Exact invariant solutions for coherent turbulent motions in Couette & Poiseuille flows

Carlo Cossu
IMFT-CNRS, Toulouse, France

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Motivation

Near-wall streaky motions

Near-wall streaks known since 1967

Scale in wall (inner) units:
$\lambda_z^+ \sim 100$, $\lambda_x^+ \sim 250\text{-}1000$

Self-sustained process main source of turbulent kinetic energy at low-moderate Re

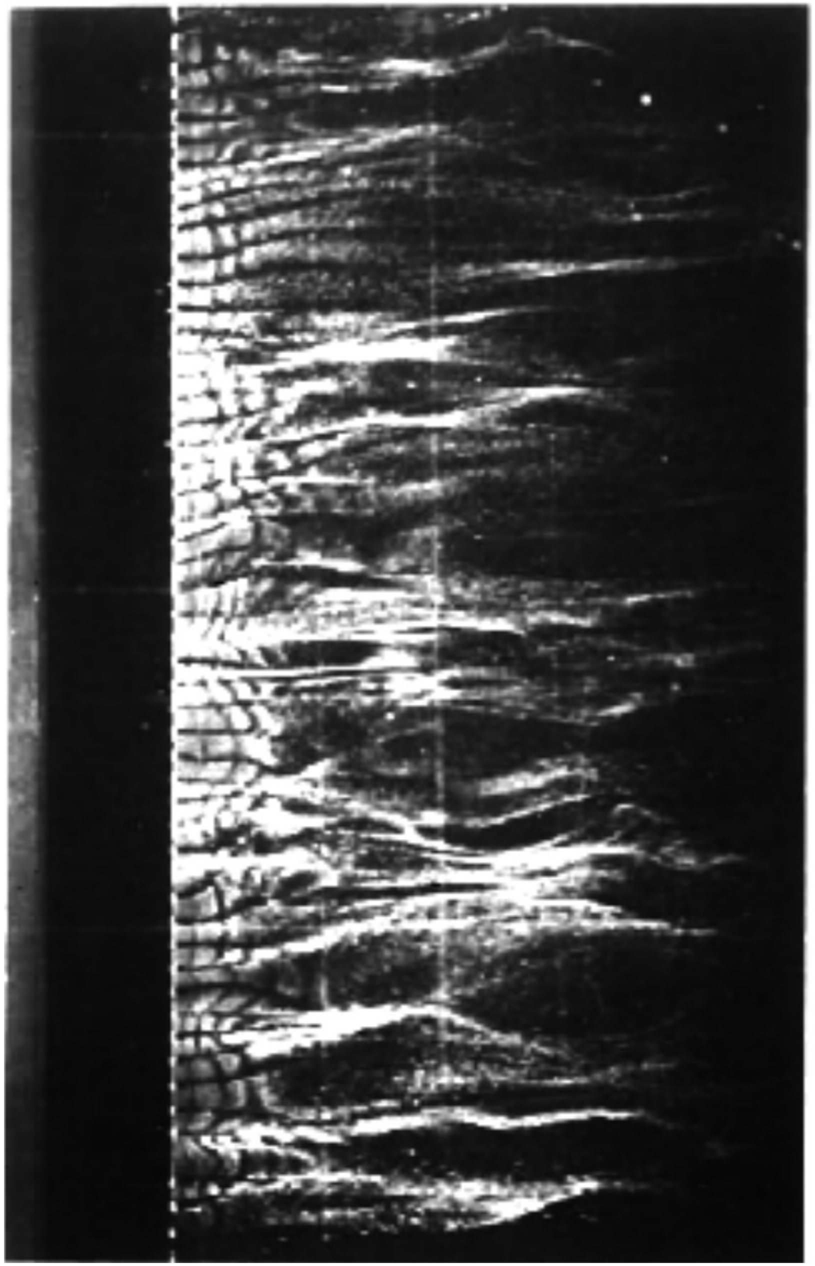


Figure from Kline *et al.* (1967)

The large-scale emerging peak

An additional peak emerges in the outer region at sufficiently high Reynolds numbers (Re_τ=7300 here). Related to motions at large scale

Near-wall energy peak at Re_τ=1010

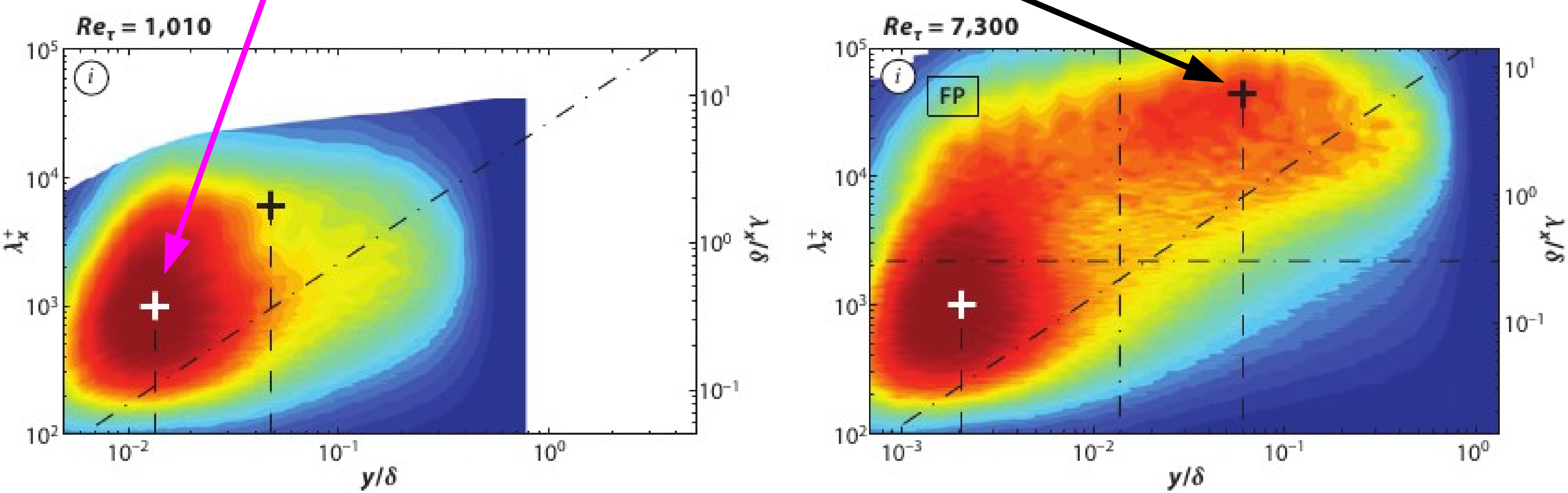


Figure 3

Contour maps showing the variation of one-dimensional premultiplied spectra with wall-normal position for two Reynolds numbers. An inner and an outer peak are noted at the higher Reynolds number. Figure taken from Hutchins & Marusic (2007a). Reprinted with permission from CUP.

from Smits et al. (ARFM, 2011)

Large and very large scale motions

Outer peak ↔ structures scaling with outer length scale δ

Typical spanwise spacing: $\lambda_z/\delta \sim 1.5\text{-}2.5$

LSM: large-scale motions (LSM) → $\lambda_x/\delta \sim 5$

VLSM very large-scale motions: $\lambda_x/\delta \sim 25$ or more

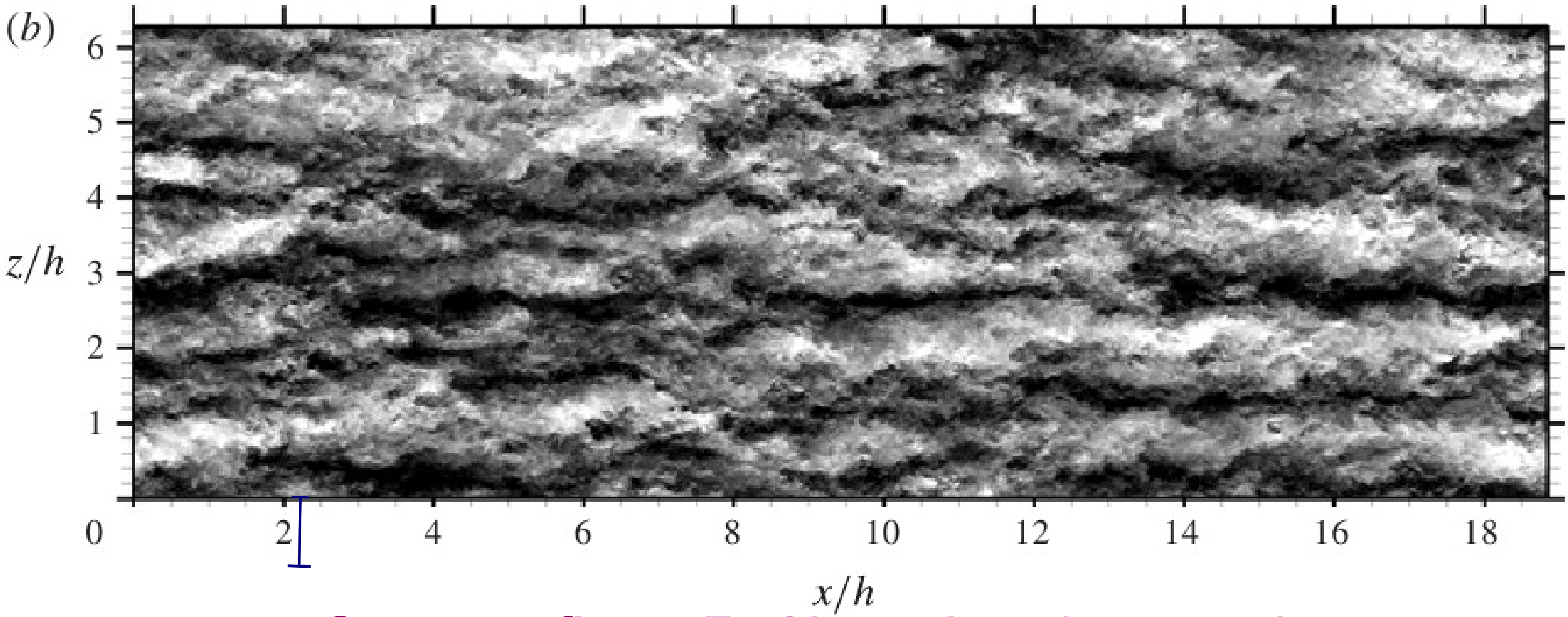


Figure from Bernardini, Pirozzoli & Orlandi, JFM 2014

Couette flow: δ=2h → λ_z ~4.5 - 5.5h,

λ_x ~10h (LSM) and 50h or more (VLSM)

Outer peak: conjectured to be dominant at very high Re (industrial, geophysical applications) →
large interest in LSM & VLSM

Origin of outer-scaling motions still unclear (and debated)

Origin/nature of LSM & VLSM

"*LSMs are believed to be created by the vortex packets formed when multiple hairpin structures travel at the same convective velocity*" (Smits et al. ARFM 2011). *See also Kim & Adrian (1999), Zhou et al. (1999), Guala et al. (2006), Balkumar & Adrian (2007)*

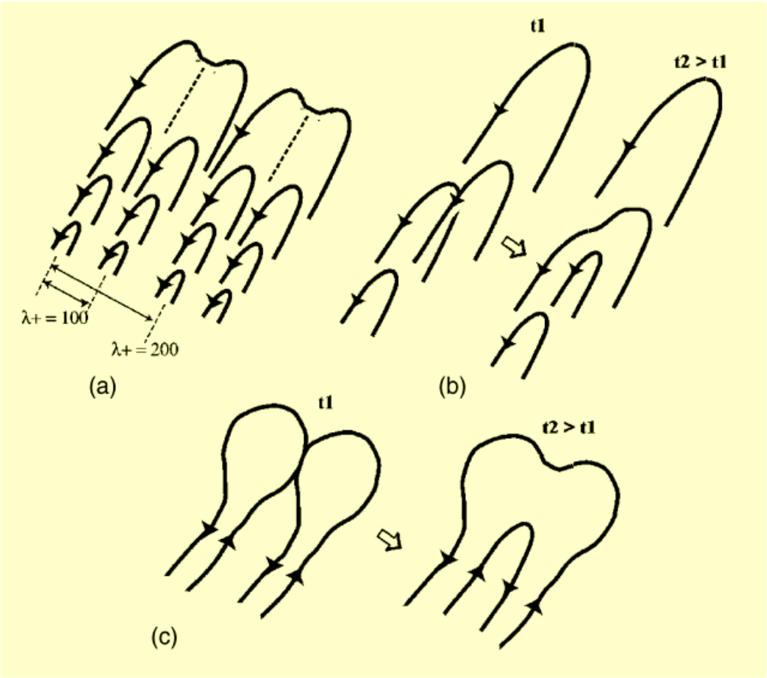


Figure from Adrian (2007)

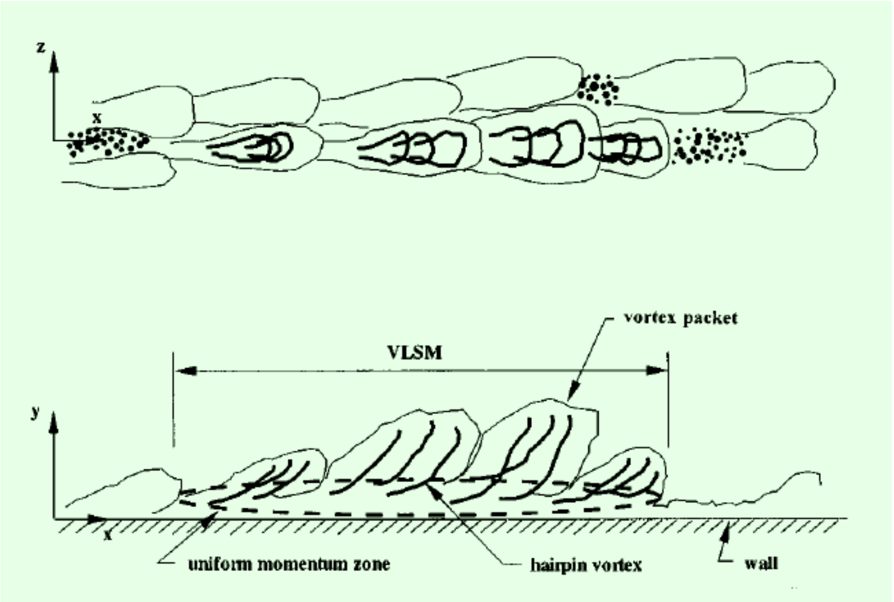


Figure from Kim & Adrian (1999)

is this the only possible explanation?

Recent results suggest other mechanisms might be at work:

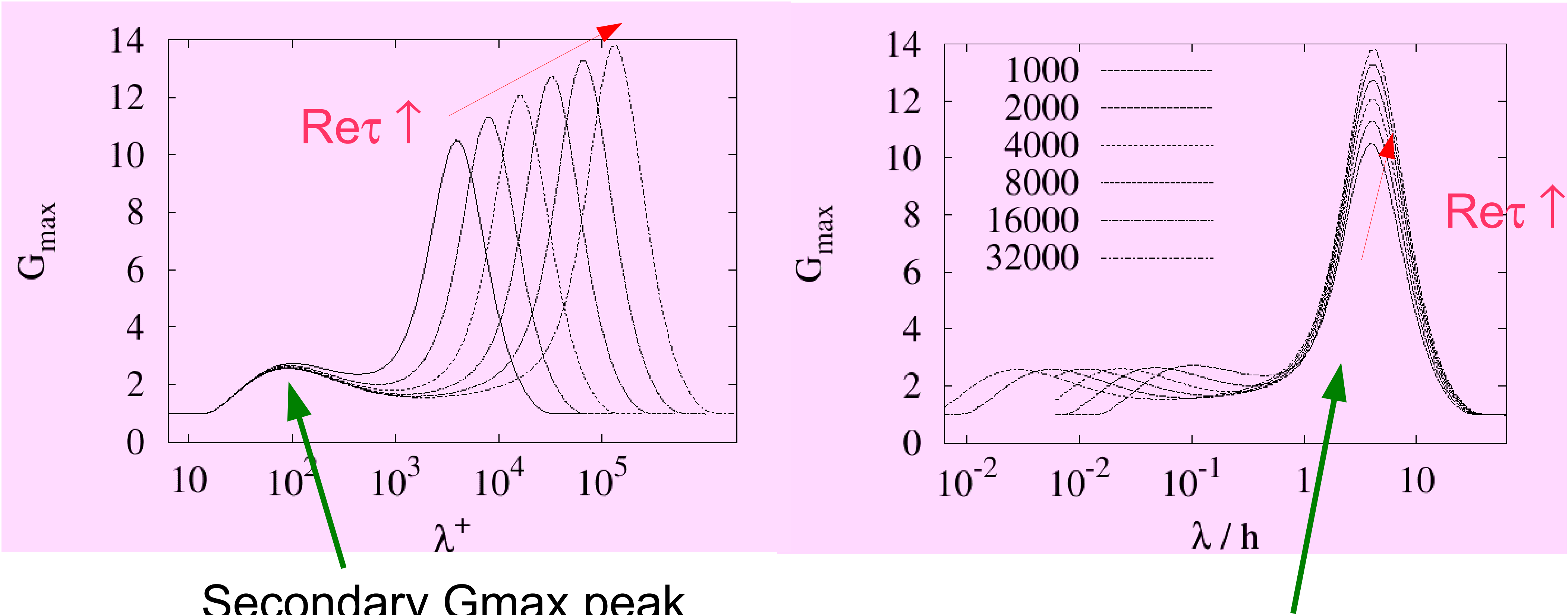
Large-scale motions independent of details of near-wall cycle (perturbed with roughness)
→ no need of near-wall cycle?
(Flores et al. 2006, 2007)

Linear optimal perturbation analysis: Large-scale coherent perturbations to turbulent mean flows can be highly amplified →
large scale structures able to efficiently extract energy from the mean flow
(del Alamo & Jiménez 2006, Cossu & Pujals 2009, Pujals et al. 2010, Hwang & Cossu 2010a,2010b, Willis et al. 2010)

Ingredients of a coherent SSP

2006-2010 in collaboration with
Grégory Pujals, Sébastien Depardon
Yongyun Hwang, Junho Park

Maximum amplification in channel flow



Secondary Gmax peak scaling in inner variables with optimal at λ+ = 92

Primary Gmax peak scaling in outer variables with optimal at λ = 4 h

Gmax increases with Re!

Pujals, Garcia-Villalba, Depardon & Cossu, *Phys. Fluids* 2009

see also Alizard et al. *J. Fluid Mech.* 2015

Optimal perturbations in channel flow

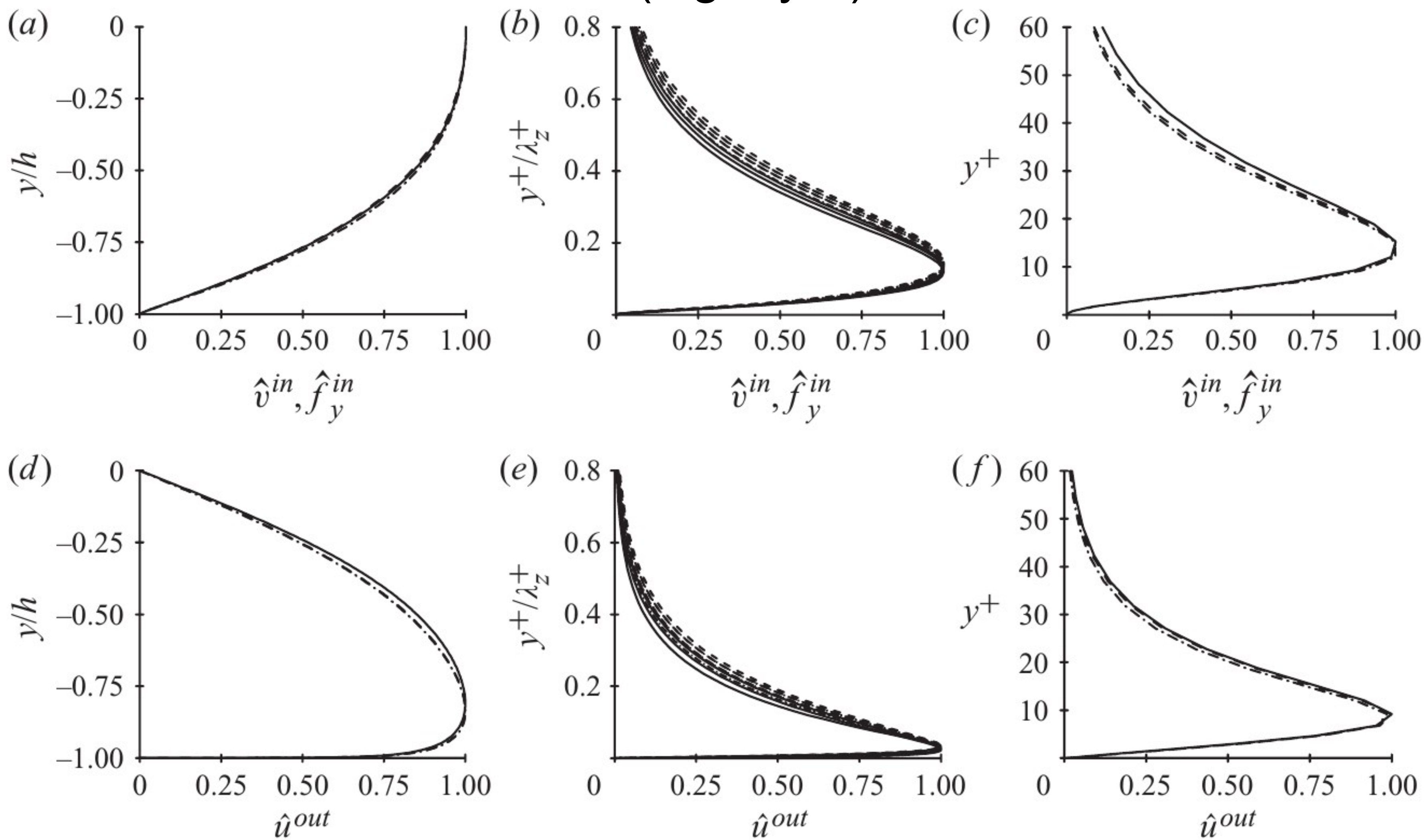
large scale peak

intermediate scales (log-layer)

small scale peak

vortices

streaks



Cossu & Hwang *J. Fluid Mech.* 2010

Large-scale peak in turbulent shear flows

Plane channel (Re_τ > ≈ 500): **$\lambda_z \approx 4h$**

del Alamo & Jiménez *JFM* 2006, Pujals et al. *Phys. Fluids* 2009

Pipe flow (Re_τ > ≈ 500): **m=1 ($\lambda_z = 2\pi R/m \approx 6R$)**

Willis, Hwang & Cossu *Phys. Rev E* 2010

Couette flow ($Re_\tau \approx 50$): **$\lambda_z = 4.5h$**

Hwang & Cossu *JFM* 2010

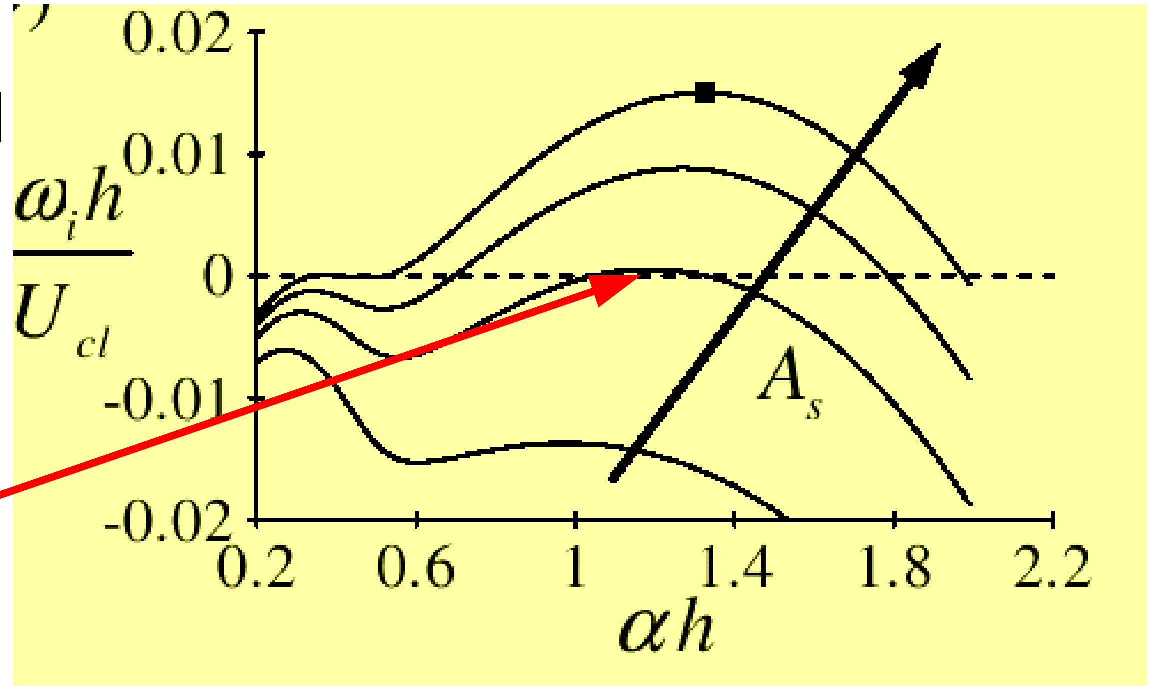
Boundary layer (ZPG) ($Re_{\delta*}$ >≈ 5000): **$\lambda_z \approx 6 - 8\delta$**

Cossu et al. *JFM* 2009

Results of optimal temporal growth (Gmax) analysis.
Similar scales obtained for stochastic forcing
Larger scales obtained for optimal harmonic forcing

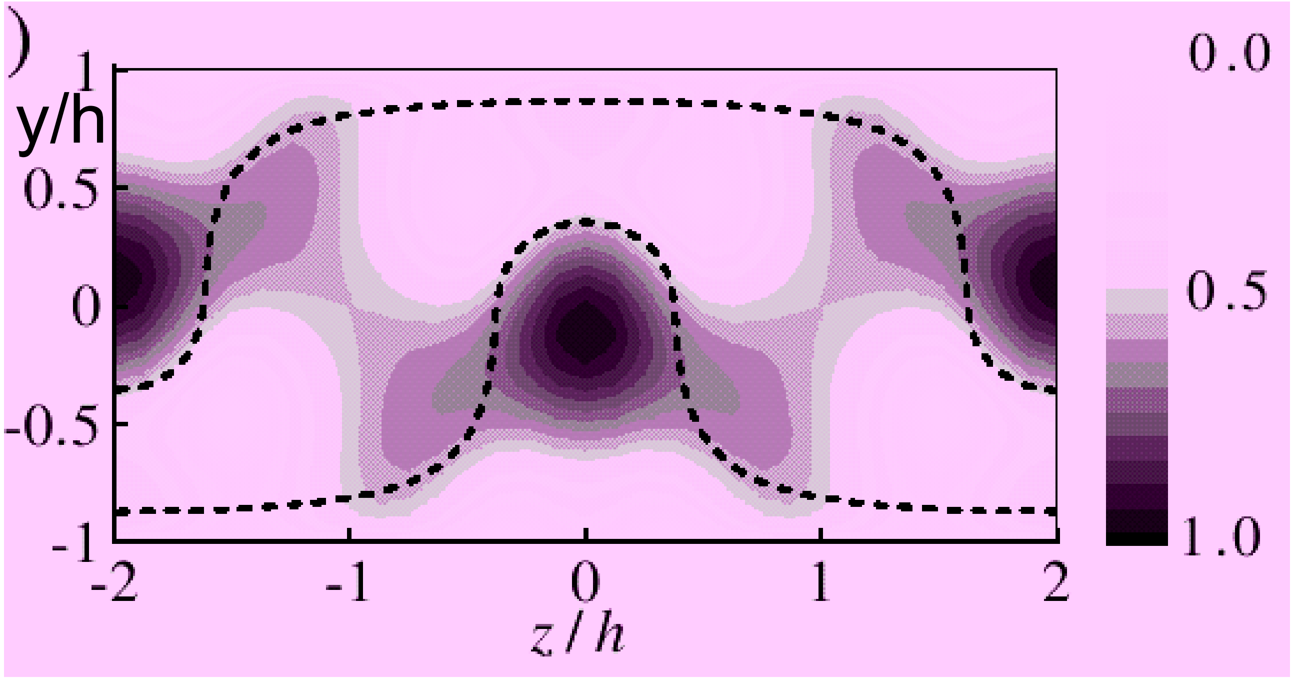
Secondary instability of coherent streaks

Temporal growth rate
of secondary fundamental
sinuous modes



Critical As = 21%Ue
(< laminar $A_{s,crit}$!)

w-component of the most unstable secondary mode



Park, Hwang & Cossu,
C.R.Ac.Sci. Méc. 2011
see also
Alizard *Phys. Fluids* 2015

→ a coherent SSP might be at play at all amplified scales

"Coherent SSP" because Reynolds stresses of (incoherent) fluctuations are accounted

...suggestive results.. BUT

Can large-scale motions really (self) sustain in the absence of active small-scale motions?

How can you prove that?

Removing small-scale active structures from the picture: coherent self-sustained motions *at all scales*

2009-2015 in collaboration with Yongyun Hwang & Subhandu Rawat

Near-wall cycle analysis:
remove potentially active large scales
by using small periodic domains →
minimal flow unit (Jiménez & Moin 1991)

Analysis of large-scale motions:
must remove active small scales to prove
that large scales are self-sustained.
How can this be done?

Idea #1: solve Navier-Stokes equations on very coarse grid larger than near-wall structures → tested: inaccurate solutions (few points) & unphysical energy production peak at grid scale → not a good idea...

Idea #2: use a `reasonable' grid (good resolution) + use filter → **quench the energy production of small scales & take into account dissipation** → no unphysical energy production peak at grid scale

The over-damped LES technique

Integrate (LES) equations for the filtered motions:

$$\frac{\partial \overline{u}_i}{\partial t} + \overline{u}_j \frac{\partial \overline{u}_i}{\partial x_j} = -\frac{\partial \overline{q}}{\partial x_i} + \nu \frac{\partial^2 \overline{u}_i}{\partial x_j^2} - \frac{\partial \overline{\tau}^r_{ij}}{\partial x_j}$$

Use purely dissipative Smagorinsky model

$$\overline{\tau}^r_{ij} = -2\nu_t \overline{S}_{ij}$$

$$\nu_t = D(C_S \overline{\Delta})^2 \overline{S}$$

Eddy viscosity

Smagorinsky constant

(mean) grid spacing

Smagorinsky mixing length for residual motions (Mason & Callen 1986)

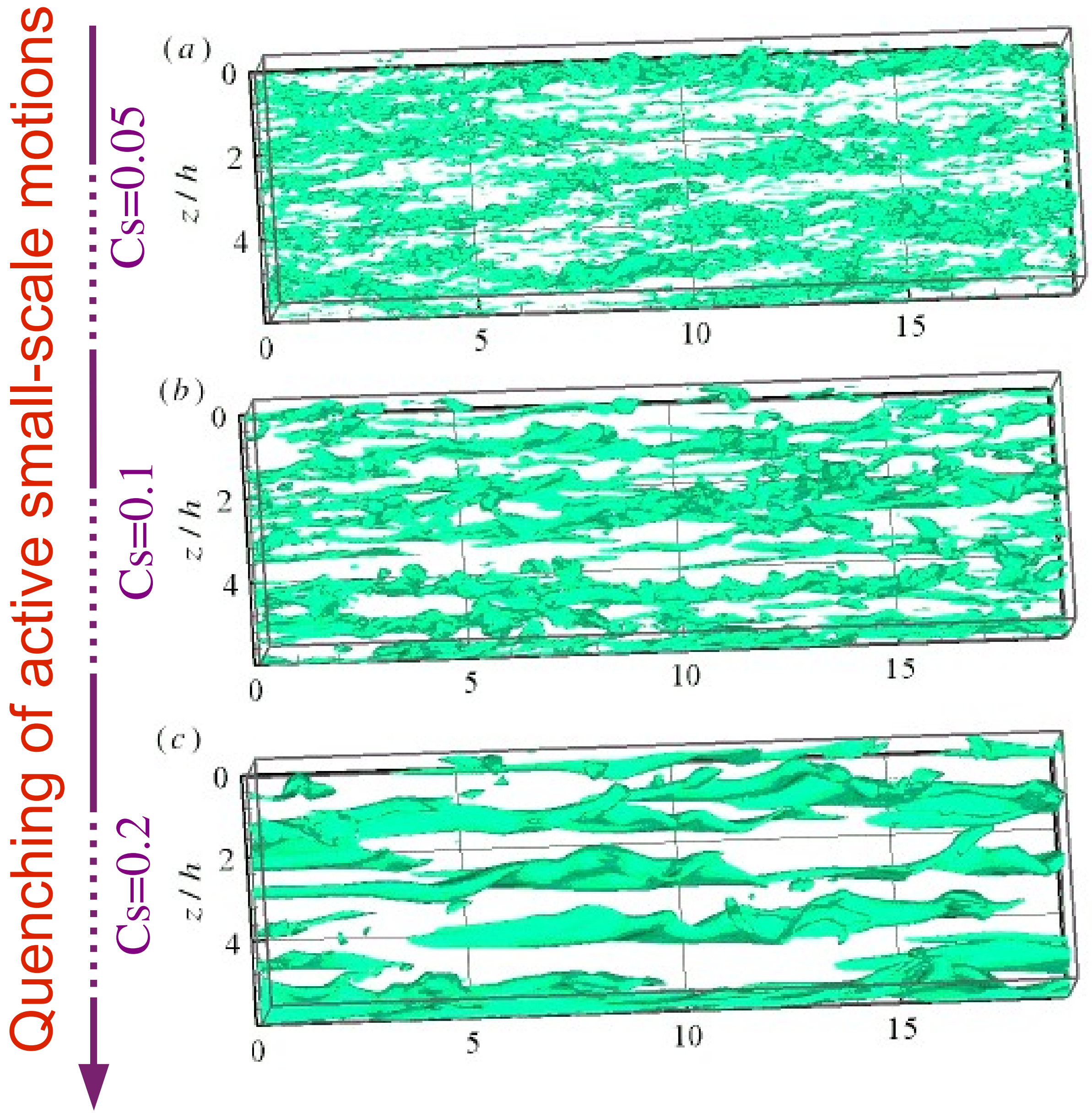
$$l_0 = C_s \Delta$$

`Passivate' increasing range of small scales → increase l_0

Idea (Cossu & Hwang 2010, 2011): **increase Cs instead of Δ (grid size) →** increase l_0 & keep a good resolution

Survival of large-scale motions : Poiseuille flow

streamwise velocity levels $u_\tau^+=-2$



Hwang & Cossu *Phys. Rev. Lett.* 2010

LSM survive when smaller-scale active structures are quenched

(Hwang & Cossu 2010c)

Channel flow Re_τ=550

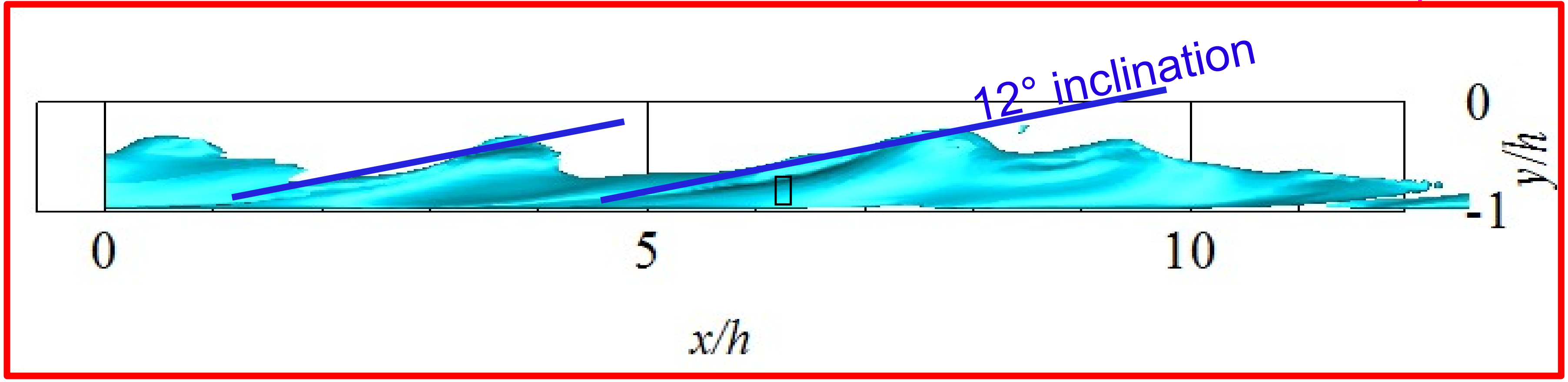
Surviving structures:

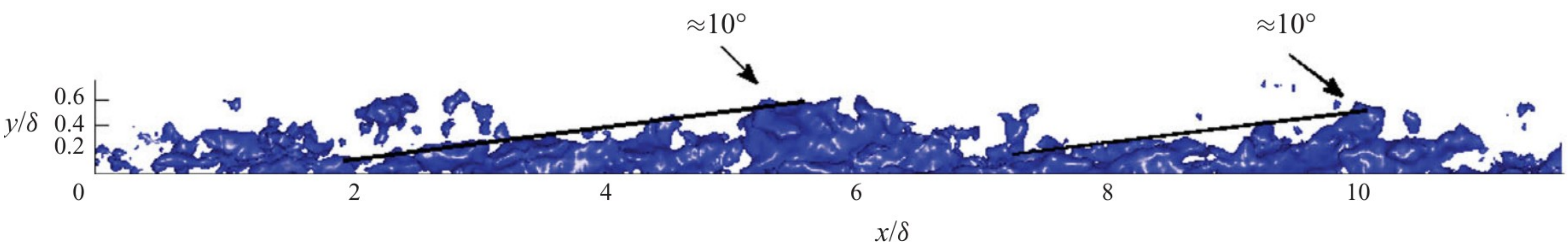
$\lambda_z \approx 1.5h$, $\lambda_x \approx 3$–$4h$ (peaks)

same size of original LSM !

Surviving large-scale motions

Zoomed side-view from extra dissipative LES





PIV data ZPG boundary layer (Dennis & Nickels JFM 2011)

Survival of large-scale motions : Couette flow

u^+=-2.5 surface (low-speed streaks)

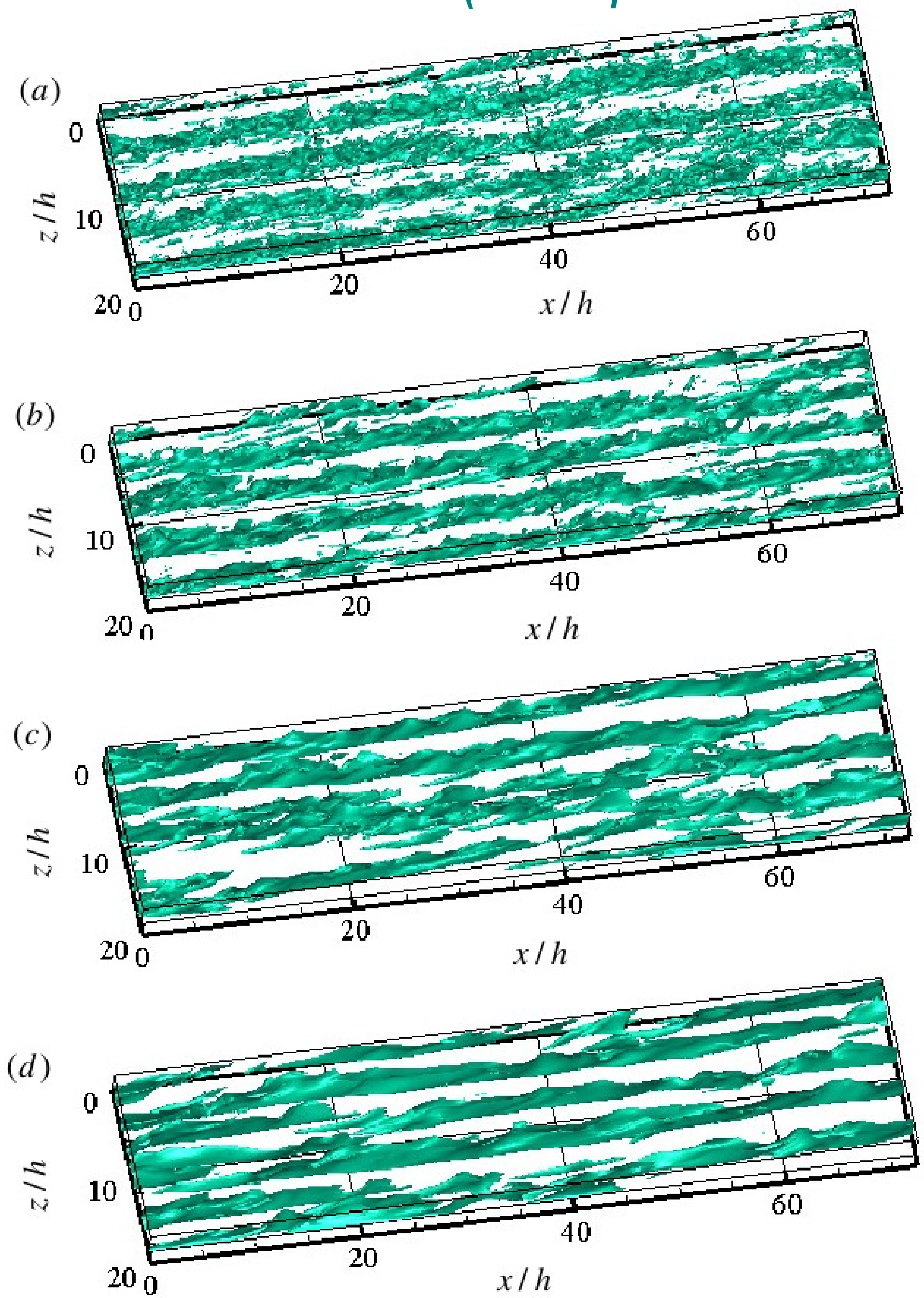
Quenching of active small-scale motions

Cs=0.05

Cs=0.10

Cs=0.14

Cs=0.18



reference LES
(Cs=0.05)

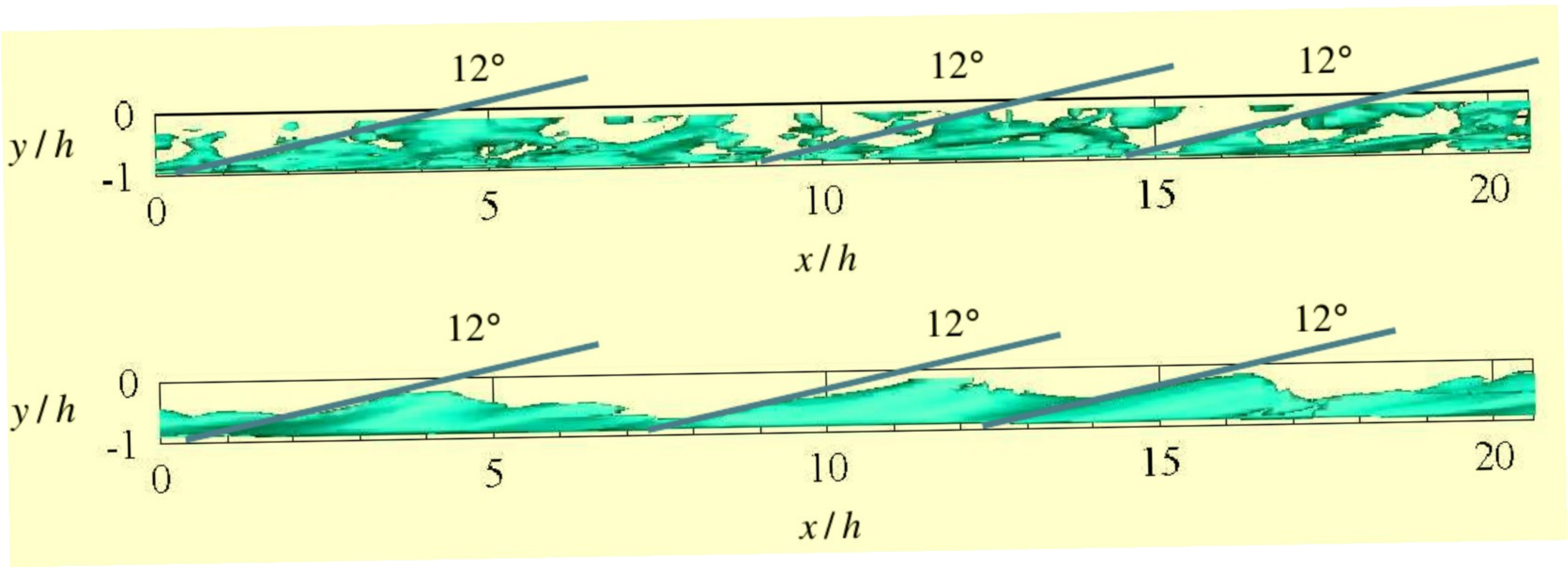
small-scales artificially damped

Large-scale motions survive without active buffer-layer processes

Rawat, Cossu & Rincon, *J. Fluid Mech.* 2015

Large-scale streaks structure

reference LES (Cs=0.05)



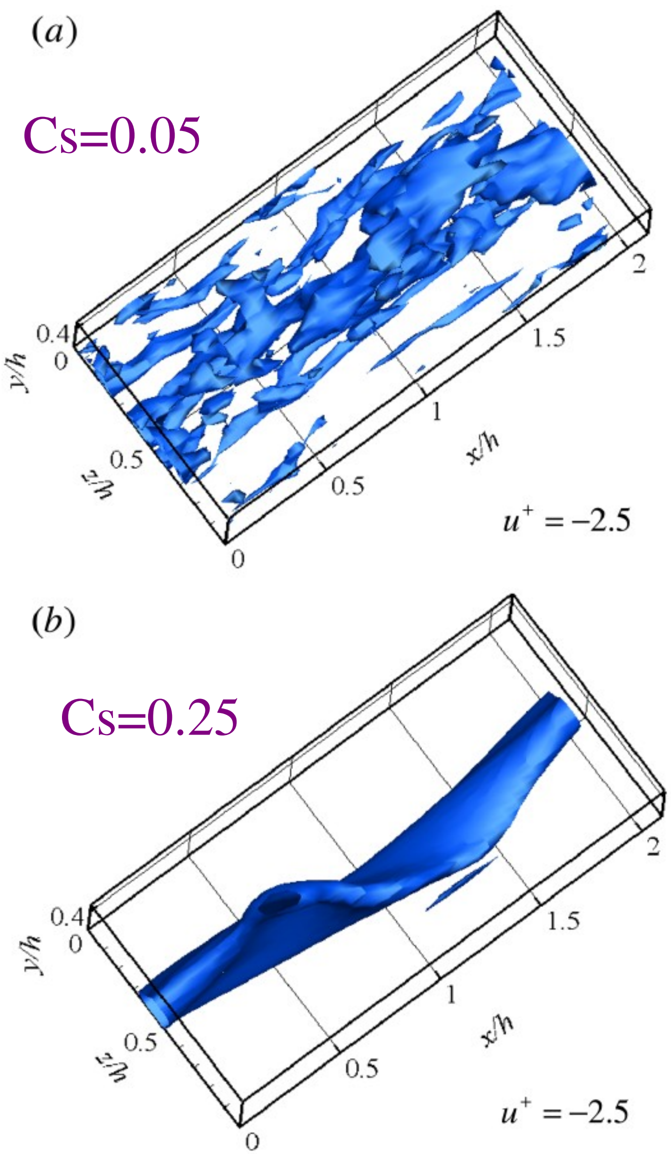
overdamped LES (Cs=0.14)

Rawat, Cossu & Rincon, *J. Fluid Mech.* 2015

Survival of log-layer motions (universal)

streamwise velocity levels u_τ^+=-2.5

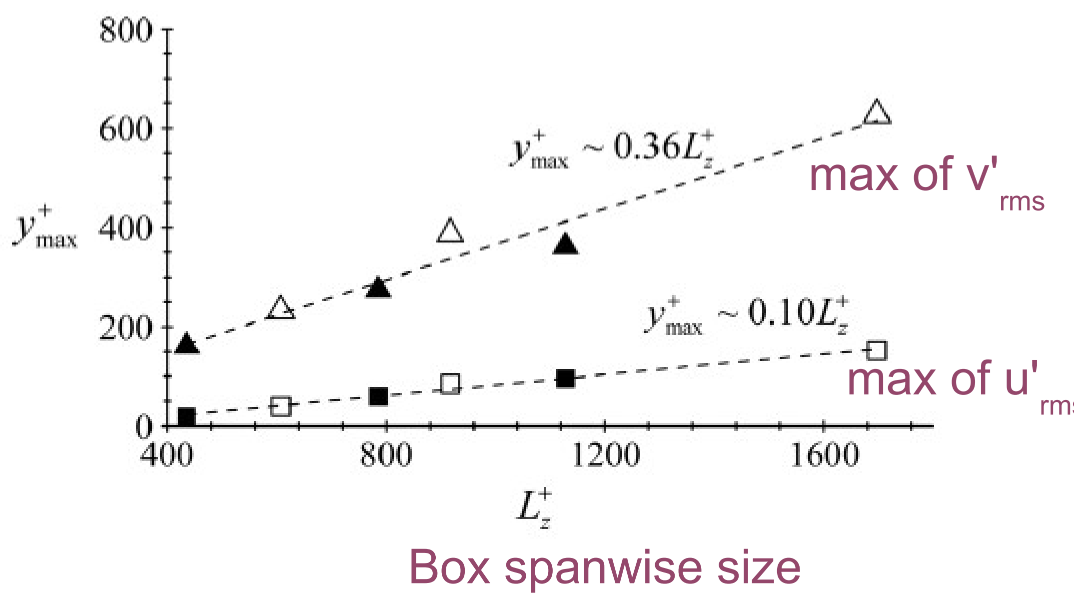
Quenching of active small-scale motions



Hwang & Cossu *Phys. Fluids* 2011

Also intermediate (log-layer) motions survive when smaller-scale active structures are quenched

Intermediate coherent motions (from overdamped LES) are **self-similar (like Townsend's attached eddies!)**



Partial summary

A **continuum of self-sustained coherent motions** exists (scales from those of buffer-layer streaks to those of LSM & VLSM).

These motions **directly extract energy from the mean flow** (coherent lift-up).

No bottom-up or top-down mechanism needed.

Different scales interact mainly via U.

Motions issued from (the overdamped LES) equations (not from a priori assumptions)

We believe that these motions *are* Towsend's attached eddies (see also work by Y Hwang)

A step further

What is the nature of these self-sustained coherent motions?

`Phase-space' interpretation?

Repeat what done in transitional flows → look for invariant solutions

Large eddy coherent solutions (LECS): Steady solutions in plane Couette flow

2011-2014 in collaboration with Subhandu Rawat & François Rincon (IRAP Toulouse)

Setting

Use the **LSM-box $L_x x L_z$=11h x 5.5h**:

- same size of most energetic LSM
- optimal size at which the NCBW steady solutions of the Navier-Stokes equations appear a the lowest Re in Couette flow (Waleffe 2003)

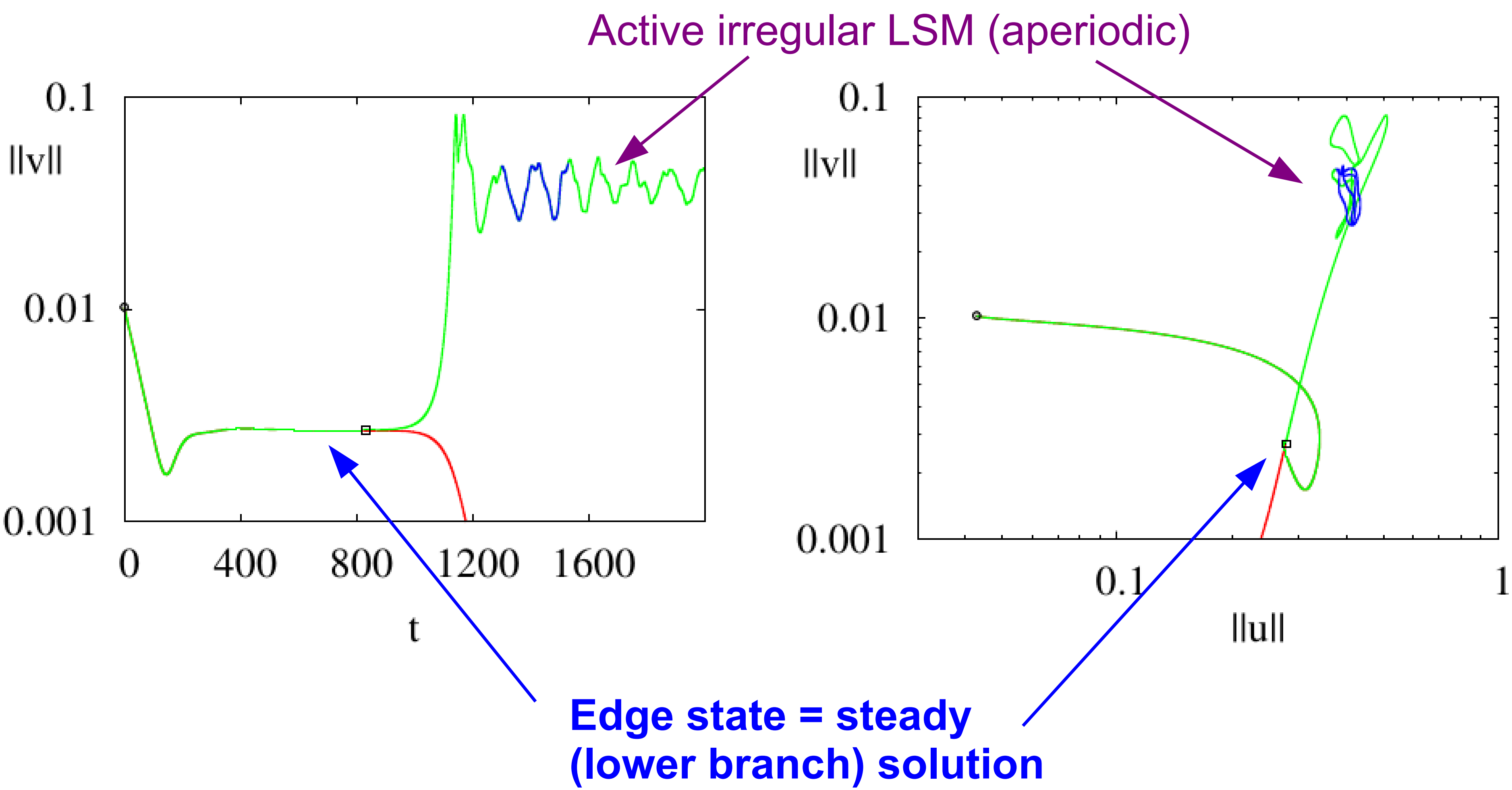
Start with surviving LSM in overdamped LES (Cs=0.14) at low but fully turbulent Re=750 (Re_τ=52)

Compute steady solutions by using Newton-based method (`peanuts`) interfaced to LES code (`diablo`) → **needs an initial guess**

Initial guess: edge state at Cs=0.14

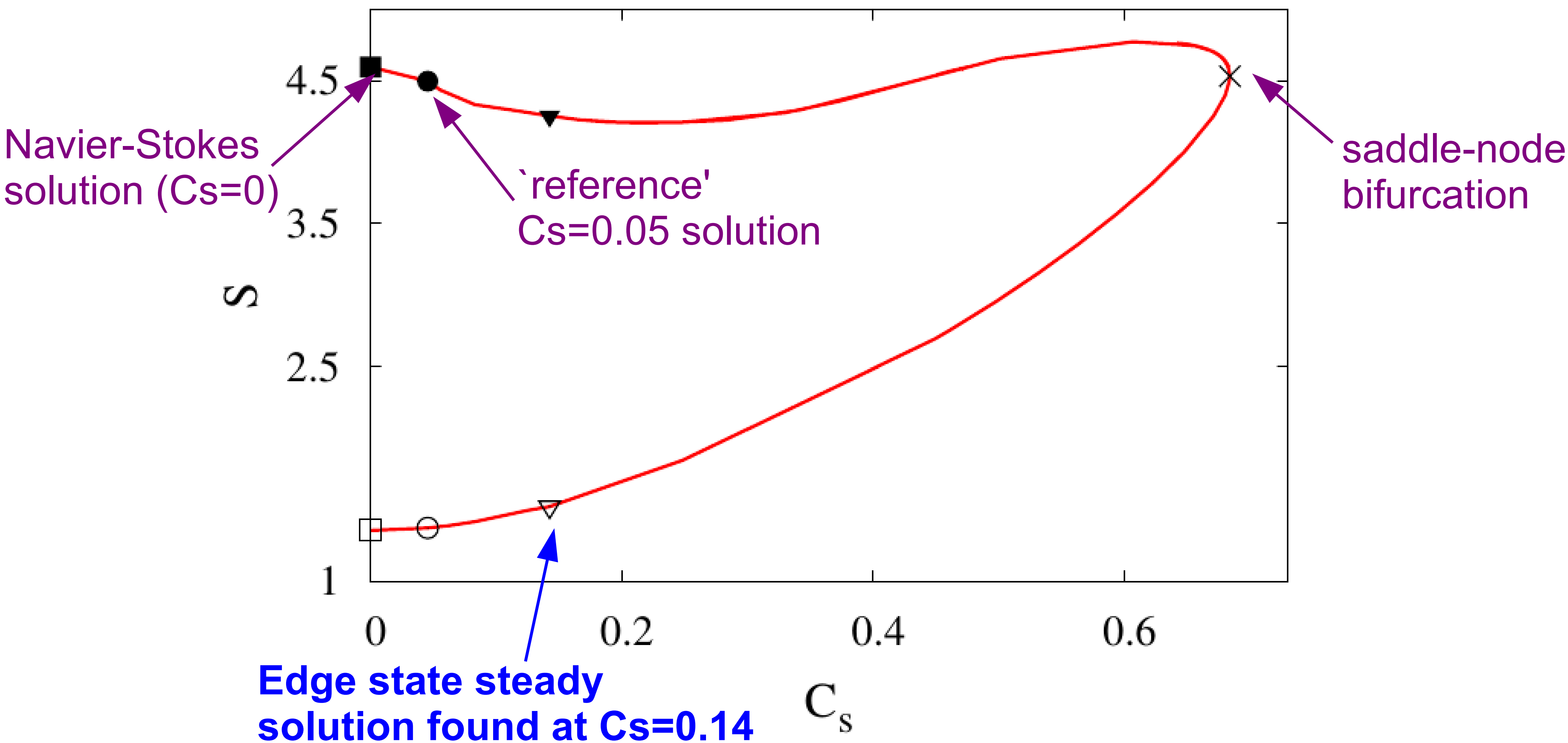
Edge tracking using mean flow + coherent large-scale perturbation (amplitude used as bisection parameter)

$$u_0 = U(y) + A_0 \, u'_0(x,y,z)$$



Continuation in Cs at Re=750

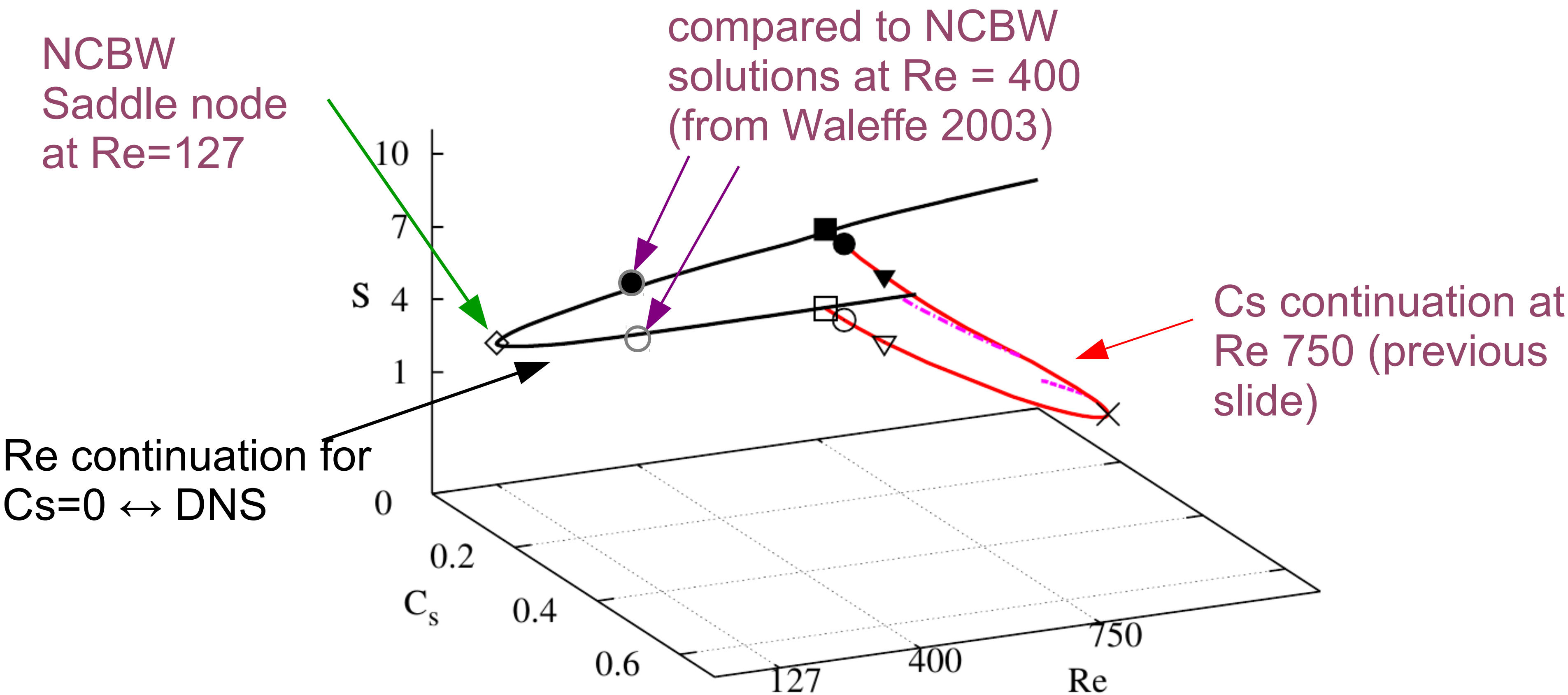
Edge state used as initial guess for Newton continuation in Cs at Re=750 in the LSM-box →
upper branch & continuation to Navier-Stokes solutions



Rawat, Cossu & Rincon, *J. Fluid Mech.* 2015

Reynolds number
continuation
in a LSM-box

Cs=0 (Navier-Stokes) continuation to **lower Re**

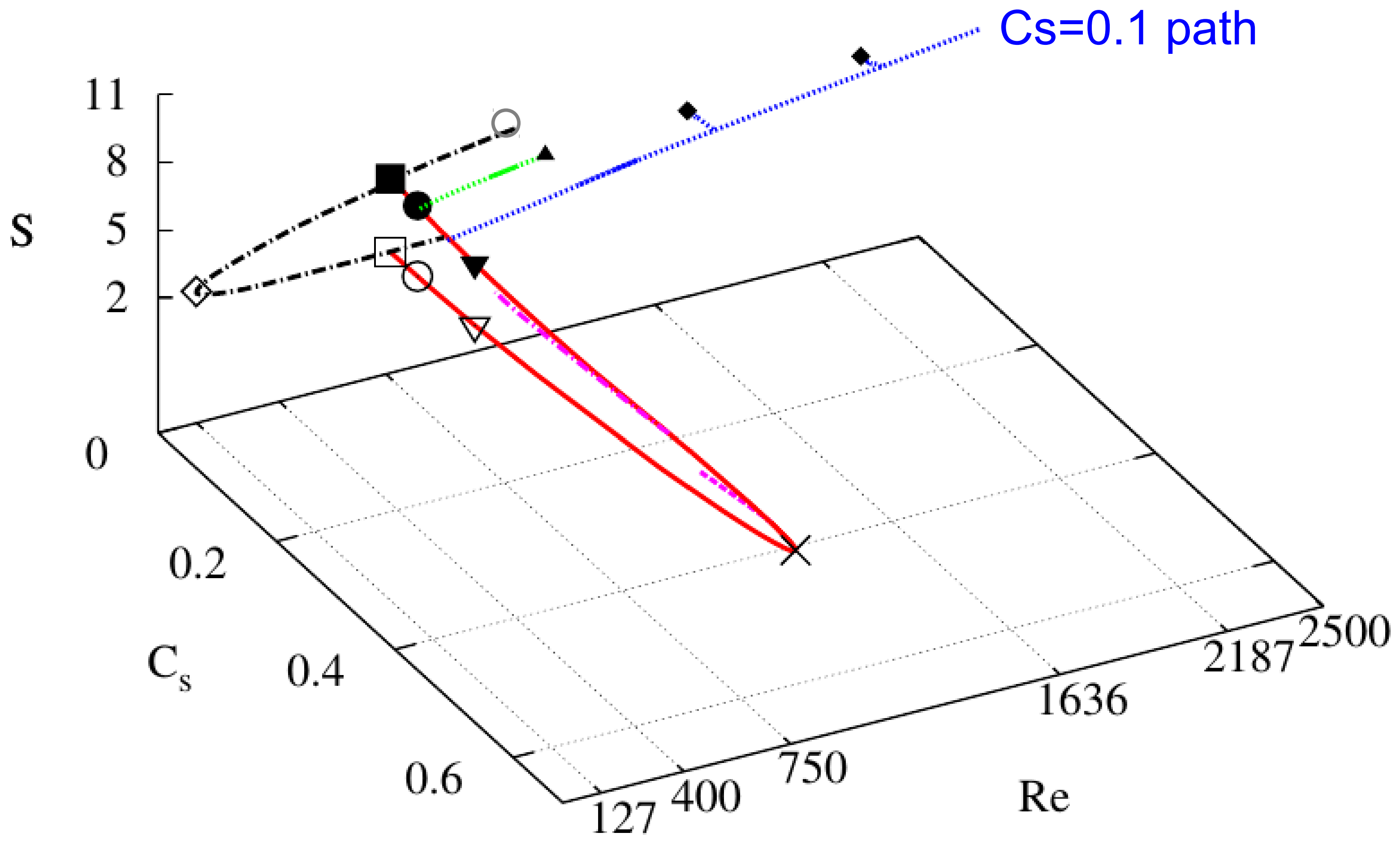


The filtered steady solutions are connected to the NCBW branch of Navier-Stokes solutions

Rawat, Cossu & Rincon, *J. Fluid Mech.* 2015

Upper branch continuation to higher Re

Cs=0 & Cs=0.05 upper branch continuation fails when Re > ~1000 → **alternative paths**

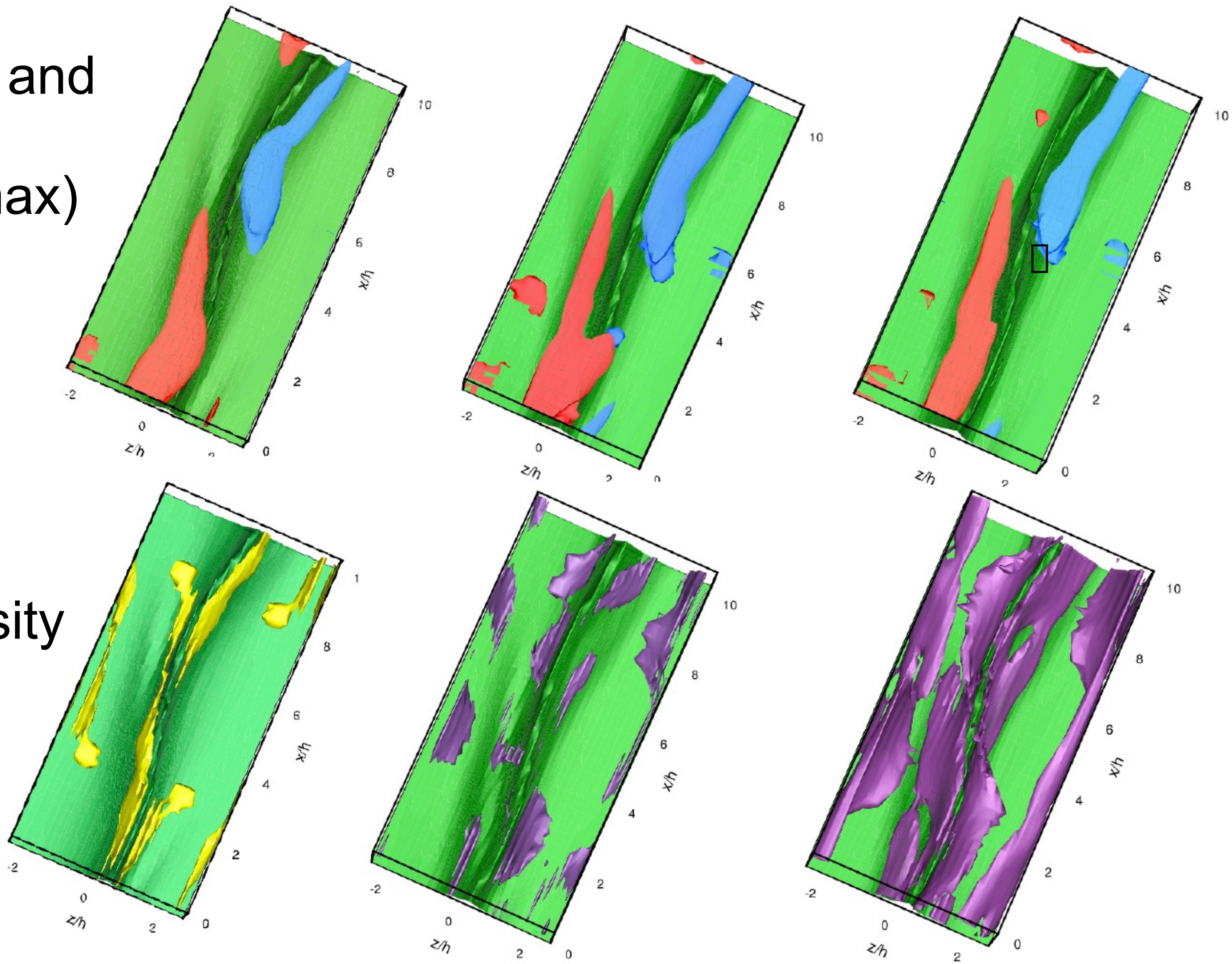


UB coherent structures for higher Re

Re=750 **Re=1650** **Re=2150**

streamwise velocity (green=50% of max) and streamwise vorticity (red/blue=±70% of max)

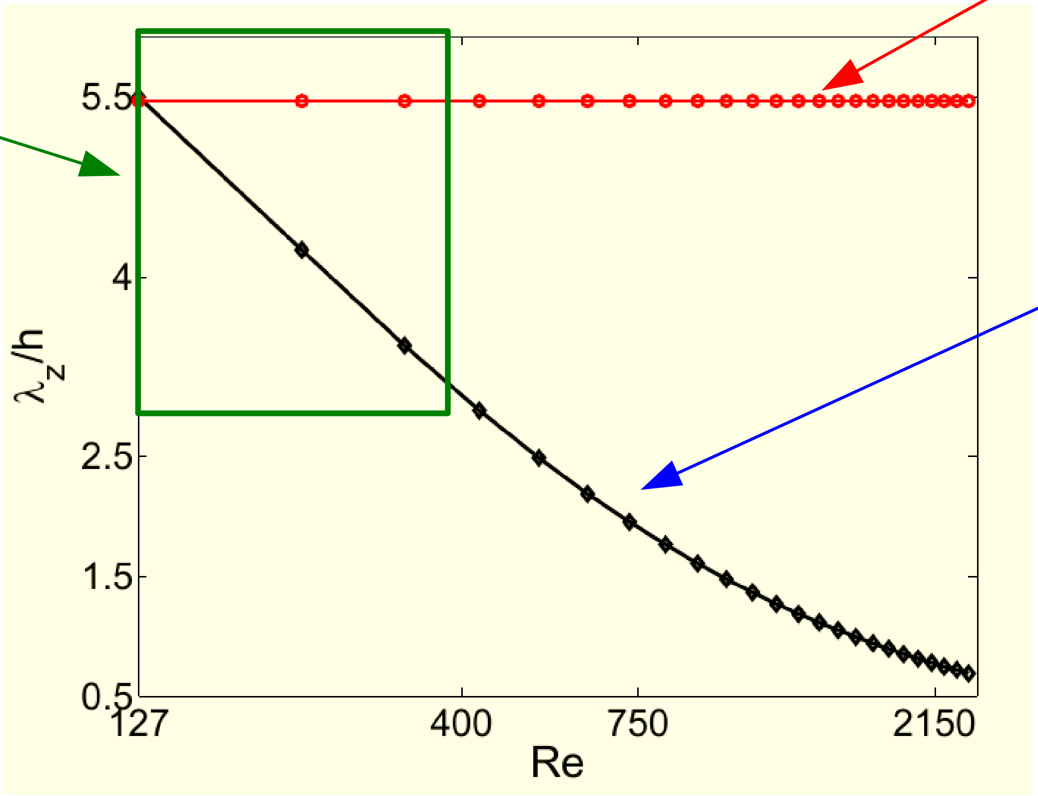
(subgrid) eddy viscosity
yellow: 10% of v
purple: 40% of v



Reynolds number continuation in a minimal flow unit

The two continuation paths

L_z-Re range where NCBW ECS are usually computed/ discussed



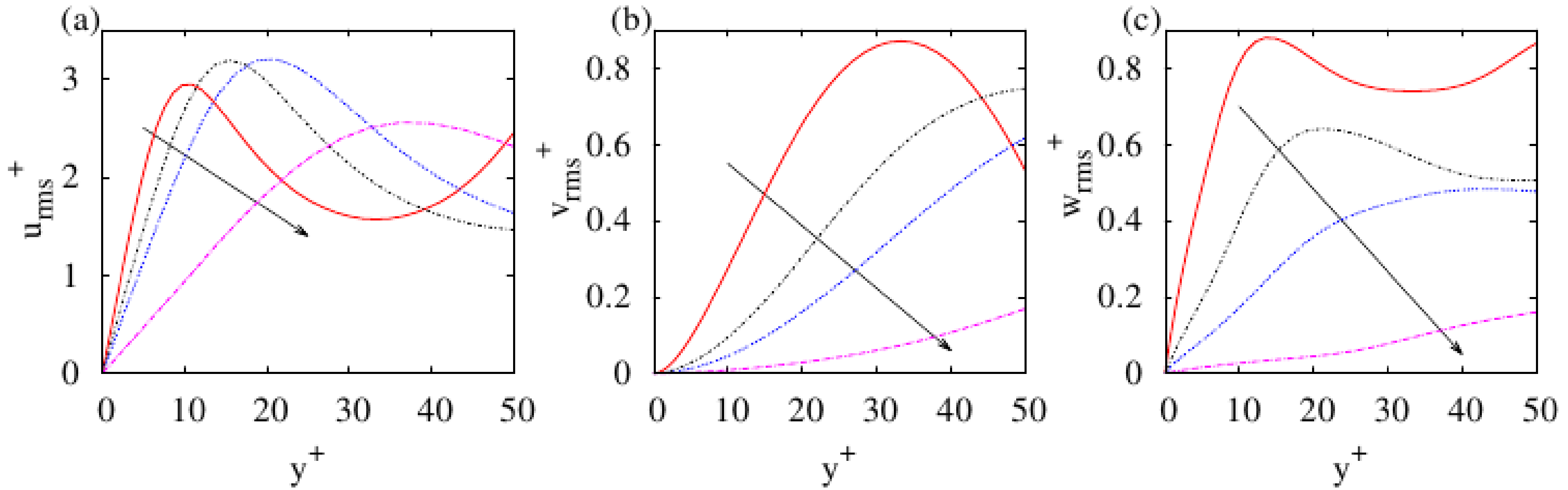
LSM-box size (L_z=5.5) where LECS solutions have been computed

Spanwise size of near-wall streaks λ_z^+=100 decreases in outer units when Re is increased

Test: continue NCBW solutions in a minimal flow unit L_x^+=250, L_z^+=100 (size shrinking in outer units)

Continuation in the minimal flow unit

rms velocity profiles of converged solutions for Re=400, 750, 1100 and 1600 expressed in wall units

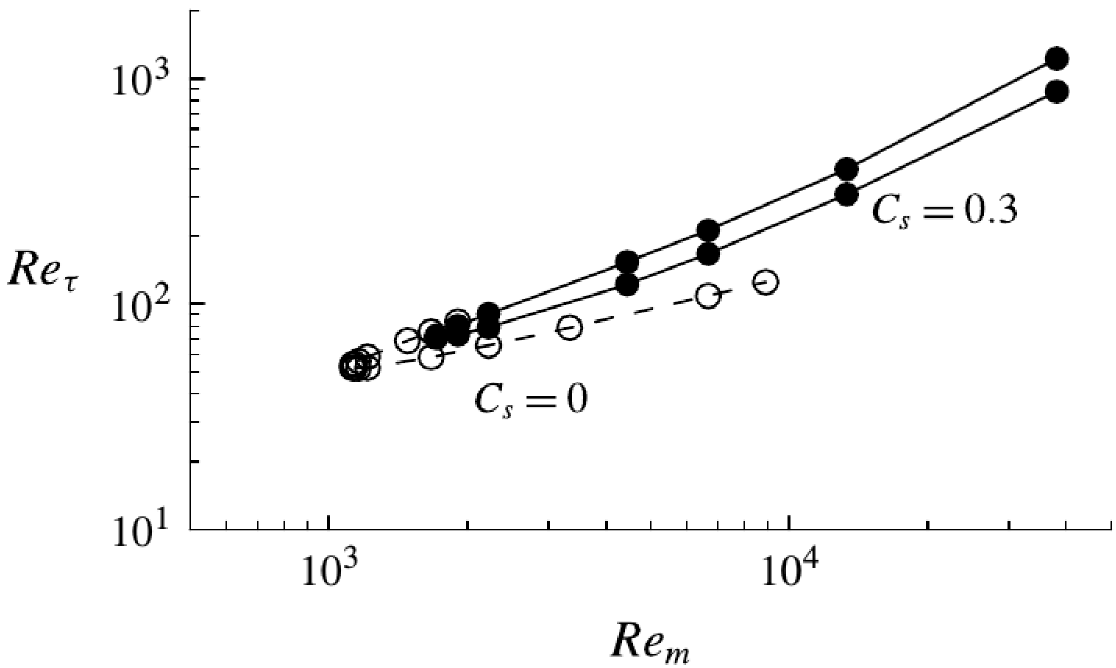


Solutions do not converge to constant shape in wall units → NCBW upper-branch solutions probably more related to LSM dynamics than to the near-wall dynamics

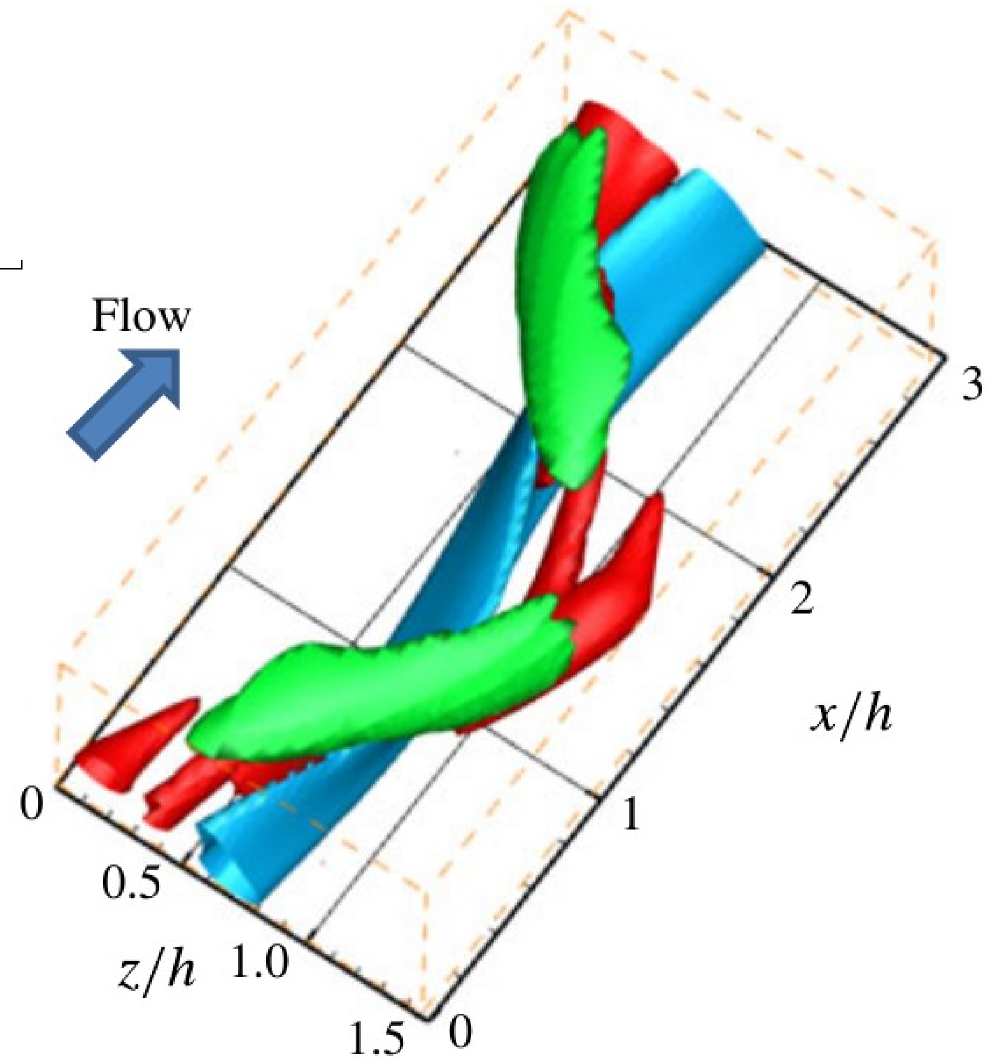
Large eddy coherent solutions (LECS): travelling wave solutions in plane Poiseuille flow

2013-2016 in collaboration with
Subhandu Rawat (IMFT, Toulouse),
François Rincon (IRAP Toulouse),
Yongyun Hwang (Imperial)
Ashley Willis (Sheffield), Jae Sung Park
& Mike Graham (Wisconsin)

Single-streak high-Re TW solutions



Plane channel flow
LSM box: Lx=3h, Lz=1.5h
continuation to high Re

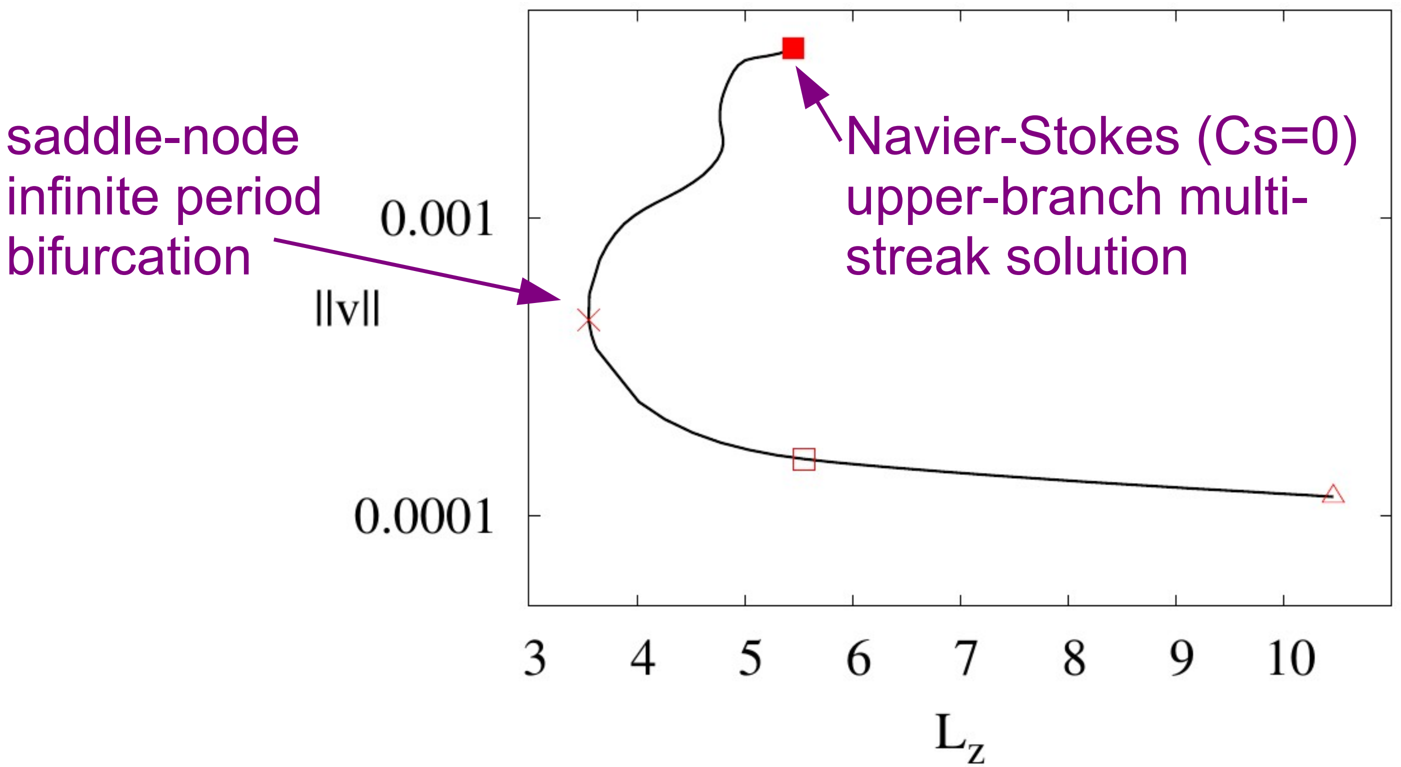


TW solutions in LSM box can be continued to very high Reynolds numbers (with Hwang & Willis, 2016)

Hwang, Willis & Cossu, *J. Fluid Mech.* 2016

Multi-streaks travelling wave solutions

Plane channel flow, Re 2000 with Lx=6.28h, Lz=5.55 h
NS solutions issued by a saddle-node infinite period bifurcation and computed by continuation in Lz



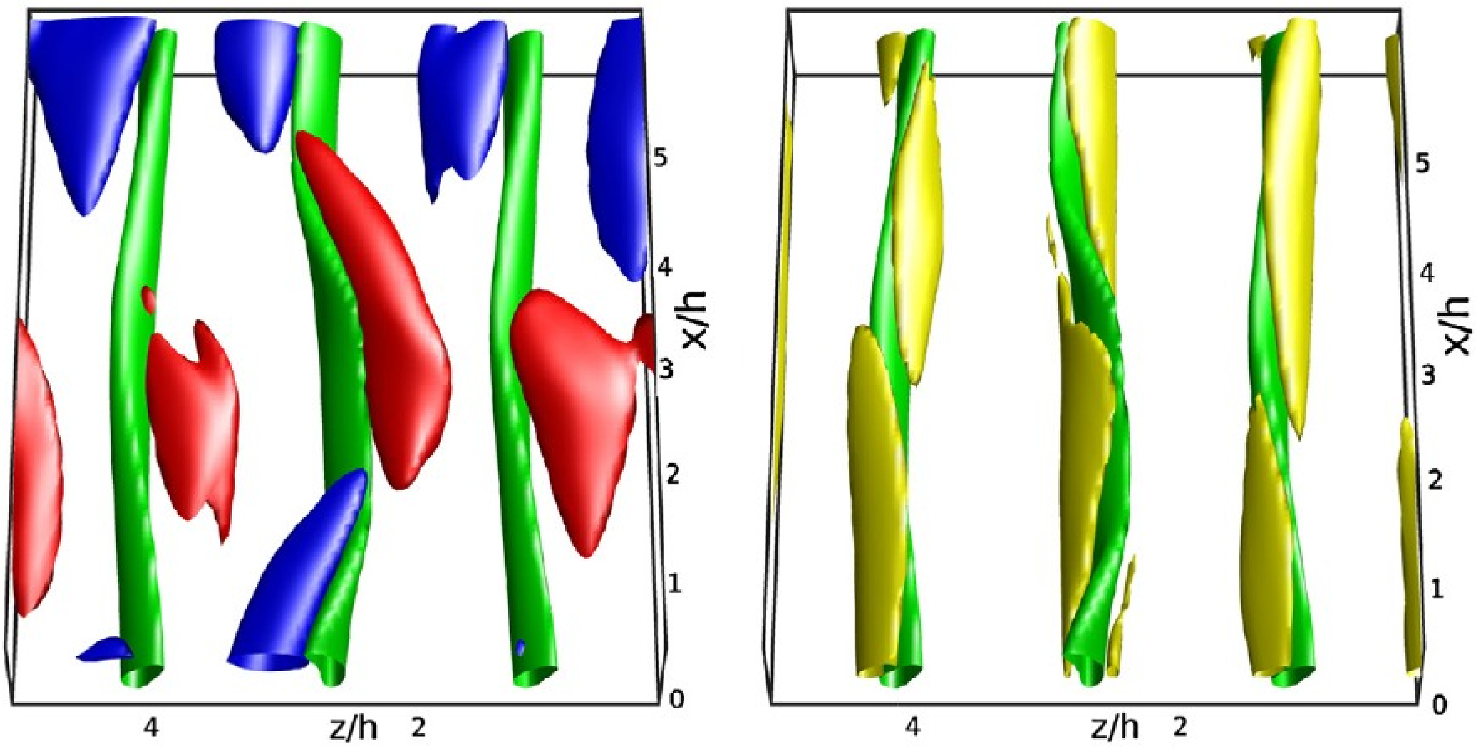
Reverse-continuation from Navier-Stokes (Cs=0) upper-branch traveling wave exact solution to Cs=0.05 (reference LES solutions)

figure from Rawat, Cossu & Rincon (2016)

Multi-streaks travelling wave solutions

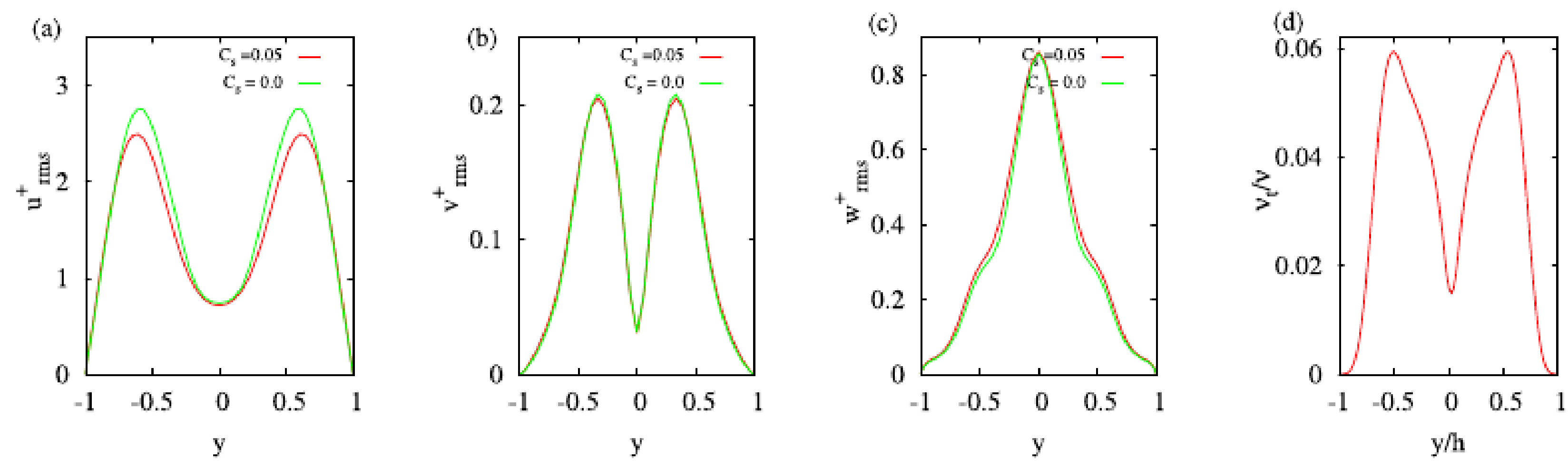
3D view

low-speed streaks (green: u^+=-2% & streamwise vorticity (red /blue=±65% of max)



low-speed streaks & eddy viscosity (yellow: ν_t/ν=6.6%)

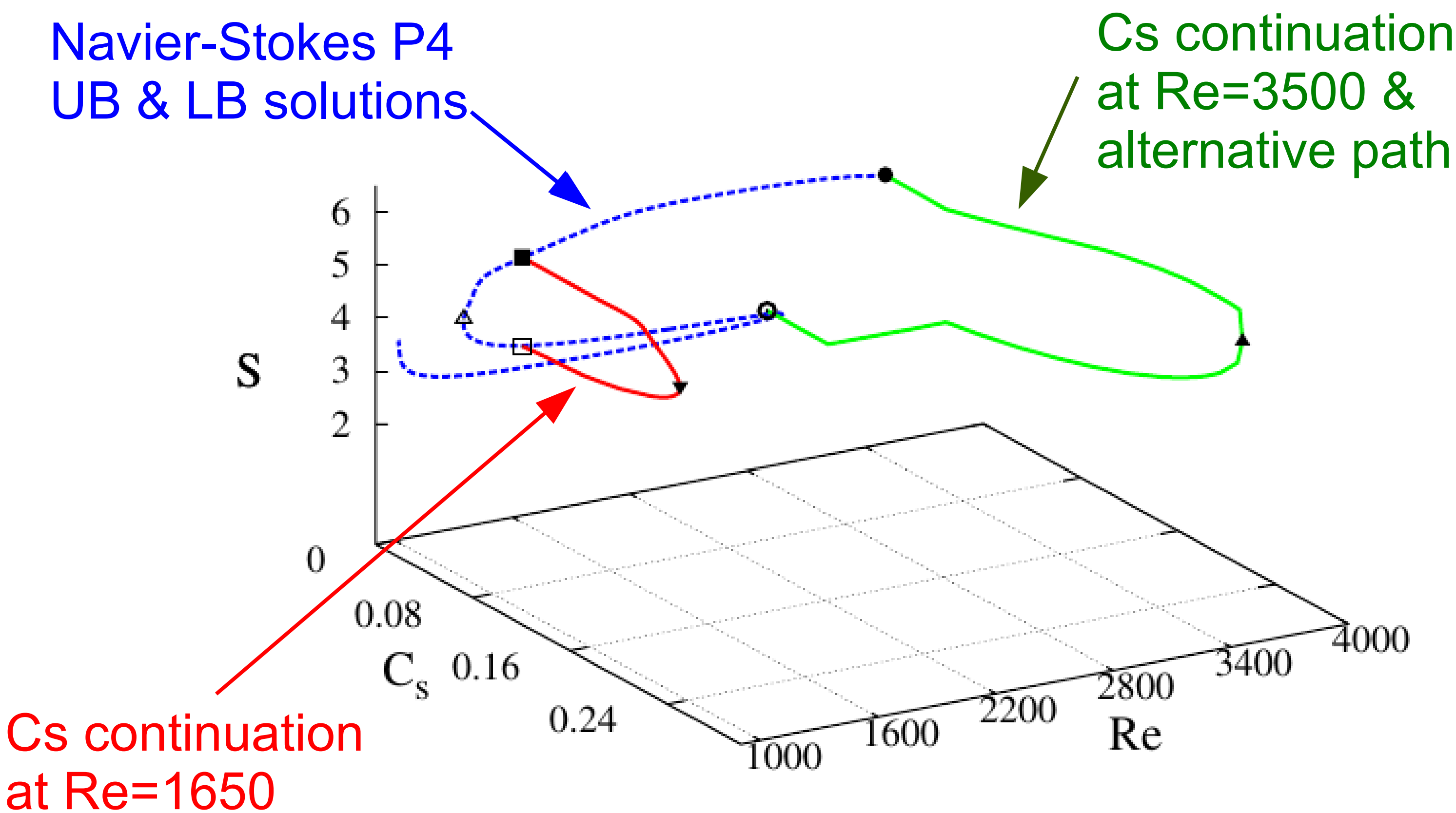
wall-normal rms profiles



figures from Rawat, Cossu & Rincon (*C.R. Méc.* 2016)

Park & Graham P4 travelling wave solutions

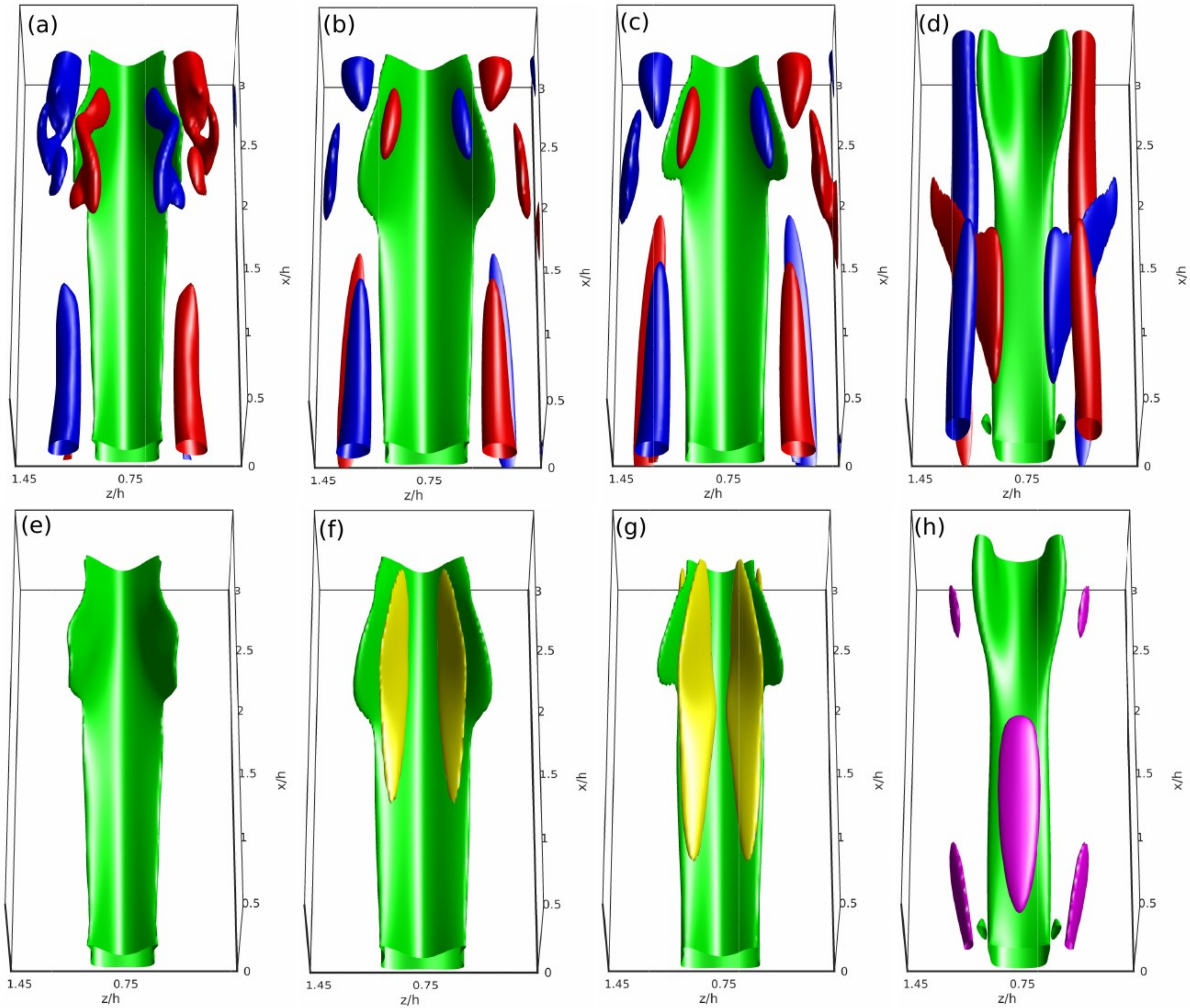
Reverse continuation from Navier-Stokes (Cs=0) P4 TW ECS solution of Park & Graham (JFM 2015)
Lx=πh, Lz=πh/2



Rawat, Park, Graham & Cossu (2016, under review)

P4-LECS UB solutions at Re=3500

Cs=0 (NS) | Cs=0.1 | Cs=0.2 | Cs=0.2 (TP)



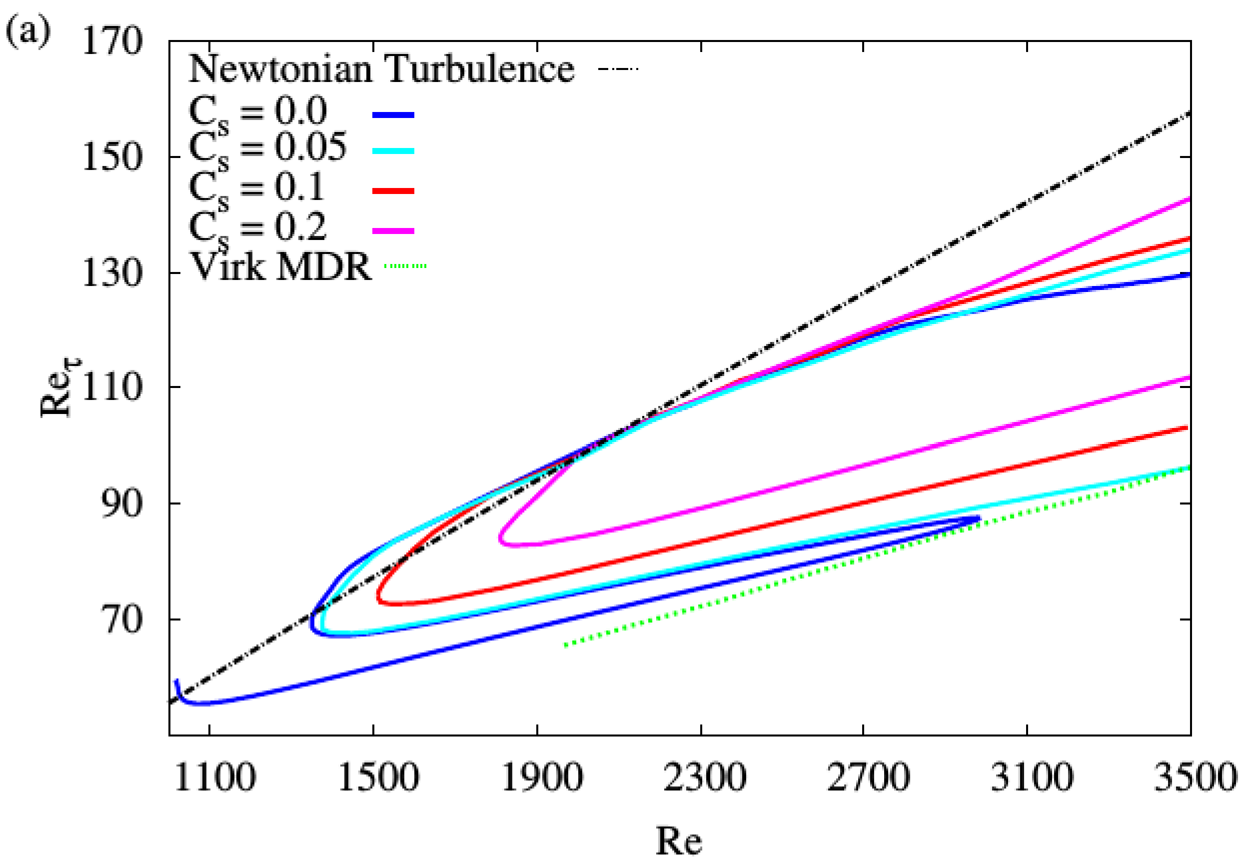
streaks & streamwise vorticity

streaks & eddy viscosity associated to unresolved (small-scale) motions

Rawat, Park, Graham & Cossu (2016, under review)

Park & Graham P4 travelling wave solutions

Higher Cs ~improves upper branch solutions
BUT deteriorates lower branch solutions if large



Cs=0.05 to stabilize the (subharmonic) turning point of the Navier-Stokes (Cs=0) LB solution

Rawat, Park, Graham & Cossu (2016, under review)

Summary

Found steady & travelling-wave `large-eddy' solutions of the filtered (coherent) large-scale motions

These solutions take into full account the effect of residual motions (inhomogeneous eddy viscosity)

Averaging of small scales $\rightarrow$ steady filtered LSM solutions even with unsteady small-scale motions

Solutions of the filtered equations can be connected to solutions of the Navier-Stokes equations. Reverse also works (but not always)

A final remark

Saddles: the high Re problem for the NS eqs.

Additional steady or TW solutions *of the Navier-Stokes equations* appear when Re increases → difficult to compute all of them in the developed turbulent regime

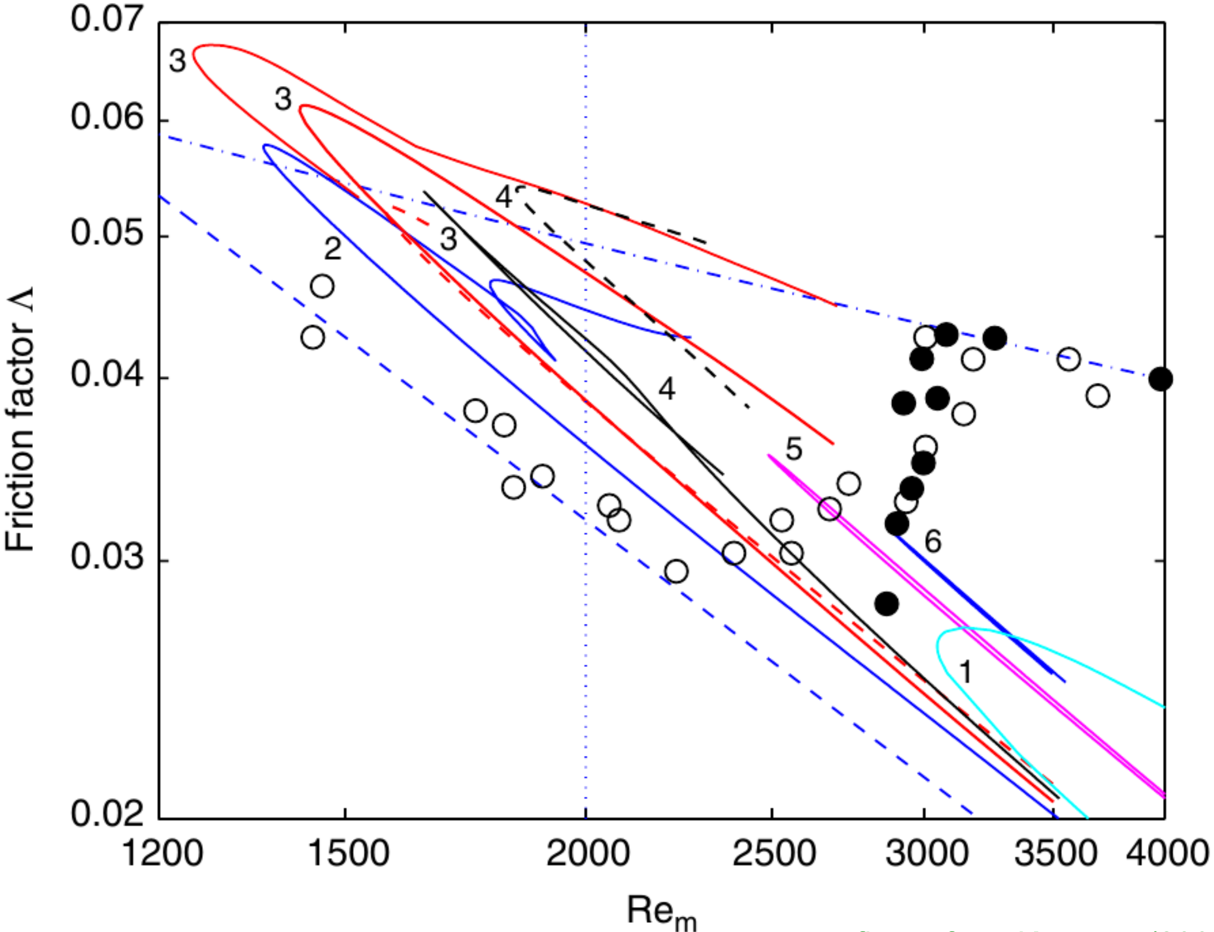


figure from Kerswell (2005)

Turbulent solutions spend only 10-20% of the time near saddles already at transitional Re (Schneider et al 2007, Kerswell & Tutty 2007)

→ look at *periodic* solutions to build turbulent statistics from averaging of `exact' solutions (not successful yet)

High Re most of the energy is in large-scale motions → Can a few exact solutions of the *filtered* equations capture the dynamics of large-scale motions at large Re? If yes, is this enough to converge meaningful turbulent statistics?

Thank you for listening

papers available on:
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and/or Google Scholar / ResearchGate / ORCID / Researcher ID
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