Self-sustaining attached eddies in wall-bounded turbulence: Pressure, skin friction & invariant solutions

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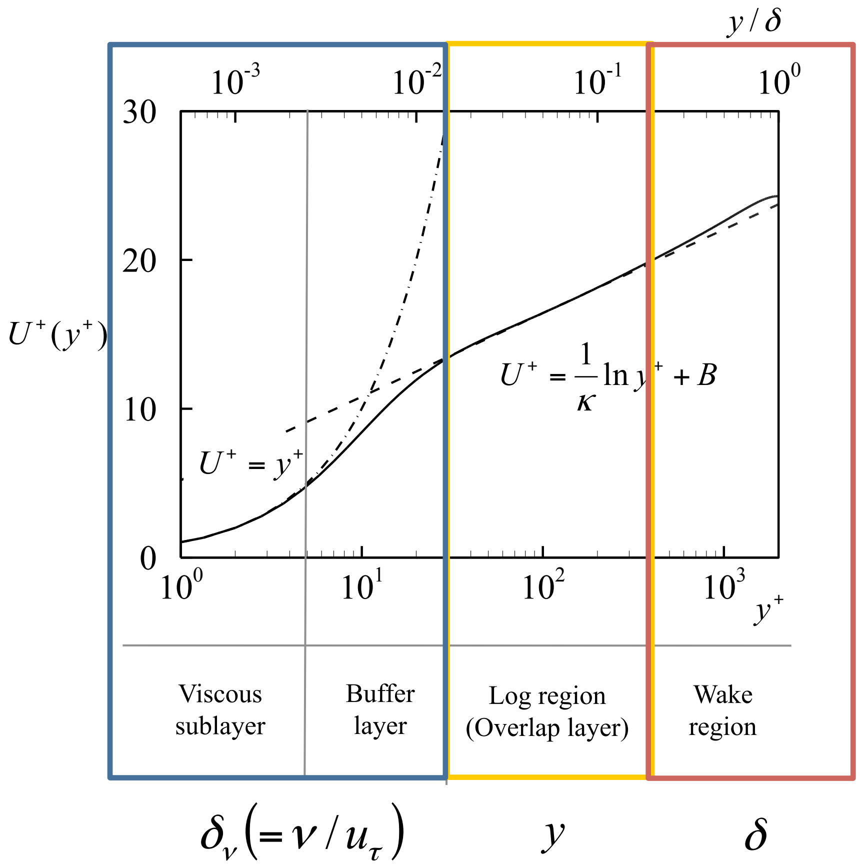
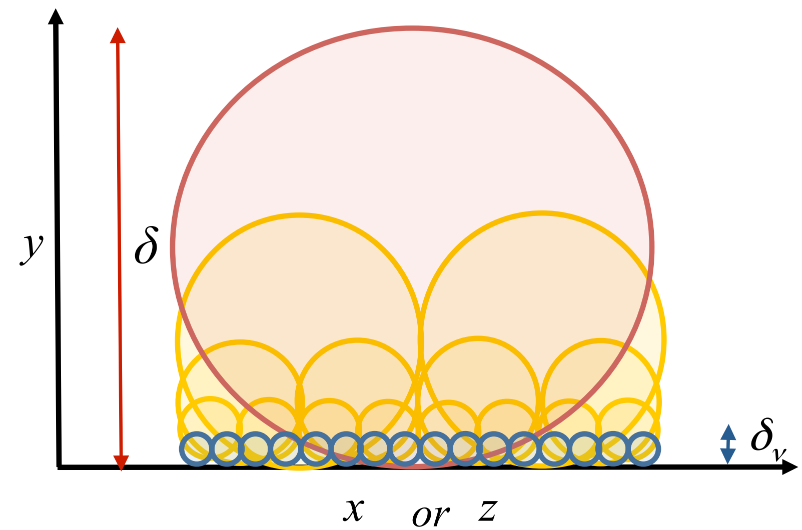
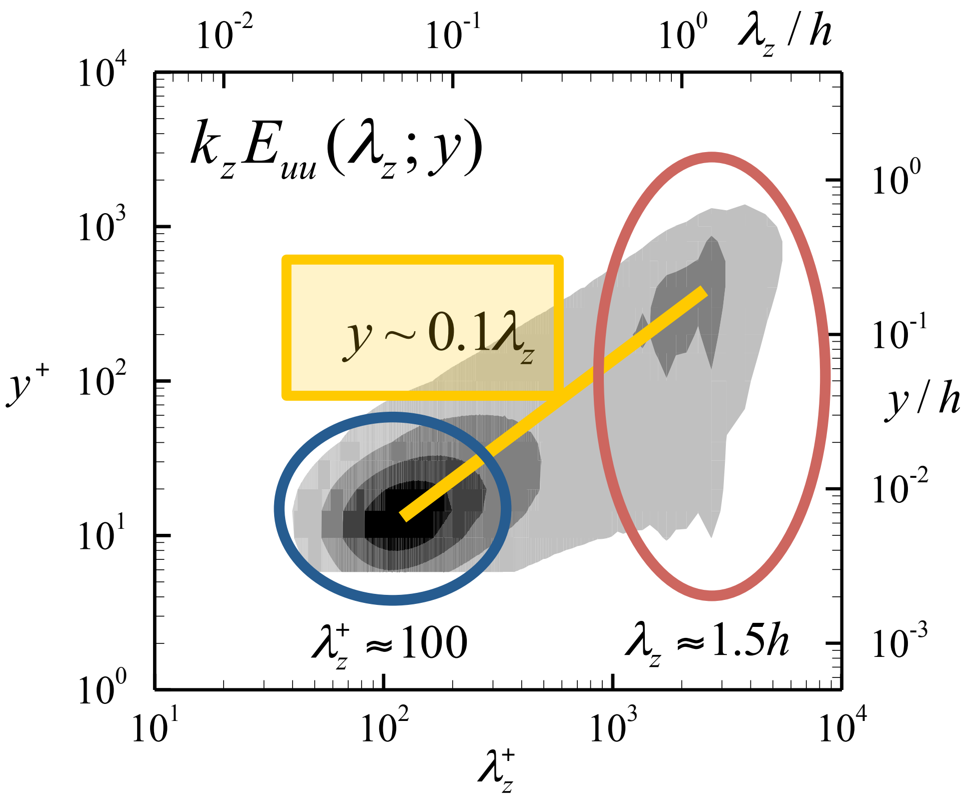
Attached eddy hypothesis – Townsend (1961, 1976)



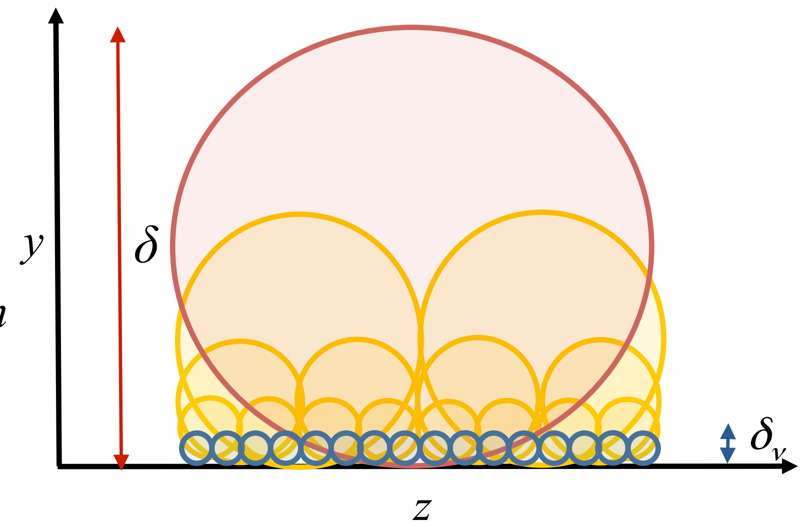


Evidence: Linear spanwise length scale growth



Hoyas & Jimenez (2006,PoF)

DNS (channel) at $\mathrm{Re}_\tau = 2003$



$\lambda_z \sim y$

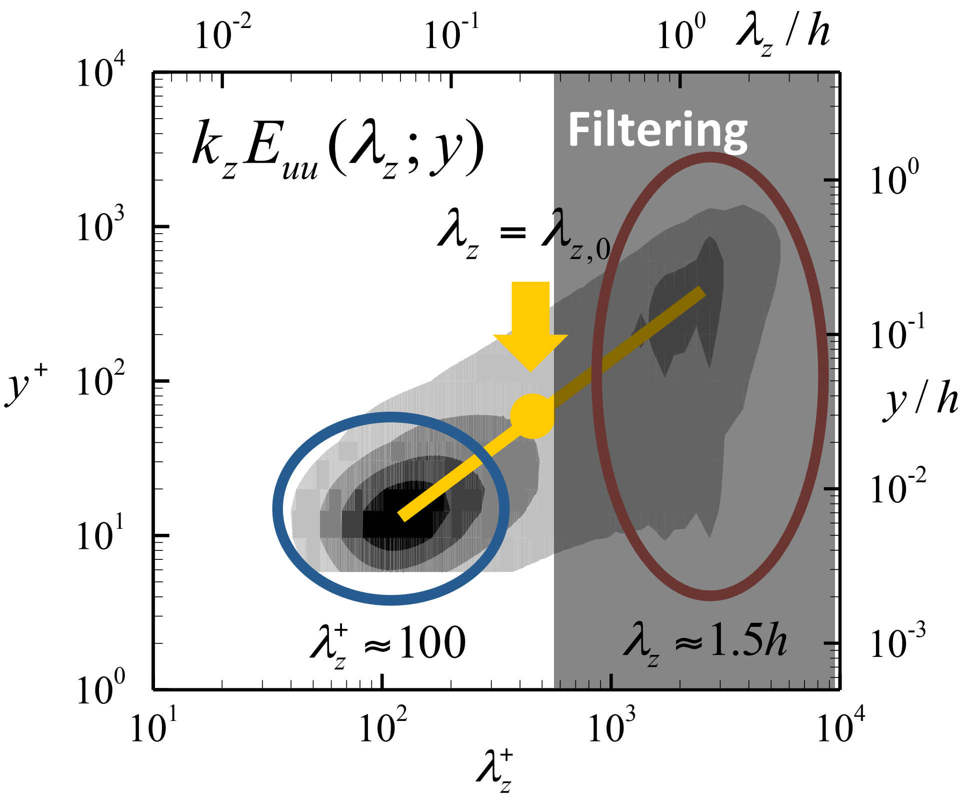
Attached eddies do exist and sustain themselves

Hwang & Cossu, 2011, *Phys. Fluids* **23** 061702

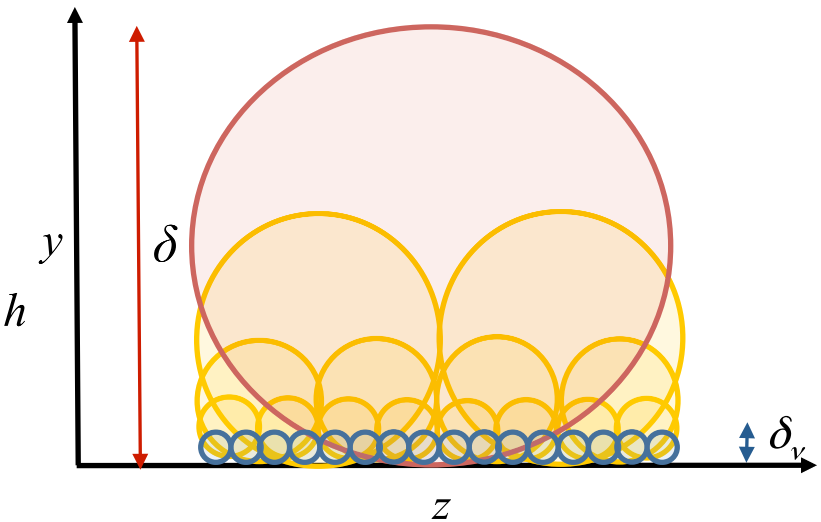
Hwang, 2015, *J. Fluid Mech.* **767** p254

Hwang & Bengana, 2016, *J. Fluid Mech.* **795** p708

Isolating the motions at a given $\lambda_{z,0}$ $(100\delta_v < \lambda_{z,0} < 1.5h)$



DNS (channel) at $\mathrm{Re}_\tau = 2003$

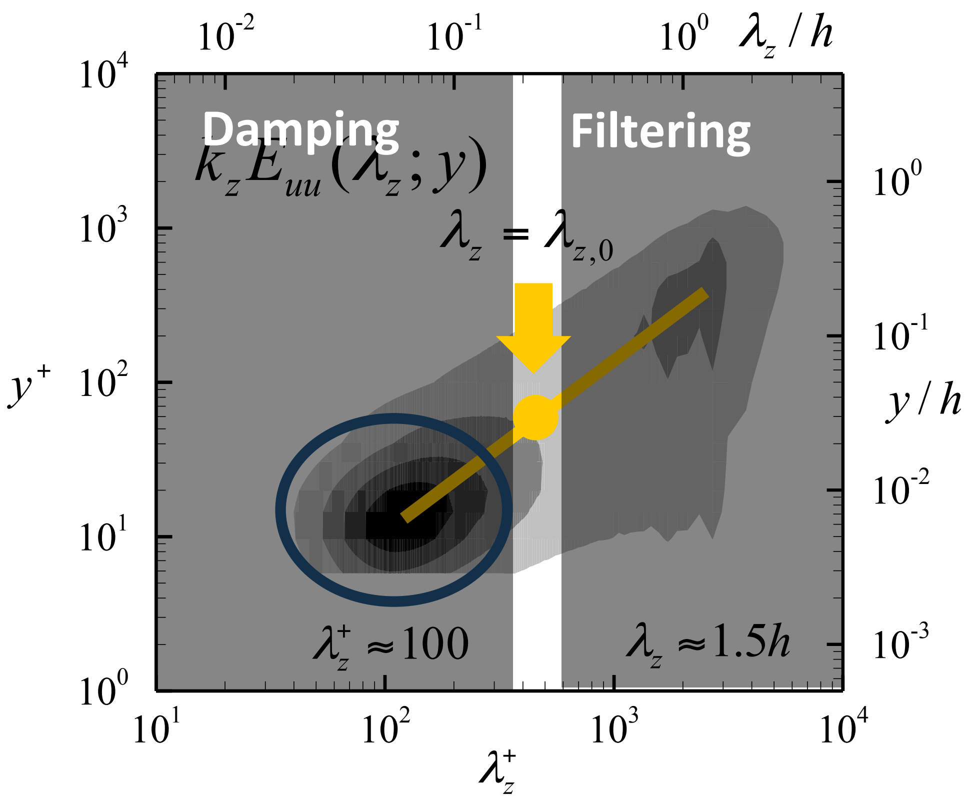


Narrow spanwise domain + additional removal of quasi 2D motion

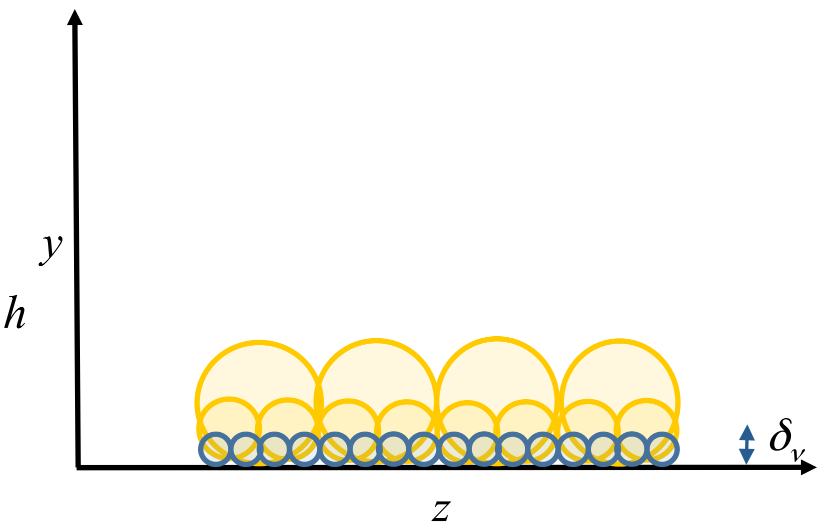
$$L_z = \lambda_{z,0}$$

Hwang, 2013, *J. Fluid Mech*. **727** p264

Isolating the motions at a given $\lambda_{z,0}$ $(100\delta_v < \lambda_{z,0} < 1.5h)$



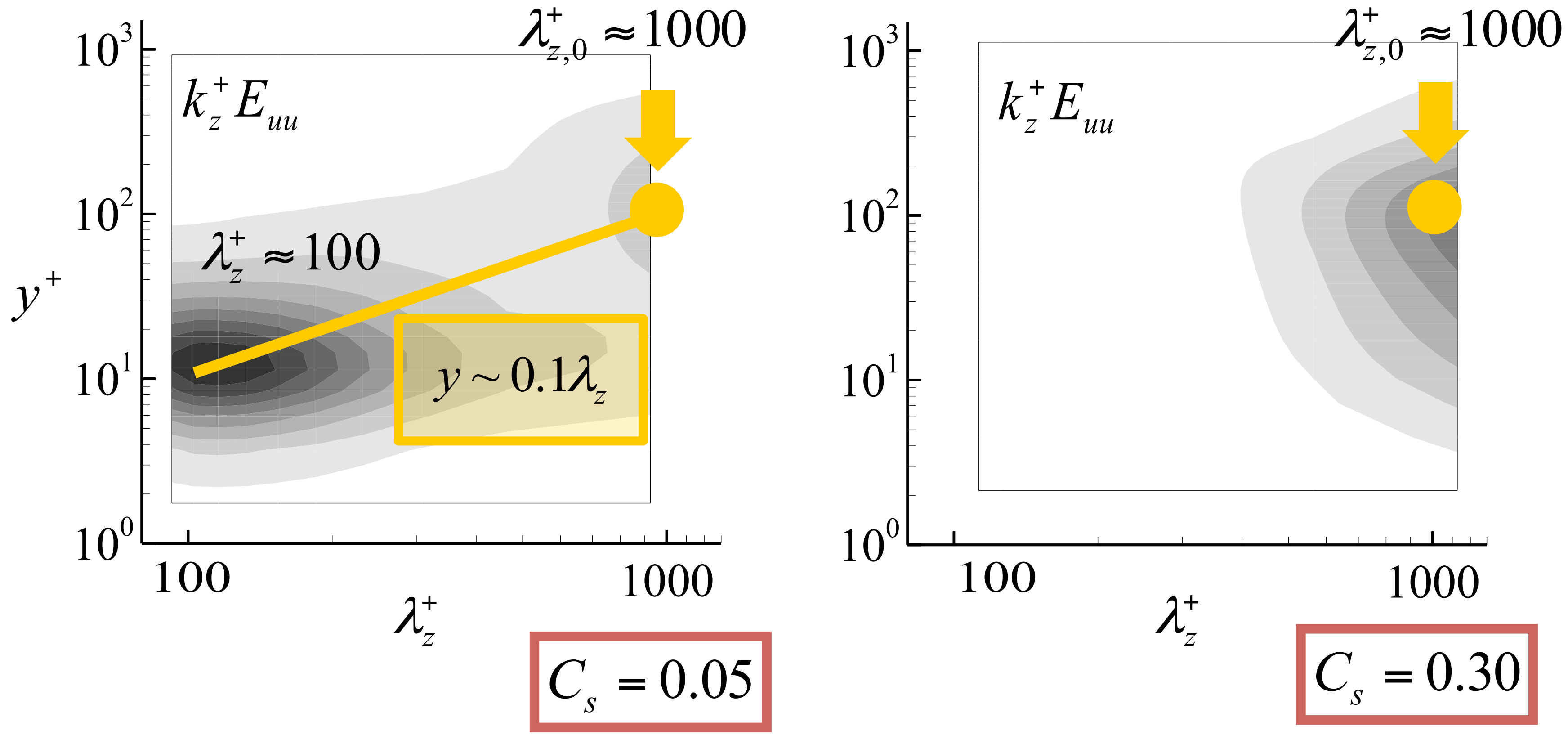
Hoyas & Jimenez (2006,PoF)



LES with an excessively large eddy viscosity (artificial elevation of C_s)

Hwang & Cossu, 2010, *Phys. Rev. Lett*. **105** 044505

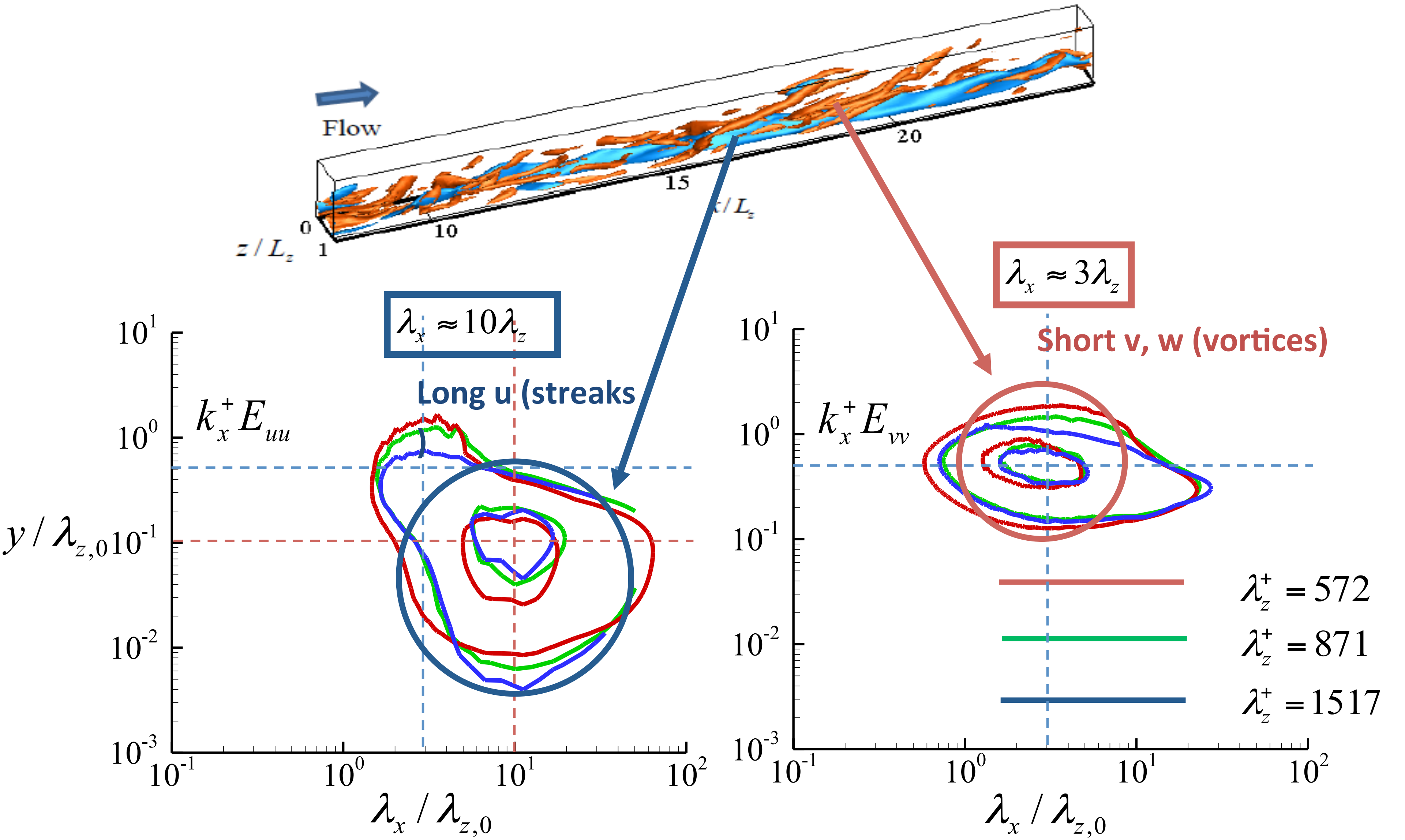
Example: isolating the motions at $\lambda_{z,0}^{+} \approx 1000 \; (\mathrm{Re}_\tau \approx 2000)$



$\lambda_z^+ > 1000$ → **Filtering**

$\lambda_z^+ < 1000$ → **Over-damped LES**

Statistics of self-sustaining Townsend's attached eddies



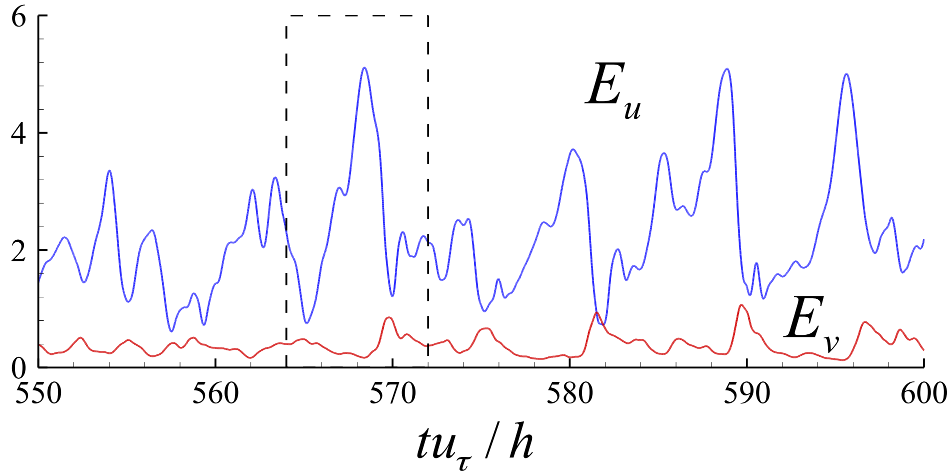
Bursting of self-sustaining attached eddies

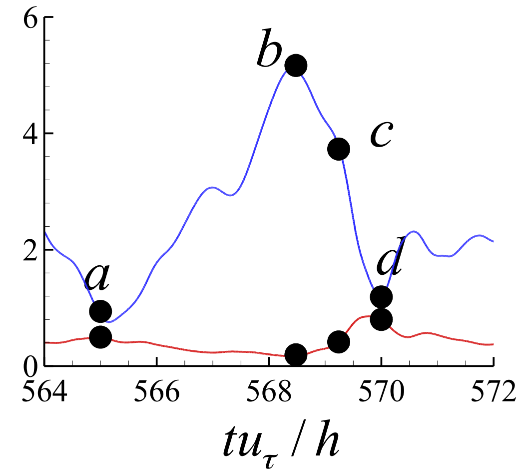
Streaks

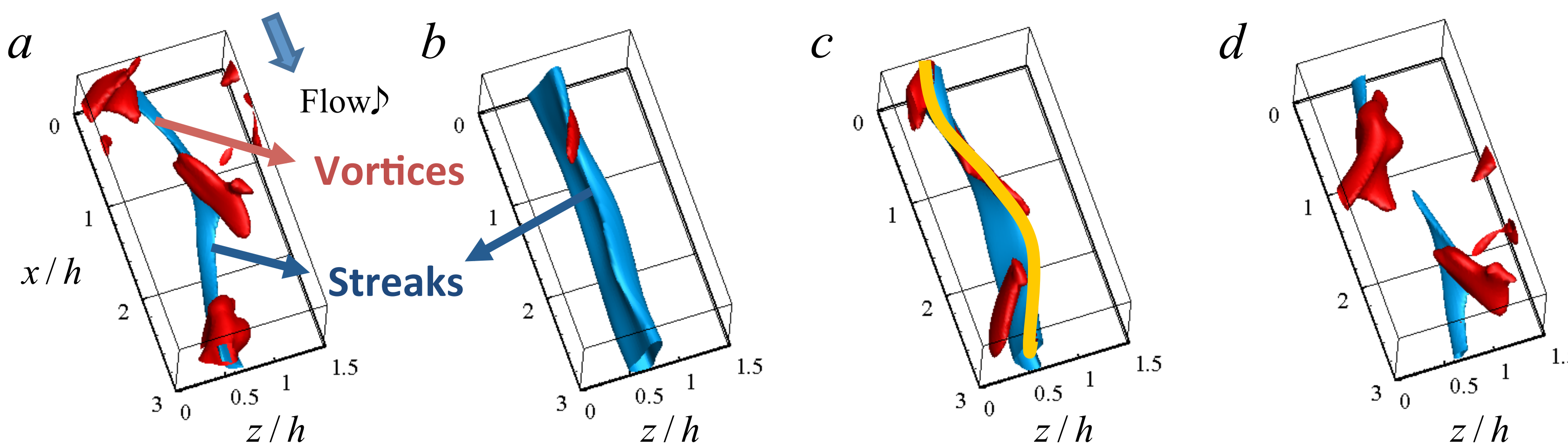
$$E_u \equiv \int_V u^2 \, dV$$

Vortices

$$E_v \equiv \int_V v^2 \, dV$$







Causality of the three elements

Streaks

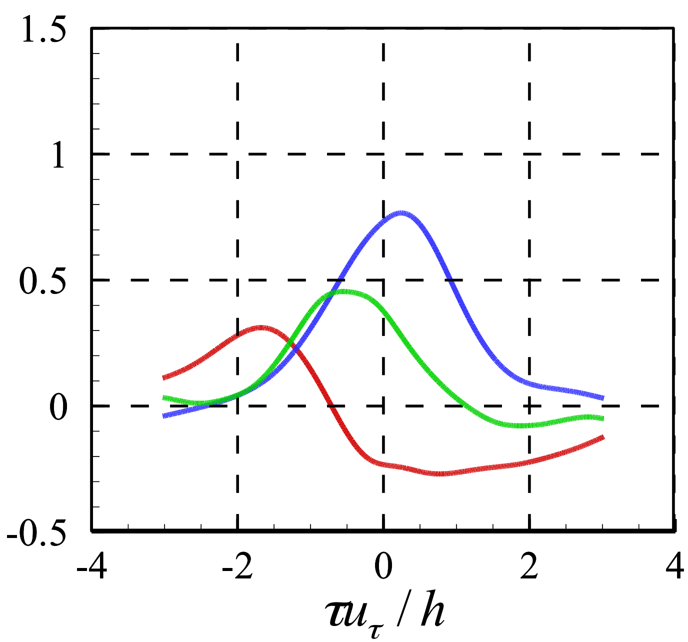
$$E_u \equiv \int_V (u^+)^2 dV$$

Vortices

$$E_v^+ \equiv \int_V (v^+)^2 dV$$

Wave (streak instability)

$$E_1 \equiv \int_V |\hat{u}|^2 + |\hat{v}|^2 + |\hat{w}|^2 dV \quad \text{for} \quad k_x = 2\pi/L_x \text{ and } k_z = 2\pi/L_z$$

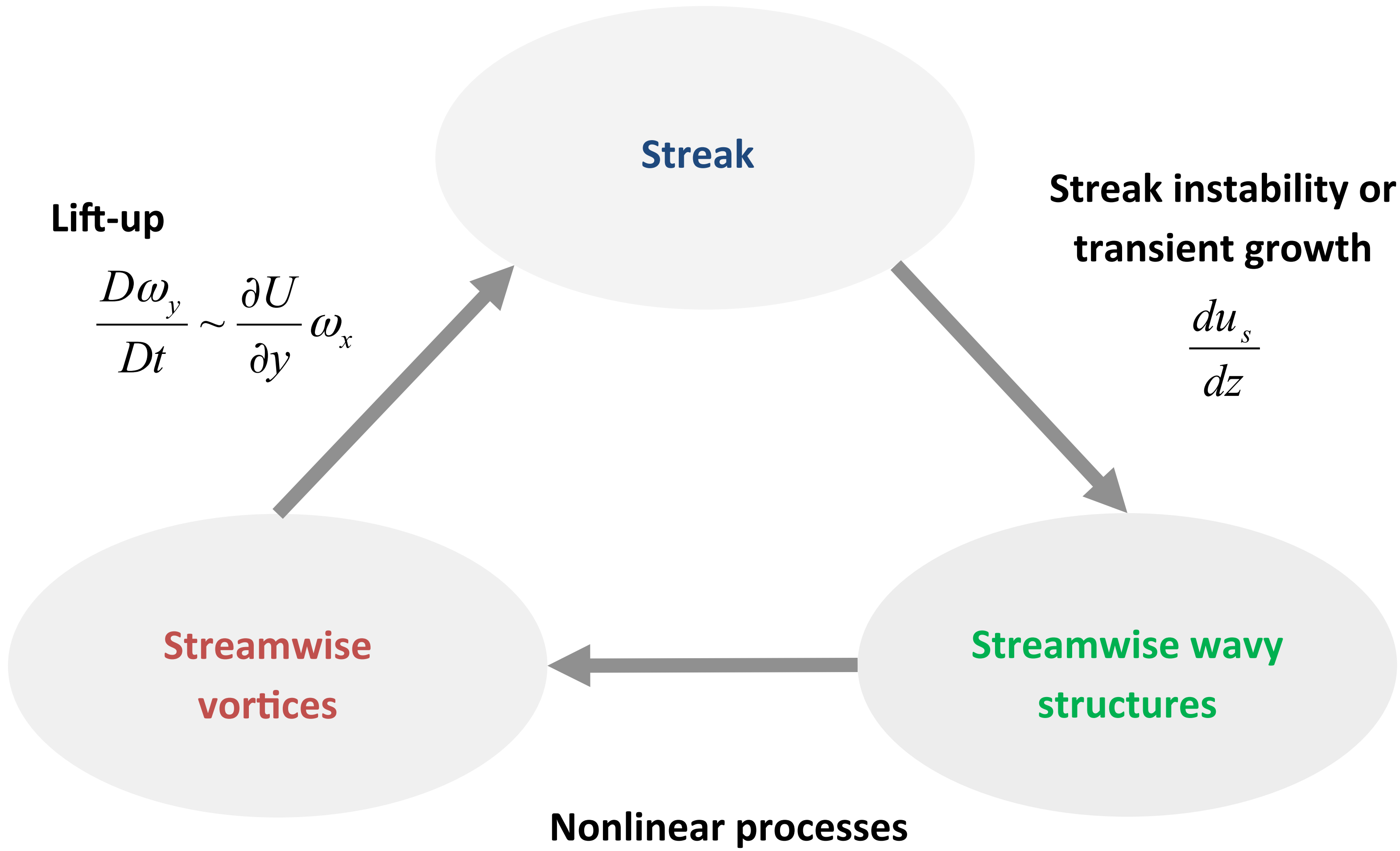


$$C_{uv}(\tau) = \frac{\langle E_u(t+\tau)E_v(t)\rangle}{\langle E_u(t)\rangle^{1/2}\langle E_v(t)\rangle^{1/2}},$$

$$C_{vw}(\tau) = \frac{\langle E_v(t+\tau)E_w(t)\rangle}{\langle E_v(t)\rangle^{1/2}\langle E_w(t)\rangle^{1/2}},$$

$$C_{1v}(\tau) = \frac{\langle E_1(t+\tau)E_v(t)\rangle}{\langle E_1(t)\rangle^{1/2}\langle E_v(t)\rangle^{1/2}},$$

Self-sustaining process of attached eddies



Hamilton, Kim & Waleffe (JFM, 1995)

Pressure fluctuation is generated by nonlinear feeding processes of vortices

Minjeong Cho (SNU, Korea)
Haecheon Choi (SNU, Korea)

Classical description on pressure in a turbulent flow

$$\nabla^2 p = -\frac{\partial u_i}{\partial x_j}\frac{\partial u_j}{\partial x_i} \quad \Leftarrow \quad u_i = U_i + u_i'$$

$$\nabla^2 p_{rapid} = -2\frac{dU}{dy}\frac{\partial v'}{\partial x}$$

Rapid (linear) pressure

: rapidly responding pressure under direct effect of mean shear

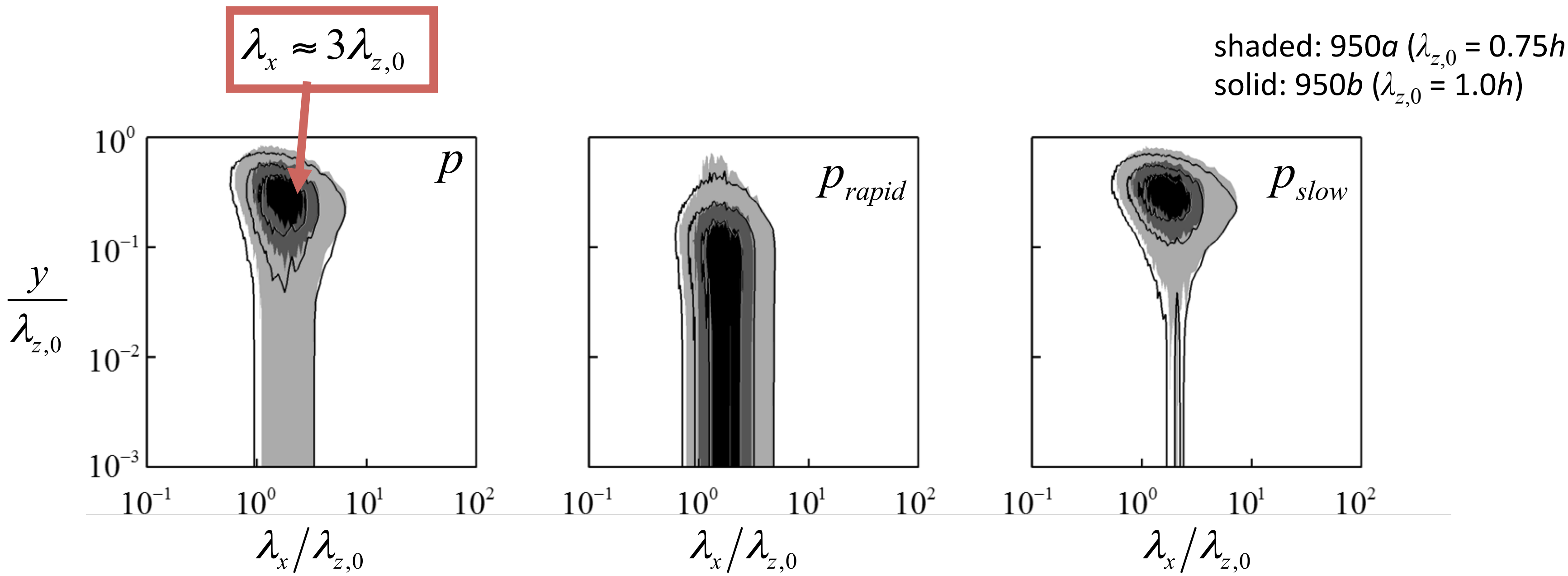
$$\nabla^2 p_{slow} = -\frac{\partial u_i'}{\partial x_j}\frac{\partial u_j'}{\partial x_i}$$

Slow (nonlinear) pressure

: slow responding pressure by the following nonlinear interaction

See e.g. Kim (1989, JFM)

Self-similar pressure of self-sustaining attached eddies



Pressure shows strong correlation with vortices (not surprisingly!).

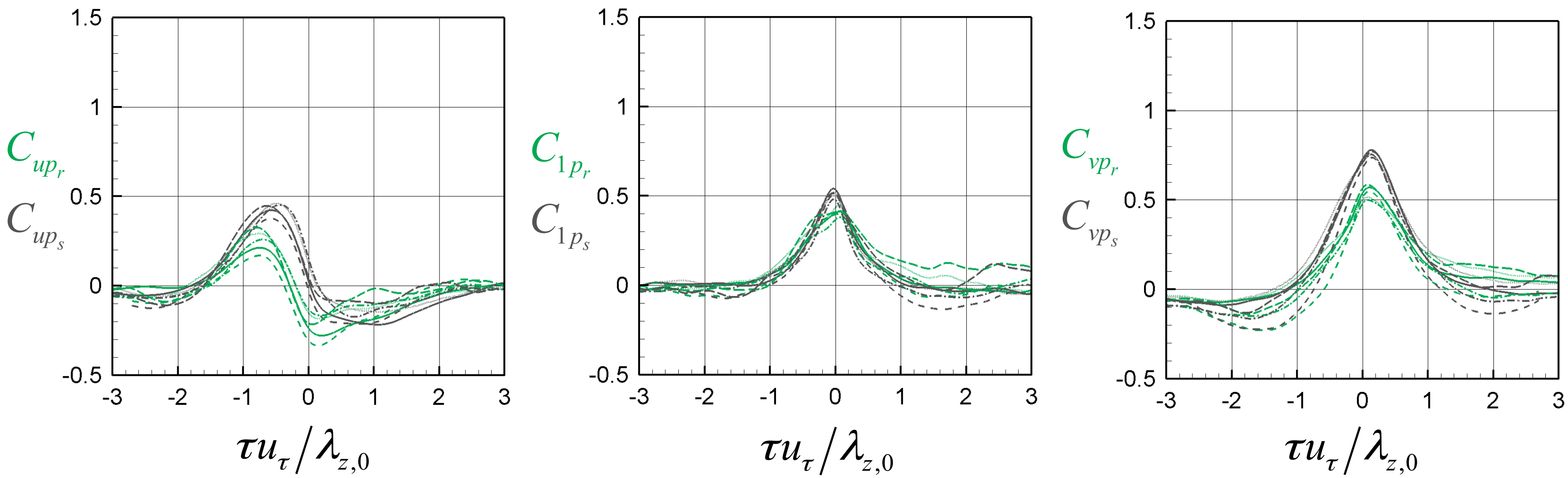
Slow pressure is about twice larger than rapid pressure.

Rapid pressure extends to the wall due to mean shear: $\nabla^2 p_{rapid} = -2\frac{dU}{dy}\frac{\partial v'}{\partial x}$

Causality of pressure in self-sustaining process

$$C_{ij}(\tau) = \frac{\langle E_i(t+\tau)E_j(t)\rangle}{\sqrt{\langle E_i^2(t)\rangle}\sqrt{\langle E_j^2(t)\rangle}} \quad \left(\begin{array}{l} i, j = u, v, 1, p_r, p_s \\ \langle\ \rangle\text{: average in time} \end{array} \right)$$

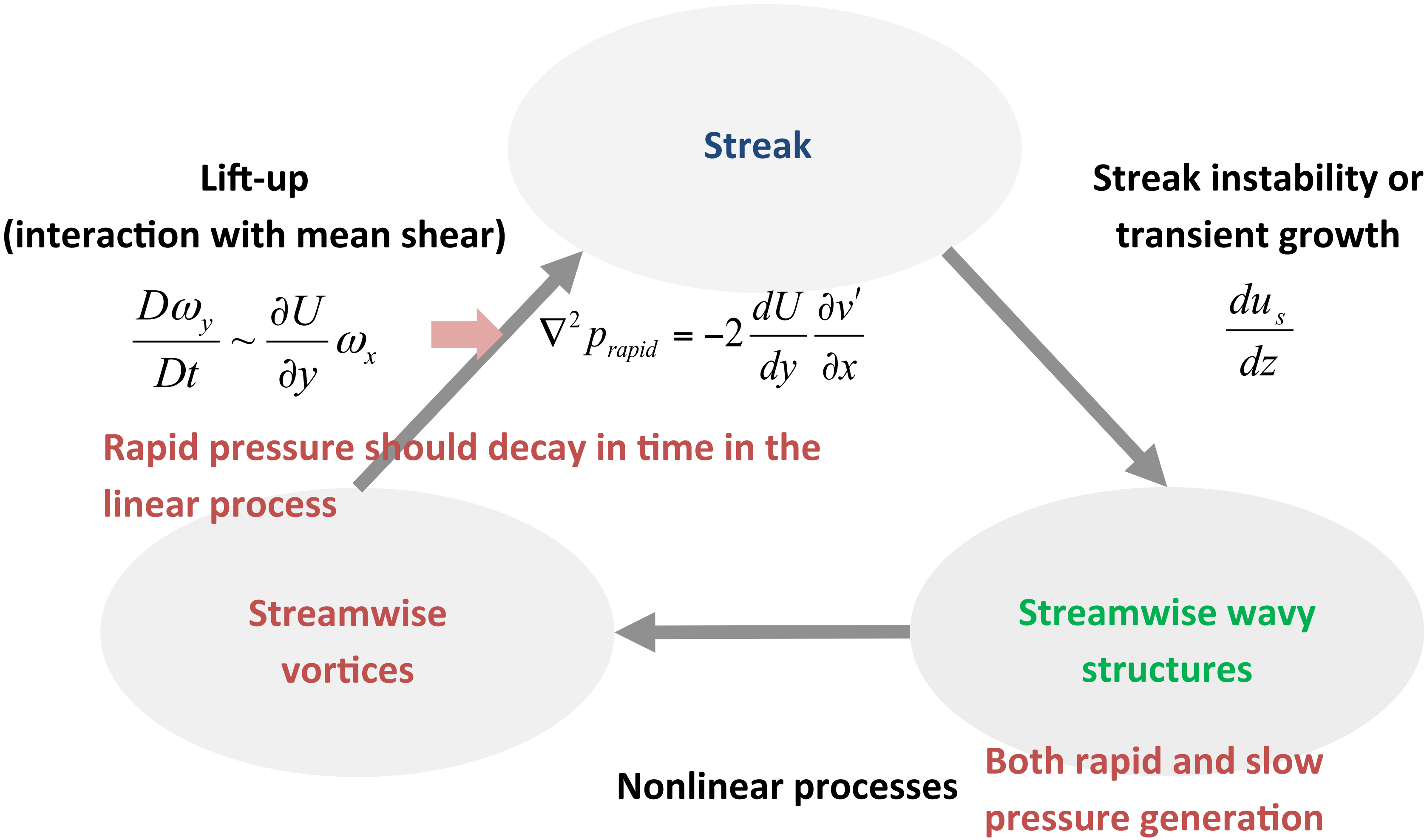
950*a* ($\lambda_{z,0}$ = 0.75*h*)
950*b* ($\lambda_{z,0}$ = 1.0*h*)
1800*a* ($\lambda_{z,0}$ = 0.375*h*)
1800*b* ($\lambda_{z,0}$ = 0.5*h*)
1800*c* ($\lambda_{z,0}$ = 0.75*h*)



Streak → **Streamwise wavy motions** → **Vortices**

Rapid & slow pressures

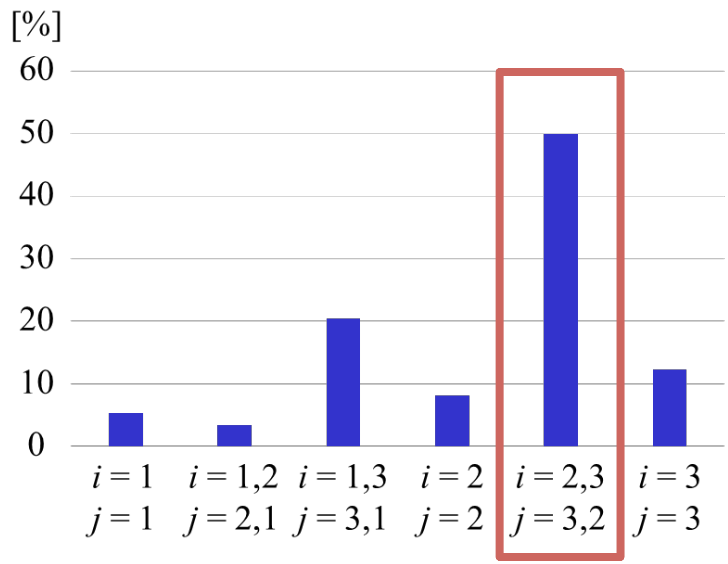
Self-sustaining process and pressure



Rapid pressure is the mediator of the lift-up effect

The mechanisms of slow pressure generation

$$\nabla^2 p_{slow} = -\frac{\partial u_i'}{\partial x_j}\frac{\partial u_j'}{\partial x_i}$$



The leading nonlinear vortex dynamics is found with

$$\frac{\partial \omega_x'}{\partial t} \sim -v'\frac{\partial \omega_x'}{\partial y} + \omega_x'\frac{\partial u'}{\partial x}$$

Nonlinear wall-normal advection

Hamilton, Kim & Waleffe (JFM, 1995)

Vortex stretching by streamwise wavy streak

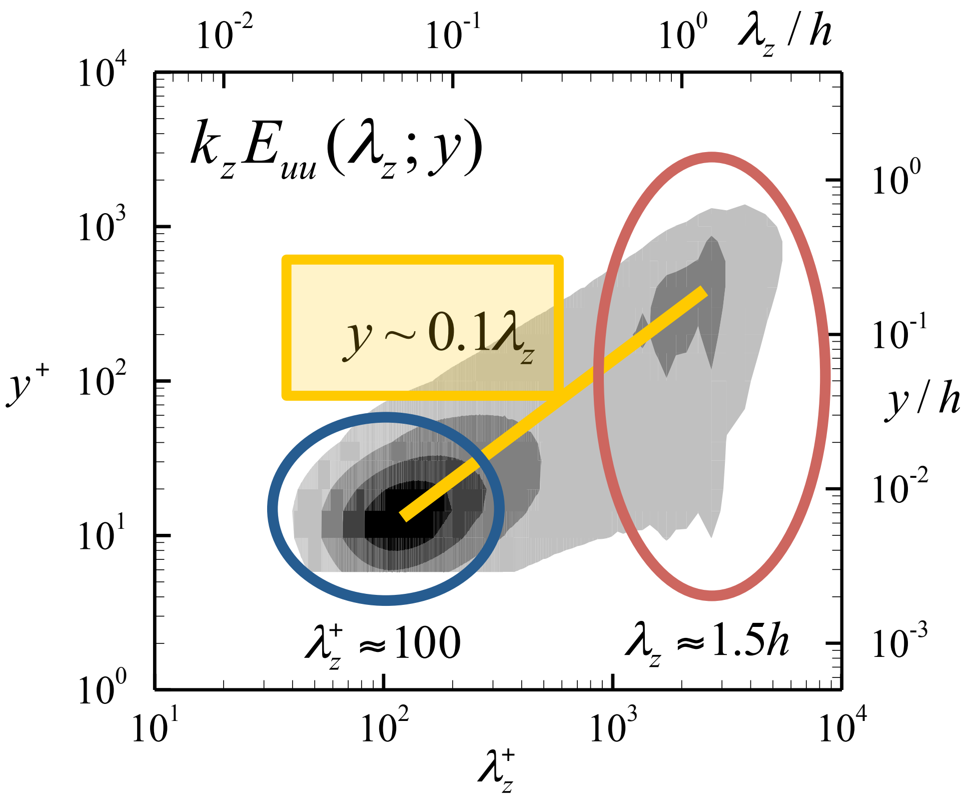
Schoppa & Hussain (JFM, 2002)

Skin friction at high Re turbulence is dominated by log-layer attached eddies

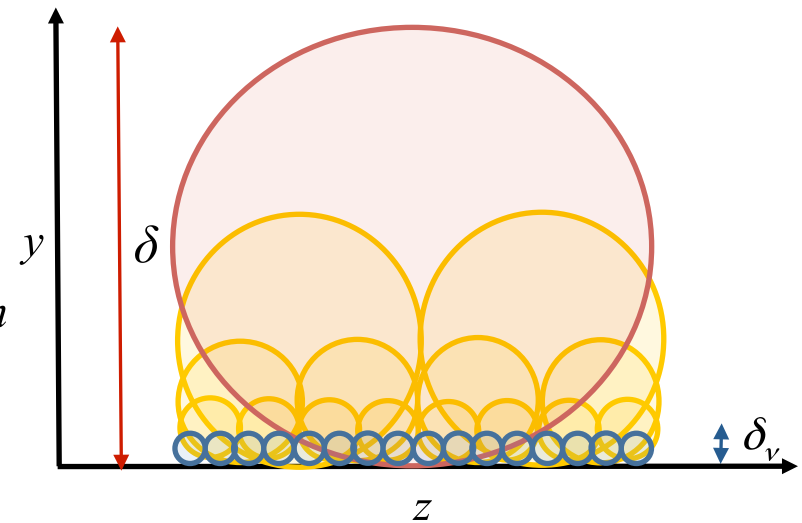
de Giovanetti, Hwang & Choi, 2016, *J. Fluid Mech.* **808** p51

Matteo de Giovanetti (Imperial)
Haecheon Choi (SNU, South Korea)

Turbulent skin-friction generation at high Re



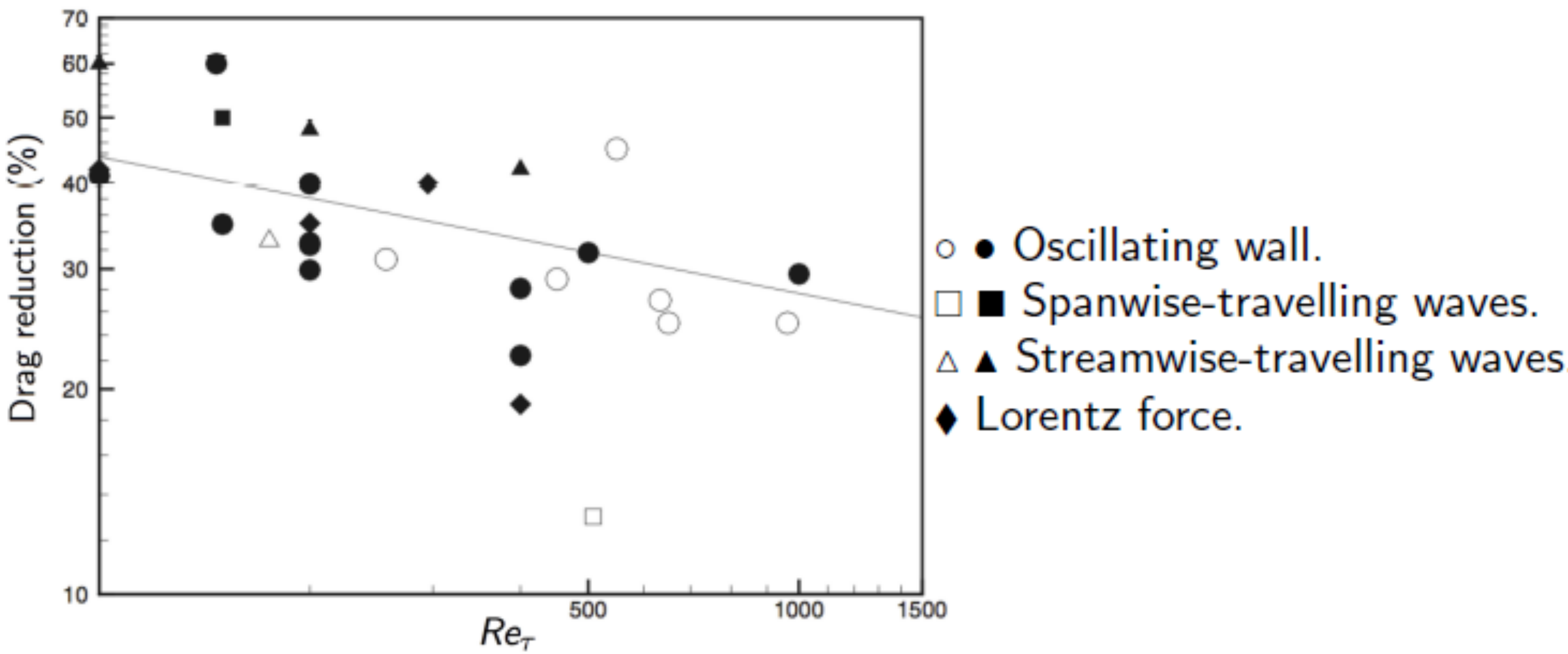
DNS (channel) at $\mathrm{Re}_\tau = 2003$



$\lambda_z \sim y$

How do they contribute to turbulent skin friction at high Re?

A fundamental issue of skin-friction control



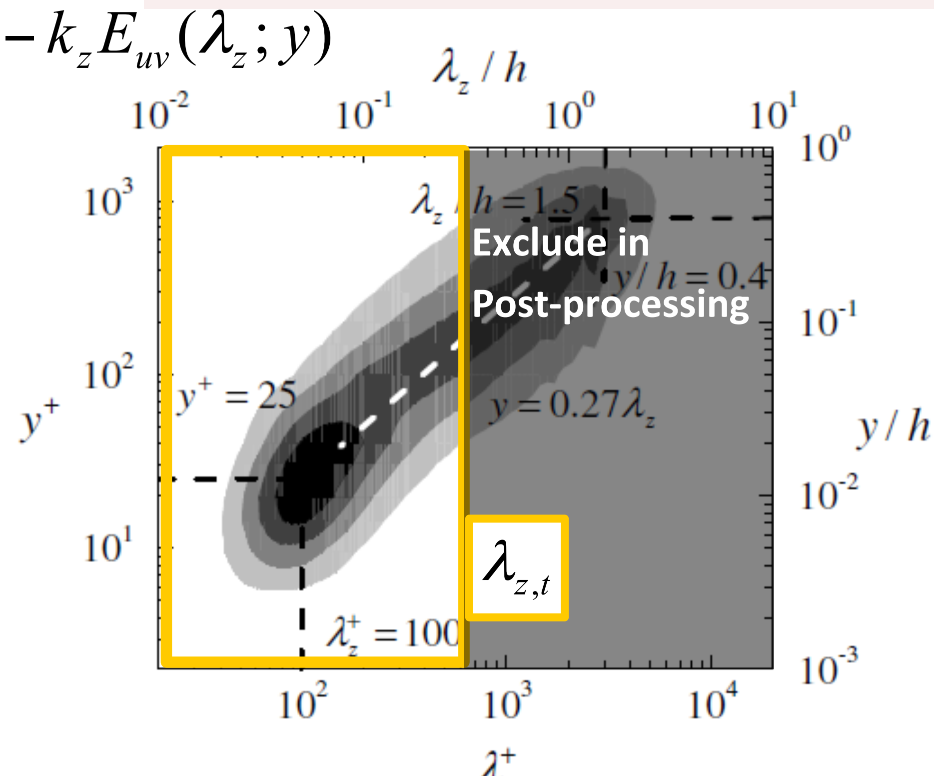
Performance of flow control for near-wall structures decays with Reynolds number.

Assessment I – FIK identity based approach

FIK (Fukagata-Iwamoto-Kasagi) identity

$$C_f(\lambda_{z,t}) = \frac{12}{Re_m} + 12\int_0^h 2\left(1-\frac{y}{h}\right)\left(-\frac{\overline{u'v'}(y;\lambda_{z,t})}{4U_m^2}\right)dy$$

$$\overline{u'v'}(y;\lambda_{z,t}) = \int_{2\pi/\lambda_{z,t}}^{\infty} E_{uv}(k_z)\,dk_z$$



Hoyas & Jimenez (2006,PoF)
DNS (channel) at $Re_\tau = 2003$

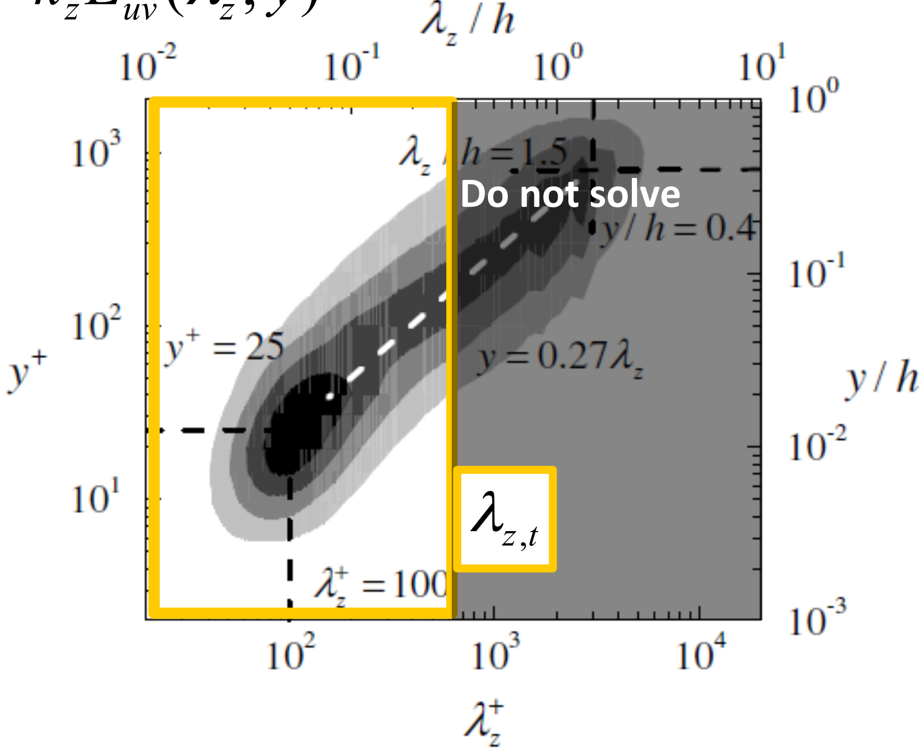
Fukagata et al. (2002, Phys. Fluids)

Assessment II – confined spanwise domain

Restrict the computational box + removal of 2D uniform motion

$$L_z = \lambda_{z,t}$$

$-k_z E_{uv}(\lambda_z; y)$



Hoyas & Jimenez (2006,PoF)
DNS (channel) at $\mathrm{Re}_\tau = 2003$

Hwang, 2013, *J. Fluid Mech*. **727** p264

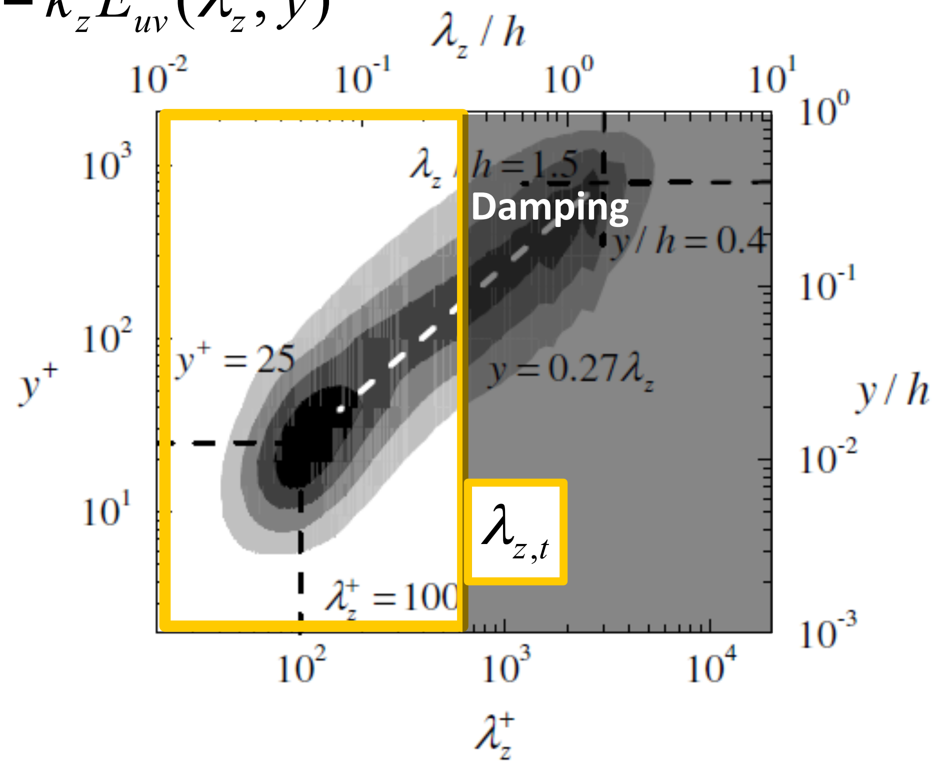
Assessment III – artificial damping of large structures

Damp out the motions at $\lambda_z > \lambda_{z,t}$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\nabla p + \frac{1}{\mathrm{Re}}\nabla^2\mathbf{u} + \mathbf{f} \quad \text{where} \quad \hat{\mathbf{f}} = \mu(\lambda_z)\hat{\mathbf{u}}$$

$$\mu(\lambda_z) = \begin{cases} 0 & \text{for } \lambda_z \le \lambda_{z,t} \\ \mu_0 & \text{for } \lambda_z > \lambda_{z,t} \end{cases}$$

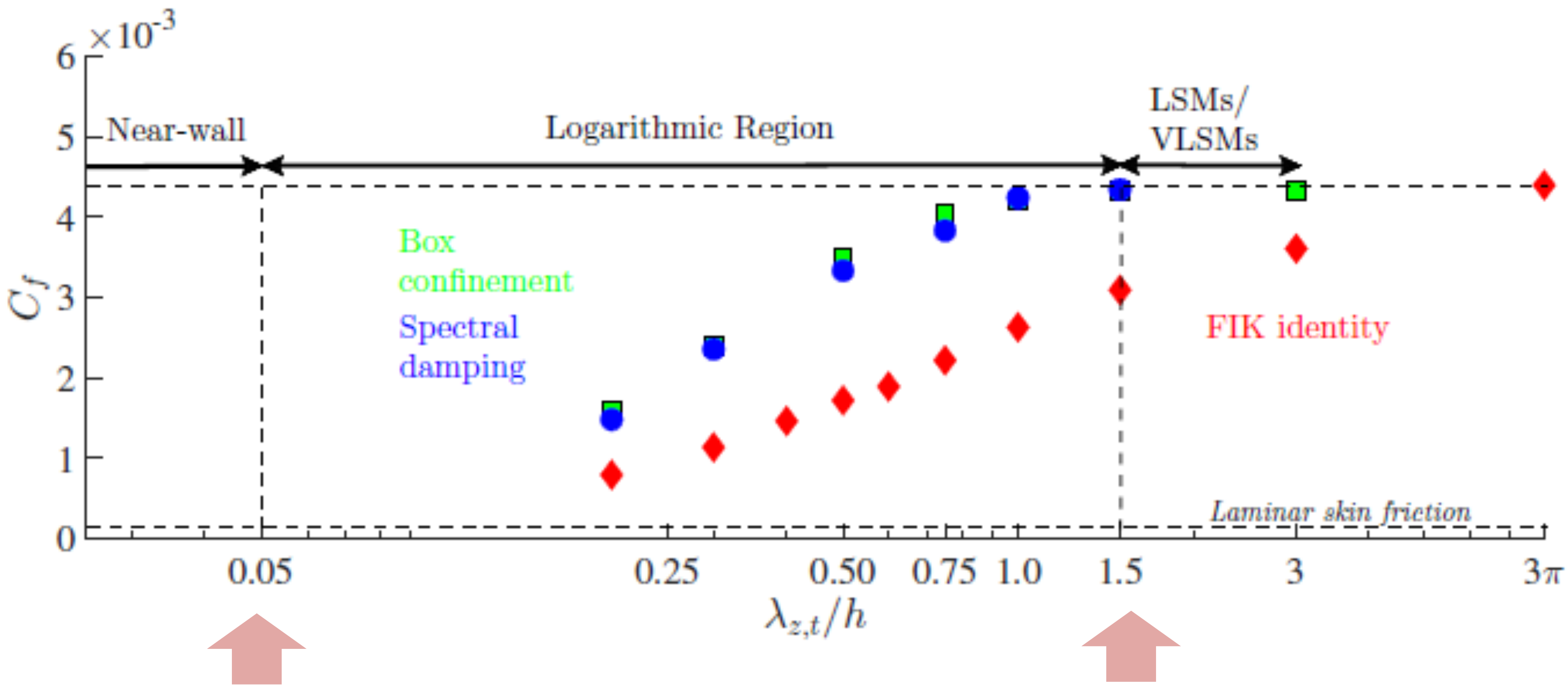
$-k_z E_{uv}(\lambda_z; y)$



Hoyas & Jimenez (2006,PoF)
DNS (channel) at $\mathrm{Re}_\tau = 2003$

Log-layer attached eddies dominate skin-friction generation

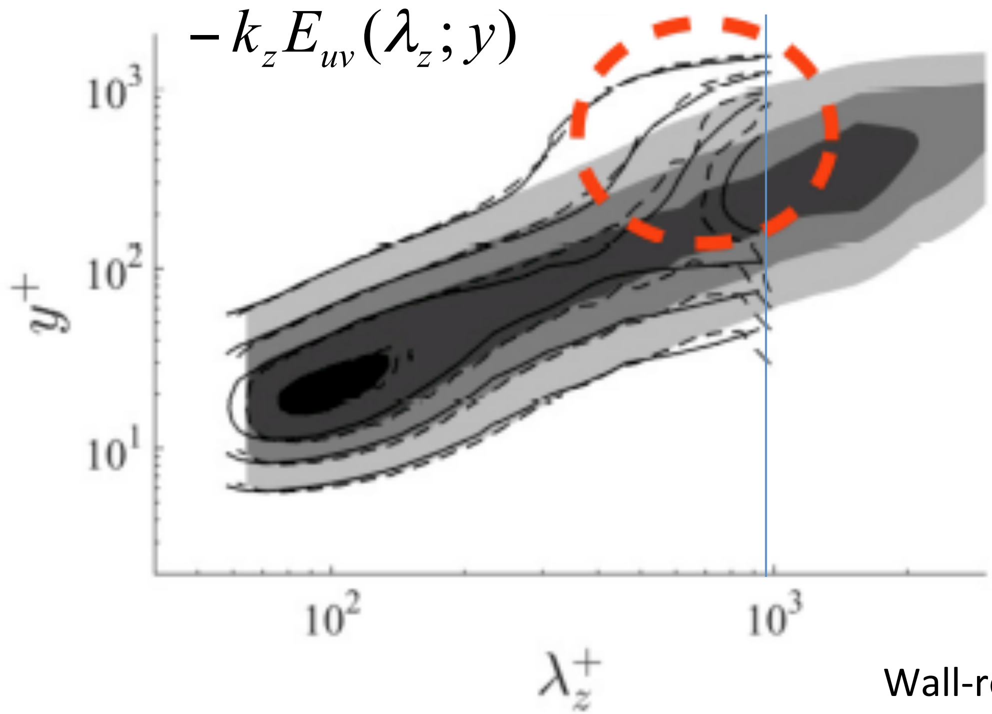
Three different assessments at $\mathrm{Re}_\tau \approx 2000$



$\lambda_z^+ = 100$

$\lambda_z = 1.5h$

Scale interaction is not trivial and important



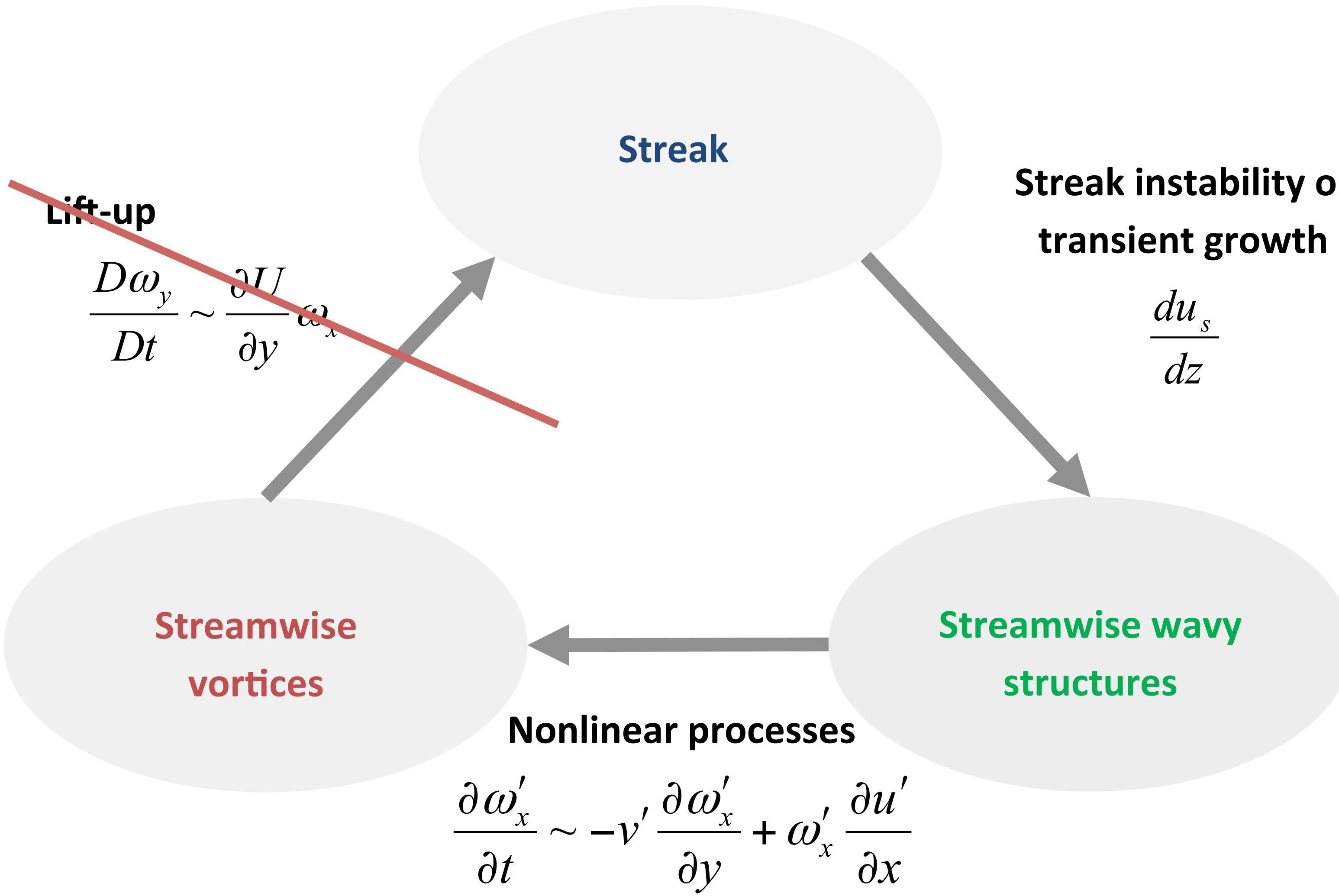
Shaded (full simulation)
Solid (box confinement)
Dashed (spectral damping)

Wall-resolved LES at $\mathrm{Re}_\tau = 2003$

Generation of new Reynolds stress in the absence of large motions

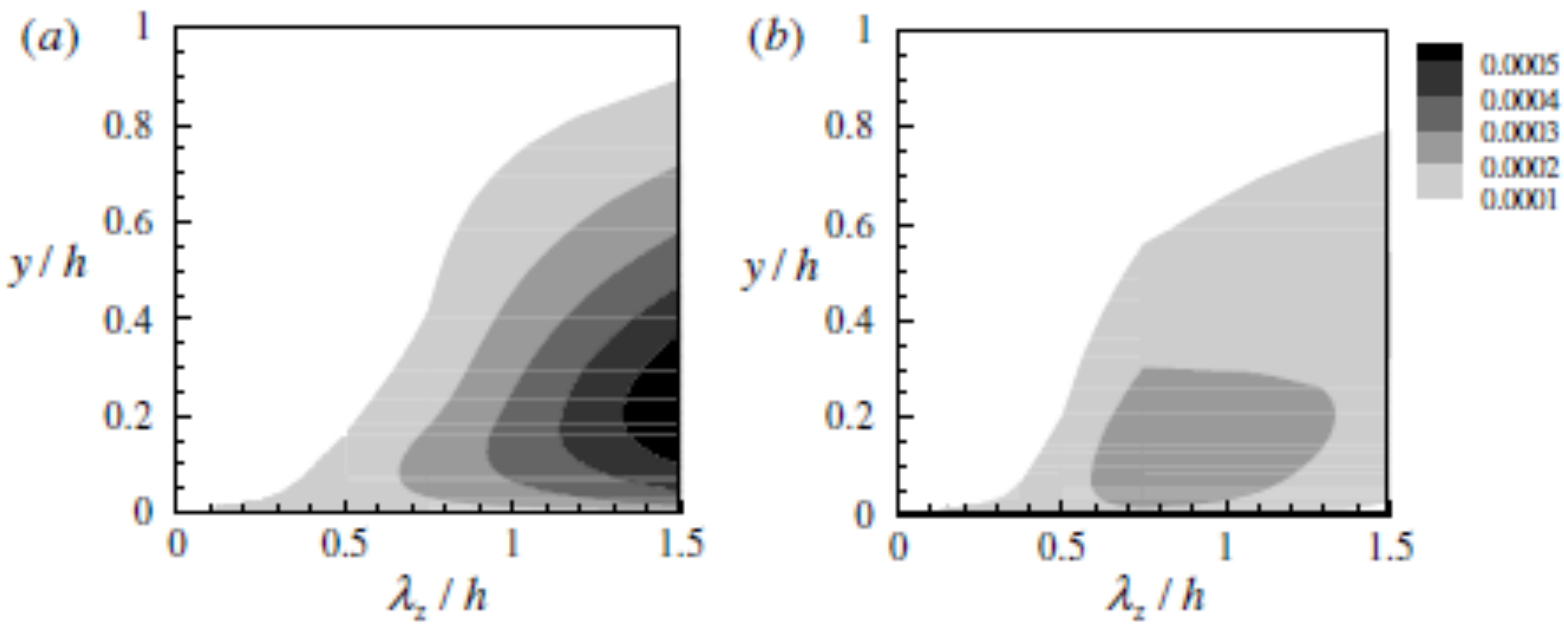
Modified actual size of inner scale

Lift-up effect is a skin-friction generation process



Hwang & Bengana, 2016, *J. Fluid Mech.* **795** p708

Suppression of lift-up effect leads to drag reduction



Full LES

LES without lift-up at $\lambda_z = 1.5h$

8% of drag reduction !!!

Hwang & Bengana, 2016, *J. Fluid Mech.* **795** p708

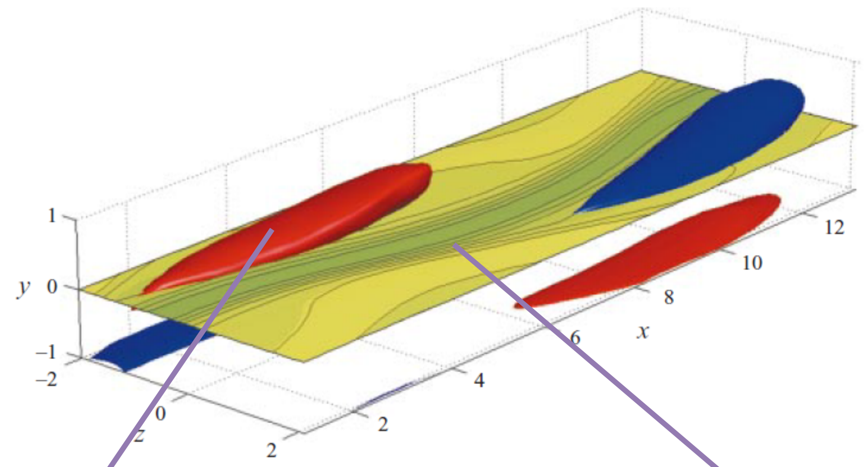
Invariant solutions of attached eddies

Rawat, Cossu, Hwang & Rincoln, 2015, *J. Fluid Mech.* **782** p515

Hwang, Willis & Cossu, 2016, *J. Fluid Mech.* **802** R1

Oliver Yang (Warwick)
Ashley Willis (Sheffield)
Carlo Cossu (IMFT)

SSP reflects the existence of invariant solutions



Streamwise Vortices

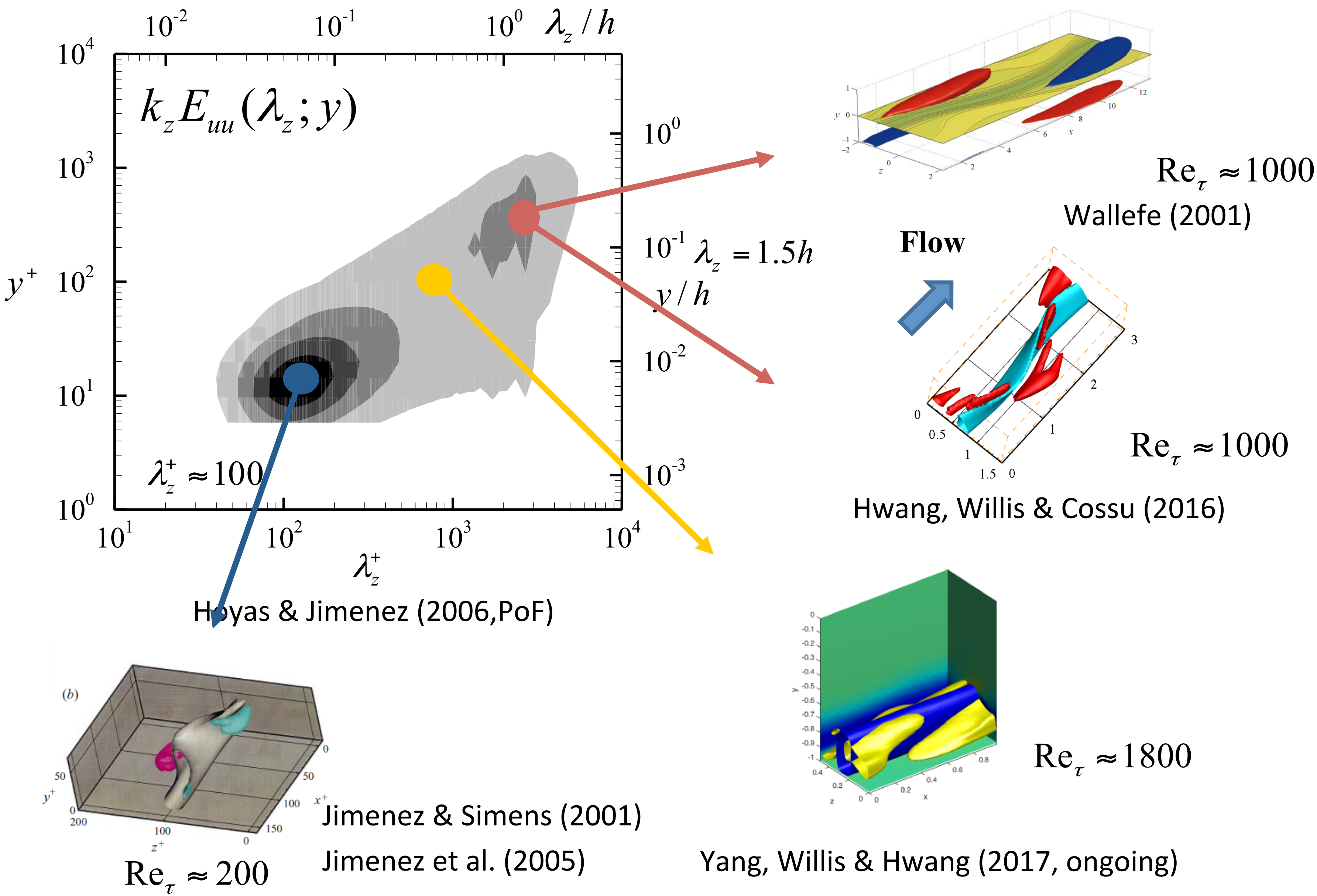
Lift up →

← **Instability & Nonlinear Feedback**

Streaks

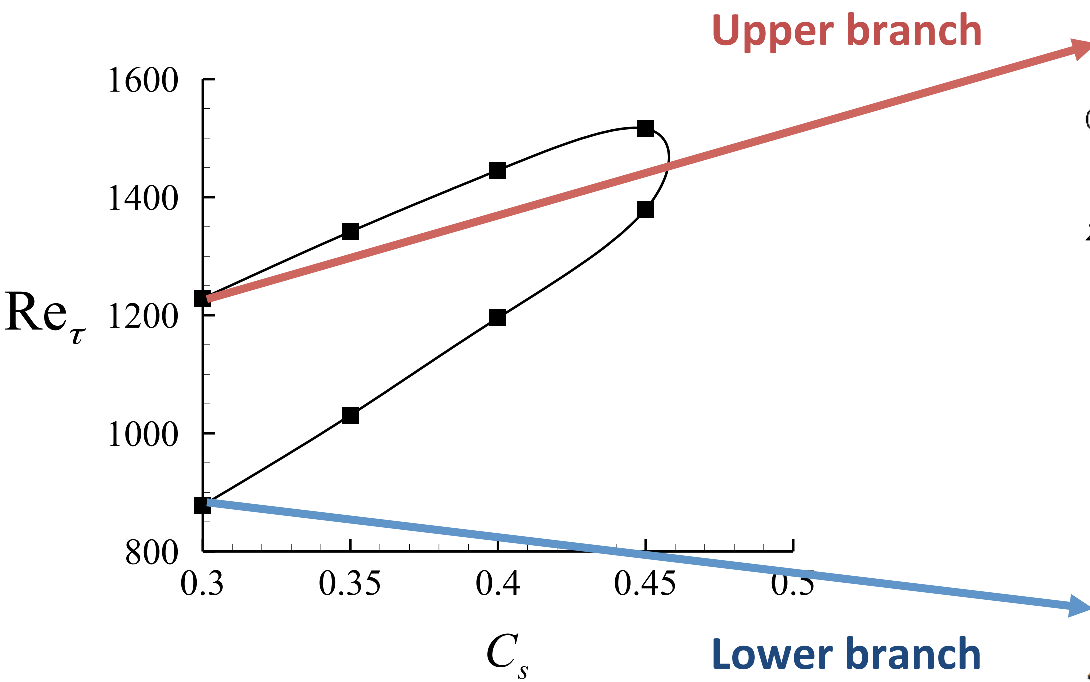
Nagata (1990), Waleffe (1998,2000,2002), Eckhardt, Kerswell, Hall, and many others.

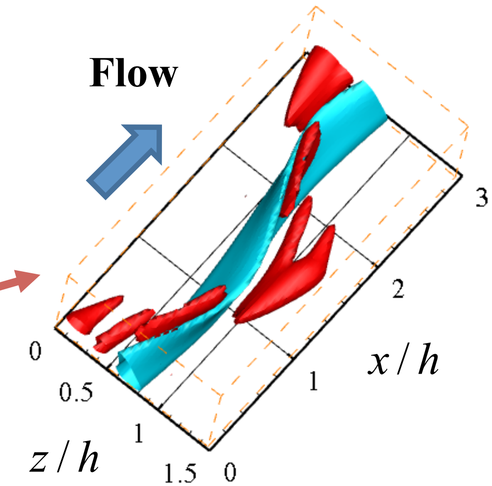
The terribly difficult puzzle to complete

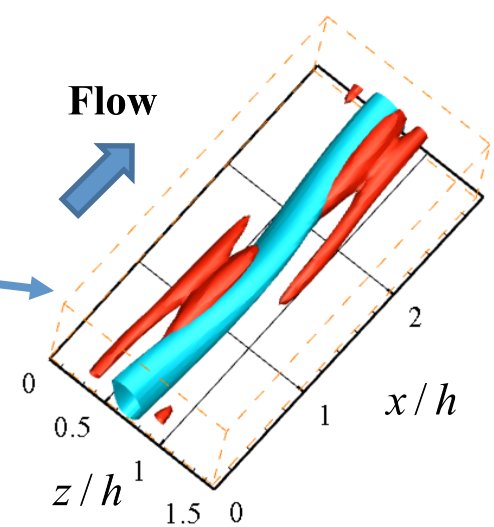


Traveling wave solutions of large-scale structures at $\mathrm{Re}_\tau \approx 1000$

Bifurcation with C_s $(\lambda_z = 1.5h)$



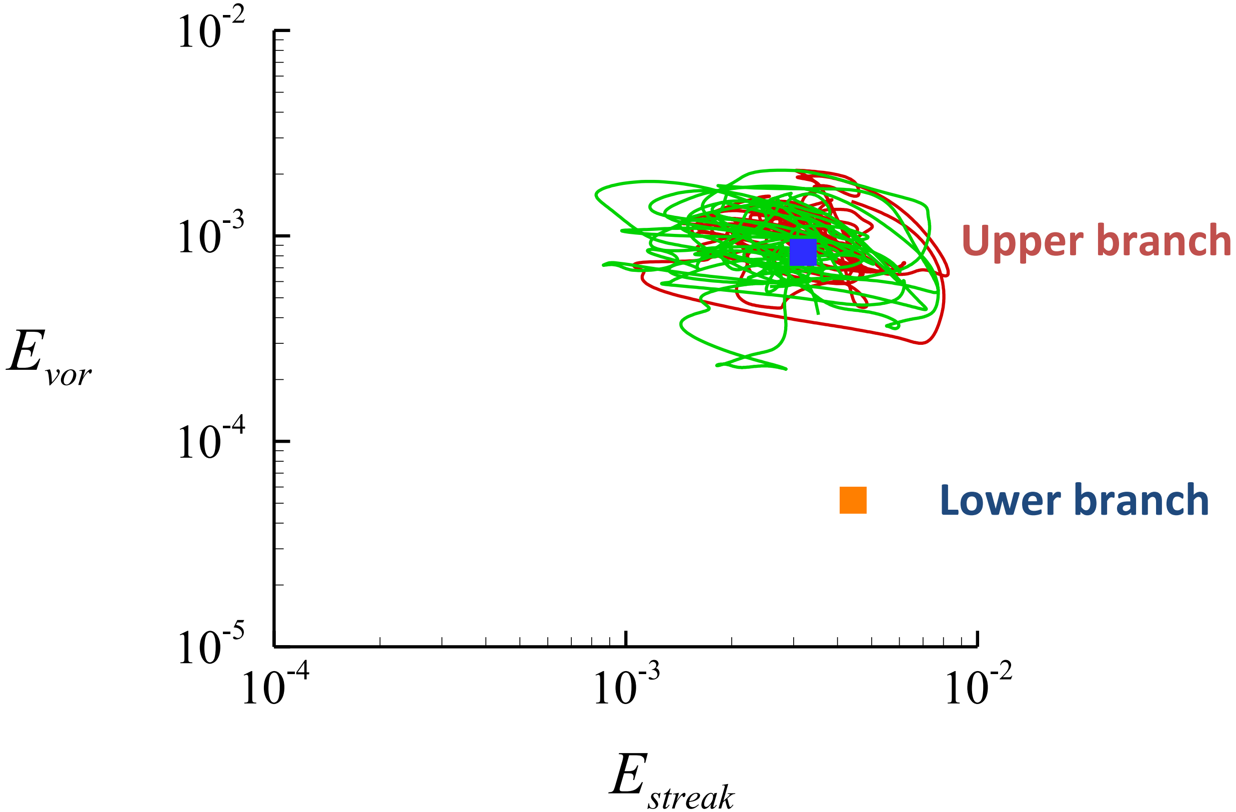




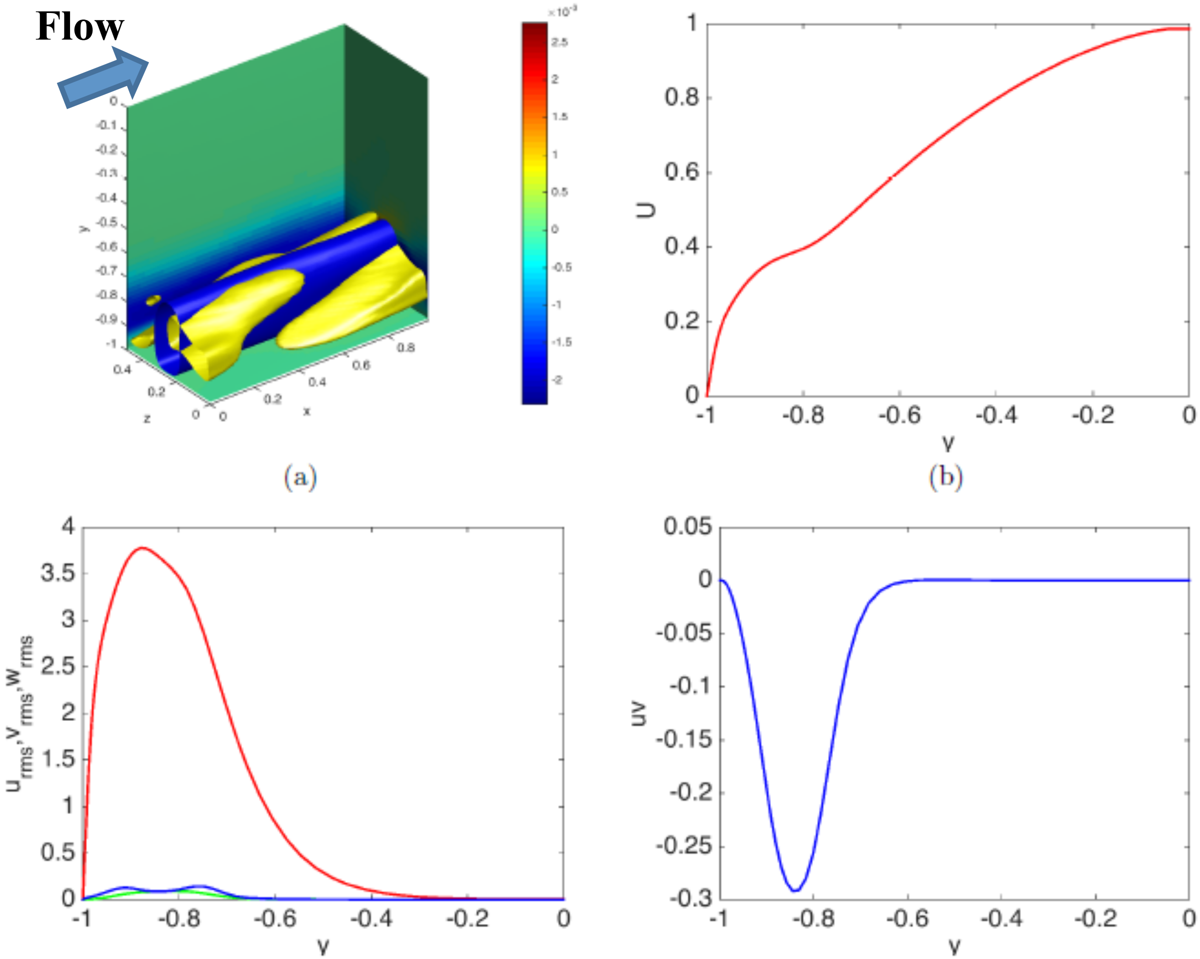
Hwang, Willis & Cossu, 2016, *J. Fluid Mech.* **802** R1

The UB solution conceptually represents large-scale structures

$$E_{streak} = \frac{1}{2V}\int_V (u')^2 \, dV \qquad E_{vor} = \frac{1}{2V}\int_V (v')^2 + (w')^2 dV$$



A glimpse of log-layer traveling wave solution at $\mathrm{Re}_\tau \approx 1800$



Any connection with the one in Gibson & Brand (2014)?

Yang, Willis & Hwang, 2017, Ongoing work

Conclusions

Townsend's attached eddies do exist and sustain with SSP.

Pressure fluctuations (both rapid and slow) are generated by nonlinear feeding processes of vortices.

Skin friction at high Re is dominated by log-layer eddies and the lift-up effect is an important mechanism of momentum transfer to the wall.

There exists traveling wave form of invariant solutions for att ached eddies, but their computation is very challenging.