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Mode Coupling among Gravito-Inertial Modes in Slowly Pulsating B Stars

Transport in Stellar Interiors conference

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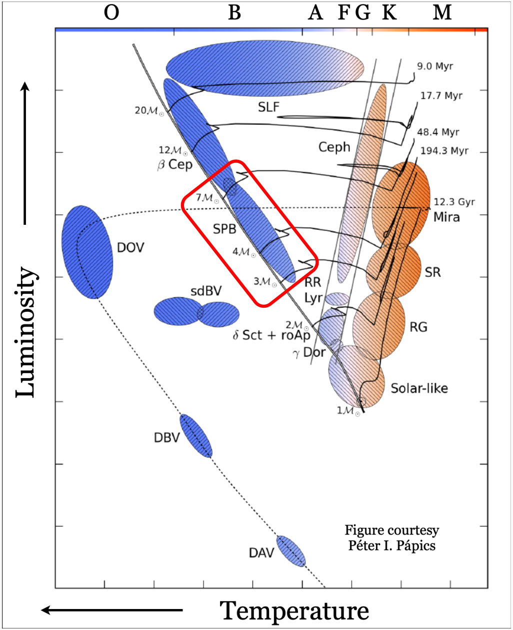
Warning!

Observational point of view

Slowly pulsating B stars (SPBs)



⟶ Asteroseismic HR Diagram

SPBs = Multiperiodic gravito-inertial mode oscillators

↓

$\sim$ 3 to $\sim$ 9 $M_\odot$

0.5 days $\leq P_{puls} \leq$ 5.0 days

low-degree, high-radial-order

varying rotation rates

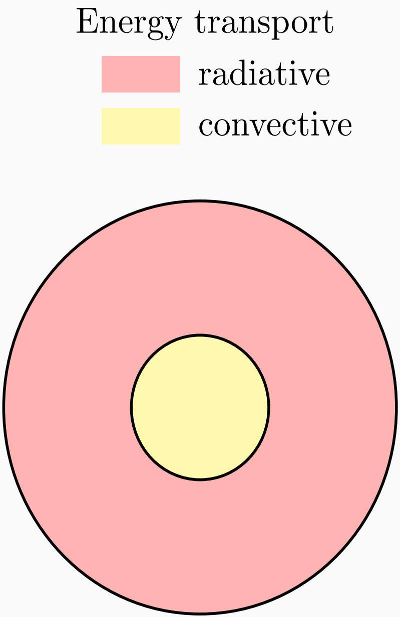
(e.g. Pedersen et al., 2021)

⇝

Fe-group opacity enhancement: κ mechanism

(Dziembowski et al., 1993; Gautschy and Saio, 1993; Pamyatnykh, 1999)

Slowly pulsating B stars (SPBs)



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Importance

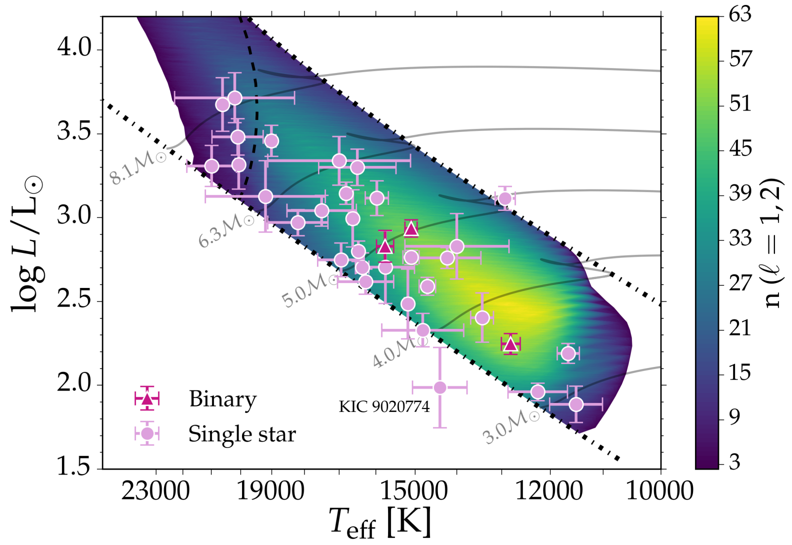
Linear asteroseismology $\rightarrow$ using frequencies to probe:

- internal rotation (e.g. Triana et al., 2015)
- internal mixing (e.g. Pedersen et al., 2021)
- mode excitation (e.g. Szewczuk et al., 2021)
- internal magnetic fields (e.g. Buysschaert et al., 2018)

Non-linear asteroseismology $\rightarrow$ also account for:

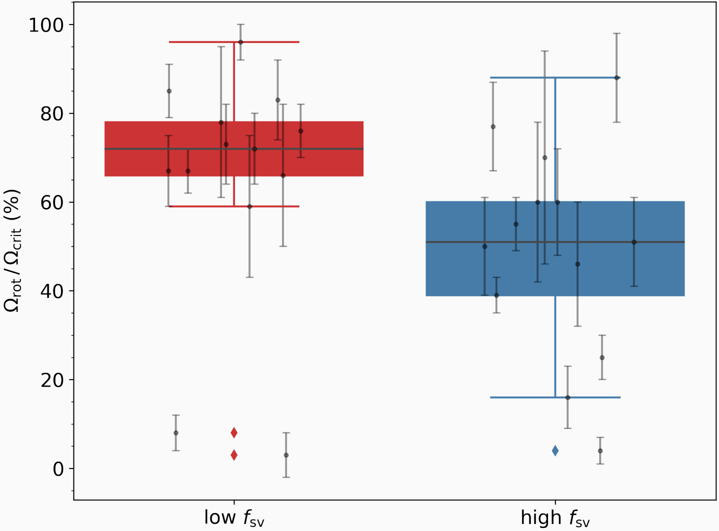
- amplitude limitation
- non-linear frequency shifts

Our Sample 38 *Kepler* SPBs



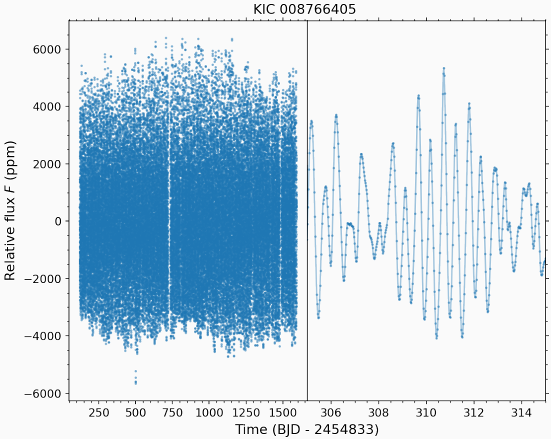
(Pedersen et al., 2020)

Our Sample 38 *Kepler* SPBs

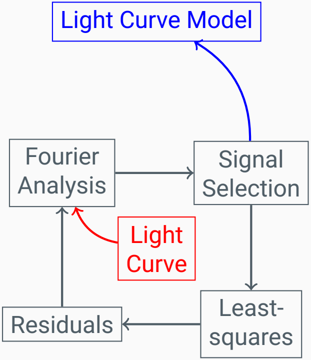


(Pedersen et al., 2021; Van Beeck et al., 2021)

Variability Analysis Pre-Whitening



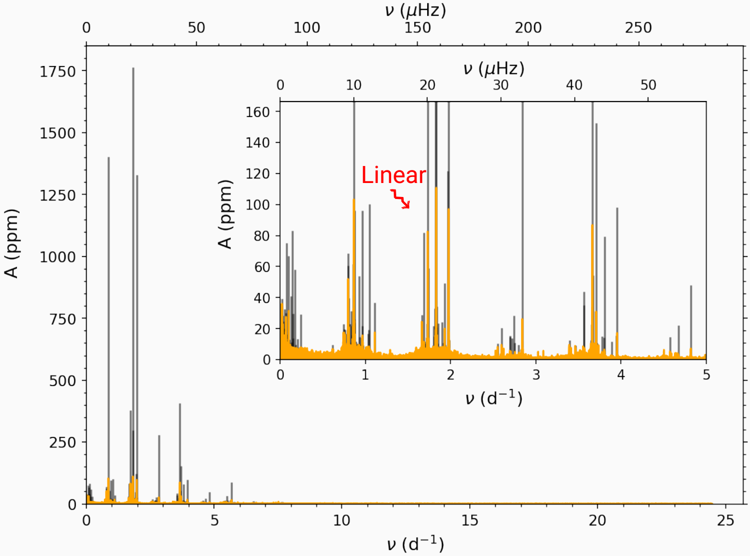
Iterative Pre-Whitening:



Fourier Analysis + Sum-of-sines fit

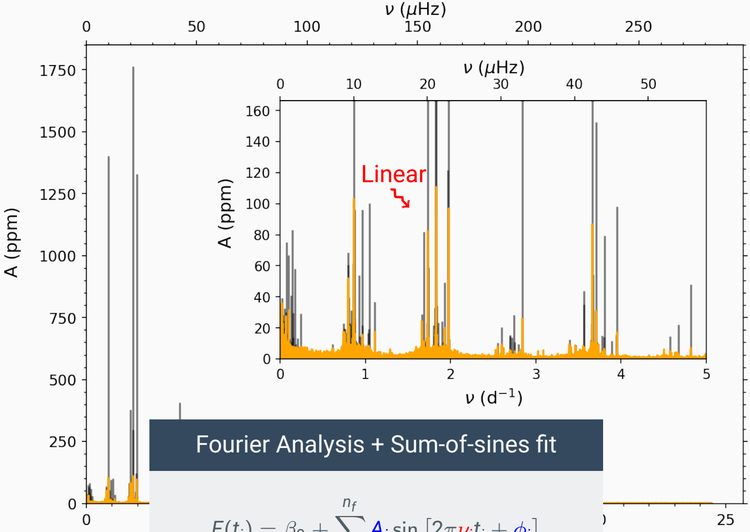
$$F(t_i) = \beta_0 + \sum_{j=1}^{n_f} A_j \sin\left[2\pi\nu_j t_i + \phi_j\right]$$

Variability Analysis Pre-Whitening



(Van Beeck et al., 2021)

Variability Analysis Pre-Whitening



Fourier Analysis + Sum-of-sines fit

$$F(t_i) = \beta_0 + \sum_{j=1}^{n_f} A_j \sin\left[2\pi\nu_j t_i + \phi_j\right]$$

Variability Analysis Pre-whitening Error Budget

Checking self-consistency:

Δstrategy !

Van Beeck et al. (2021) : 5 strategies

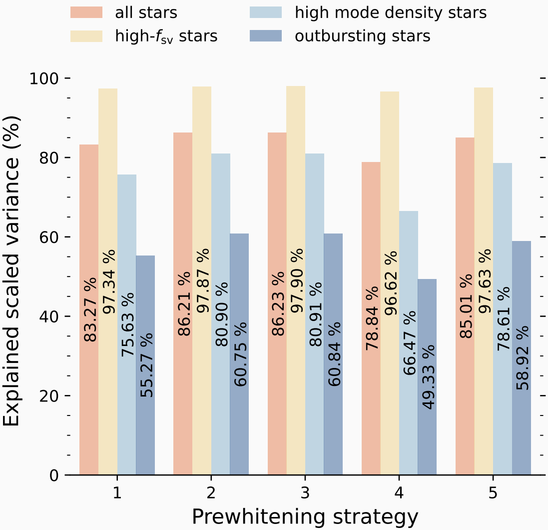
$\rightarrow$ Δ signal selection

$\rightarrow$ Δ least-squares

Model Comparison

$$f_{\mathrm{sv}} \equiv 1 - \left(\frac{\sum_i^{n_t} (y_i - F(t_i))^2}{\sum_i^{n_t} (y_i - \bar{y})^2} \right) \left\{ \frac{n_t - 1}{n_t - n_p} \right\}$$

Self-consistency



(Van Beeck et al., 2021)

(weakly) Non-linear Mode Coupling

Observational Characteristics

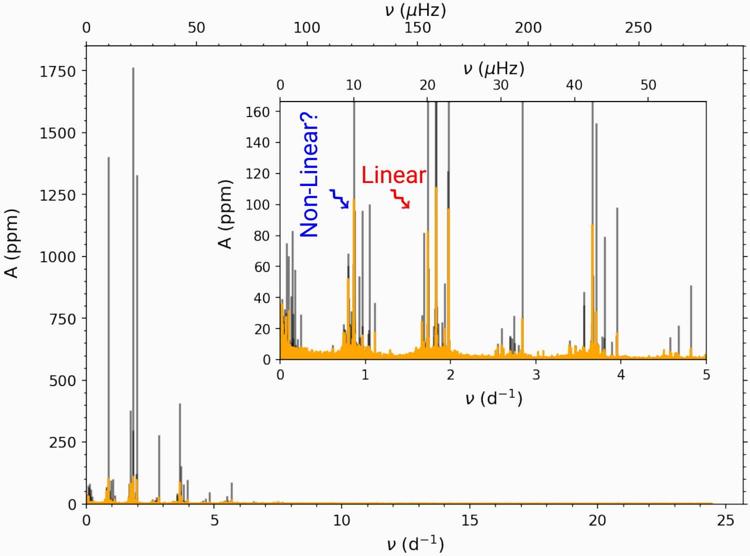
(Resonant) Three-wave coupling:

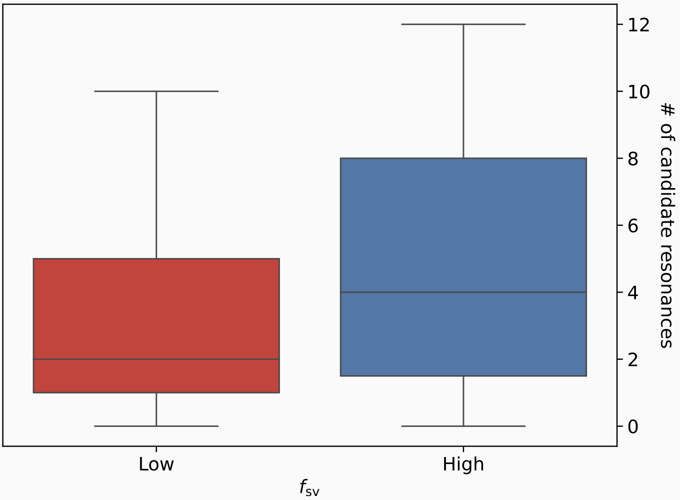
$$\delta\nu \equiv \nu_{\mathrm{daughter}} - n_1\,\nu_{\mathrm{parent},\,1} - n_2\,\nu_{\mathrm{parent},\,2} \approx 0\,\mathrm{d}^{-1}$$

$$\Phi \equiv \phi_{\mathrm{daughter}} - n_1\,\phi_{\mathrm{parent},\,1} - n_2\,\phi_{\mathrm{parent},\,2} \approx k \cdot \frac{\pi}{2}$$

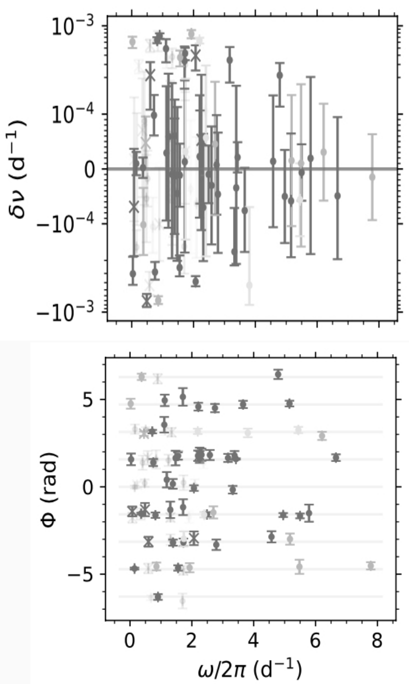
(inspired by e.g. Van Hoolst, 1995; Buchler et al., 1997)

(weakly) Non-linear Mode Coupling





(Van Beeck et al., 2021)



Resonant Couplings = Ubiquitous

(Van Beeck et al., 2021)

What now?

Gravito-inertial modes linear asteroseismology

(Linear) equations of motion: $\frac{\partial^2 \boldsymbol{\xi}}{\partial t^2} + \mathbf{B}\left(\frac{\partial \boldsymbol{\xi}}{\partial t}\right) + \mathbf{C}\left(\boldsymbol{\xi}\right) = 0$

Assumptions:

- **uniform rotation**
- **no centrifugal deformation**

Coriolis force $\Rightarrow$ separation of variables $\neq$ possible!

$\rightarrow$ Traditional Approximation for Rotation (TAR)

- *neglect* horizontal component angular frequency vector for rotation ($\Omega \sin \theta$, Bildsten et al., 1996; Lee and Saio, 1997)

Gravito-inertial modes linear asteroseismology

Driving

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Gravito-inertial modes linear asteroseismology

Driving Coriolis

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Gravito-inertial modes linear asteroseismology

Driving

Coriolis

(Linear) equations of motion: $\frac{\partial^2 \xi}{\partial t^2} + \mathbf{B}\left(\frac{\partial \xi}{\partial t}\right) + \mathbf{C}\left(\xi\right) = 0$

Pressure + Buoyancy

Assumptions:

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Wave coupling weakly non-linear asteroseismology

Coupled-mode Equations: nonlinear acceleration

$$\boxed{\frac{\partial^2 \xi}{\partial t^2} + \mathbf{B}\left(\frac{\partial \xi}{\partial t}\right) + \mathbf{C}(\xi) = \boldsymbol{a}^{(2)}(\xi, \xi) + \mathcal{O}\left(\xi^3\right)}$$

Wave coupling weakly non-linear asteroseismology

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$$\downarrow$$

Expansion in linear eigenmodes: amplitudes

$$\begin{bmatrix} \xi \\ \frac{\partial \xi}{\partial t} \end{bmatrix} = \sum_a c_a(t) \begin{bmatrix} \xi_a \\ \frac{\partial \xi_a}{\partial t} \end{bmatrix}$$

(e.g. Schenk et al., 2001; Lee, 2012; Weinberg et al., 2021)

Wave coupling amplitude equations

Coupled-mode equations **+** Expansion **+** (Modified) Orthogonality:

Amplitude Equations

$$\dot{c}_a + (i\omega_a + \gamma_a)c_a = i\omega_a \sum_{bc} \kappa^*_{abc} c^*_b c^*_c$$

$(\omega_a, \boldsymbol{\xi}_a)$ / $\gamma_a(\ll \omega_a)$ $\leftarrow$ adiabatic / non-adiabatic linear solutions

(e.g. Schenk et al., 2001; Lee, 2012; Weinberg et al., 2021)

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Linear growth rate

(ω_a, ξ_a) / $\gamma_a (\ll \omega_a)$ $\leftarrow$ adiabatic / non-adiabatic linear solutions

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Linear growth rate

Coupling Coefficient

(ω_a, ξ_a) / $\gamma_a (\ll \omega_a)$ $\leftarrow$ adiabatic / non-adiabatic linear solutions

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Wave coupling amplitude equations

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Linear growth rate

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(e.g. Schenk et al., 2001; Lee, 2012; Weinberg et al., 2021)

Analogous to non-rotating case: **Stationary Solutions**

(e.g. Van Hoolst, 1995; Buchler et al., 1997)

↳ $(\delta\nu, \Phi) \leftarrow$ observations!

Conclusions & Outlook

Conclusions:

based on 38 *Kepler* SPBs

$\rightarrow$ exploit more information $\rightarrow$ error budget!

$\rightarrow$ resonant couplings $=$ ubiquitous among SPB gravito-inertial modes!

Prospects/Outlook:

$\rightarrow$ modeling three-wave coupling among gravito-inertial modes in SPBs

$\rightarrow$ equivalent non-linear frequency shift for rotating stars? (e.g. Nayfeh and Mook, 1979)
↳ Δ Period Spacing Pattern?

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