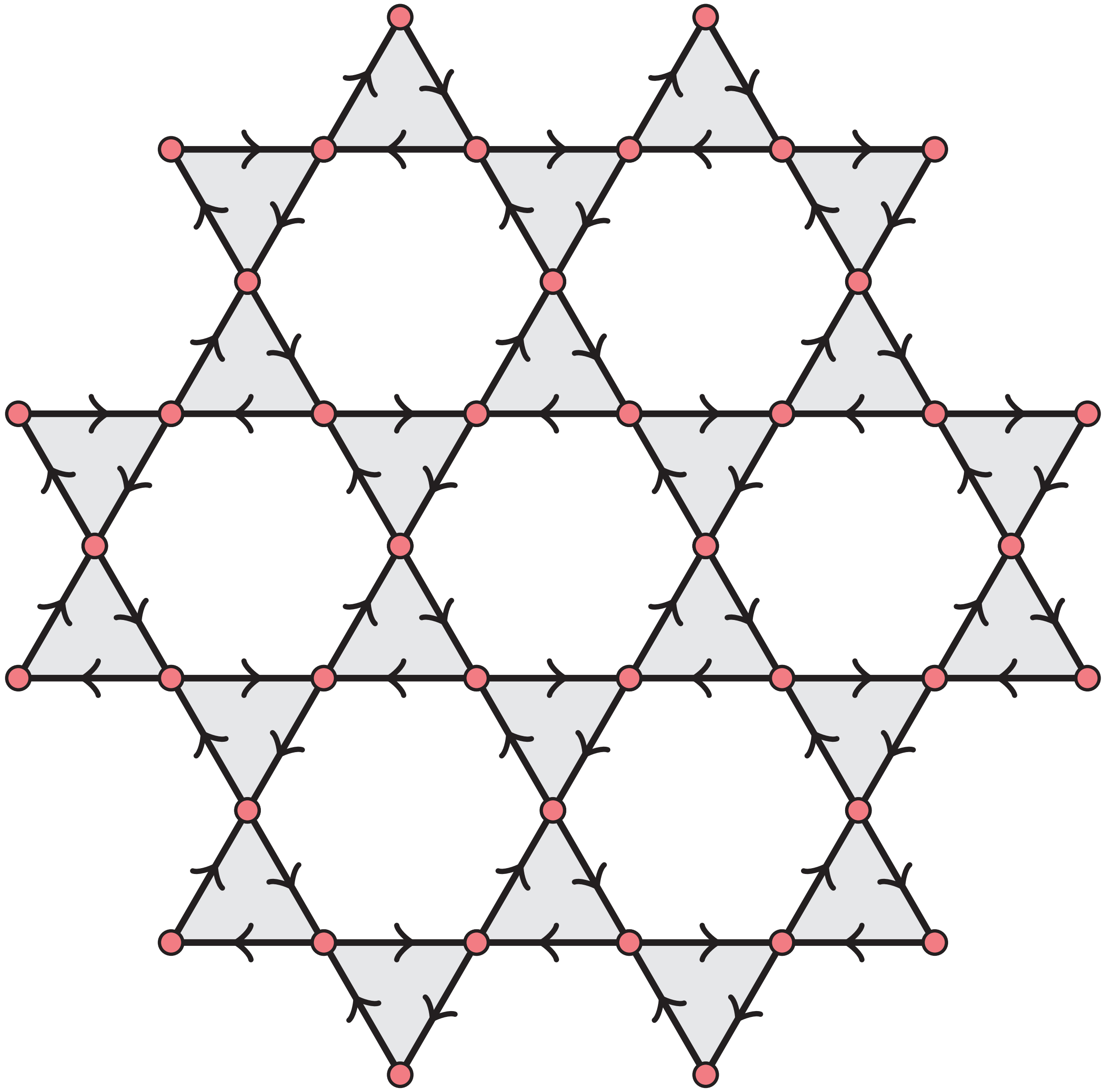


$$\mathcal{S}_f=\int d^2r d\tau \sum_{\alpha=1}^{2}\left[\overline{f}_\alpha\gamma^\mu(\partial_\mu-iA_\mu)f_\alpha+m_\chi\overline{f}_\alpha f_\alpha\right]$$

Quantum critical theory $+\ J_\chi$



$H = \sum_{\text{n.n.}} J_1 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \sum_{\text{n.n.n.}} J_2 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k)$
$\frac{J_\chi}{J_1} \propto \boldsymbol{B}_\perp$
Chiral spin liquid (CSL)
with
semion topological order
Infinitesimal J_χ induces a CSL
Néel
VBS
J_2/J_1
$\mathcal{S}_f = \int d^2 r d\tau \sum_{\alpha=1}^{2} \left[\overline{f}_\alpha \gamma^\mu (\partial_\mu - i A_\mu) f_\alpha + m_\chi \overline{f}_\alpha f_\alpha \right]$
Quantum critical theory + J_χ



$$H = H_1 + H_\chi$$

$$H_1 = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \ldots$$

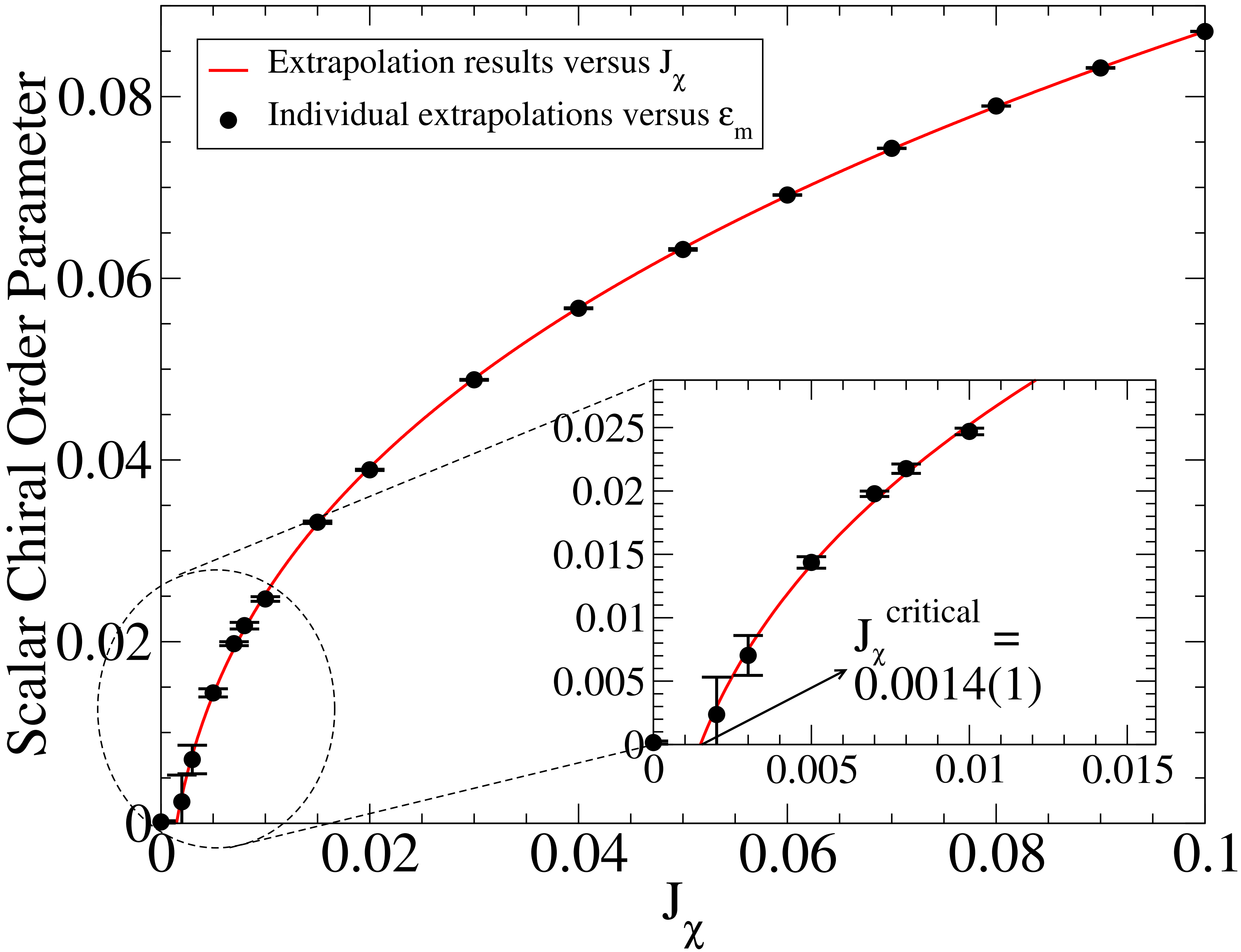
$$H_\chi = J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k)$$

B. Bauer, L. Cincio, B.P. Keller, M. Dolfi, G. Vidal, S. Trebst and A.W.W. Ludwig, Nature Communications **5**, 5137 (2014)

Semion topological order,
i.e. the Kalmeyer-Laughlin chiral spin liquid,
appears for $J_\chi/J > 0.01$.

R. Haghshenas, Shou-Shu Gong, and D.N. Sheng, arXiv:1812.11436

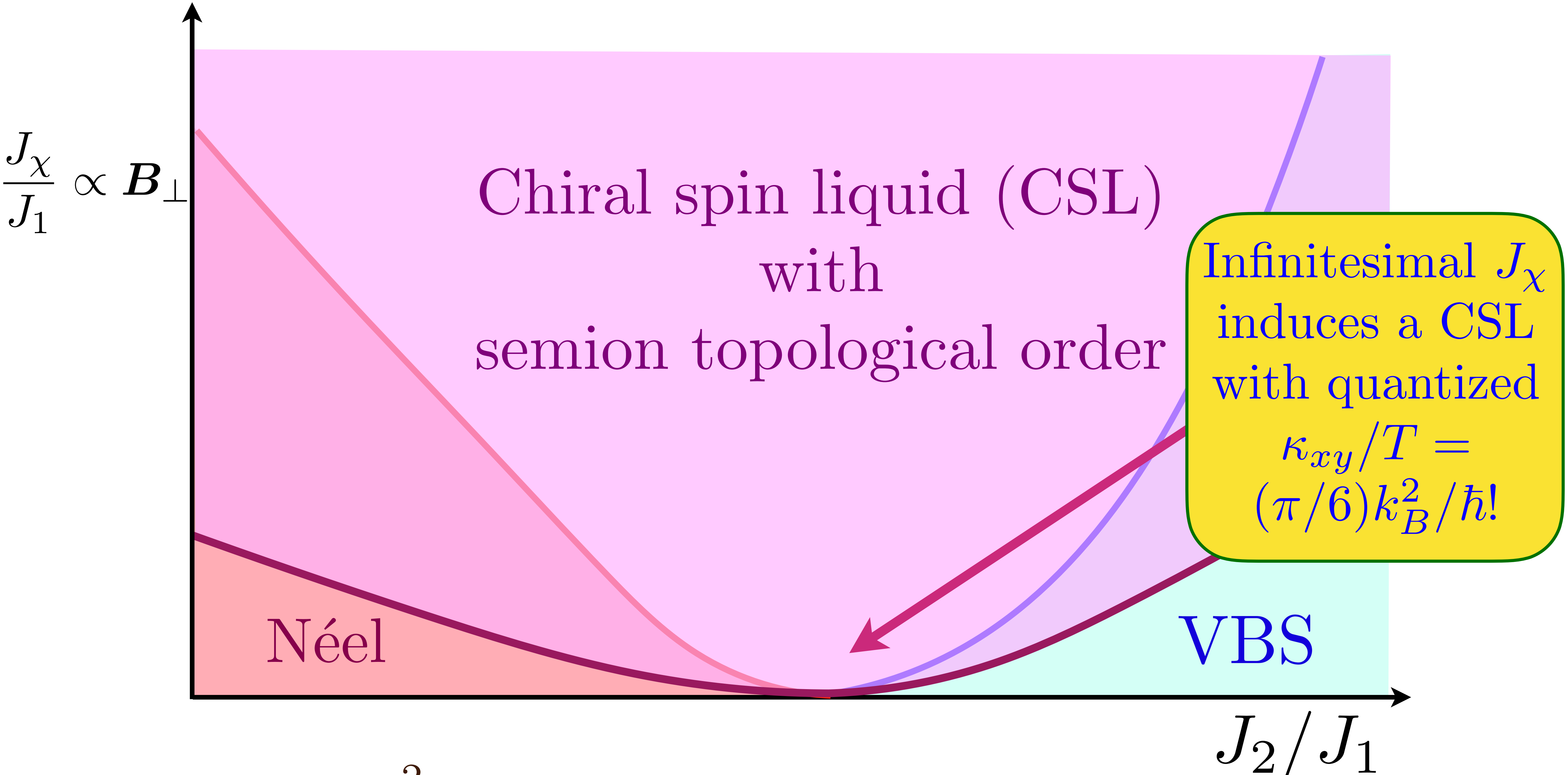
Triangular lattice antiferromagnet



$$J_2/J_1 = 1/8;\ \text{critical } J_\chi = 0.0014$$

S. N. Saadatmand and I. P. McCulloch, PRB **96**, 075117 (2017)

$$H = \sum_{\text{n.n.}} J_1 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \sum_{\text{n.n.n.}} J_2 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k)$$



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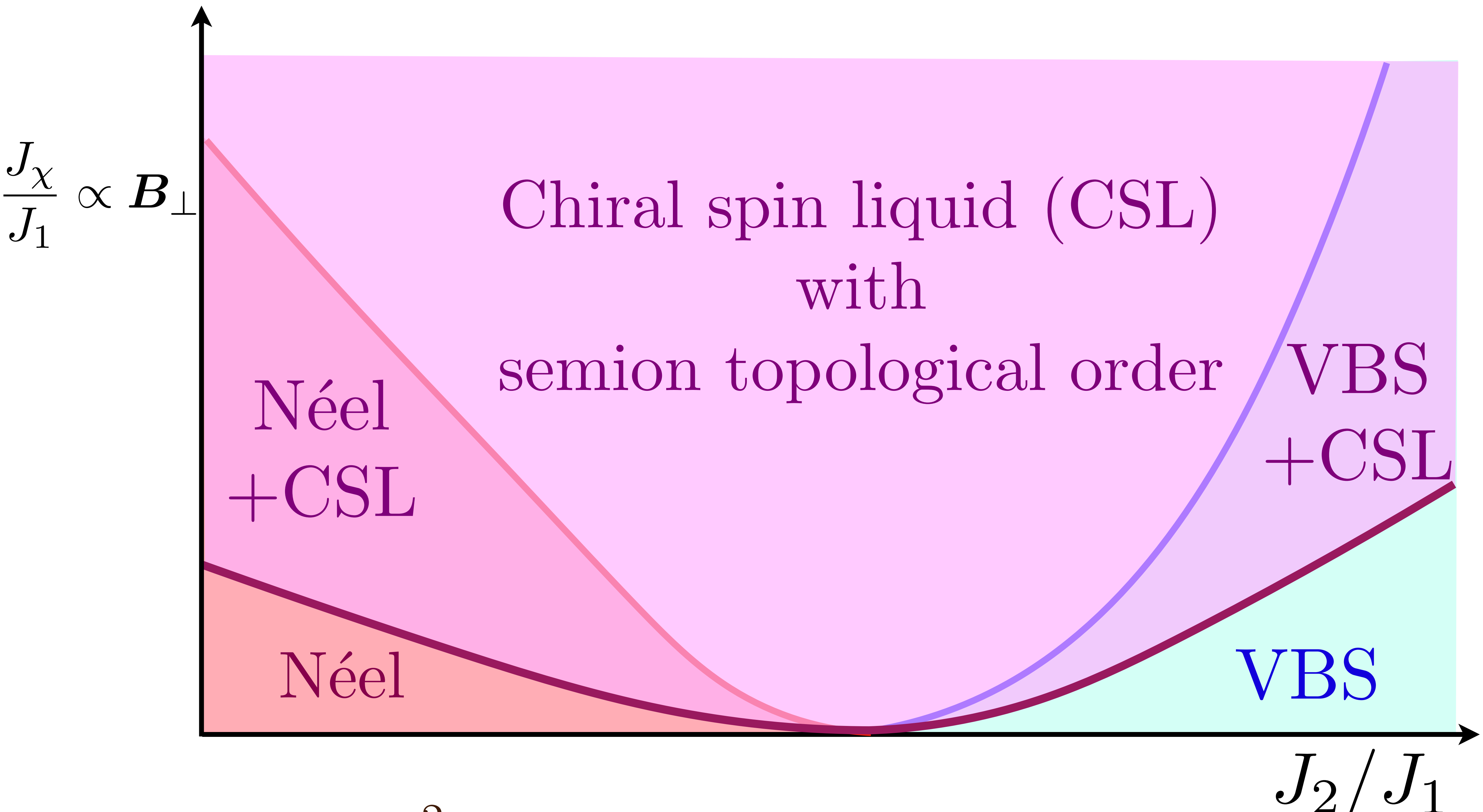

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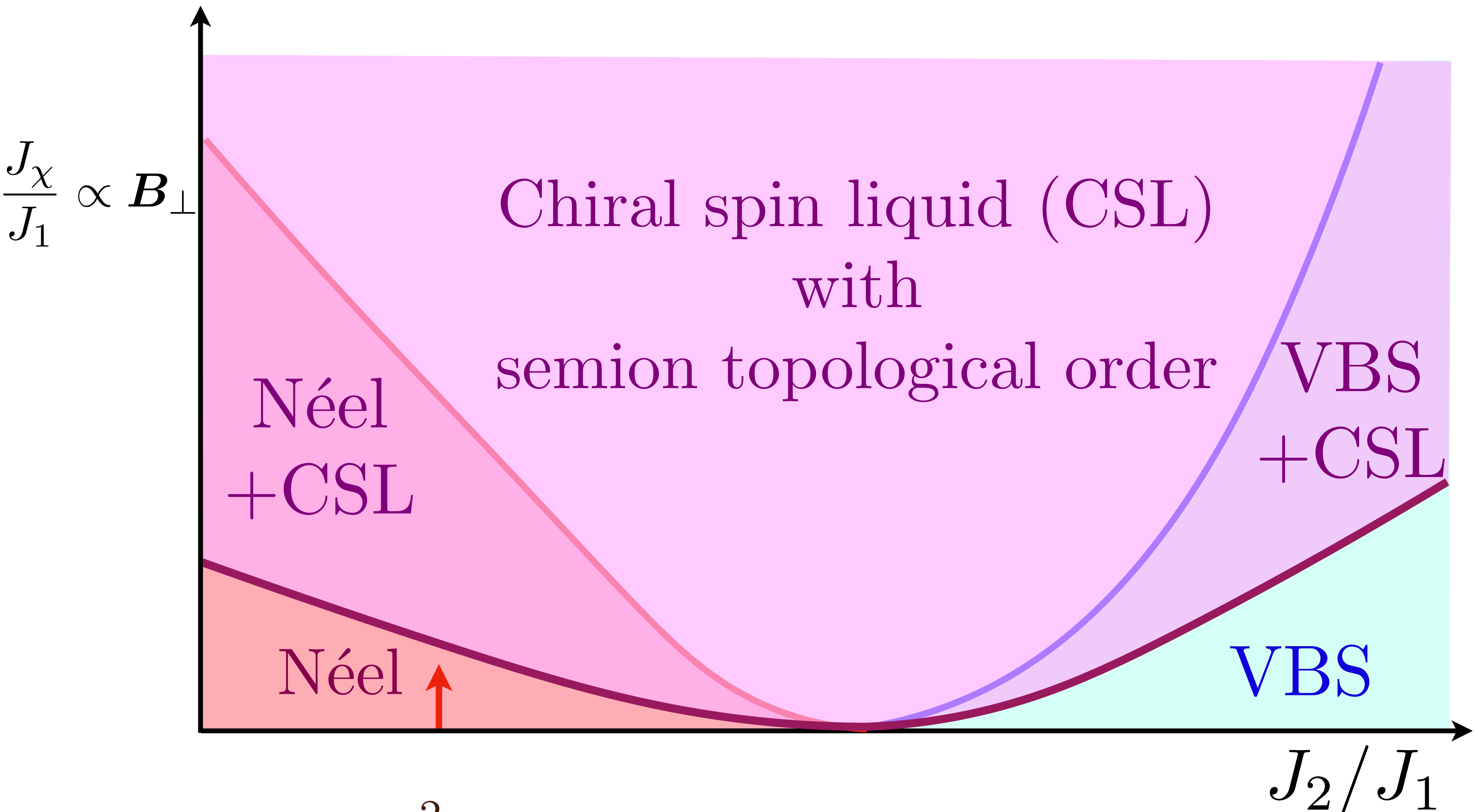
Quantum critical theory + J_χ + Néel order

$$H = \sum_{\text{n.n.}} J_1 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \sum_{\text{n.n.n.}} J_2 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k)$$



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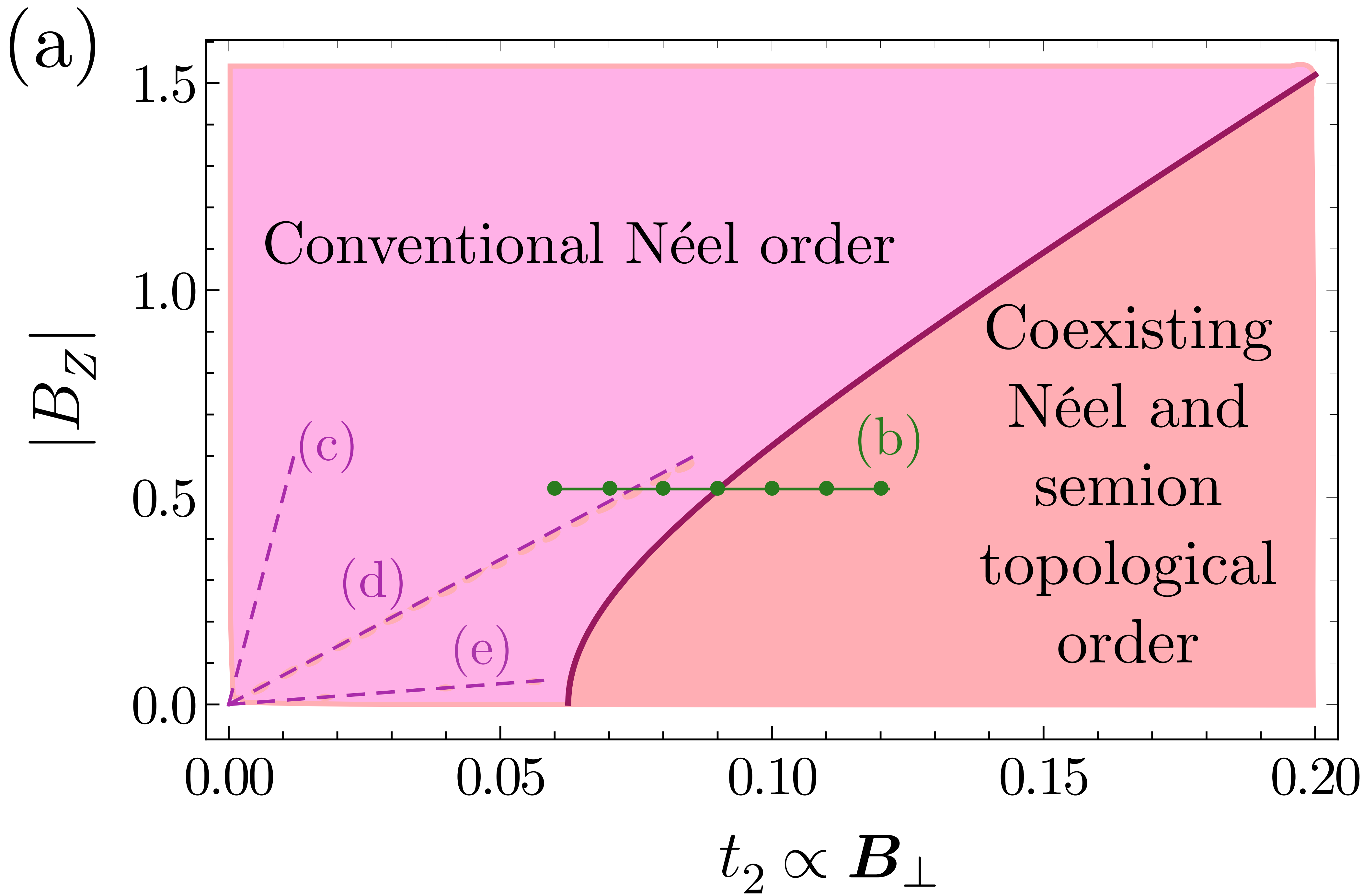

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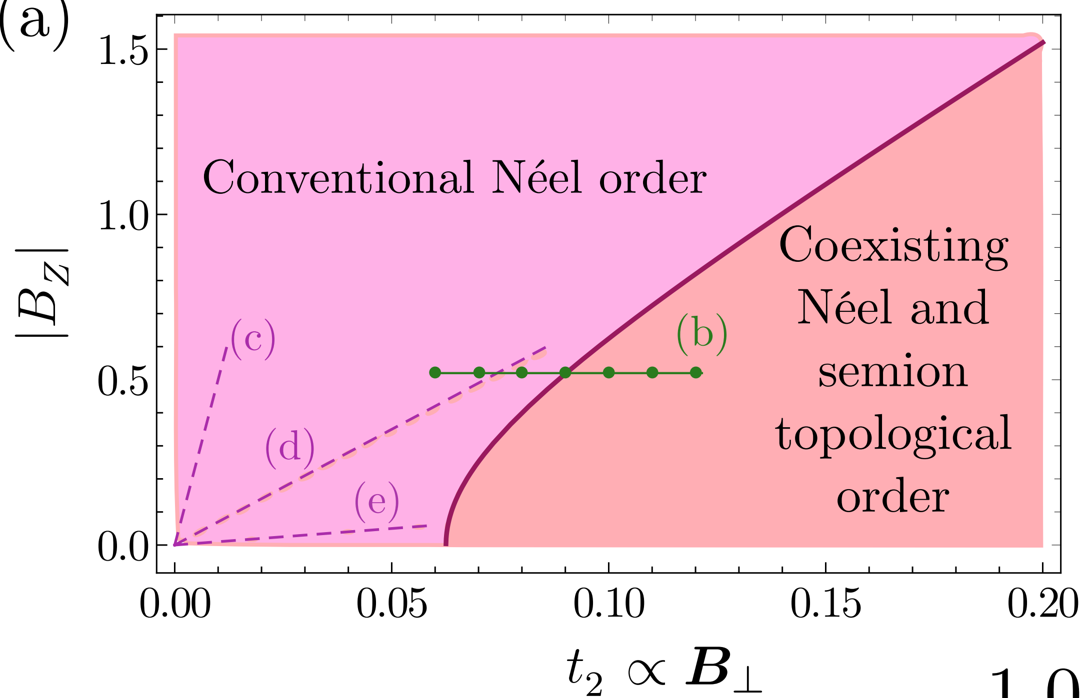

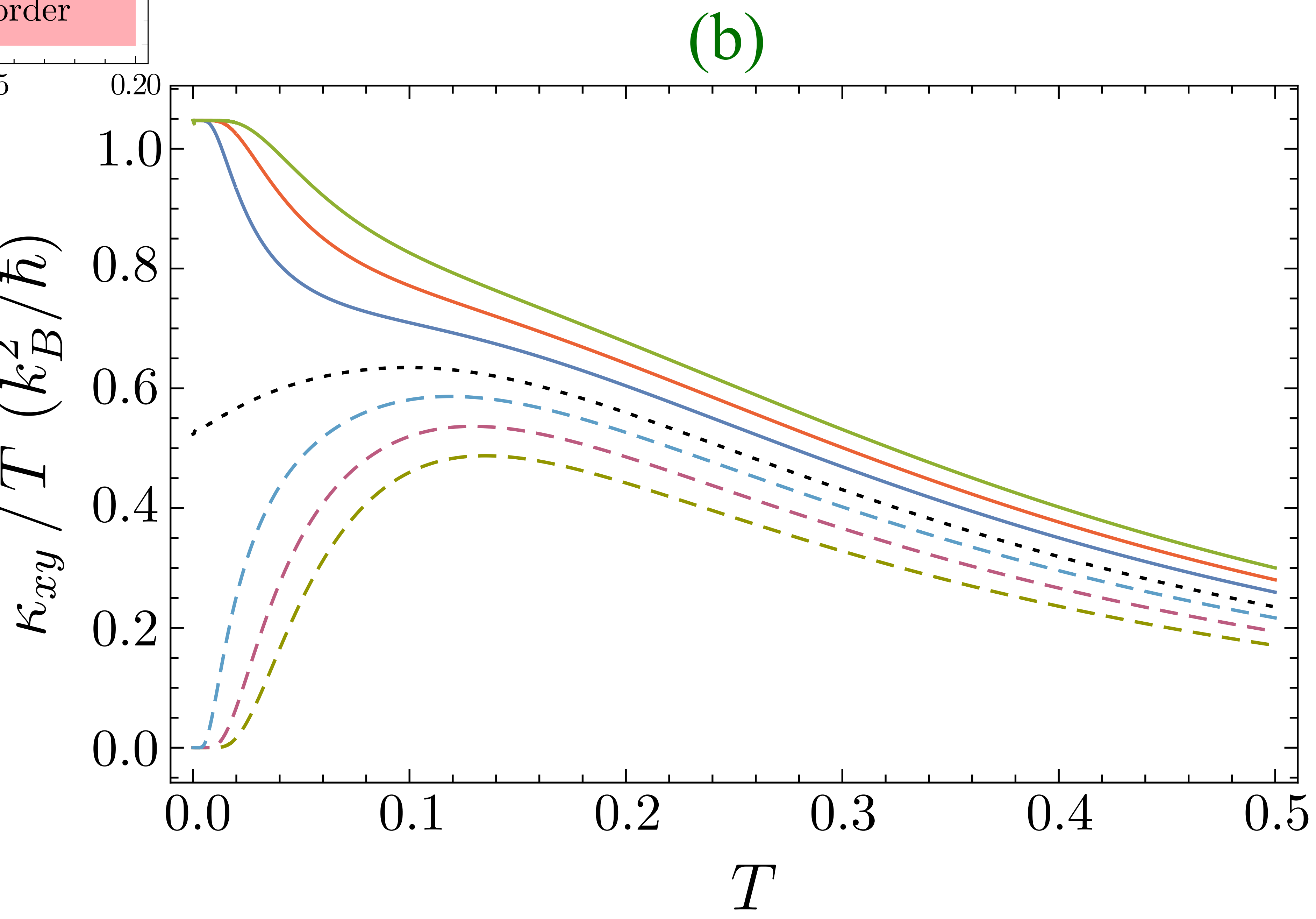

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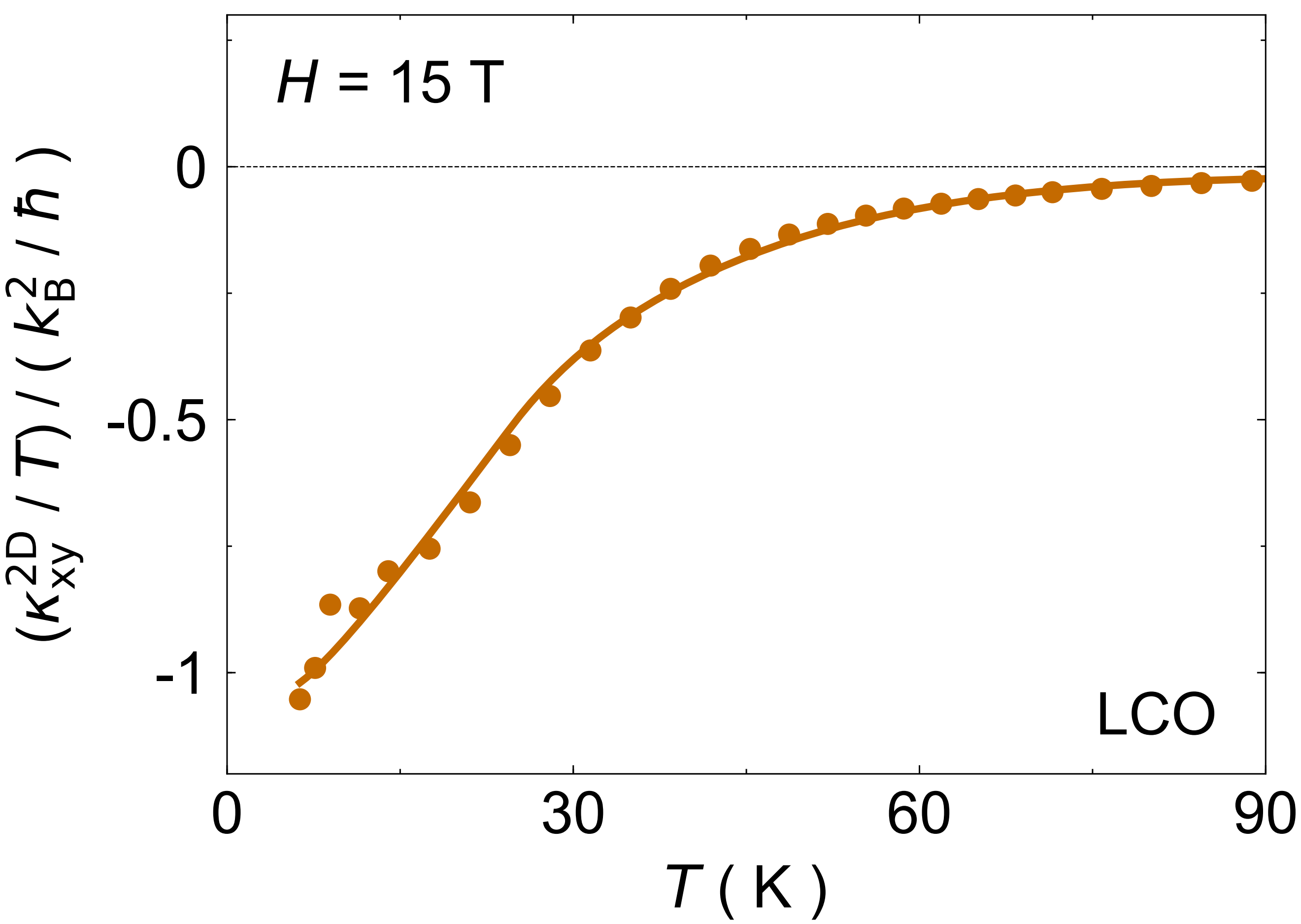

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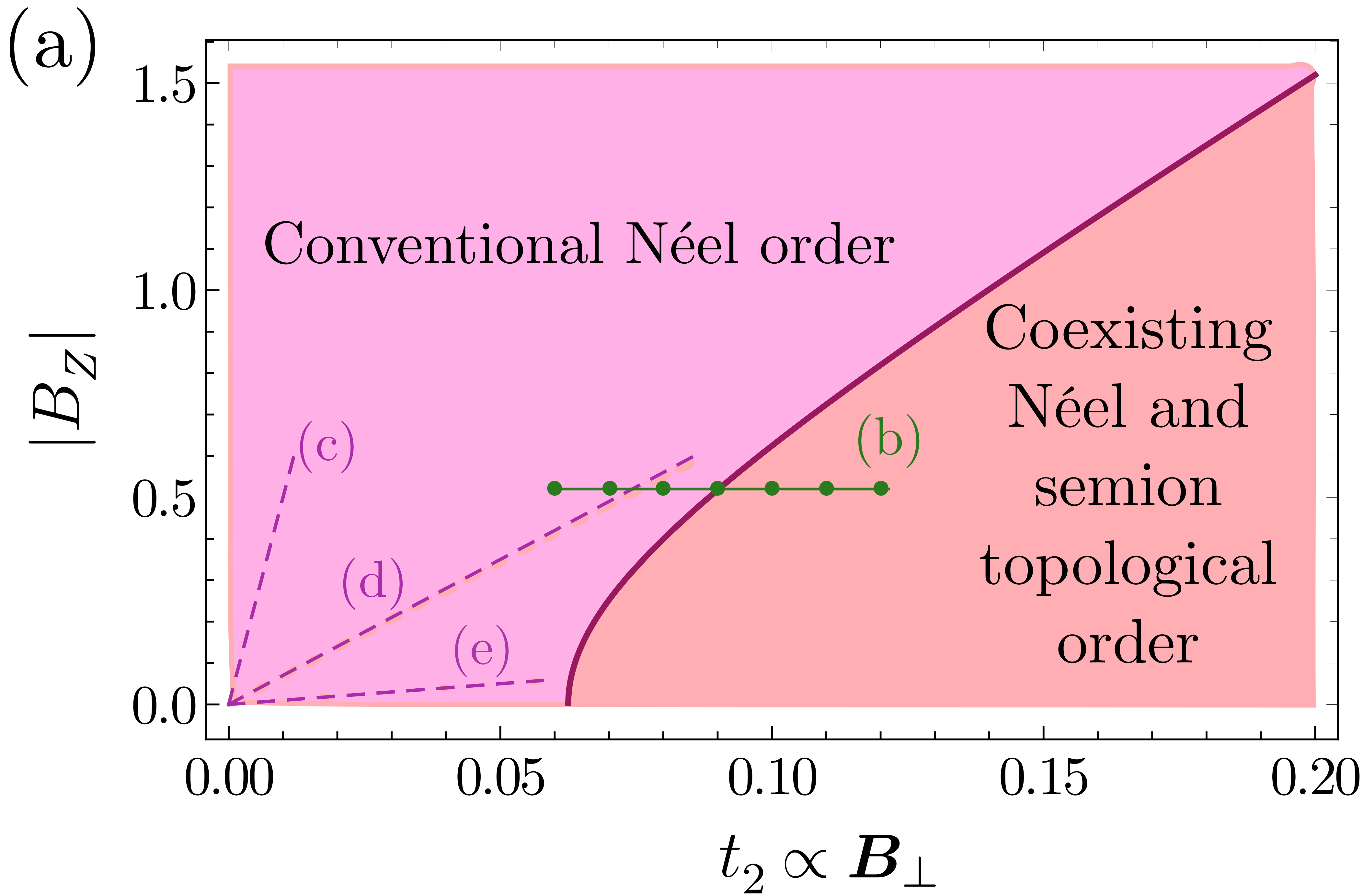

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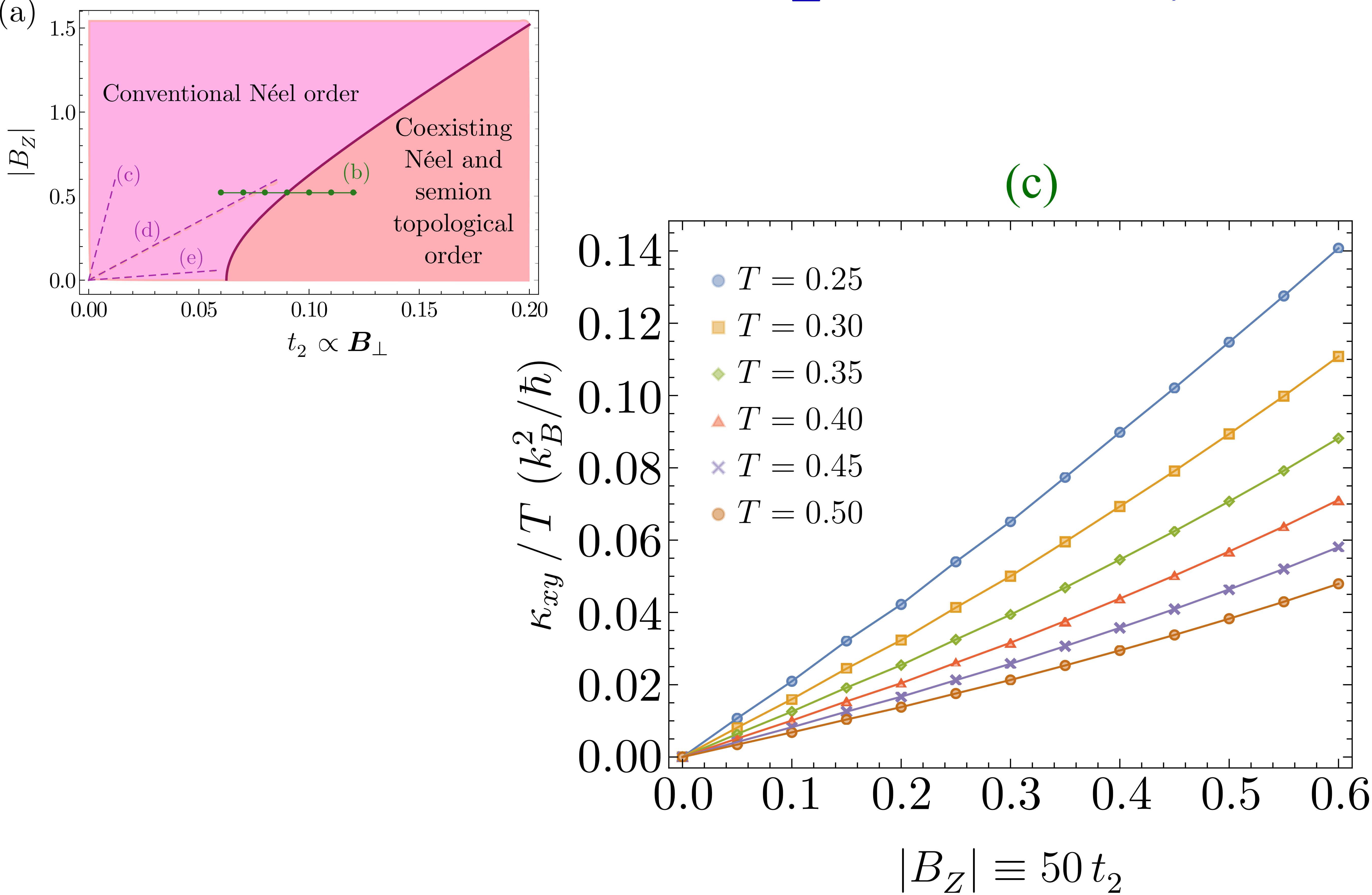

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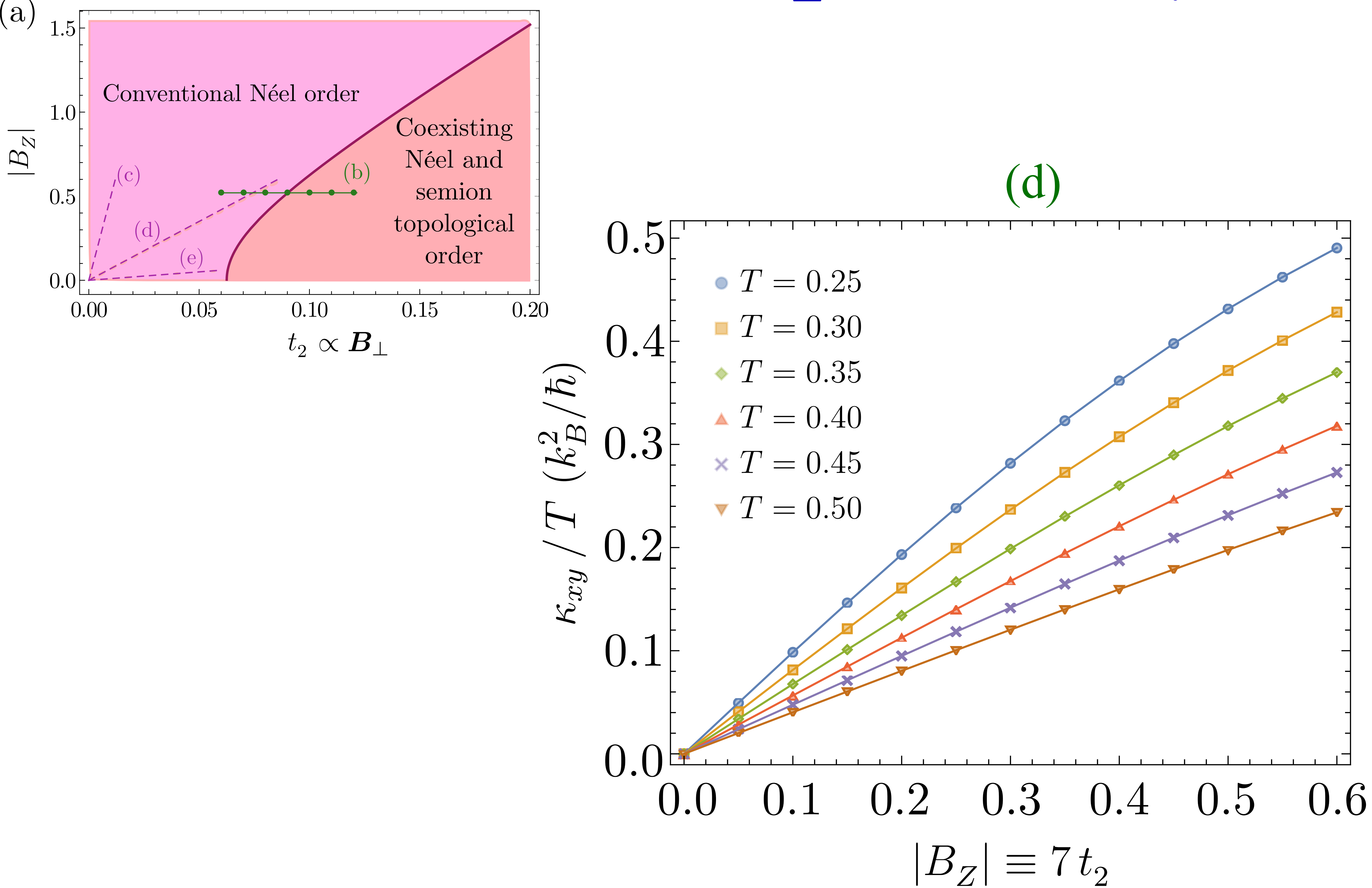

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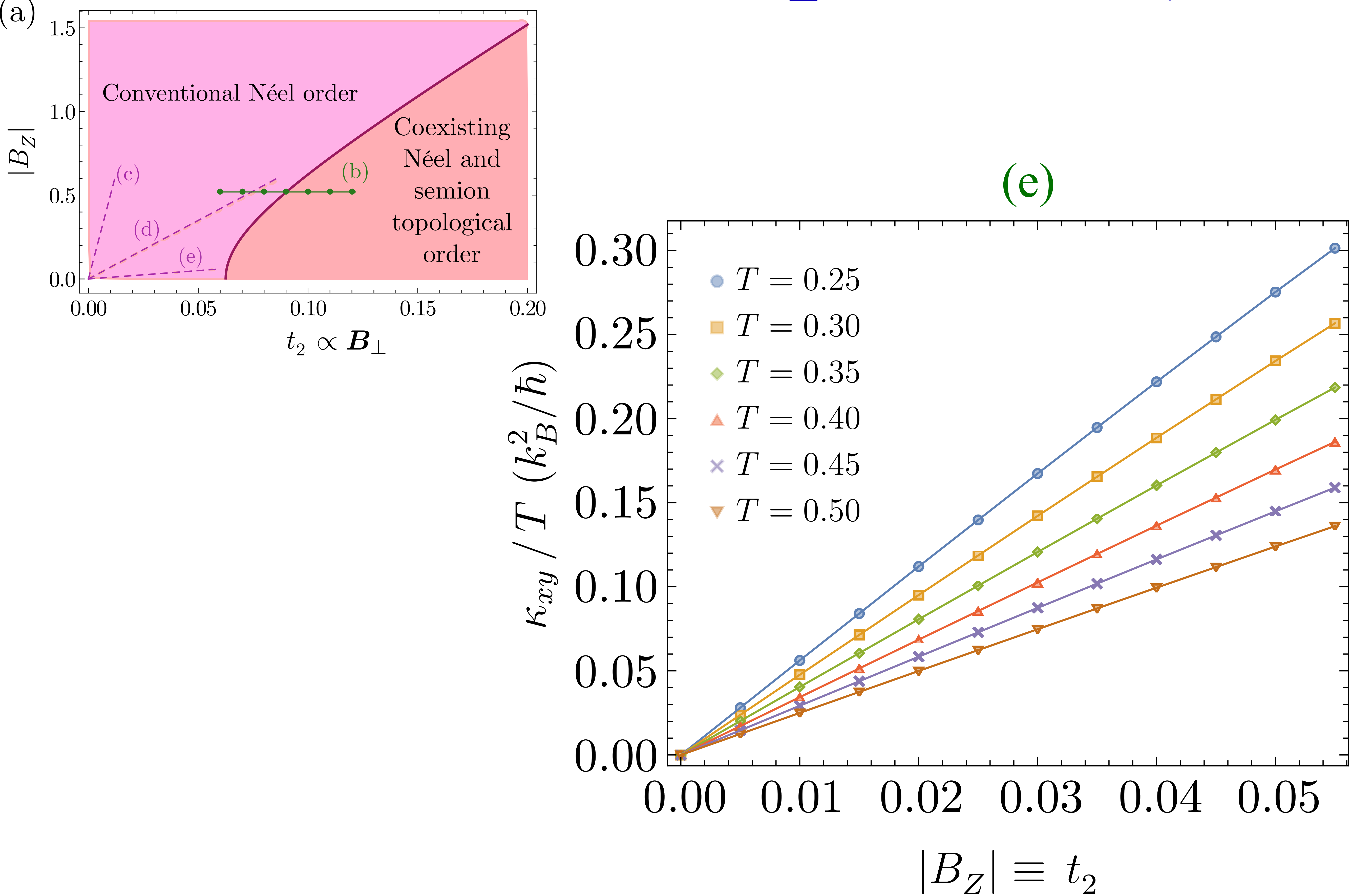

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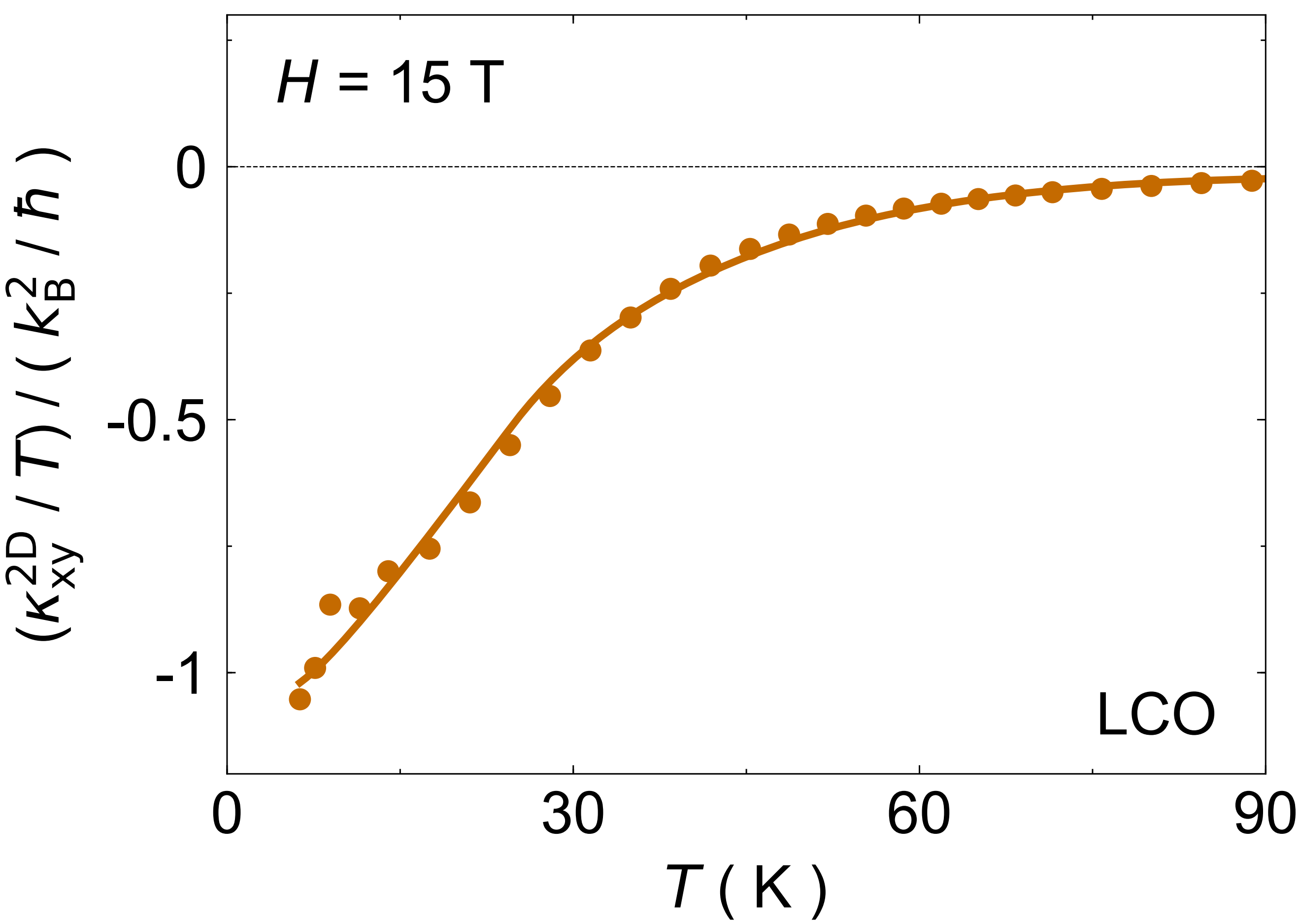

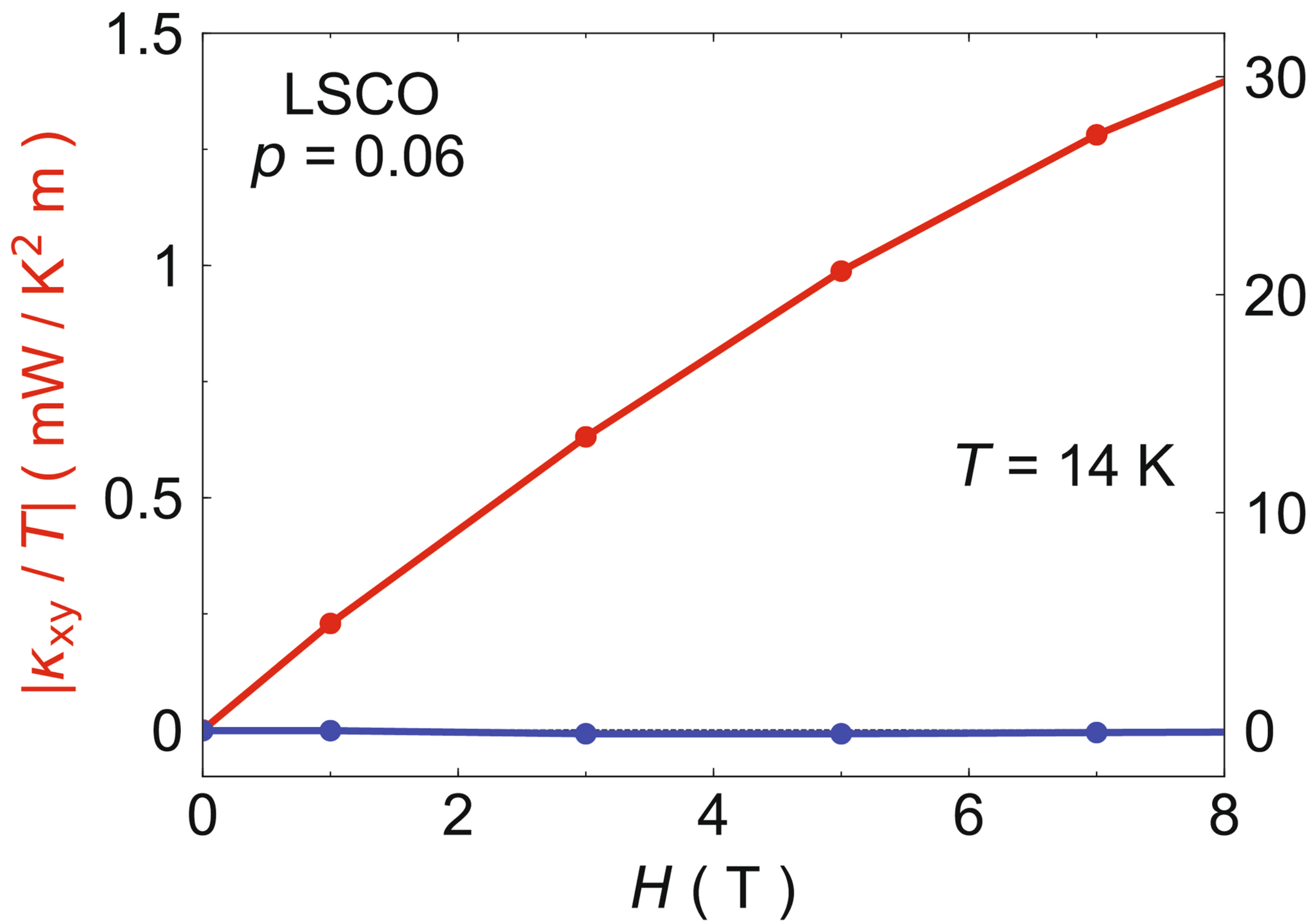

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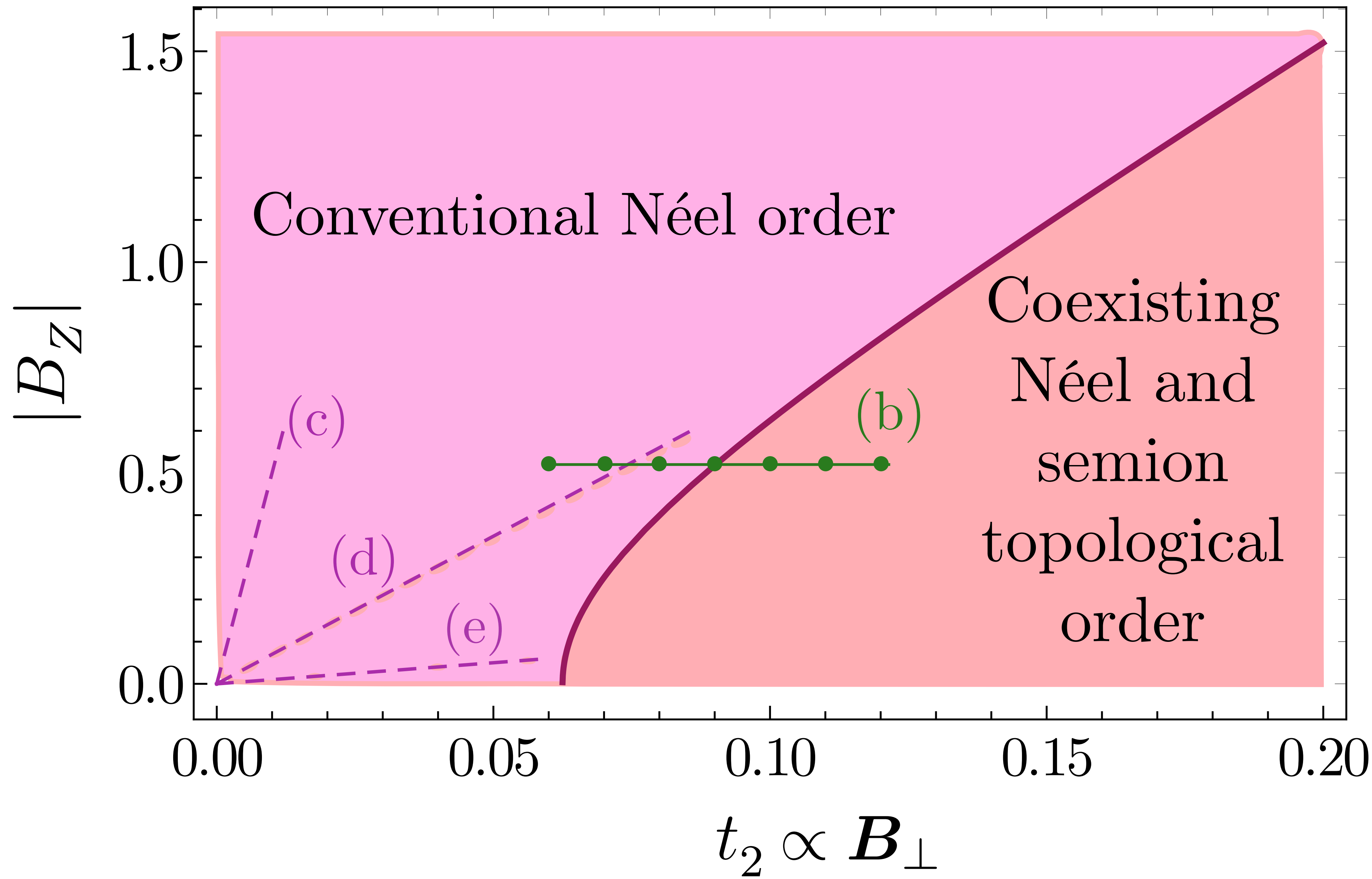

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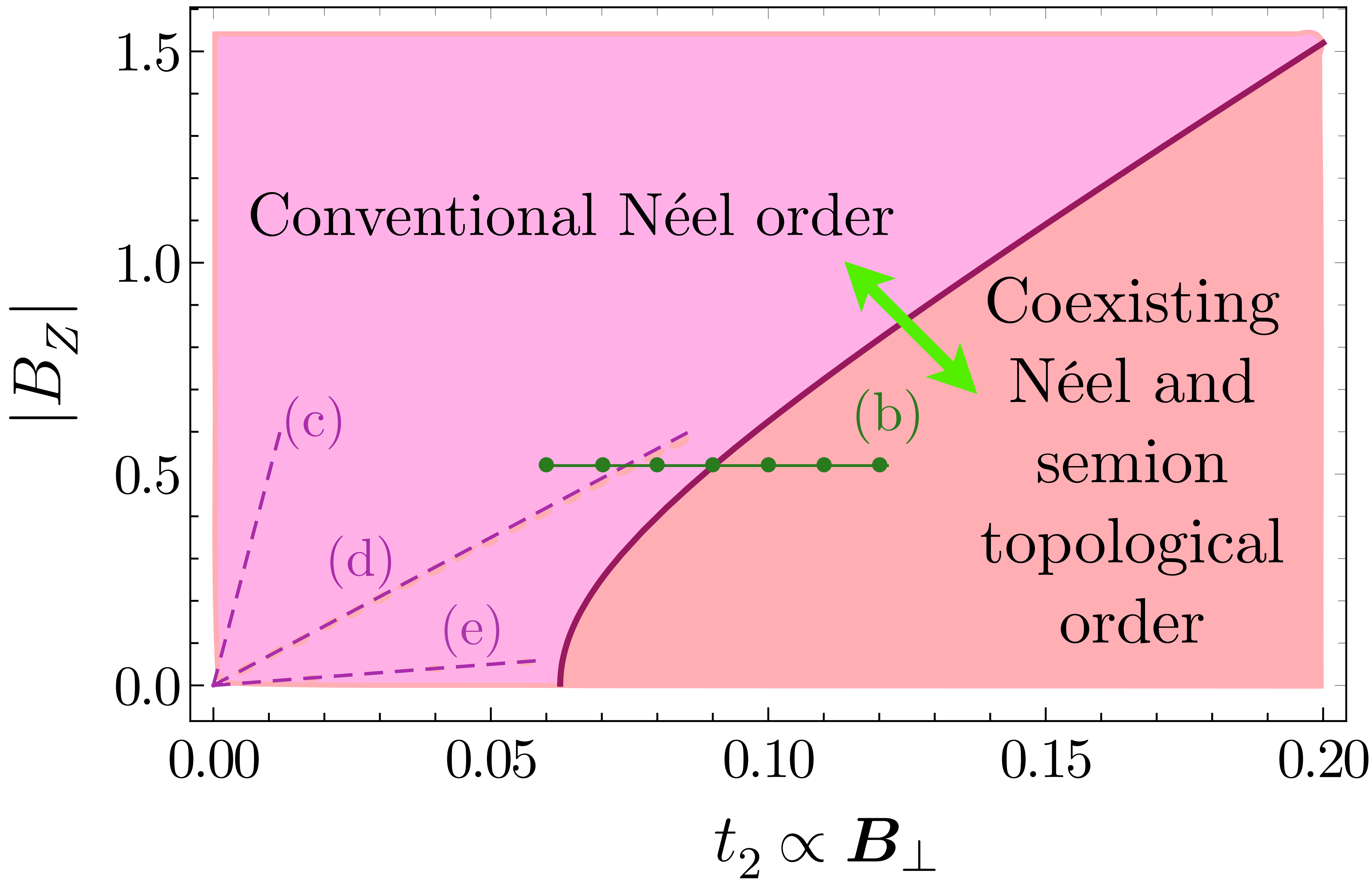
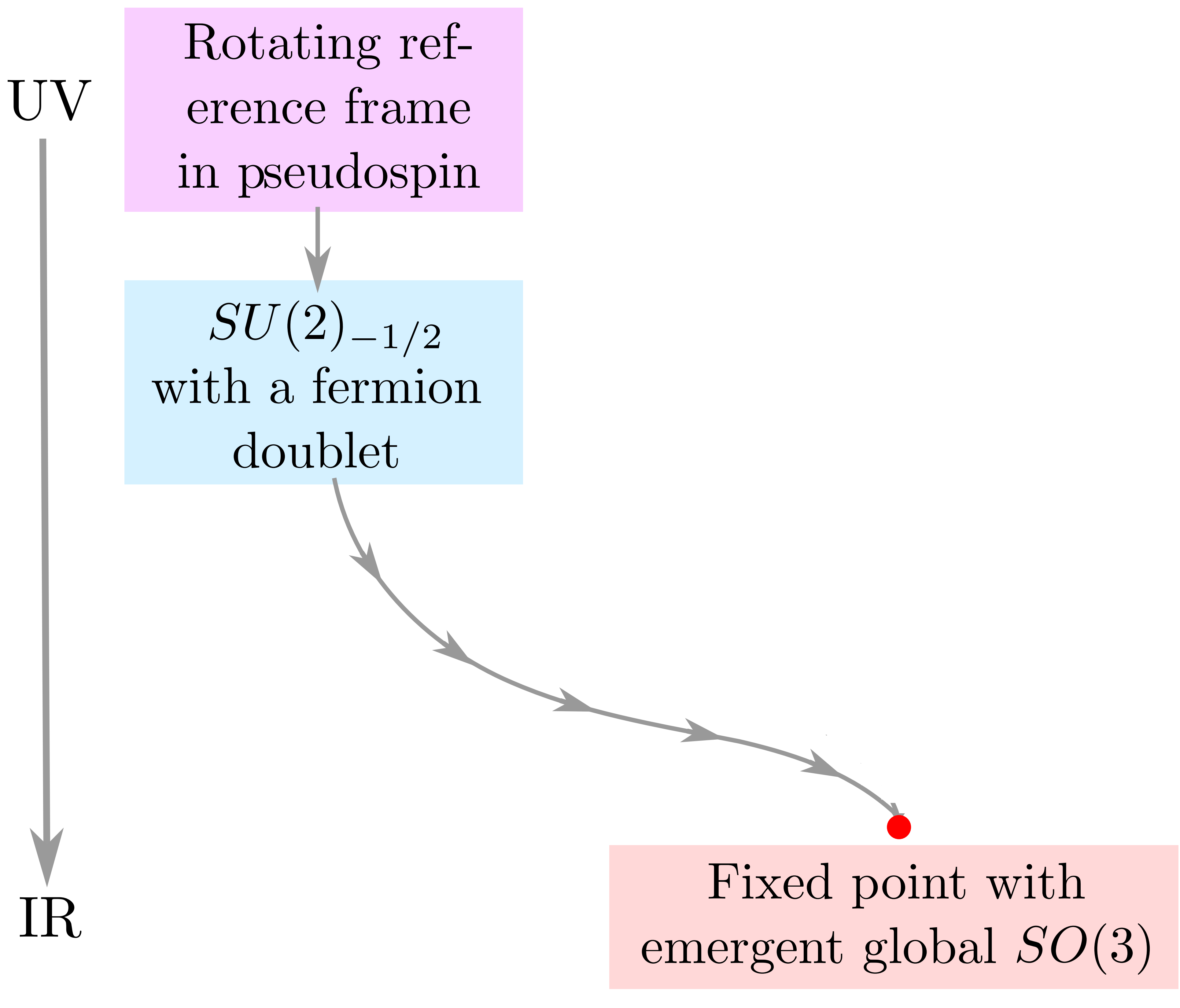
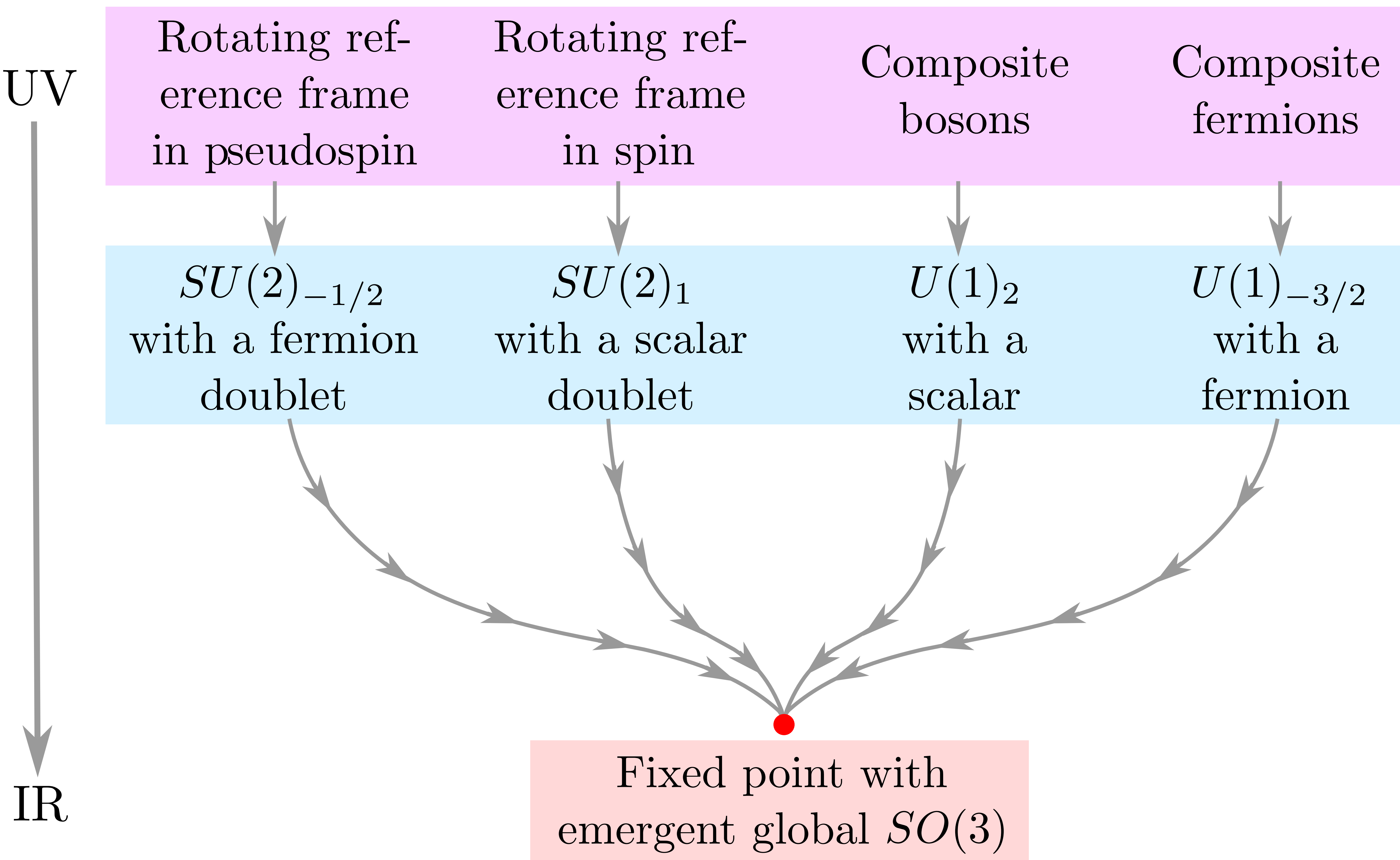


$$\mathcal{L}_f = \overline{f}\gamma^\mu(\partial_\mu - iA_\mu)f + m\overline{f}f - \frac{1}{2}\mathrm{CS}[A_\mu]$$


UV
IR
Rotating reference frame in pseudospin
$SU(2)_{-1/2}$ with a fermion doublet
Fixed point with emergent global $SO(3)$



Francesco Benini, Po-Shen Hsin, and Nathan Seiberg, arXiv:1702.07035

A quadrilarity

$$\mathcal{L}_z = |(\partial_\mu - iA_\mu)z|^2 + s|z|^2 + u(|z|^2)^2 + \mathrm{CS}[A_\mu]$$

$$\mathcal{L}_f = \overline{f}\gamma^\mu(\partial_\mu - iA_\mu)f + m\overline{f}f - \frac{1}{2}\mathrm{CS}[A_\mu]$$

$$\mathcal{L}_\phi = |(\partial_\mu - ia_\mu)\phi|^2 + s|\phi|^2 + u(|\phi|^2)^2 + 2\mathrm{CS}[a_\mu]$$

$$\mathcal{L}_g = \overline{g}\gamma^\mu(\partial_\mu - ia_\mu)g + m\overline{g}g - \frac{3}{2}\mathrm{CS}[a_\mu]$$

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Thermal Hall conductivity of N_f fermions f_α, $\alpha = 1 \ldots N_f$ coupled to SU(2) gauge field A_μ at Chern-Simons level k

$$\mathcal{L}_f = \overline{f}_\alpha \gamma^\mu (\partial_\mu - i A_\mu) f_\alpha + m \overline{f}_\alpha f_\alpha + k \, \mathrm{CS}[A_\mu]$$

The thermal Hall conductivity is a universal function of m/T, which can be computed in an expansion in $1/N_f$ with $k \propto N_f$.

$$\kappa_{xy} = \frac{k_B^2 T}{\hbar} \mathcal{F}(m/T)$$

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where the integer $\hat{k}$ is defined by

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In the large N_f limit, the first term is the fermion contribution, while the gauge field contributes the second term with the opposite sign.

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$$\kappa_{xy} = \frac{k_B^2 T}{\hbar} \mathcal{F}(m/T)$$

- In the limit $T \ll |m|$, we obtain two gapped topological phases in which $\mathcal{F}$ is a rational number. These are described by SU(2) gauge theory at integer level $k - (N_f/2)\mathrm{sgn}(m)$.
- For $T \gg |m|$, we have the quantum critical region, where $\mathcal{F}$ is not rational, and κ_{xy} requires a bulk 2+1 dimensional description.

1. Insulator

A. Thermal Hall conductivity across the Neel/Neel+CSL quantum transition

B. Quantum criticality and non-Abelian dualities

2. Pseudogap at non-zero doping

Giant thermal Hall conductivity in the pseudogap phase of cuprate superconductors

G. Grissonnanche, A. Legros, S. Badoux, E. Lefrancois, V. Zatko,
M. Lizaire, F. Laliberte, A. Gourgout, J. Zhou, S. Pyon, T. Takayama,
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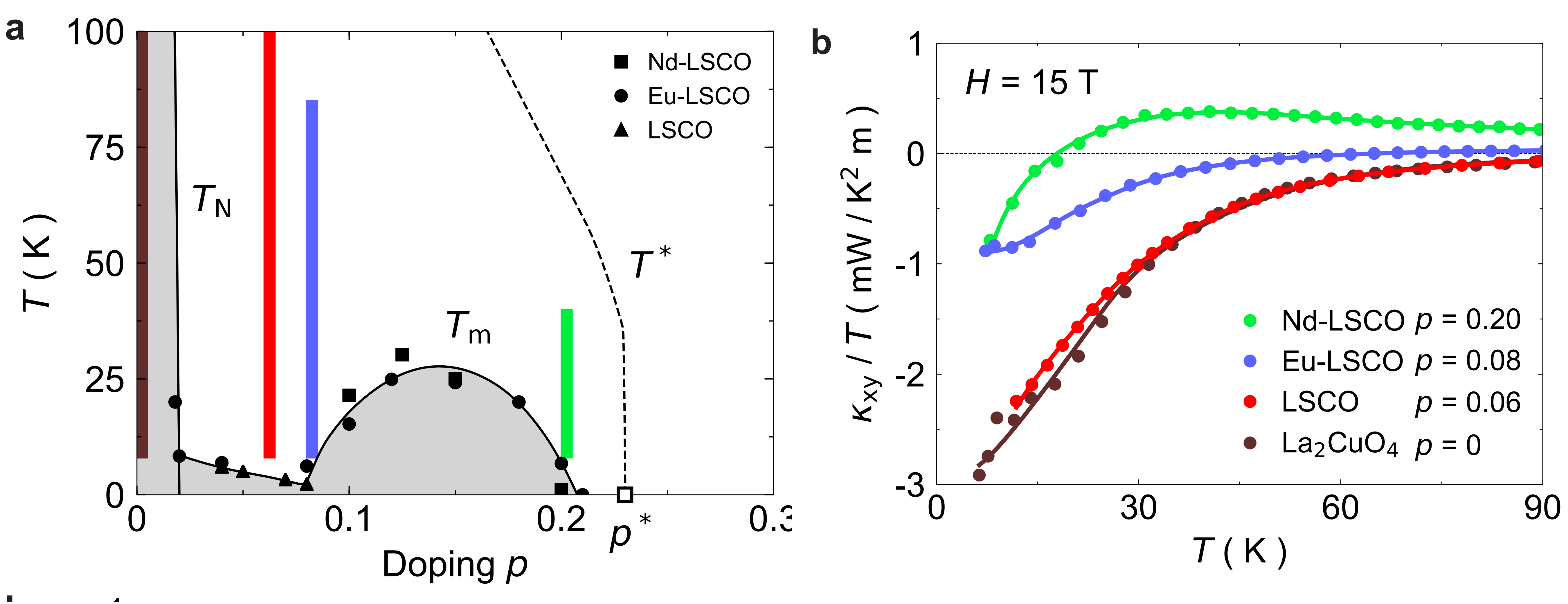

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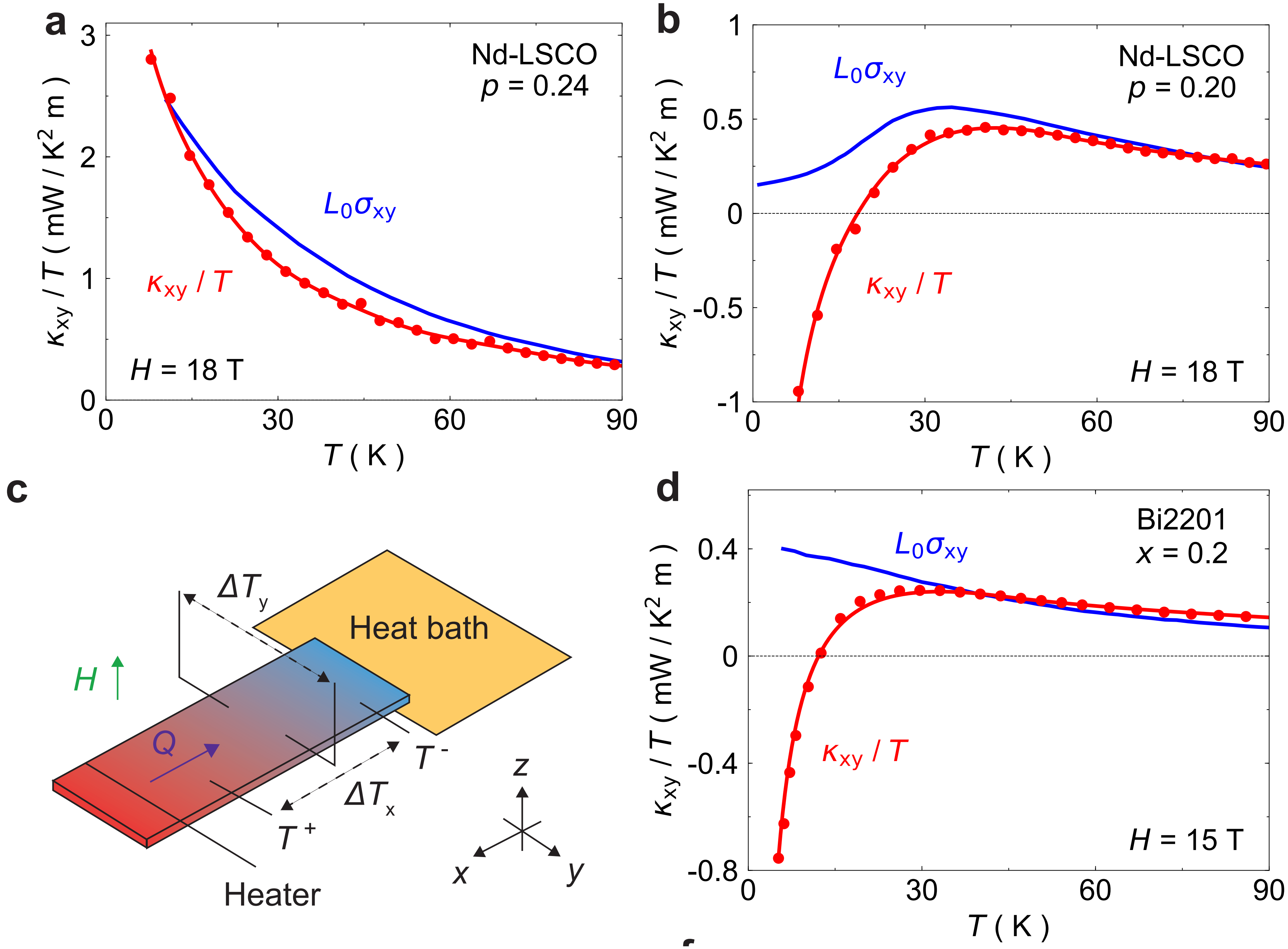
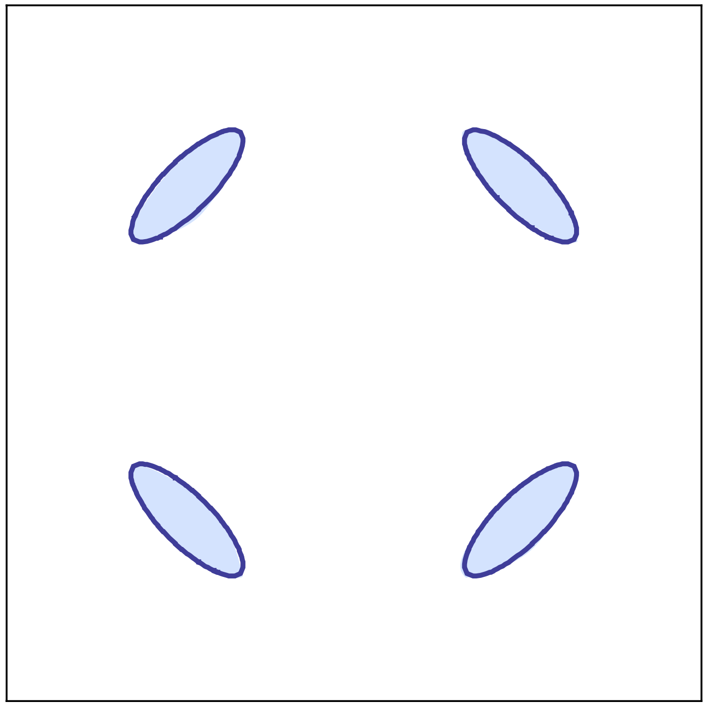


Model for the pseudogap

Fermionic 'chargons' of density δ in hole pockets.
The fermions carry electromagnetic gauge charge $+e$,
and charges $p = \pm 1$ under an emergent U(1) gauge field.
v is a valley index, $v_{\rm dis}$ is an impurity potential.

$$\mathcal{L}_f = \sum_{v=1,2} \sum_{p=\pm 1} f_{pv}^\dagger \left(\frac{\partial}{\partial \tau} - \mu - i p a_\tau - \frac{(\boldsymbol{\nabla} - i p \boldsymbol{a} - i e \boldsymbol{A}_{\rm em})^2}{2m^*} \right) f_{pv} + v_{\rm dis}(\boldsymbol{r}) f_{pv}^\dagger f_{pv}$$



Thermal Hall conductivity

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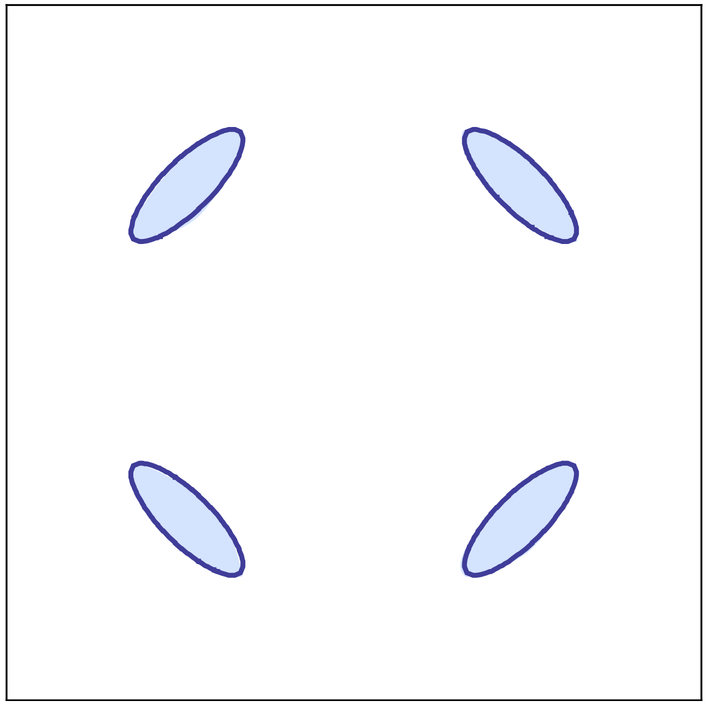

Leading order fermionic contribution is that implied by the Wiedemann-Franz law.

$$\sigma_{xy} = \left(\frac{\delta e^2 \tau}{m^*} \right) \omega_c \tau$$

$$\kappa_{xy}^0 = \frac{\pi^2 T}{3} \left(\frac{k_B}{e} \right)^2 \sigma_{xy}$$

Thermal Hall conductivity

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Integrating out the fermions leads to an effective action for the emergent U(1) gauge field

$$\mathcal{S}_a = \int d^2x d\tau \left[\frac{K_1(\boldsymbol{x})}{2} (\boldsymbol{\nabla} \times \boldsymbol{a})^2 + \frac{K_2(\boldsymbol{x})}{2} (\boldsymbol{\nabla} a_\tau - \partial_\tau \boldsymbol{a})^2 - \frac{i\sigma_{xy}(\boldsymbol{x})}{2e^2} \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda \right] + \int \frac{d^2k d\omega}{8\pi^3} \gamma_k |\omega| \left[\boldsymbol{a}^T(k,\omega) \right]^2$$

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The gauge field contributes a thermal Hall conductivity, κ_{xy}^1, which has the *opposite sign* from the Wiedemann-Franz term determined from σ_{xy}.

Summary: Insulator

- The ground state of the square lattice antiferromagnet is a conventional Neel state.

- In a sufficiently large orbital magnetic field, there is a quantum transition to a "chiral spin liquid" co-existing with conventional Neel order.

- Proximity to this quantum transition can enhance the thermal Hall effect at non-zero temperatures, even though the ground state is conventional.

- Can identify two contributions to the thermal Hall effect: from the fermionic matter (spinons) , and from the emergent gauge field with the opposite sign.

Summary: Pseudogap at non-zero doping

- Model the pseudogap by pockets of fermionic "chargons" carrying gauge charges of an emergent U(1) gauge field

- One contribution to the thermal Hall effect is the Wiedemann-Franz law of a disordered Fermi liquid

- There is an additional emergent gauge field contribution which has the opposite sign.