MHD effects on fingering convection in stars: the problem with parasites

Adrian Fraser, Pascale Garaud

UC Santa Cruz

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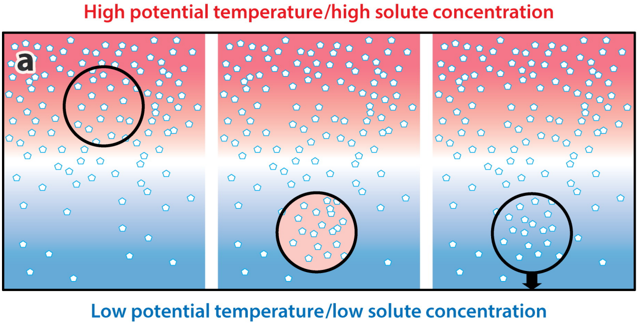
My email: adfraser@ucsc.edu

Fingering convection: a (double-) diffusive instability

Consider **Ledoux-stable** fluid with competing T and C (or "μ") gradients



Garaud 2018, Ann Rev Fluid Mech

Larger thermal diffusion $\Rightarrow$ high-μ parcel buoyantly sinks
$\rightarrow$ **Fingering instability**, driven by ∇_μ, competing against $\nabla_T - \nabla_{\mathrm{ad}}$

(AKA thermohaline mixing)

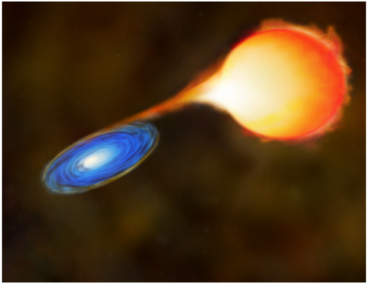
Some examples in stars

Polluted WDs: thermohaline mixing enhances inferred accretion rates [Bauer & Bildsten 2018, 2019]

Massive accretor stars: thermohaline mixing due to accreted material can dominate over other processes [Renzo & Götberg 2021]

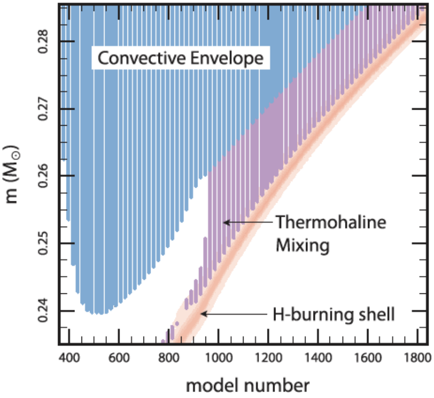
RGB stars at L bump: anomalous mixing beneath CZ post-dredge-up [Shetrone, Tayar, et al. 2019]

→ What drives the mixing?



[NASA APOD]

Turbulent mixing in stars – “missing mixing” problem



Thermohaline mixing added to MESA [Cantiello & Langer 2010; Paxton et al. 2013]

→ **Agreement with observations depends on mixing prescription**

Hydro simulations: **insufficient mixing to explain observations** [e.g. Denissenkov 2010, Brown et al. 2013]

Harrington & Garaud 2019 (HG19): MHD enhances mixing dramatically

Model: local box with fixed gradients, Boussinesq

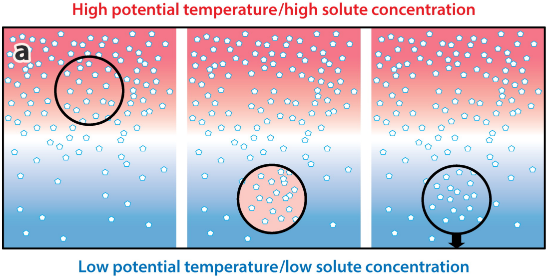
Linearize EOS: $\rho'/\rho_{\rm m} = -\alpha T' + \beta C'$

Perturb about constant gradients:

$T' = \frac{dT_0}{dz} z + \tilde{T}$,

$C' = \frac{dC_0}{dz} z + \tilde{C}$

Periodic BCs for $\tilde{T}$, $\tilde{C}$



Non-dimensionalize in terms of:

$$[x] = d \equiv \left(\frac{\kappa_T \nu}{\alpha g \left| \frac{dT_0}{dz} - \frac{dT_{\rm ad}}{dz} \right|} \right)^{1/4}, \quad [u] = \frac{\kappa_T}{d}$$

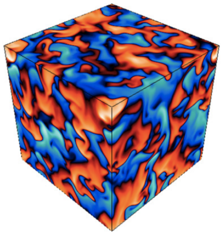
Model: local box with fixed gradients, Boussinesq

$$\frac{1}{\mathrm{Pr}}\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + (T-C)\mathbf{e}_z + \nabla^2\mathbf{u}, \qquad \nabla\cdot\mathbf{u} = 0,$$
$$\frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T + w = \nabla^2 T, \qquad \frac{\partial C}{\partial t} + \mathbf{u}\cdot\nabla C + \frac{w}{R_0} = \tau\nabla^2 C$$

Where $\mathrm{Pr} = \frac{\nu}{\kappa_T}$, $\tau = \frac{\kappa_C}{\kappa_T}$, $R_0 = \frac{\alpha|dT_0/dz - dT_{\mathrm{ad}}/dz|}{\beta|dC_0/dz|}$

$1 < R_0 < 1/\tau \Rightarrow$ fingering convection

Goal: predict mixing, i.e.,
$\mathrm{Nu}_C = \frac{\text{total flux}}{\text{diffusive flux}}$, or $D_{\mathrm{turb}} \sim \mathrm{Nu}_C \kappa_C$



Brown et al. 2013

This work: study magnetic fields

Following HG19, add MHD:

$$\frac{1}{\mathrm{Pr}}\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + (T-C)\mathbf{e}_z + \nabla^2\mathbf{u} + H_B(\nabla\times\mathbf{B})\times\mathbf{B}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{u}\times\mathbf{B}) + D_B\nabla^2\mathbf{B}, \quad \nabla\cdot\mathbf{B} = 0, \quad \nabla\cdot\mathbf{u} = 0$$

$$\frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T + w = \nabla^2 T, \qquad \frac{\partial C}{\partial t} + \mathbf{u}\cdot\nabla C + \frac{w}{R_0} = \tau\nabla^2 C$$

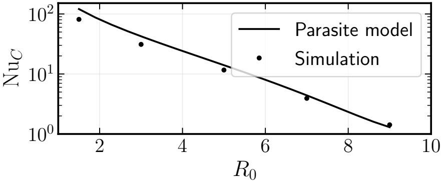
Where $\mathrm{Pr} = \nu/\kappa_T$, $\tau = \kappa_C/\kappa_T$, $R_0 = \frac{\alpha|dT_0/dz - dT_{\mathrm{ad}}/dz|}{\beta|dC_0/dz|}$, $D_B = \eta/\kappa_T$, $H_B = v_A^2/[u]^2 \propto B_0^2$

Study vertical, uniform B_0

"Parasitic saturation" models – 2 key ingredients

Thermohaline mixing well-described by "parasitic saturation" models (*cf.* GSF, MRI)

Model consistent with hydro simulations [Brown et al. 2013]

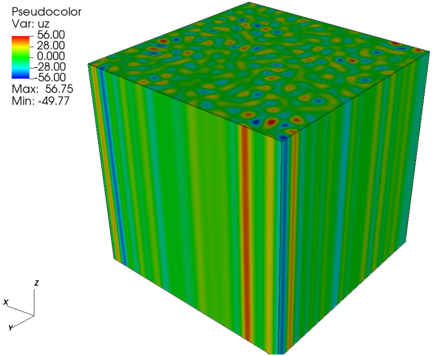


Model ingredients:

(1)

(2)

Fastest-growing modes: "elevator modes", elongated in z



Left: vertical velocity w during instability growth

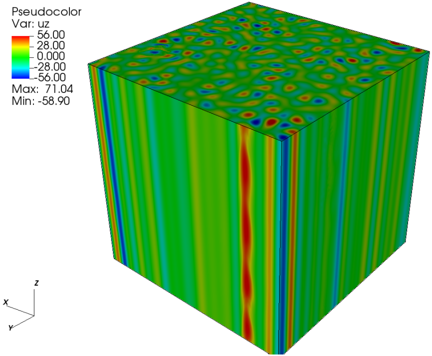
"**Elevator modes**" are fastest-growing $w_f \sim e^{\lambda_f t}$
$\rightarrow$ assume they determine mixing

Model ingredients:

(1) Mixing $\propto$ elevator mode amplitude w_f

(2)

Elevator modes become unstable to shear-flow "parasites"



Shear drives KH modes $e^{\sigma_{\mathrm{KH}} t}$

σ_{KH} increases with w_f

Modes grow ($w_f \sim e^{\lambda_f t}$) *until KH disrupts them*

$\rightarrow$ assume timescales match, $\sigma_{\mathrm{KH}}(w_f) \sim \lambda_f$

Model ingredients:

(1) Mixing $\propto$ elevator mode amplitude w_f

(2) w_f determined by parasitic growth condition: $\sigma_{\mathrm{KH}} \sim \lambda_f$

Reduced model: 2 key assumptions

Model ingredients:

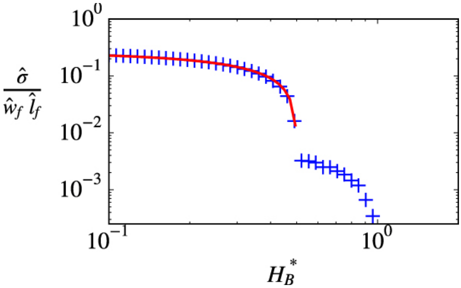
(1) Mixing $\propto$ elevator mode amplitude w_f

(2) w_f determined by parasitic growth condition: $\sigma_{\mathrm{KH}} \sim \lambda_f$

Ideal MHD:

- $\mathbf{B}_0$ reduces σ_{KH}
- To compensate, w_f must increase for $\sigma_{\mathrm{KH}} \sim \lambda_f$

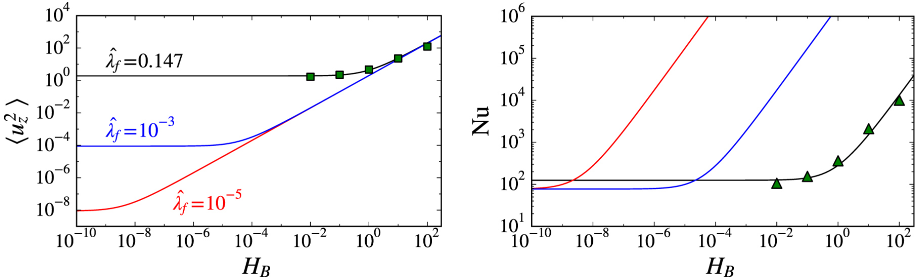
$\Rightarrow$ **increases mixing**



Harrington & Garaud 2019

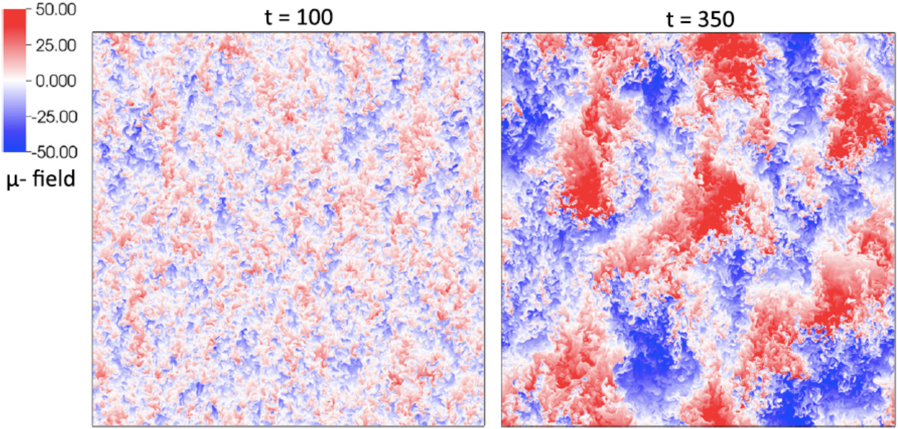
B_0 trends, model/DNS agreement at low R_0

Simulations show excellent agreement with parasite model at
$\mathrm{Pr} = \tau = 0.1$, $R_0 = 1.45$, $\mathrm{Pm} = 1$
[Harrington & Garaud 2019]



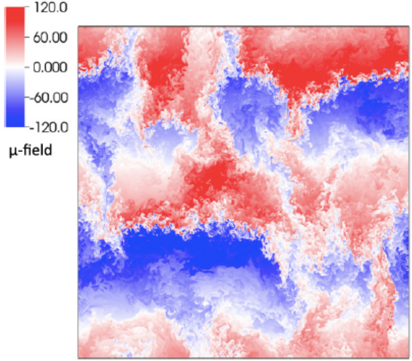
Unexplored: higher R_0, $\mathrm{Pm} < 1$ (realistic in stars)

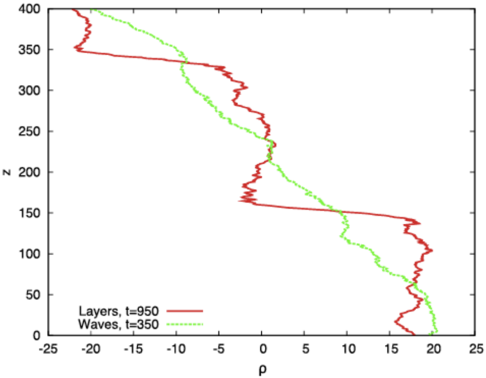
Thermohaline mixing: source of IGWs & convective layers?



[Garaud et al. 2015]

Thermohaline mixing: source of IGWs & convective layers?



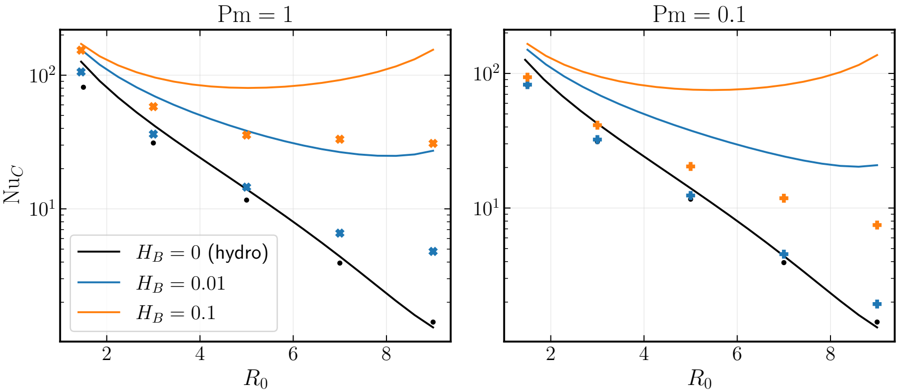


Garaud et al. 2015: thermohaline mixing typically too inefficient to drive IGWs & convective layers

→ HG19 model predicts convective layers for intermediate range of $\mathbf{B}_0$!

→ Possible "smoking gun"?

Parasite model fails at larger R_0, low Pm

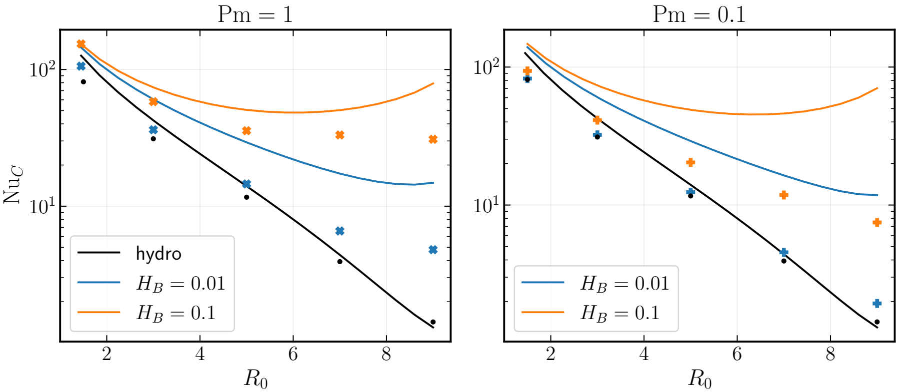


Left: higher R_0 at $\mathrm{Pm} = 1$ shows worrying model inaccuracies

Right: bad becomes worse for $\mathrm{Pm} < 1$

We've scrutinized every inch of the model...

Parasite model fails at larger R_0, low Pm



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And only made slight improvements

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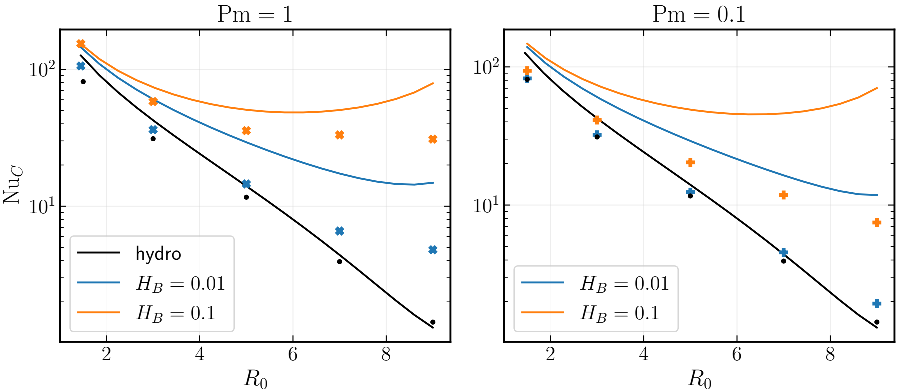

Left: higher R_0 at $\mathrm{Pm} = 1$ shows worrying model inaccuracies

Right: bad becomes worse for $\mathrm{Pm} < 1$

We've scrutinized every inch of the model...

And only made slight improvements → **parasitic saturation model misses significant physics**

Conclusions

Key take-aways:

- MHD enhances mixing $\rightarrow$ might provide "smoking guns"
- HG19 model fails at moderate R_0 and $\mathrm{Pm} < 1 \rightarrow$
- Ongoing work needed to determine what key physics is missing in model
  - $\rightarrow$ *KH saturation details [with I.G. Cresswell & P. Garaud]*
  - $\rightarrow$ *Proper accounting of Maxwell stress [with P. Garaud]*
  - $\rightarrow$ *Nonmodal growth [with J.S. Oishi & A.K. Kaminski]*