Non-Hermitian Fermionic Superfluidity in Dissipative Ultracold Atoms

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Outline of this talk

1. Introduction

Motivation ← experiment and theory

Open quantum systems ← master equation
non-Hermitian Hamiltonian

2. Model and Formulation

Non-Hermitian BCS model

Non-Hermitian mean-field theory

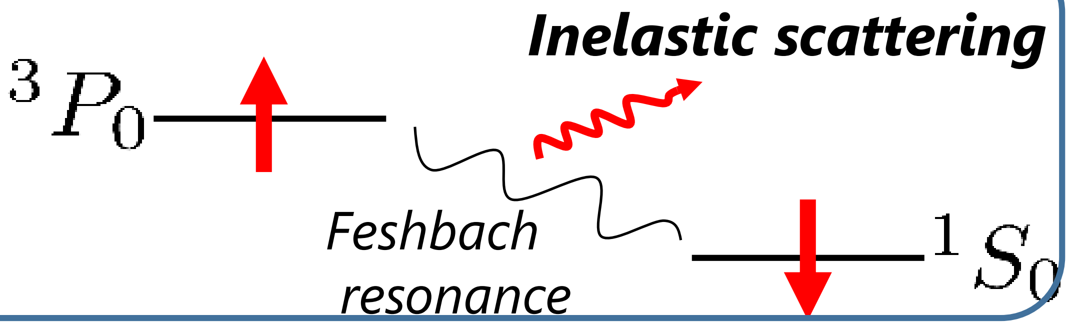
3. Results

4. Summary

Motivation : Experiments

Ultracold atoms (with dissipation)

^{173}Yb
Orbital Feshbach
resonance



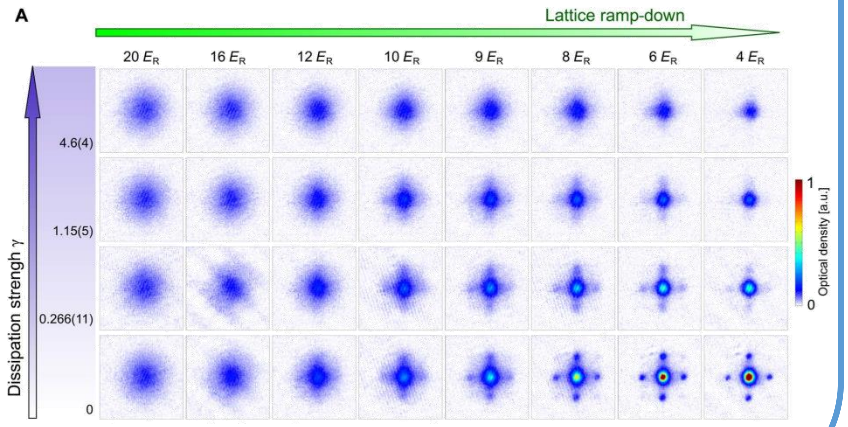
Superfluid-Mott transition via **dissipation**

→ dissipation engineering using **photoassociation**

interference pattern of superfluid

dissipation

unclear



T. Tomita et al. *Science Advances* **3** (2017) e1701513.

Recent Advances in NH Quantum Systems

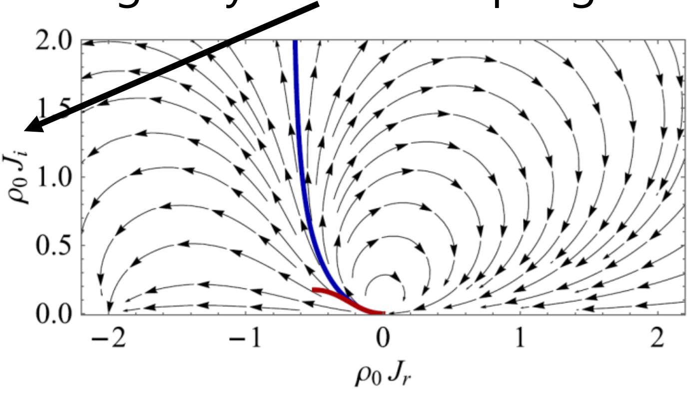
NH Kondo effect

M. Nakagawa et al. Phys. Rev. Lett. 121 (2018) 203001.

Exotic renormalization group flow

Non-Hermiticity suppresses Kondo effect

Imaginary Kondo coupling



Real Kondo coupling

NH Sine-Gordon model (1D)

Y. Ashida et al. Nature Commun. **8** (2017) 15791.

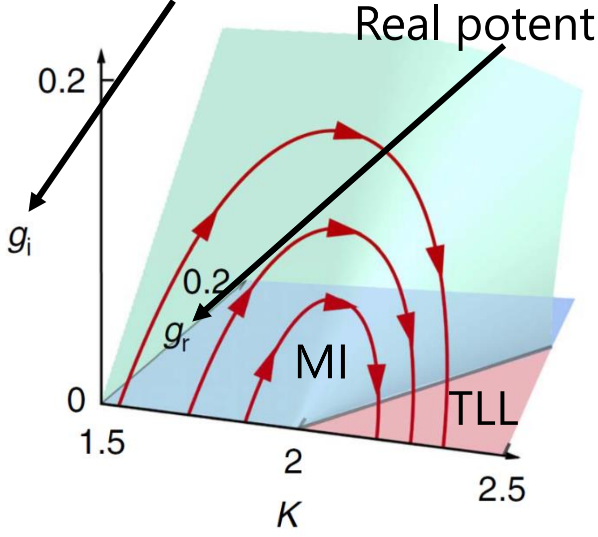
Non-Hermiticity **enhances superfluid correlation**

In stark contrast to the Hermitian case

Stable MI phase for $K<2$

Imaginary potential

Real potential



TLL parameter

Open Quantum Systems

open quantum system → **dissipation** (1, 2, 3-body loss), driving

quantum master equation

Von Neuman equation

coupling to the environment

$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] - \frac{1}{2}\gamma \sum_i \left(L_i^\dagger L_i \rho + \rho L_i^\dagger L_i - 2 L_i \rho L_i^\dagger \right)$$

$$= -i(H_{\text{eff}}\rho - \rho H_{\text{eff}}^\dagger) + \gamma \sum_i L_i \rho L_i^\dagger,$$

NH term

Jump term

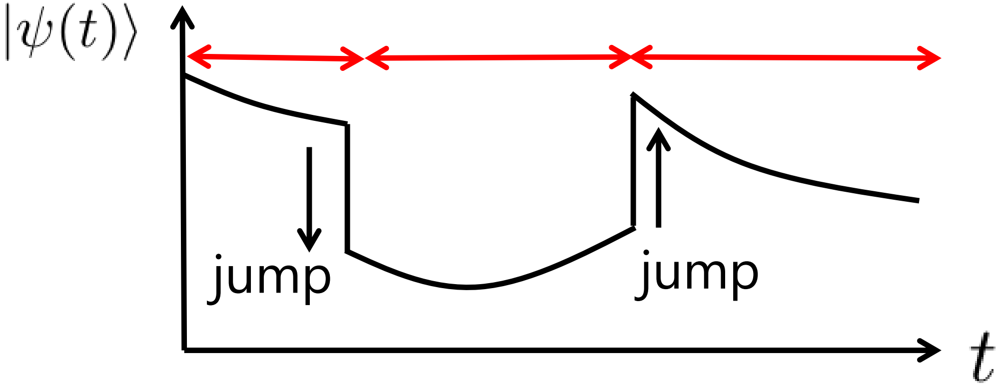
Effective **non-Hermitian (NH) Hamiltonian**

$$H_{\text{eff}} = H - \tfrac{i}{2}\gamma \textstyle\sum_i L_i^\dagger L_i$$

Non-Hermitian Quantum Systems

Physical meaning of NH Quantum Systems

(quantum trajectory method)



NH Hamiltonian ← Dynamics in each quantum trajectory

Eigenvalue of H_{eff} : **complex**

$\mathrm{Re}\,E$ → Effective energy

$\mathrm{Im}\,E$ → **Decay rate** of each eigenstate

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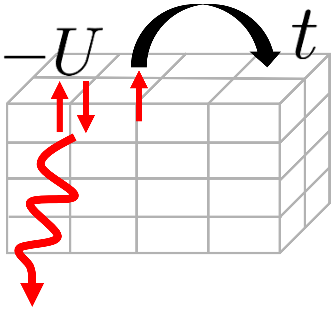
Model: NH BCS Superfluid

Non-Hermitian fermionic superfluidity

Model : BCS Hamiltonian + **Complex-valued interaction**

$$H_{\text{eff}} = \sum_{\boldsymbol{k}\sigma} \epsilon_{\boldsymbol{k}} c^{\dagger}_{\boldsymbol{k}\sigma} c_{\boldsymbol{k}\sigma} - U \sum_{i} c^{\dagger}_{i\uparrow} c^{\dagger}_{i\downarrow} c_{i\downarrow} c_{i\uparrow}$$

↑ complex



$$U \equiv U_1 + i\gamma/2 \quad (U_1, \gamma > 0)$$

$L_i = c_{i\downarrow} c_{i\uparrow}$ two-body loss of Cooper pairs

Question: How is the fermionic superfluidity altered under *complex-valued* interactions?

We develop a non-Hermitian **mean-field theory** and elucidate it

Formulation: NH Mean-Field Theory (1)

Partition function

left eigenstate

right eigenstate

$$Z = \sum_n e^{-\beta E_n} = \sum_n {}_L\langle E_n|\, e^{-\beta H_{\mathrm{eff}}}\, |E_n\rangle_R$$

normalization condition

$${}_L\langle E_n|E_m\rangle_R = \delta_{nm}$$

completeness

$$\sum_n |E_n\rangle_R\, {}_L\langle E_n| = 1$$

Path-integral and **Hubbard-Stratonovich transformation**

$$S_{\mathrm{eff}}(\bar{\Delta},\Delta) = -\sum_{\omega_n,\boldsymbol{k}} \log(\omega_n^2 + \epsilon_{\boldsymbol{k}}^2 + \bar{\Delta}\Delta) + \frac{\beta N}{U}\bar{\Delta}\Delta$$

Saddle point condition $\partial S_{\mathrm{eff}}/\partial\Delta = \partial S_{\mathrm{eff}}/\partial\bar{\Delta} = 0$

NH gap equation

$$\frac{N}{U} = \sum_{\boldsymbol{k}} \frac{1}{2\sqrt{\epsilon_{\boldsymbol{k}}^2 + \bar{\Delta}\Delta}} \tanh\frac{\beta\sqrt{\epsilon_{\boldsymbol{k}}^2 + \bar{\Delta}\Delta}}{2}$$

Complex

Complex

Different from the Hermitian case

Formulation: NH Mean-Field Theory (2)

Relation between order parameters?

$$\Delta = -\frac{U}{N}\sum_{\boldsymbol{k}} {}_L\langle c_{-\boldsymbol{k}\downarrow} c_{\boldsymbol{k}\uparrow}\rangle_R$$

$$\bar{\Delta} = -\frac{U}{N}\sum_{\boldsymbol{k}} {}_L\langle c^\dagger_{\boldsymbol{k}\uparrow} c^\dagger_{-\boldsymbol{k}\downarrow}\rangle_R$$

Δ and $\bar{\Delta}$ are **no longer complex conjugate** to each other!

$$\Delta(\theta) = \Delta_0 e^{i\theta}$$

$$\bar{\Delta}(\theta) = \Delta_0 e^{-i\theta}$$

Δ_0: **complex**

$$H_{\mathrm{MF}} = \sum_{\boldsymbol{k}} \begin{pmatrix} c^\dagger_{\boldsymbol{k}\uparrow} & c_{-\boldsymbol{k}\downarrow} \end{pmatrix} \begin{pmatrix} \epsilon_{\boldsymbol{k}} & \Delta \\ \bar{\Delta} & -\epsilon_{\boldsymbol{k}} \end{pmatrix} \begin{pmatrix} c_{\boldsymbol{k}\uparrow} \\ c^\dagger_{-\boldsymbol{k}\downarrow} \end{pmatrix}$$

Bogoliubov transformation

~~unitary~~

$$\bar{\gamma}_{\boldsymbol{k}\uparrow} = u_{\boldsymbol{k}} c^\dagger_{\boldsymbol{k}\uparrow} - \bar{v}_{\boldsymbol{k}} c_{-\boldsymbol{k}\downarrow}$$

$$\gamma_{\boldsymbol{k}\uparrow} = u_{\boldsymbol{k}} c_{\boldsymbol{k}\uparrow} - v_{\boldsymbol{k}} c^\dagger_{-\boldsymbol{k}\downarrow}$$

$$\{\gamma_{\boldsymbol{k}\sigma}, \bar{\gamma}_{\boldsymbol{k}'\sigma'}\} = \delta_{\boldsymbol{k}\boldsymbol{k}'}\delta_{\sigma\sigma'}$$

$$|\mathrm{BCS}\rangle_R = \prod_{\boldsymbol{k}} \left(u_{\boldsymbol{k}} + v_{\boldsymbol{k}} c^\dagger_{\boldsymbol{k}\uparrow} c^\dagger_{-\boldsymbol{k}\downarrow}\right)|0\rangle$$

$$|\mathrm{BCS}\rangle_L = \prod_{\boldsymbol{k}} \left(u^*_{\boldsymbol{k}} + \bar{v}^*_{\boldsymbol{k}} c^\dagger_{\boldsymbol{k}\uparrow} c^\dagger_{-\boldsymbol{k}\downarrow}\right)|0\rangle$$

Quasiparticles obey **neither Fermi nor Bose statistics**

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Theory of non-Hermitian mean-field theory

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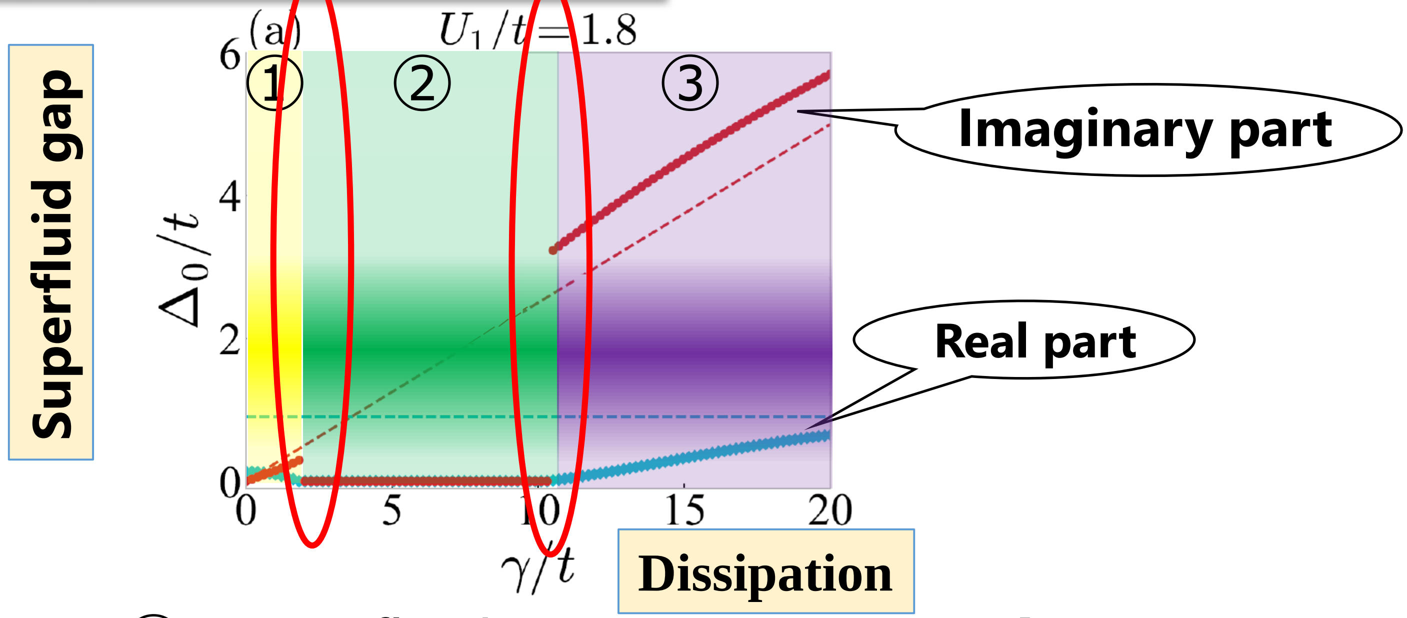
Results: Reentrant Superfluidity

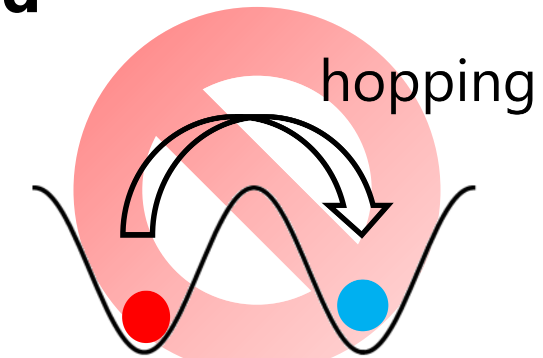
Superfluid gap: Weak attraction



① Superfluid gap is **suppressed**
② Superfluid gap **vanishes**
③ ***Reentrant*** **Superfluid gap**

Reentrant?

Continuous quantum Zeno effect



Localization of particles → **effective molecular pairs**

Results: Emergence of Exceptional Manifolds

Breakdown and **restoration** → **Exceptional points**

Exceptional points: Hamiltonian cannot be diagonalized

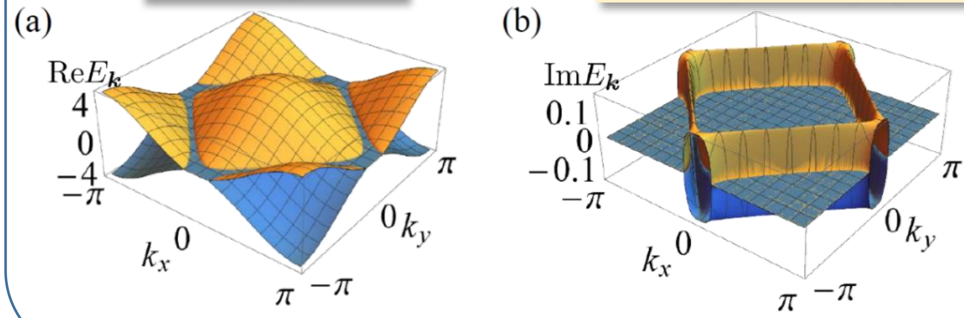
$$H_{\mathrm{MF}} = \sum_{\boldsymbol{k}} \begin{pmatrix} c^{\dagger}_{\boldsymbol{k}\uparrow} & c_{-\boldsymbol{k}\downarrow} \end{pmatrix} \begin{pmatrix} \epsilon_{\boldsymbol{k}} & \Delta \\ \bar{\Delta} & -\epsilon_{\boldsymbol{k}} \end{pmatrix} \begin{pmatrix} c_{\boldsymbol{k}\uparrow} \\ c^{\dagger}_{-\boldsymbol{k}\downarrow} \end{pmatrix}$$

Non-Hermitian matrix
Defective

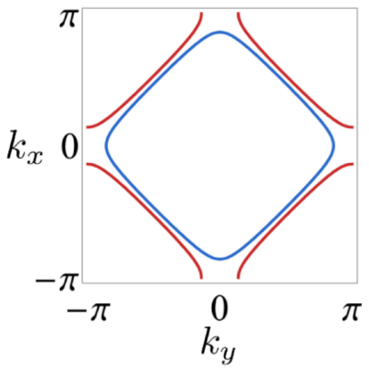
Energy spectrum (2D)

Real part

Imaginary part



Exceptional line (2D)



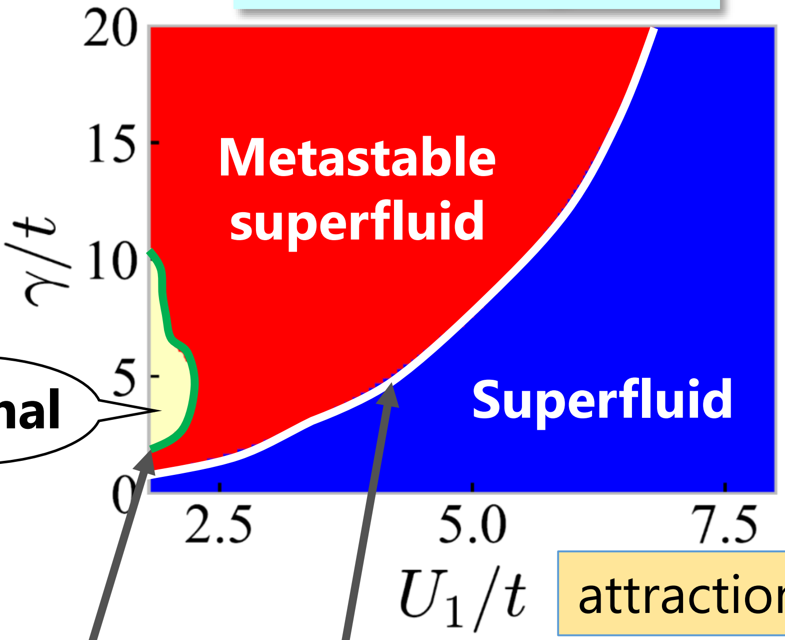
Unconventional phase transitions

attributed to **exceptional points, lines, and surfaces**

Results: Phase Diagram

Phase diagram

dissipation



attraction

Normal

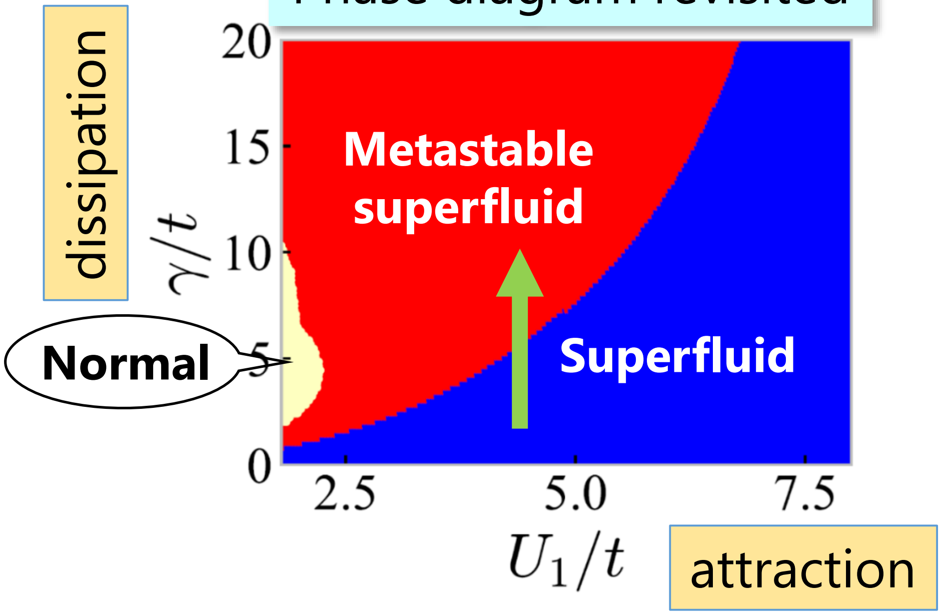
Metastable superfluid
: local energy minimum

First-order phase transition

Unconventional phase transition
: Exceptional points

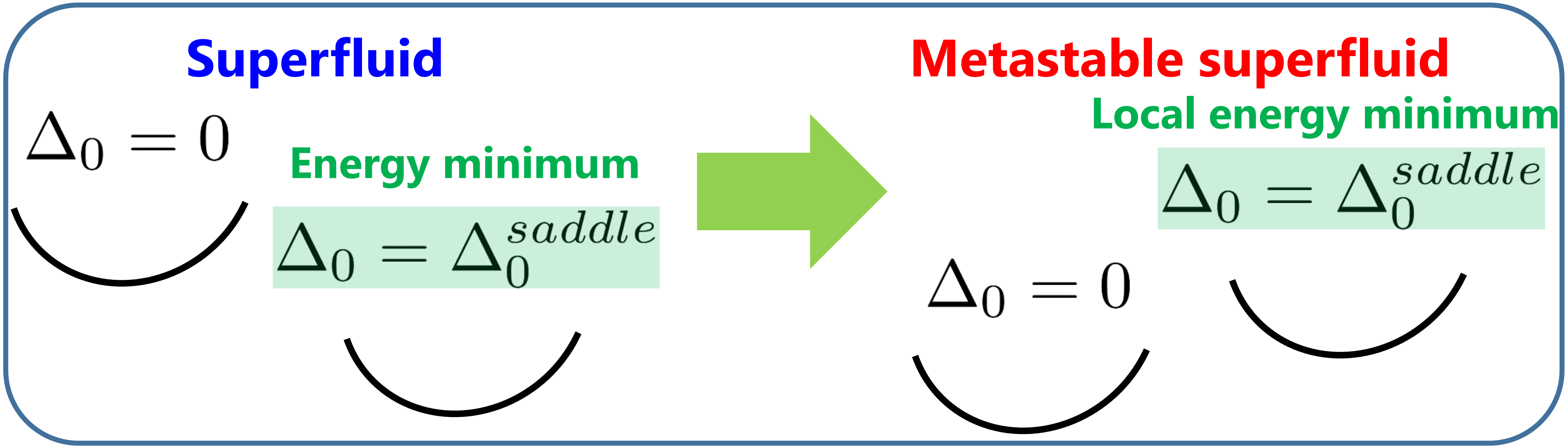
First-Order Phase Transition

Phase diagram revisited



Metastable superfluid

gradually increasing dissipation from the blue region



Superfluid

$\Delta_0 = 0$

Energy minimum

$\Delta_0 = \Delta_0^{saddle}$

Metastable superfluid

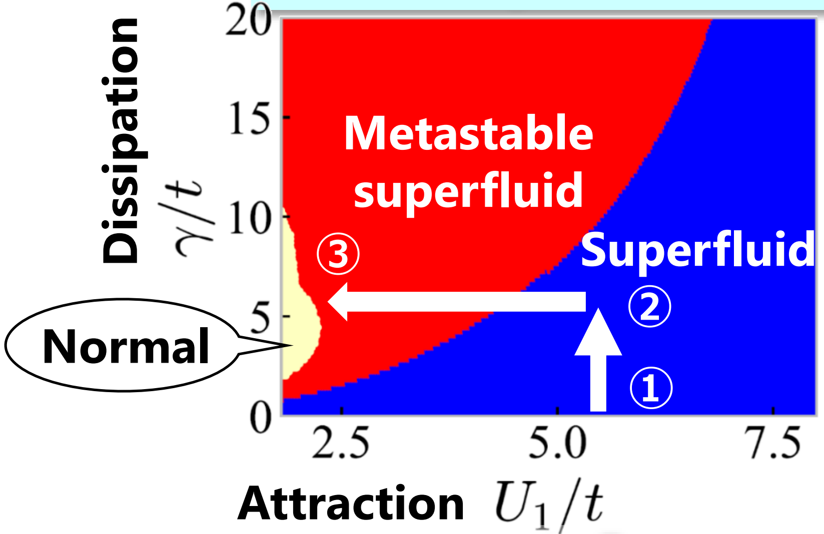
Local energy minimum

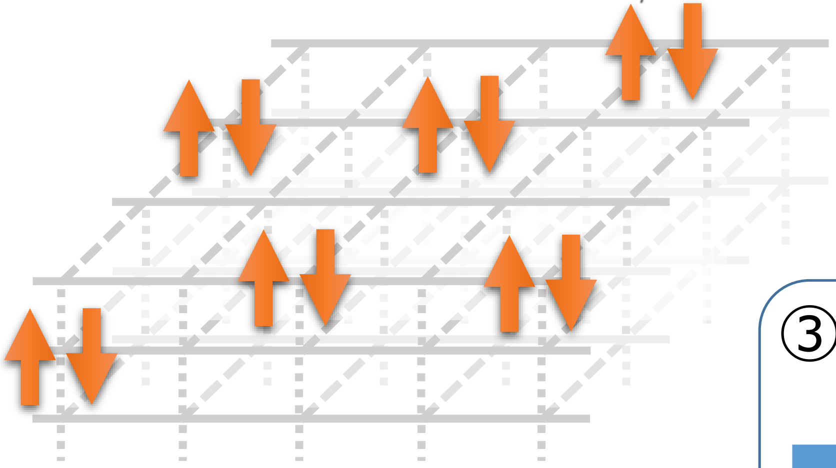
$\Delta_0 = \Delta_0^{saddle}$

$\Delta_0 = 0$

Metastable Superfluid for Large Dissipation

Phase diagram revisited





① On-site Cooper pairs for strong attraction U_1

Introduce strong dissipation

② **Decreasing attraction U_1**

Strong on-site Cooper pairs
Molecular bosons by Zeno effect

③ **Delay of dissociation** of Cooper pairs

→ molecules
Feature of the *reentrant superfluidity*

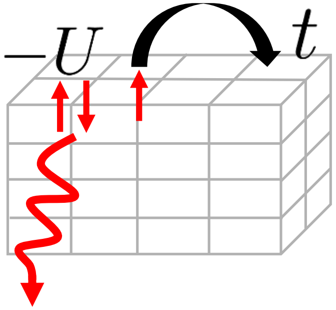
Summary

Model : BCS Hamiltonian + **Complex-valued interaction**

Formulation of the **non-Hermitian mean field theory**

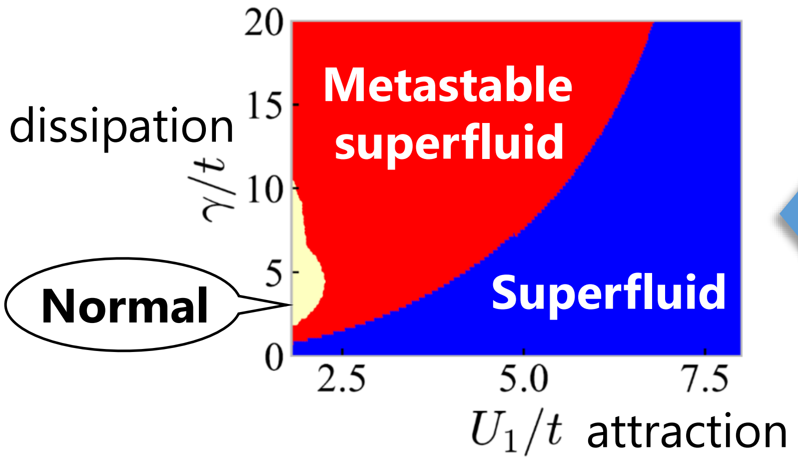
NH gap equation

$$\frac{N}{U} = \sum_{\boldsymbol{k}} \frac{1}{2\sqrt{\epsilon_{\boldsymbol{k}}^2+\bar{\Delta}\Delta}} \tanh \frac{\beta\sqrt{\epsilon_{\boldsymbol{k}}^2+\bar{\Delta}\Delta}}{2}$$

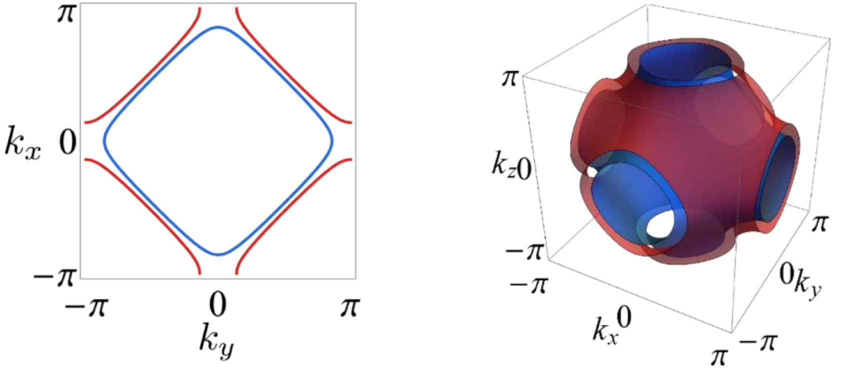


Quasiparticles unique to non-Hermitian systems

Reentrant superfluidity



Unconventional phase transition



Exceptional line (2D) Exceptional surface (3D)