Chemical Mixing By Internal Gravity Waves in Stars

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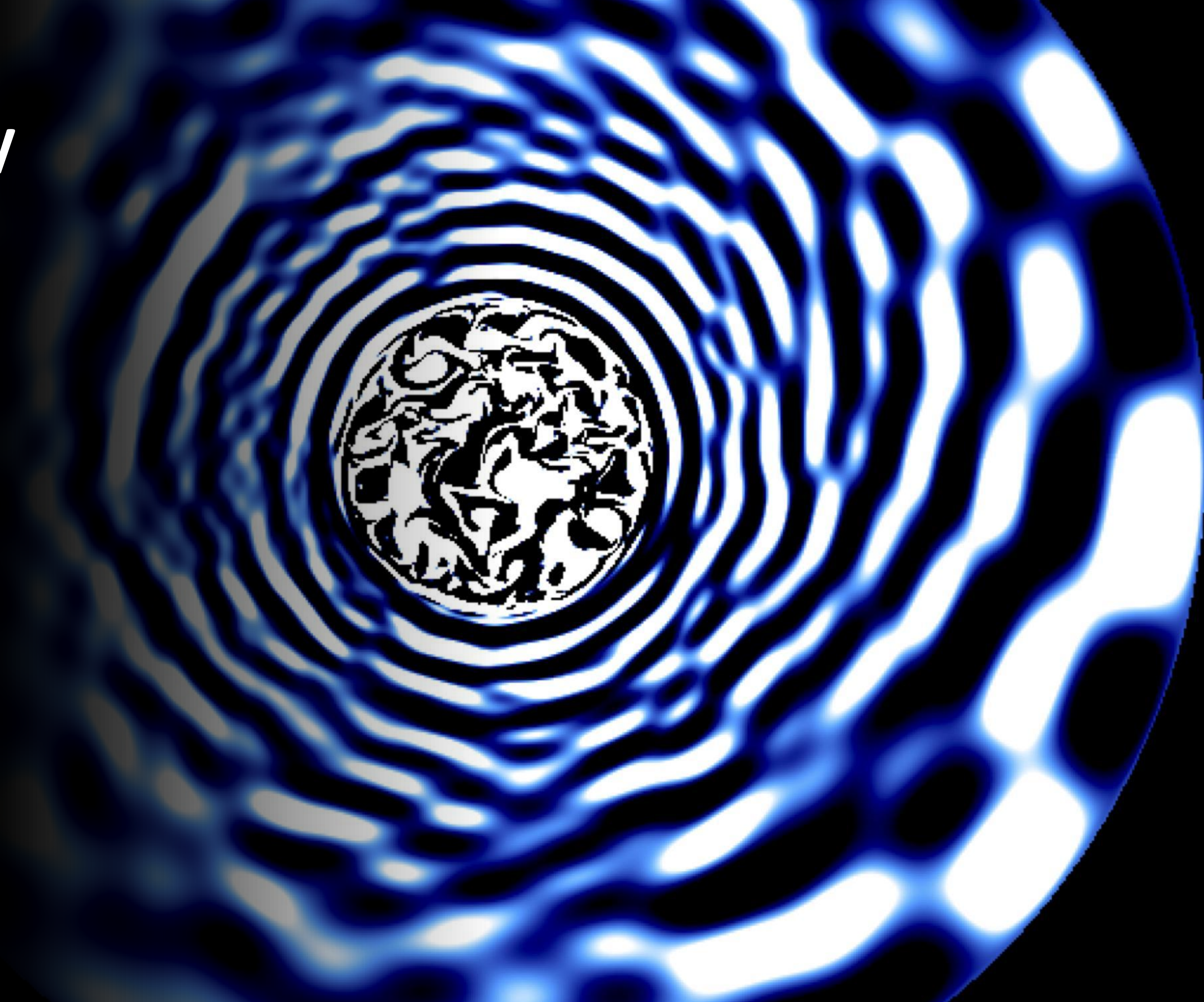
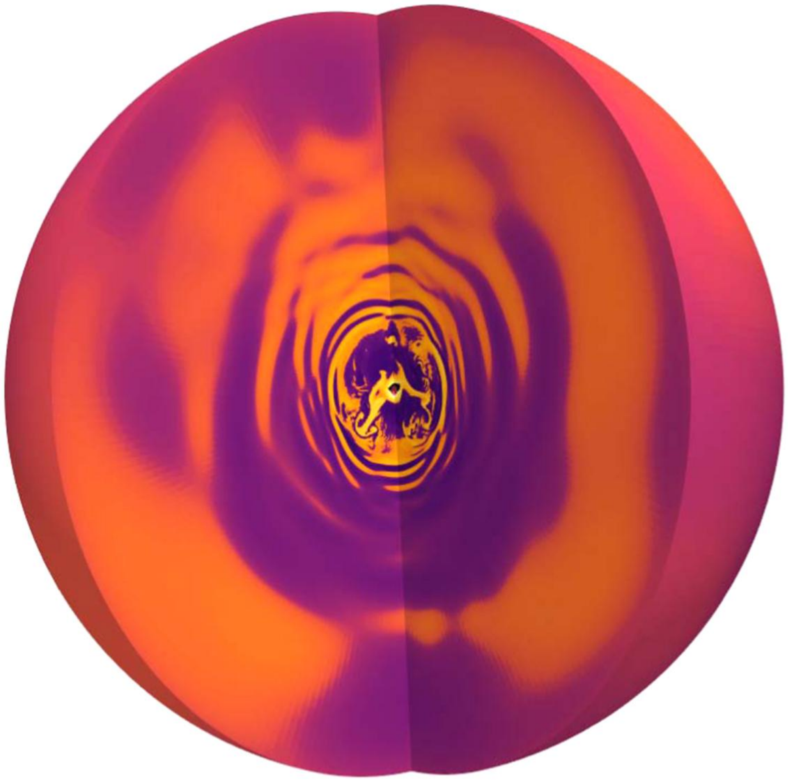


KITP: TRANSPORT IN STELLAR INTERIORS

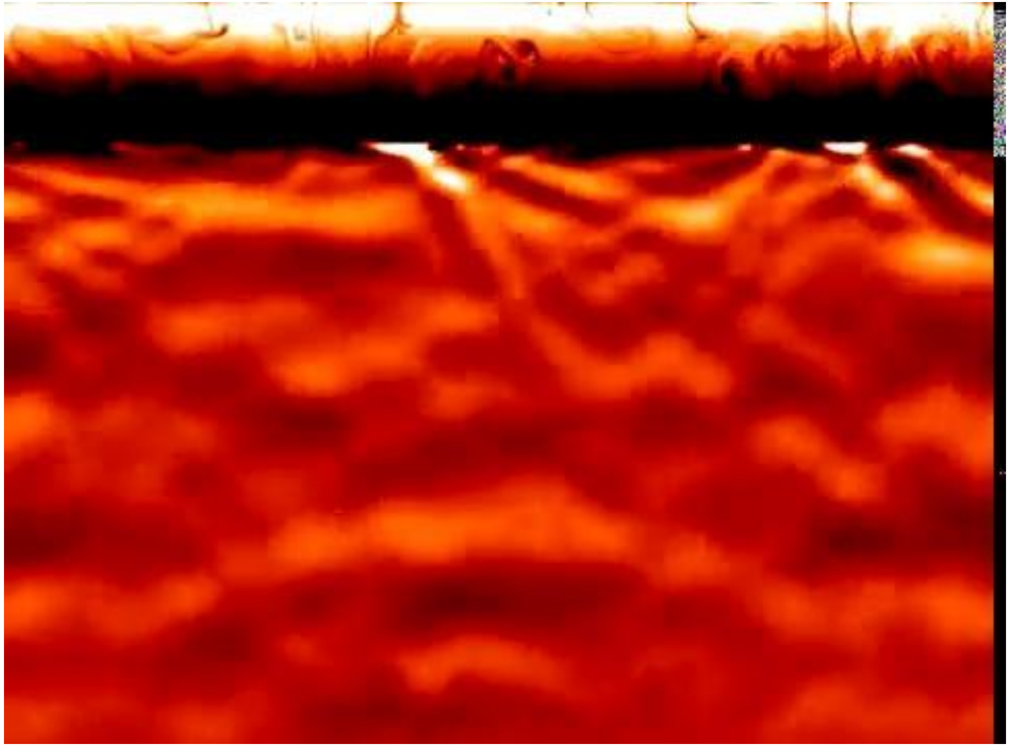
‘Mixing’ in stars !

- Increases main - sequence lifetime
- Modify Surface Abundances
- Mixing at the Convective boundary and in the radiation zone
- Focus on Wave Mixing

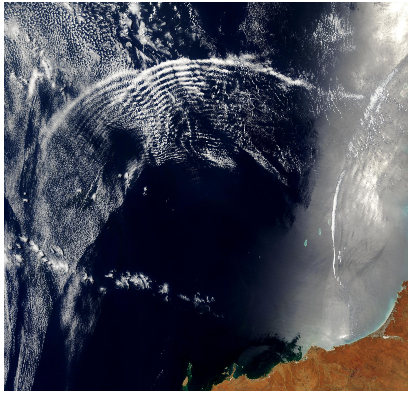


Edelmann et al. 2019

Internal Gravity Waves



Rogers et al. 2013



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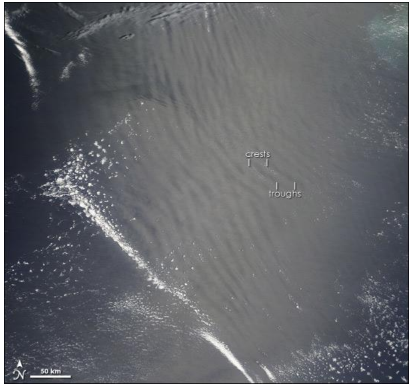
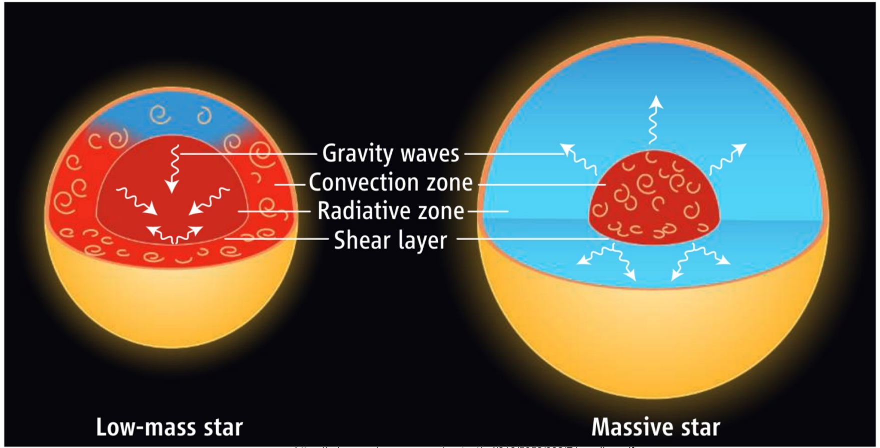


By Jeff Schmaltz MODIS Rapid Response Team, NASA-GSFC - "Atmospheric Gravity Waves over Arabian Sea", Public Domain, https://commons.wikimedia.org/w/index.php?curid=773266



https://science.sciencemag.org/content/sci/318/5852/922/F1.medium.gif

How did we study the wave mixing ?

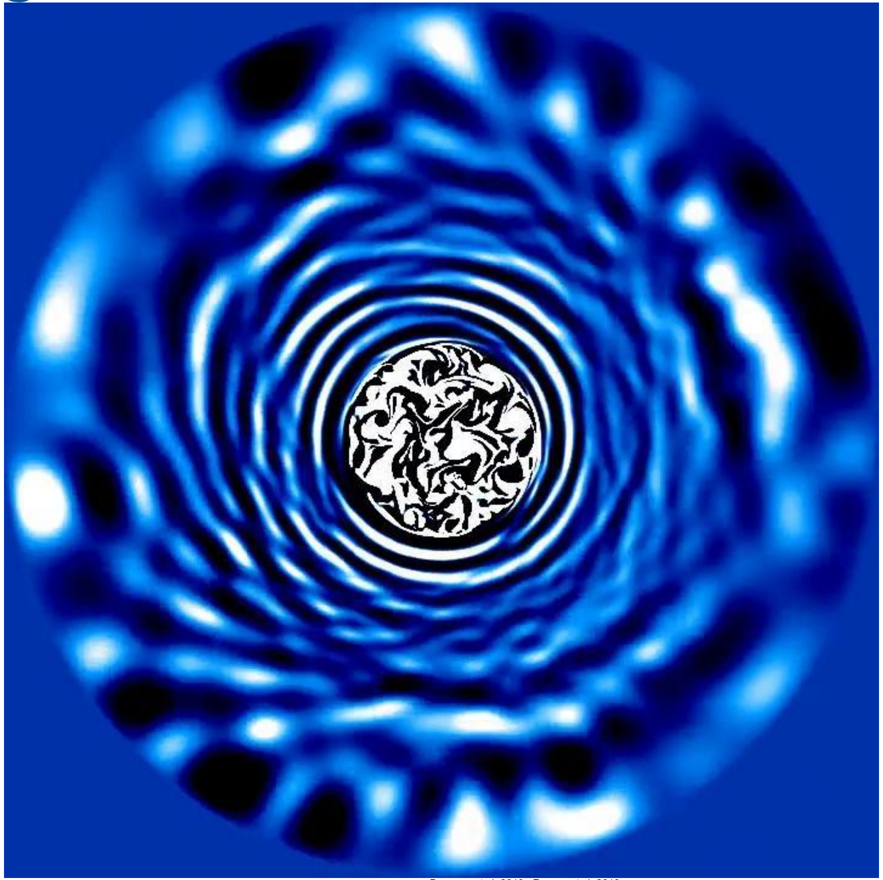
Solve 2D hydrodynamic equations in the anelastic approximation

$$\nabla.\bar{\rho}v = 0$$

$$\frac{\partial v}{\partial t} + (v.\nabla)v = -\nabla P - Cg\hat{r} + 2(v \times \Omega) + \bar{\nu}\left(\nabla^2 v + \frac{1}{3}\nabla(\nabla.v)\right)$$

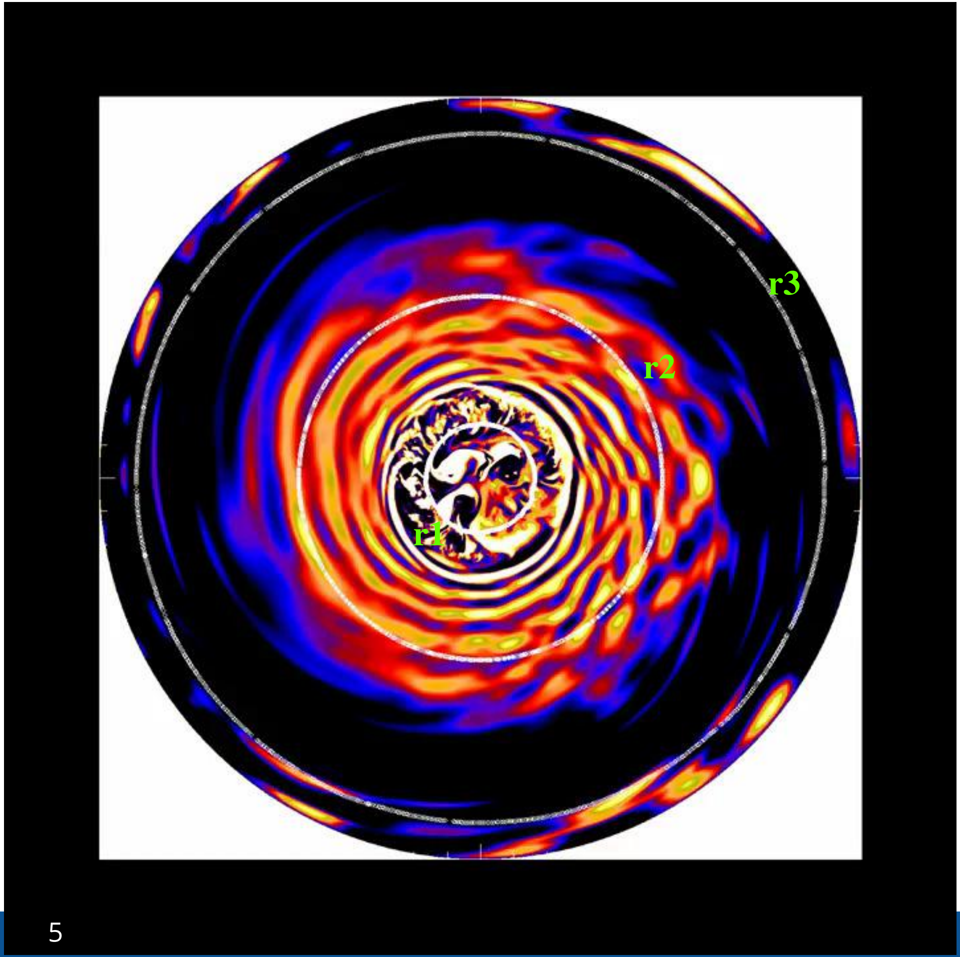
$$\frac{\partial T}{\partial t} + (v.\nabla)T = -v_r\left(\frac{\partial \bar{T}}{r} - (\gamma - 1)\bar{T}h_\rho\right) + (\gamma - 1)Th_\rho v_r + \gamma\bar{K}\left[\nabla^2 T + h_\rho\frac{\partial T}{\partial r}\right] + \gamma\bar{K}\left[\nabla^2\bar{T} + h_\rho\frac{\partial \bar{T}}{\partial r}\right] + \frac{\bar{Q}}{c_\nu}$$

Background reference state model is from 1D stellar evolution code MESA.



Bowman et al. 2019; Rogers et al. 2013.

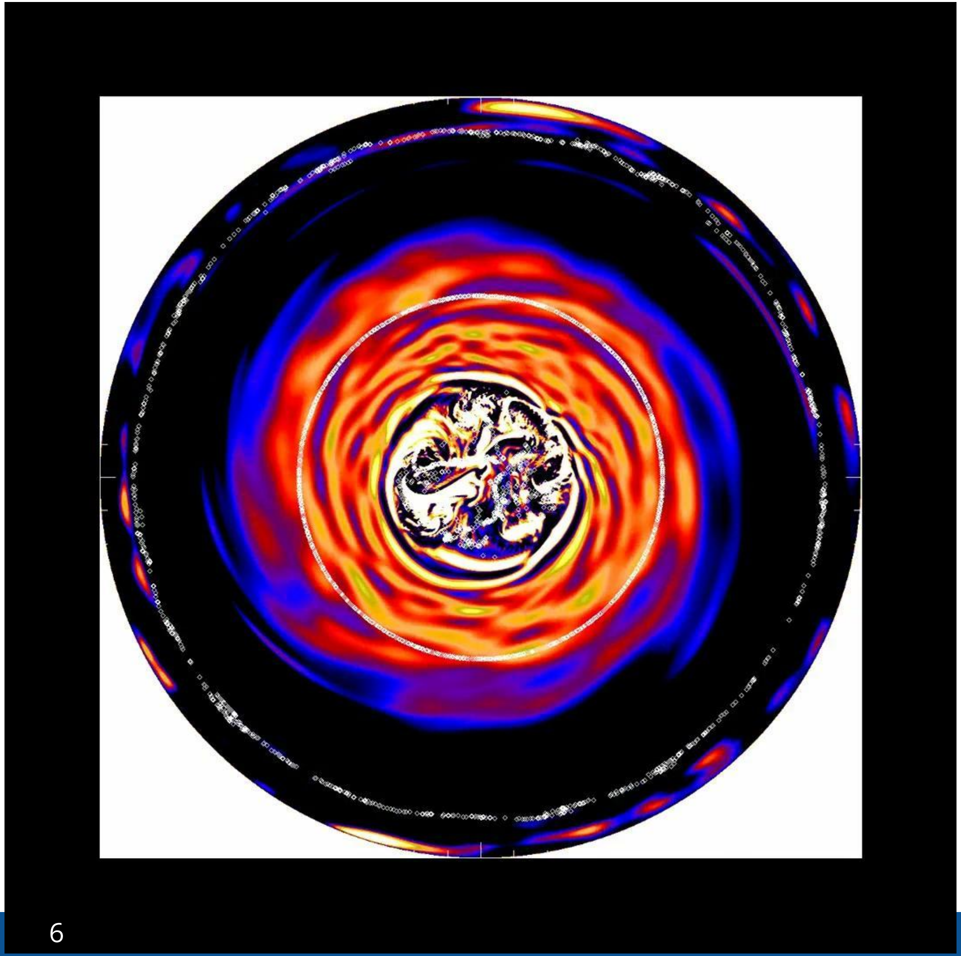
Can mixing by IGWs treated as a diffusive process ?

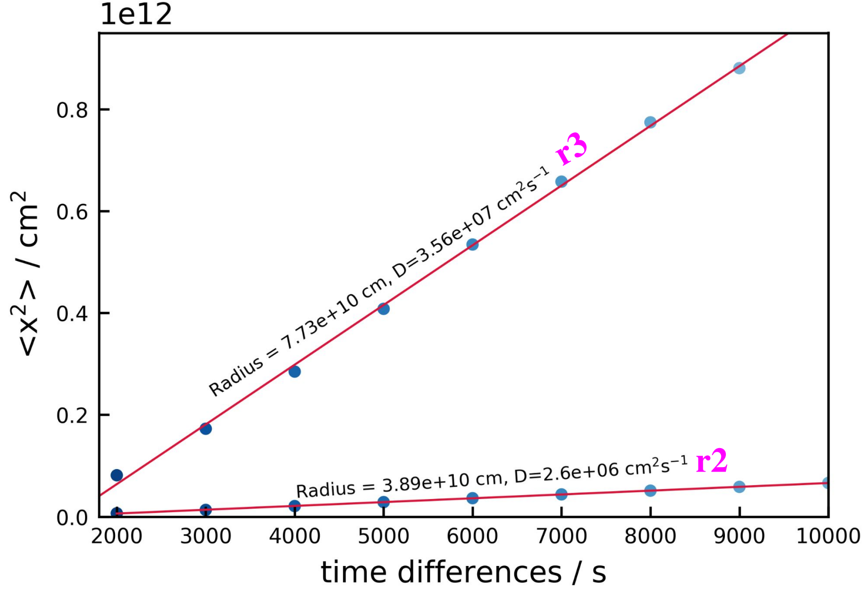


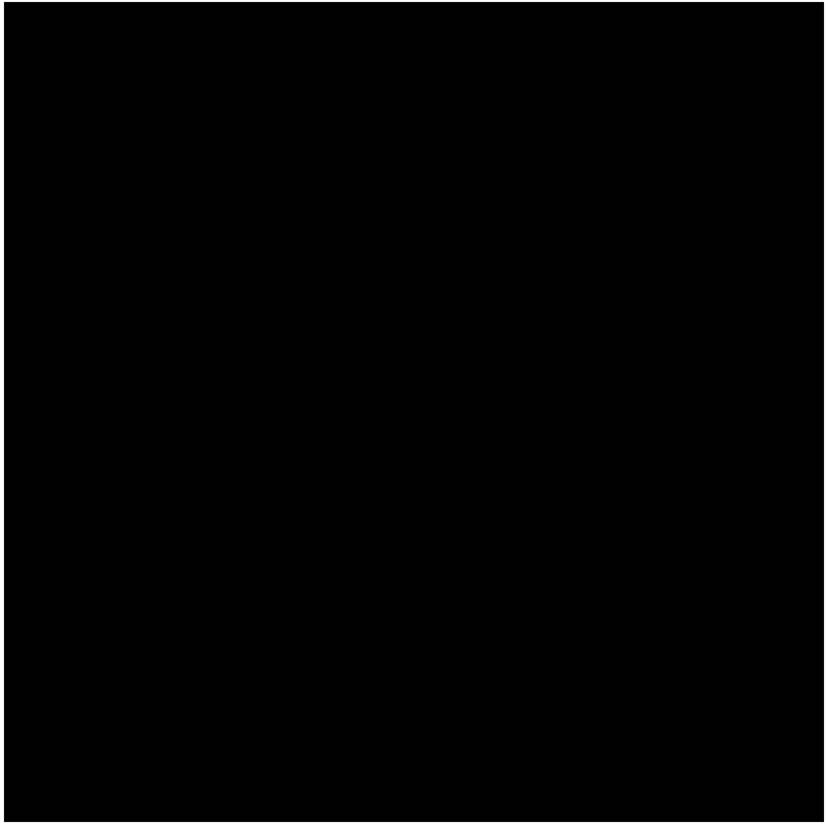
Efficient mixing expected in the convective zone

In Radiative Zone ?

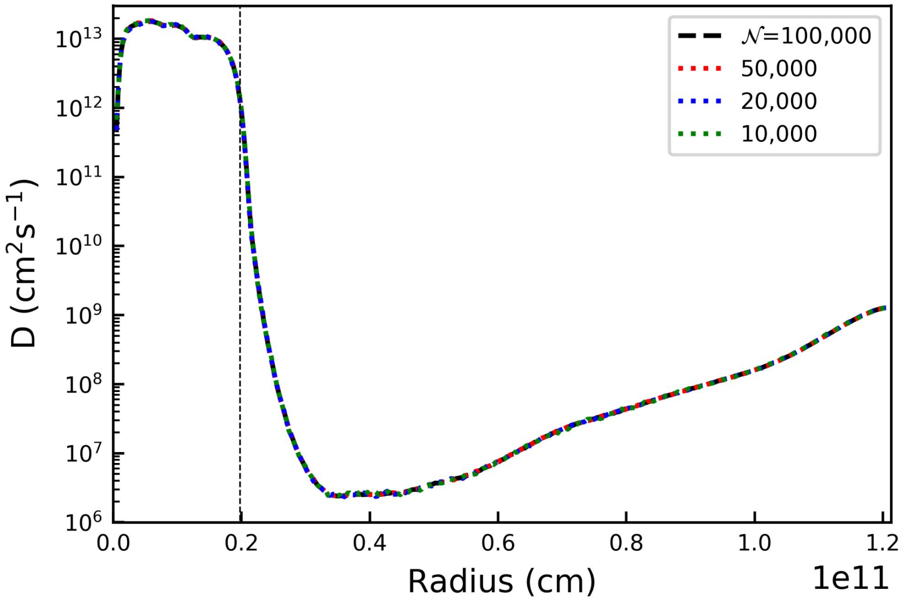
Can mixing by IGWs treated as a diffusive process ?







Radial Diffusion profile is robust



We also varied other parameters in our simulation such as:

- **Radial grid size**
- **Time resolution**

Parameterization of the Diffusion Coefficient

- **Using theory from Rogers and McElwaine (2017)**

$$D = Av_{wave}^2(\omega, l, r)$$

Ratnasingam et al. (2019):

$$v_{wave}(\omega, l, r) = v_{rms} \underbrace{\left(\frac{\rho}{\rho_0}\right)^{\frac{1}{2}}}_{\text{Density}} \underbrace{\left(\frac{r}{r_0}\right)^{-1}}_{\text{Geometric term}} \underbrace{\left(\frac{(N^2-\omega^2)}{(N_0^2-\omega^2)}\right)^{-\frac{1}{4}}}_{\text{Brunt-Väisälä frequency}} \underbrace{e^{-\frac{\mathcal{T}}{2}}}_{\text{Radiative Damping}}$$

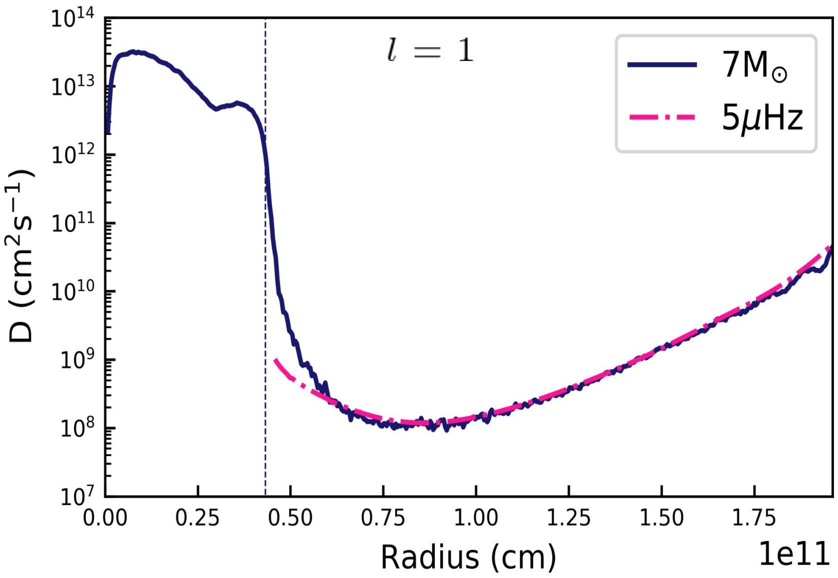
Density

Geometric term

Brunt-Väisälä frequency

Radiative Damping

ρ_0 and N_0 are the density and Brunt-Väisälä frequency at the initial reference point r_0.



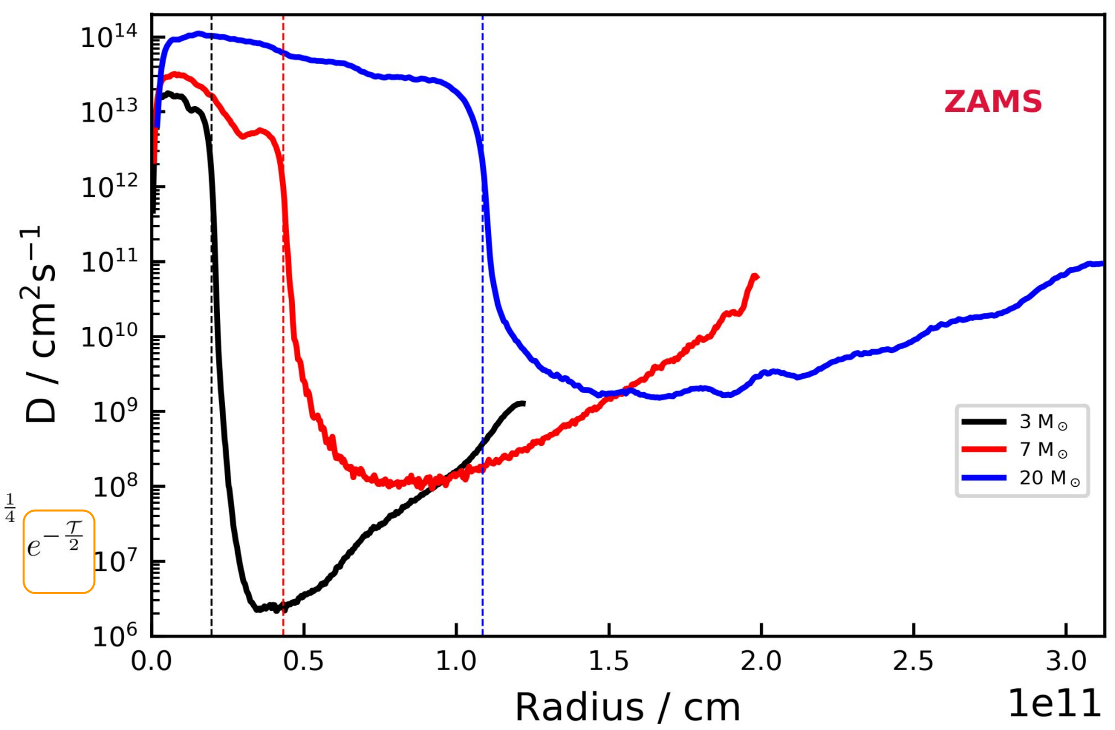
Damping coefficient by Kumar et al. (1999):

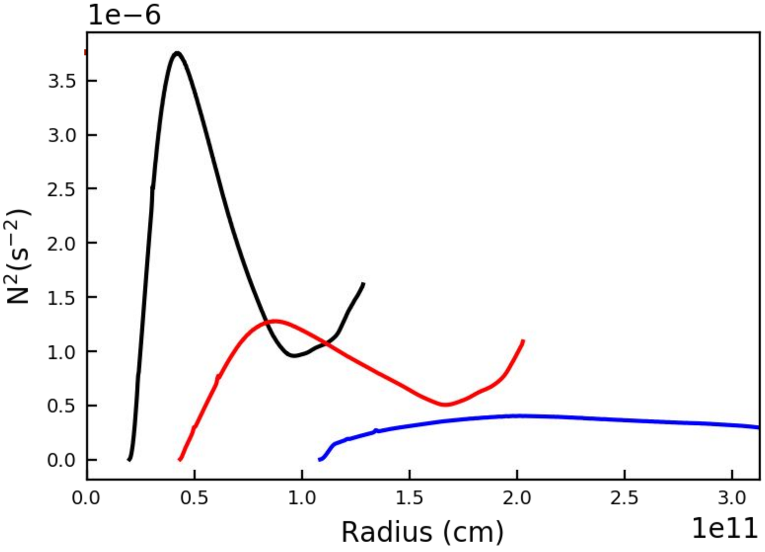
$$\mathcal{T}(\omega, l, r) = \int \frac{16\sigma T^3}{3\rho^2\kappa c_p}\left(\frac{(l(l+1))^{\frac{3}{2}}N^3}{r^3\omega^4}\right)\left(1-\frac{\omega^2}{N^2}\right)$$

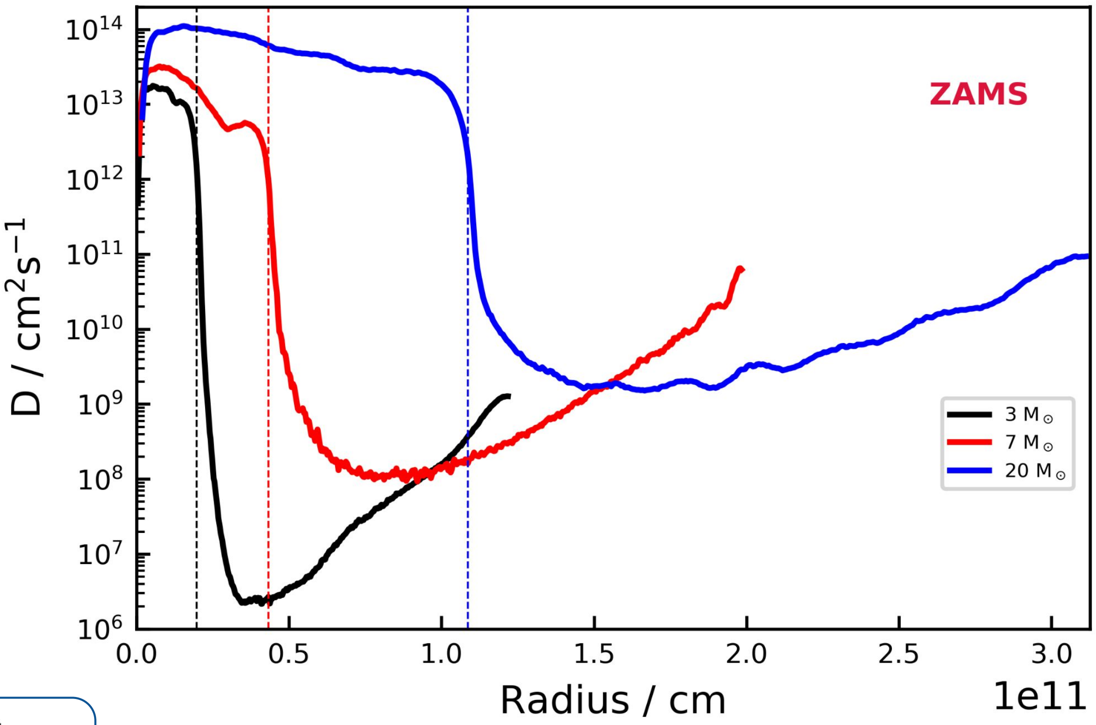
The profile variation:

- **With MASS**

$$v_{wave}(\omega, l, r) = v_{rms} \left(\frac{\rho}{\rho_0}\right)^{\frac{1}{2}} \left(\frac{r}{r_0}\right)^{-1} \left(\frac{(N^2-\omega^2)}{(N_0^2-\omega^2)}\right)^{-\frac{1}{4}} e^{-\frac{\mathcal{T}}{2}}$$





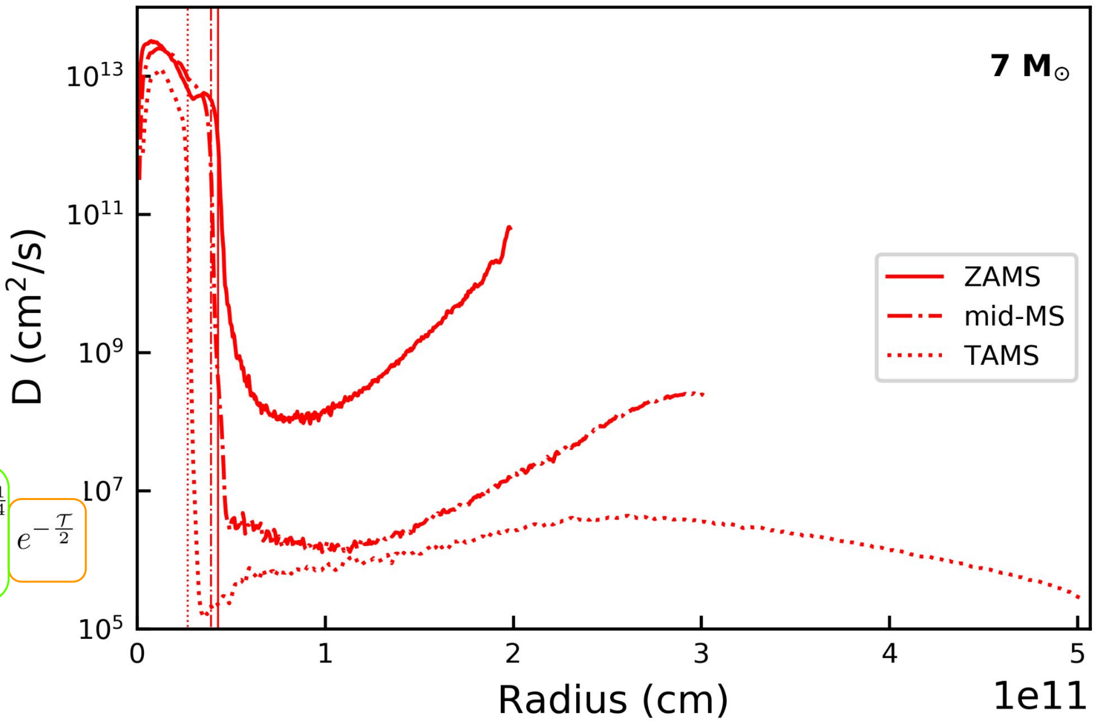


$$v_{wave}(\omega, l, r) = v_{rms}\left(\frac{\rho}{\rho_0}\right)^{\frac{1}{2}}\left(\frac{r}{r_0}\right)^{-1}\left(\frac{(N^2-\omega^2)}{(N_0^2-\omega^2)}\right)^{-\frac{1}{4}} e^{-\frac{\mathcal{T}}{2}}$$

The profile variation:

- With MASS
- **With AGE**

$$v_{wave}(\omega, l, r) = v_{rms} \left(\frac{\rho}{\rho_0}\right)^{\frac{1}{2}} \left(\frac{r}{r_0}\right)^{-1} \left(\frac{(N^2-\omega^2)}{(N_0^2-\omega^2)}\right)^{-\frac{1}{4}} e^{-\frac{\mathcal{T}}{2}}$$



Why is the profile different for older stars?

Linearised 2D equation reduced to a second order differential equation

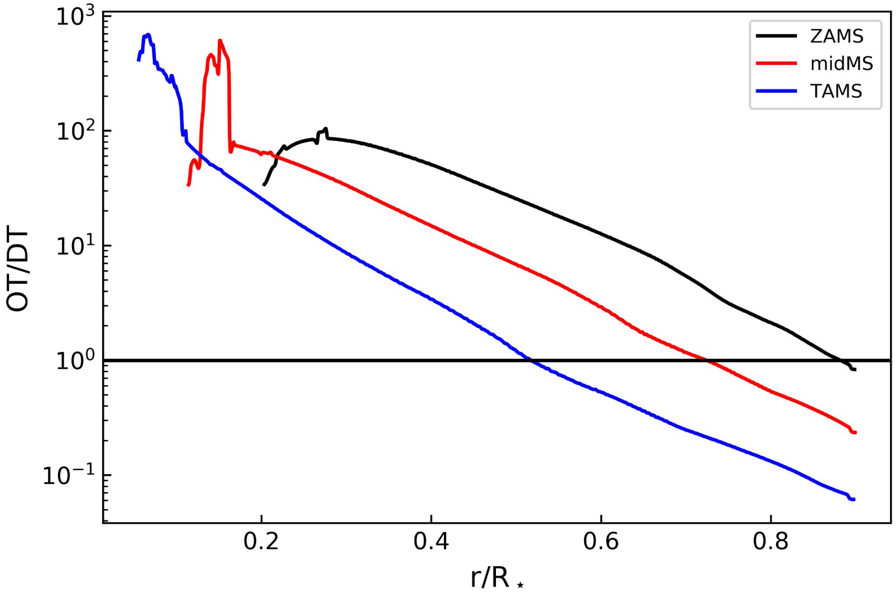
$$0 = \frac{\partial^2 \alpha}{\partial r^2} + \left(\frac{N^2}{\omega^2} - 1\right)\frac{m^2}{r^2}\alpha + \left[-\bar{\rho}^{-\frac{1}{2}}\frac{\partial^2(\bar{\rho}^{\frac{1}{2}})}{\partial r^2} + \frac{\partial h_p}{\partial r}\right]\alpha + \frac{1}{4r^2}\alpha$$

Oscillatory term (OT)

Density term (DT)

$$\alpha = v_r \bar{\rho}^{\frac{1}{2}} r^{\frac{3}{2}}$$

Turning Point effect on diffusion



Approximate radius where the ratio

of the oscillatory term (OT)

$$\left(\frac{N^2}{\omega^2} - 1\right)\frac{l^2}{r^2},$$

to the density term (DT):

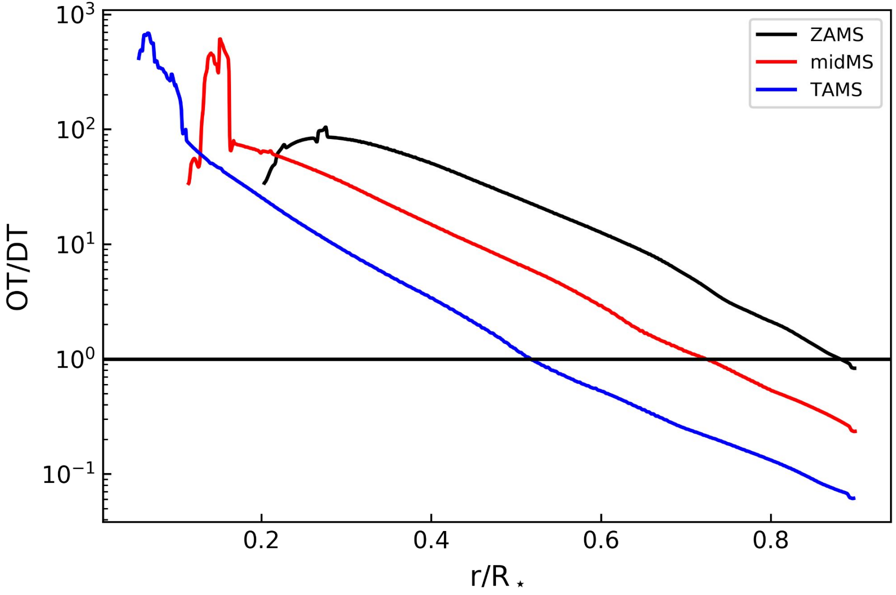
$$-\rho^{-\frac{1}{2}}\frac{\partial^2 \rho^{-\frac{1}{2}}}{\partial r^2} + \frac{\partial h_\rho}{\partial r}.$$

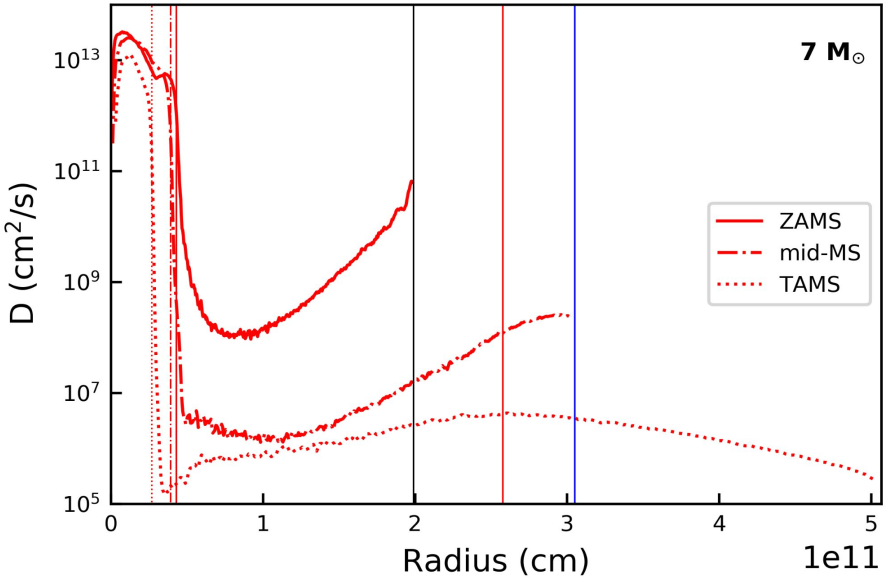
is equal to 1.

(Ratnasingam et al. 2020)

In older stars the density term (damping term) is dominant

Turning Point effect on diffusion

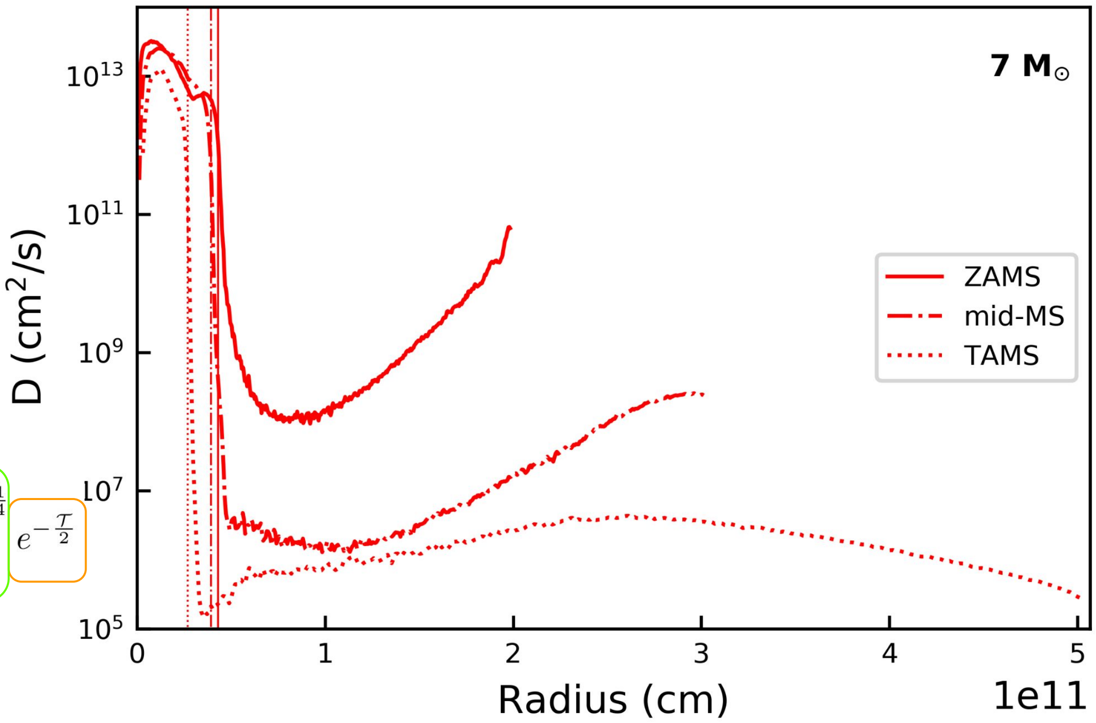


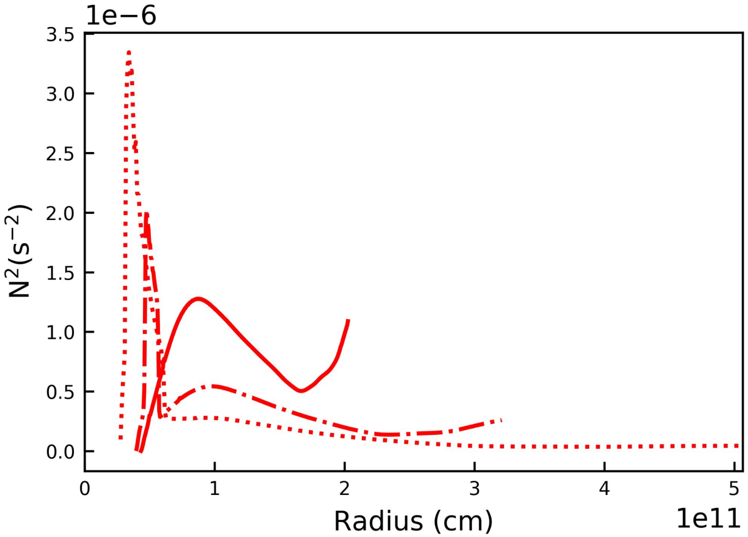


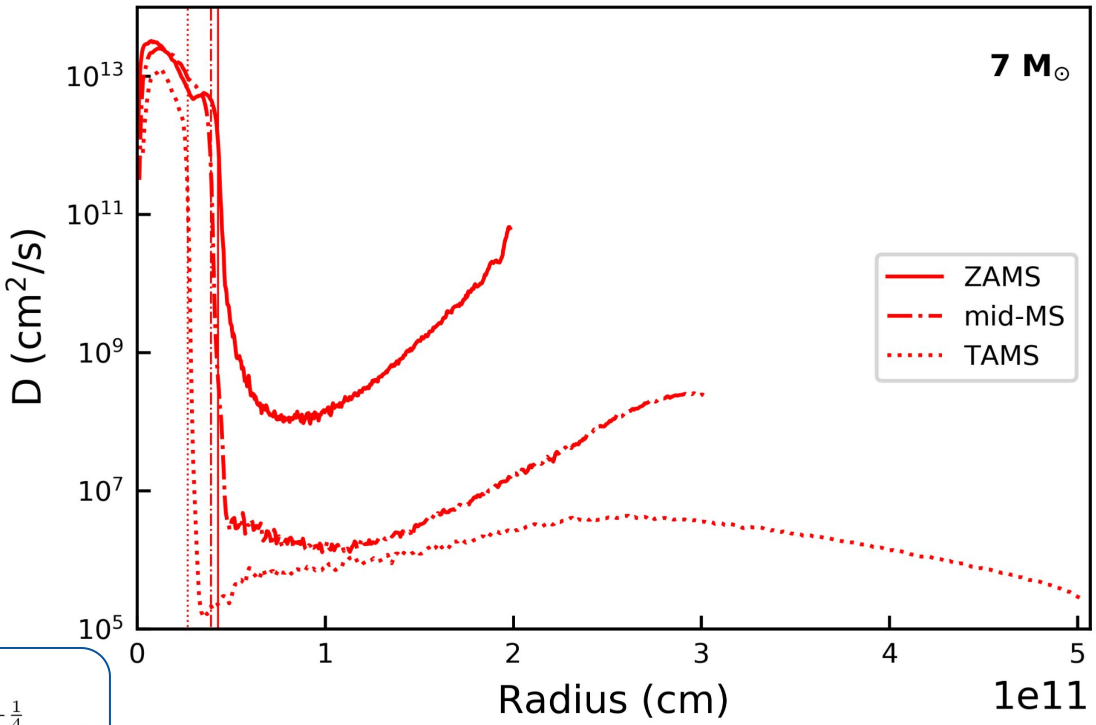
The profile variation:

- With MASS
- **With AGE**

$$v_{wave}(\omega, l, r) = v_{rms} \left(\frac{\rho}{\rho_0}\right)^{\frac{1}{2}} \left(\frac{r}{r_0}\right)^{-1} \left(\frac{(N^2-\omega^2)}{(N_0^2-\omega^2)}\right)^{-\frac{1}{4}} e^{-\frac{\mathcal{T}}{2}}$$



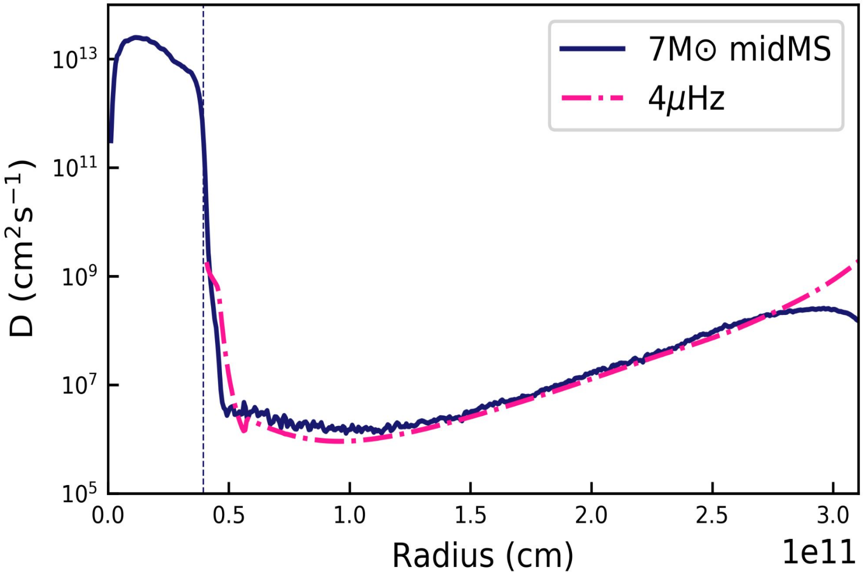




$$v_{wave}(\omega, l, r) = v_{rms}\left(\frac{\rho}{\rho_0}\right)^{\frac{1}{2}}\left(\frac{r}{r_0}\right)^{-1}\left(\frac{(N^2-\omega^2)}{(N_0^2-\omega^2)}\right)^{-\frac{1}{4}} e^{-\frac{\mathcal{T}}{2}}$$

Parameterization of the Diffusion Coefficient

- **We found the linear theory to agree with all the ZAMS and midMS models**
- **The dominant frequencies contributing to the mixing profile found to be in the range of 3 - 9 μHz**
- **Dominant frequencies were lower for more massive stars**

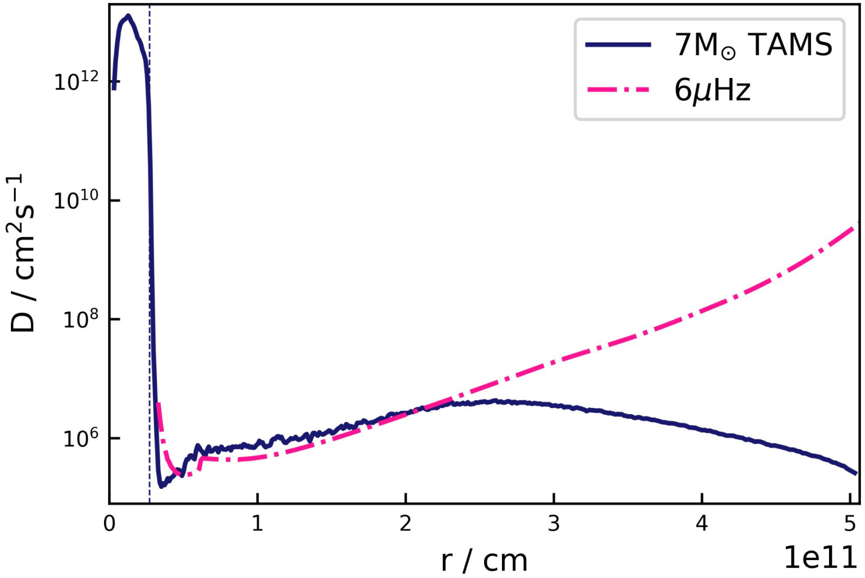


$$D = A v_{wave}^2(\omega, l, r)$$

$$v_{wave}(\omega, l, r) = v_{rms} \left(\frac{\rho}{\rho_0}\right)^{\frac{1}{2}} \left(\frac{r}{r_0}\right)^{-1} \left(\frac{(N^2 - \omega^2)}{(N_0^2 - \omega^2)}\right)^{-\frac{1}{4}} e^{-\frac{\mathcal{T}}{2}}$$

Parameterization of the Diffusion Coefficient

- **Linear Theory failed to explain the TAMS models**



Take Away

- More massive stars have higher mixing

- Mixing due to IGWS changes substantially with age
  - Turning Point
  - Brunt-Väisälä frequency

- Younger stars have higher mixing, reducing markedly with age

- The dominant frequencies contributing to the mixing are between 3-9 μHz, decreasing with mass

THANK YOU