Coherent structures in boundary layers in the quasilinear approximation

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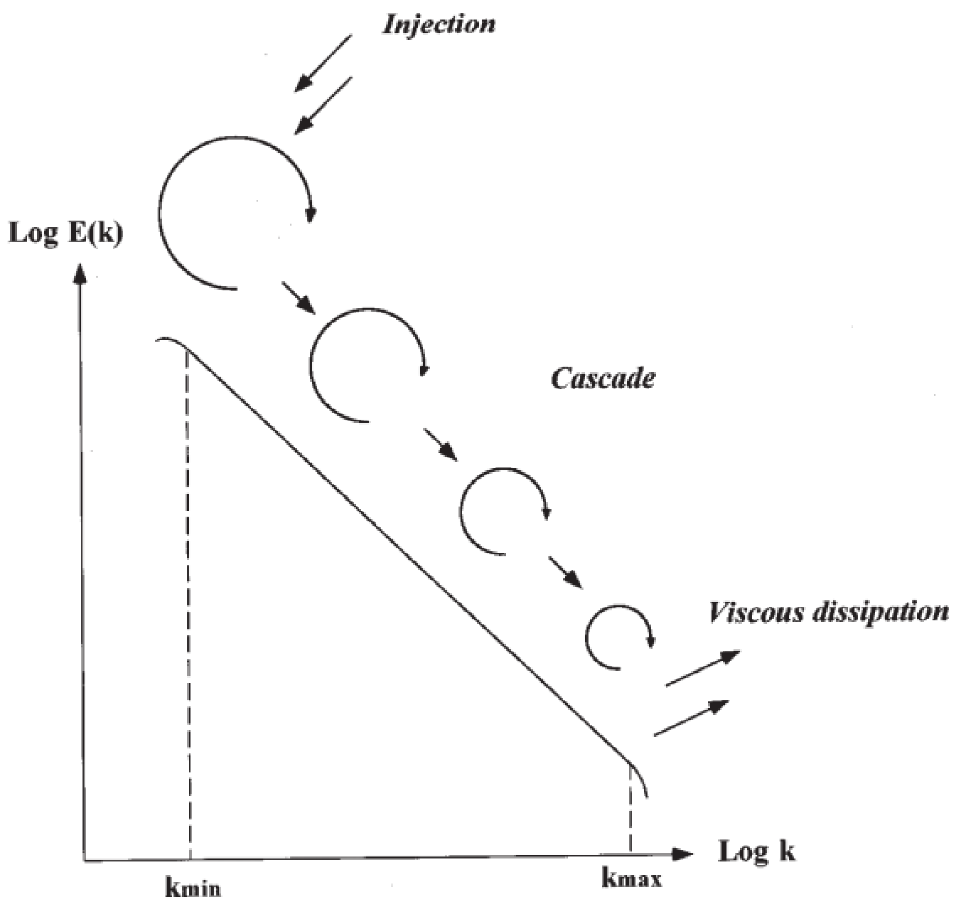
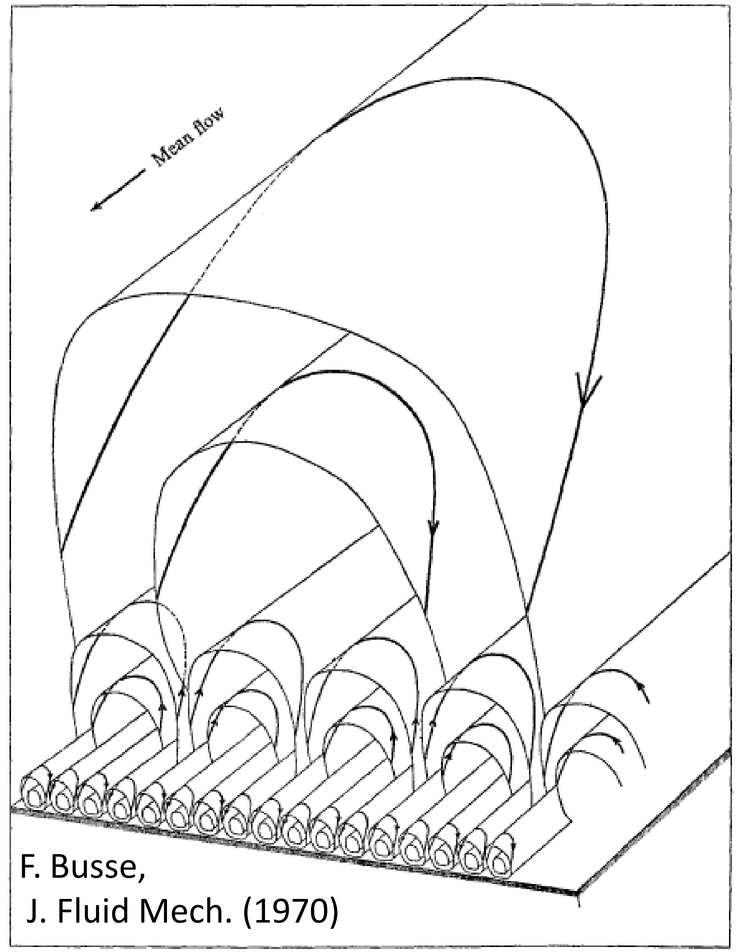
Philipps Universität Marburg

TU Delft

Motivation

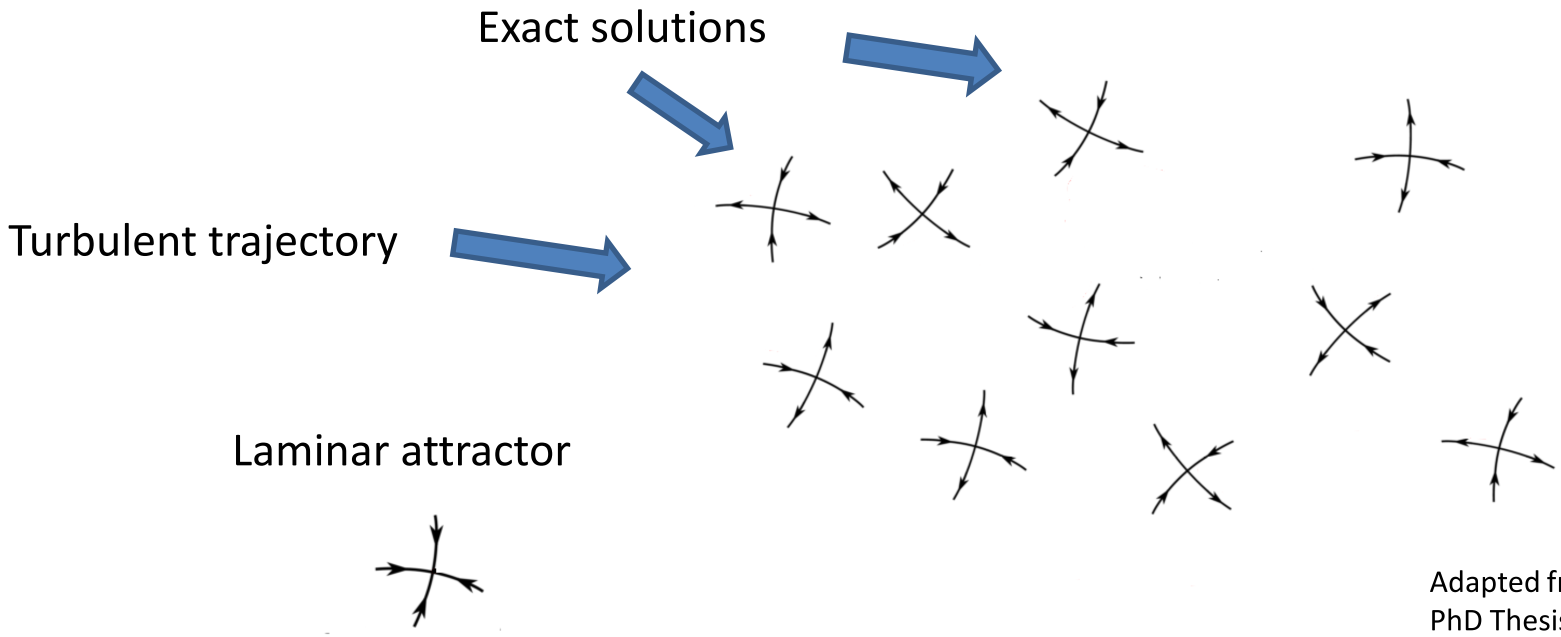


Seuront et al.,
J Plankton Res (1999)



F. Busse,
J. Fluid Mech. (1970)

The dynamical systems view on turbulence



Adapted from T. Kreilos 2014,
PhD Thesis, Uni Marburg

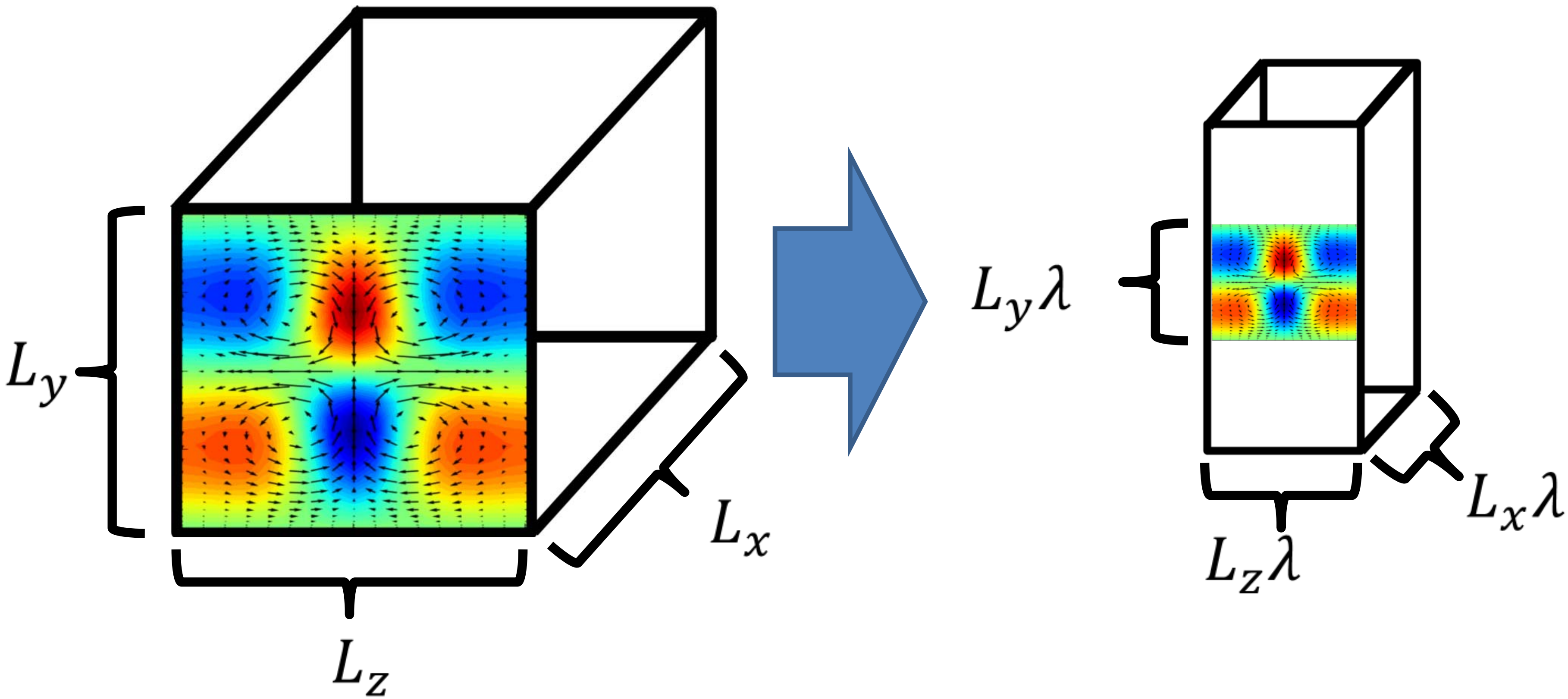
Exact solutions with smaller scales or multiple scales are necessary

How to get them?

Wall-normal localized ECS

System: **Plane Couette flow**

DNS using *Channelflow* (www.channelflow.org)



Spatially extended in x,y and z

Scaling solutions for linear shear

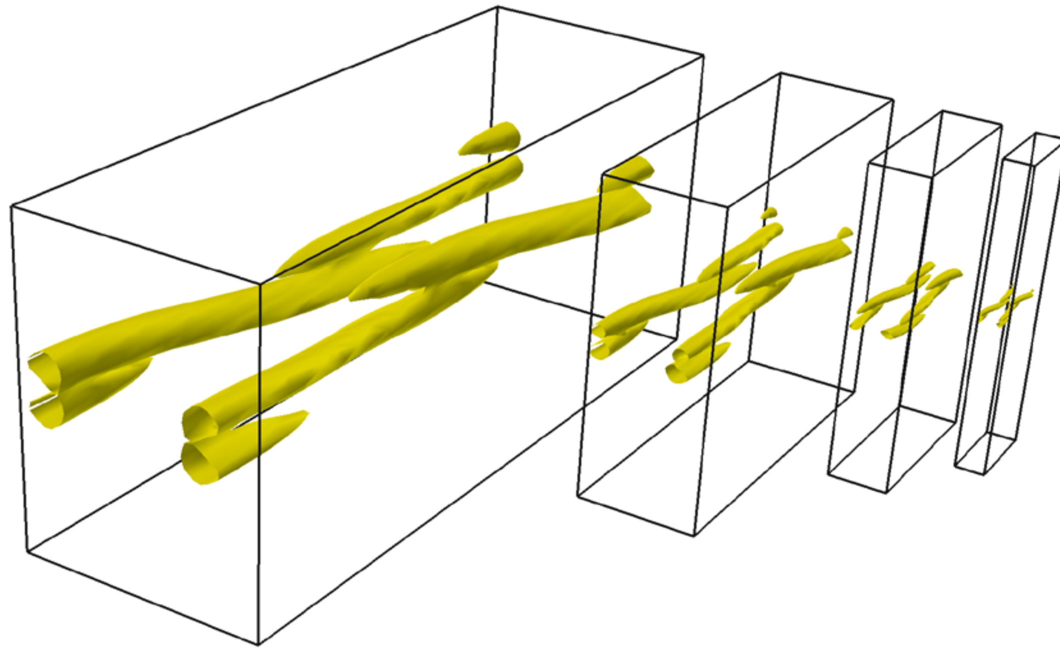
$$\vec{u}_\lambda = \lambda \vec{u}(\vec{x}/\lambda)$$

A solution u on scale 1 at Reynolds number Re

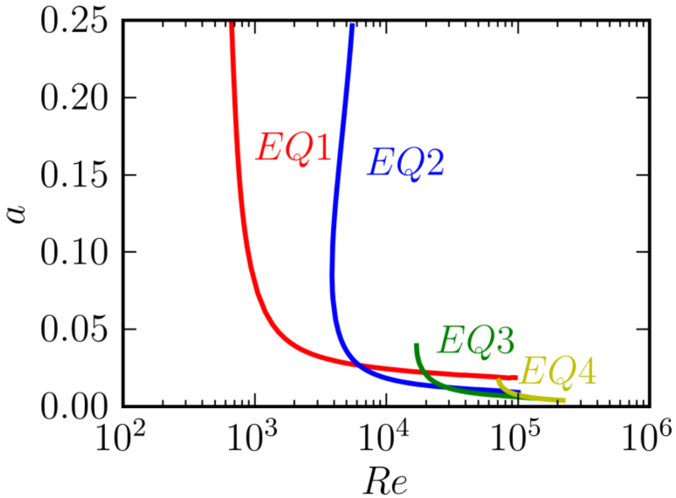
is related to

a solution u_λ on scale λ at Reynolds number Re/λ^2

Exact without walls!



Similar to states in Deguchi JFM 2015

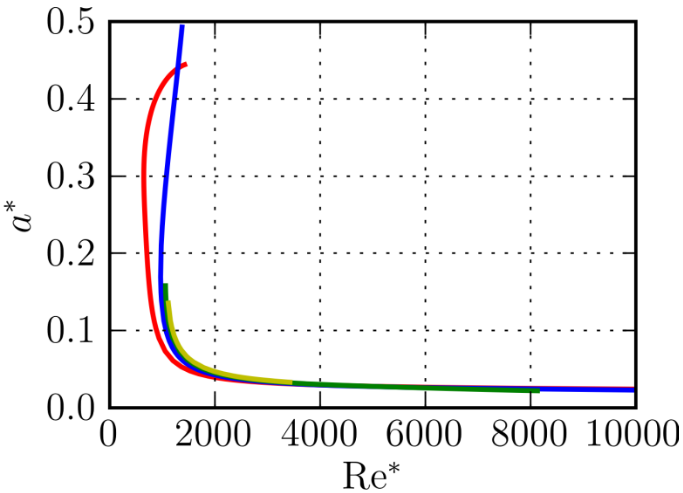


$$a(\vec{u}) = \sqrt{\frac{1}{L_x L_y L_z} \iiint \vec{u}^2 dV}$$

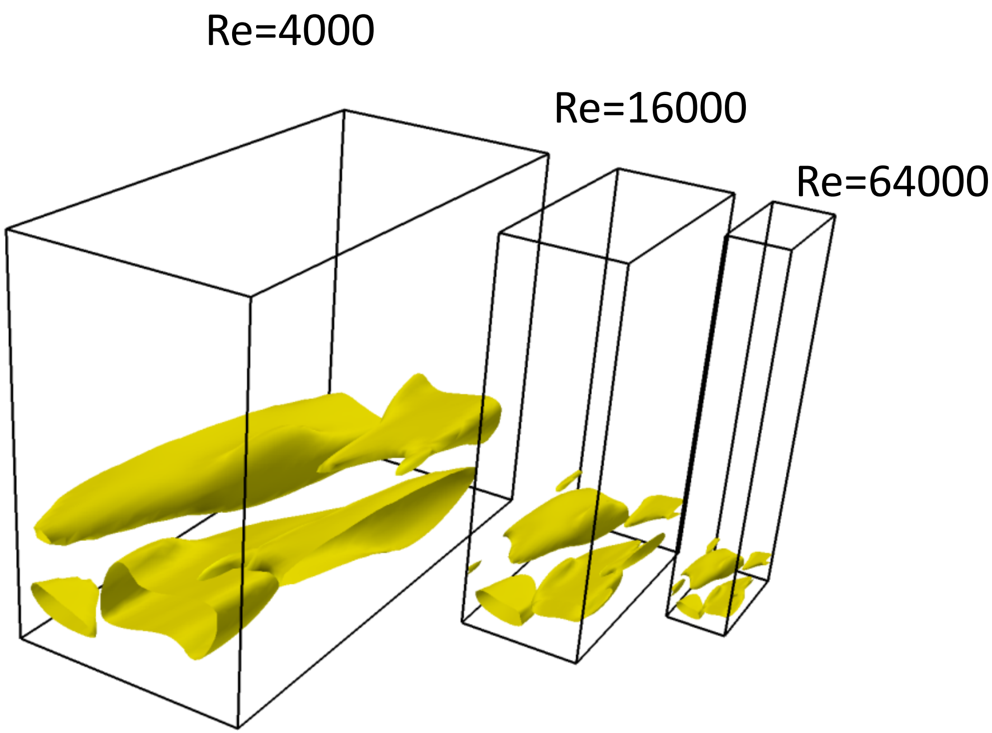
Rescaling

$a^* = a/\lambda$

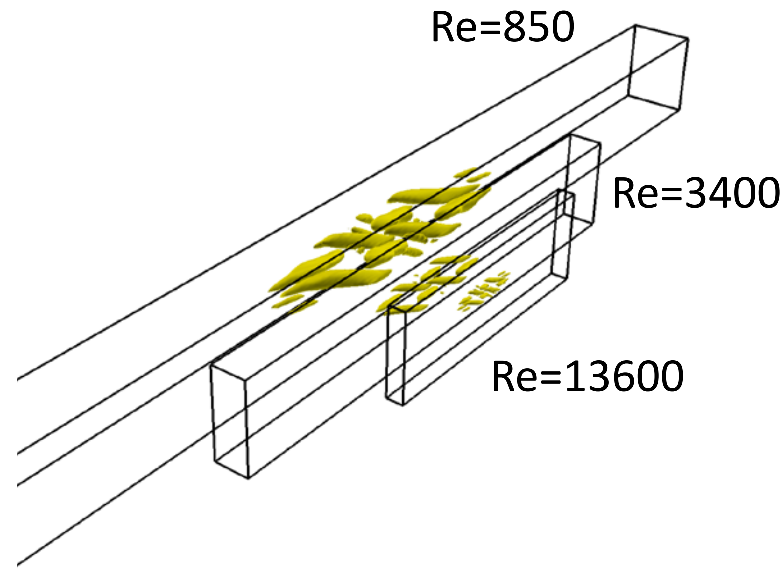
$Re^* = Re\lambda^2$



Wall solutions & Localized states



Iso-surfaces Q=0.001



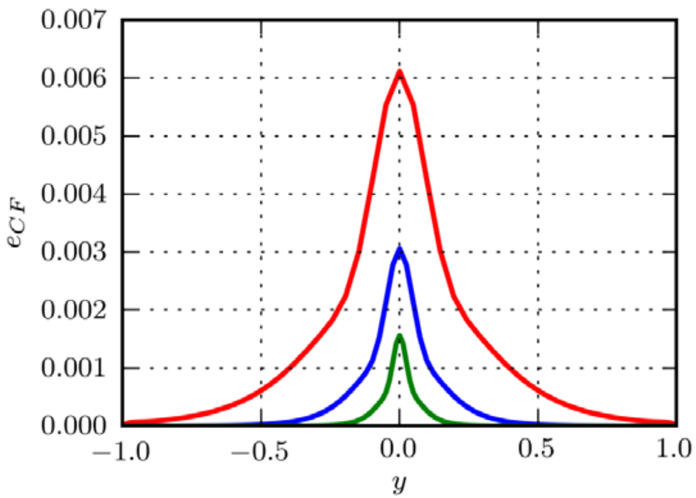
Iso-surfaces Q=0.001

Attached eddy states?

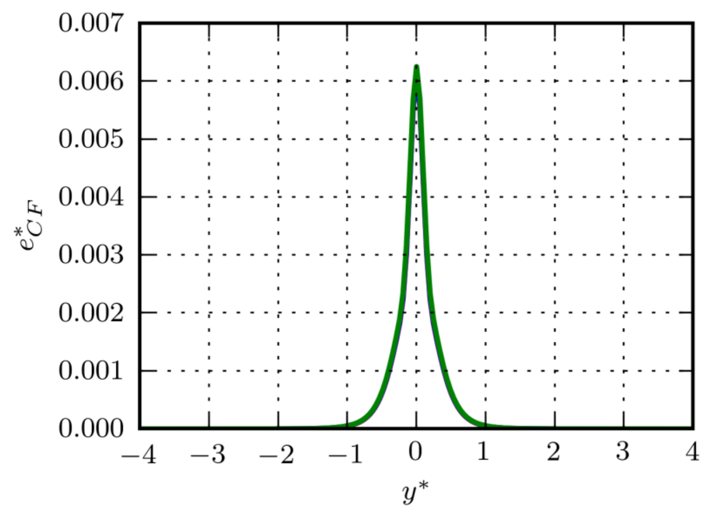
Wall-normal localization

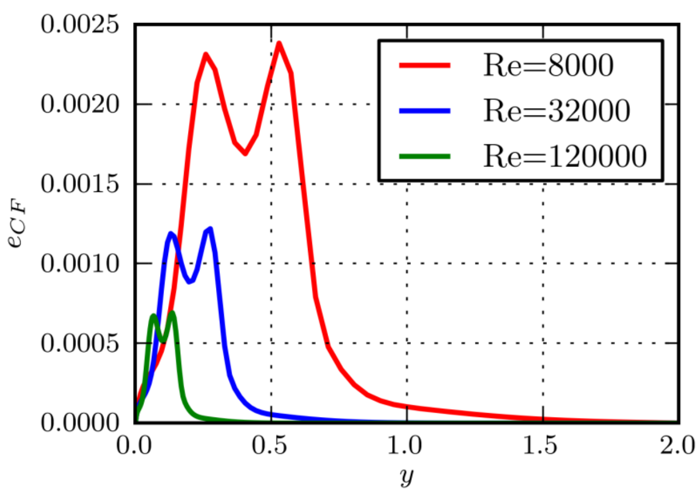
Cross-flow energy density:
(measure for vortices)

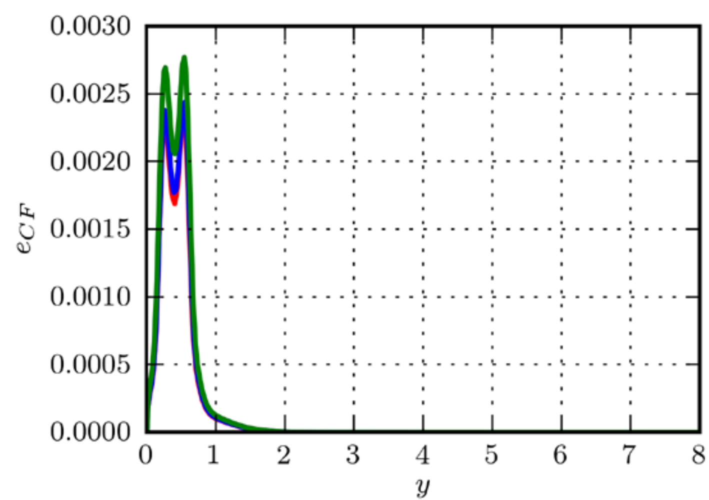
$$e_{CF}(\vec{u}) = \sqrt{\frac{1}{L_x L_z}\iiint v^2 + w^2 dV}$$



Rescaling
$y^* = y/\lambda$
$e^*_{CF} = e_{CF}/\lambda$

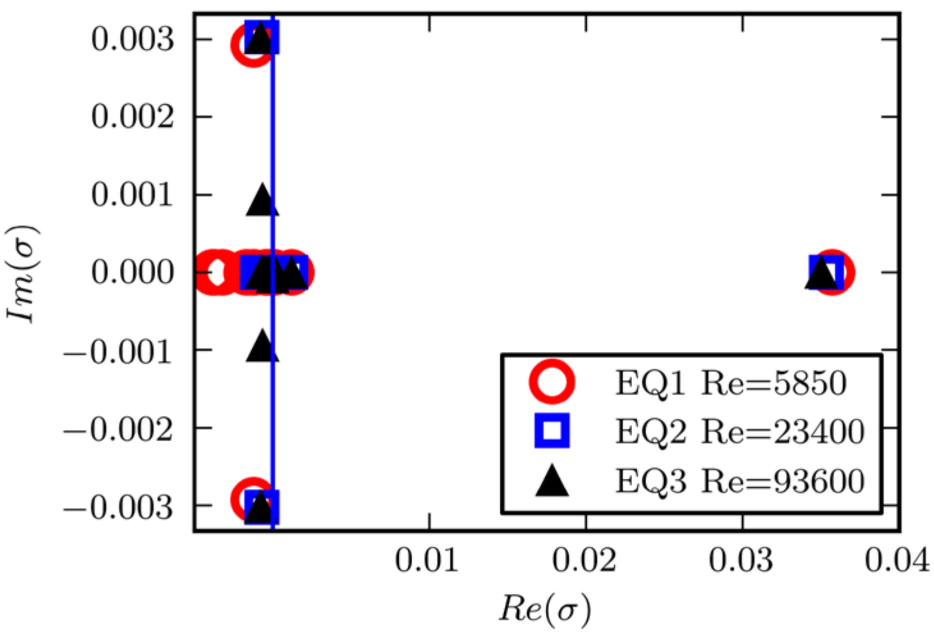




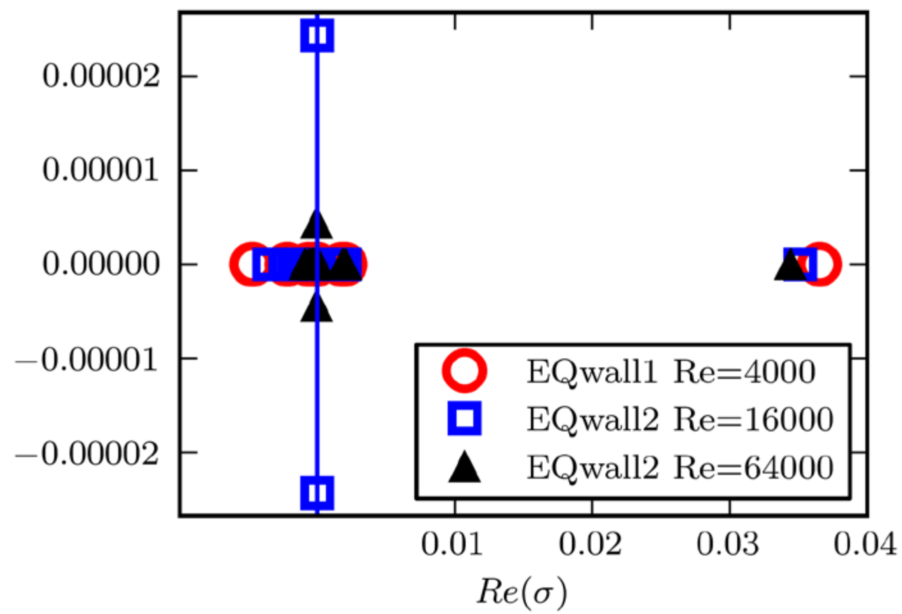


Stability

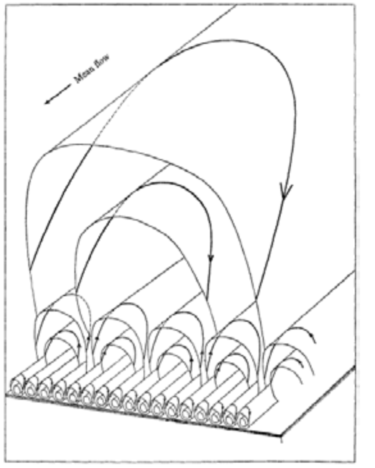
Center-solutions



Wall-solutions



Missing among ECS:



- Equilibrium states
  involving two or more scales

or

- Periodic states connecting flow structures
  with different scales

Proposal: study dynamics in quasilinear approximation

Then:
different scales interact with the mean flow only but not with each other

Rotating plane Couette flow (RPCF)

- Simple flow (2D, 3 components)
- Linear instability
  - Well understood transition
  - Easy access to exact solutions

Equations:

$$\partial_t u + (\vec{u} \cdot \nabla)u = -\partial_x p - Ro\, v + \frac{1}{Re}\Delta u$$

$$\partial_t v + (\vec{u} \cdot \nabla)v = -\partial_y p - Ro\, u + \frac{1}{Re}\Delta v$$

$$\partial_t w + (\vec{u} \cdot \nabla)w = -\partial_z p + \frac{1}{Re}\Delta w$$

$$\nabla \vec{u} = 0$$

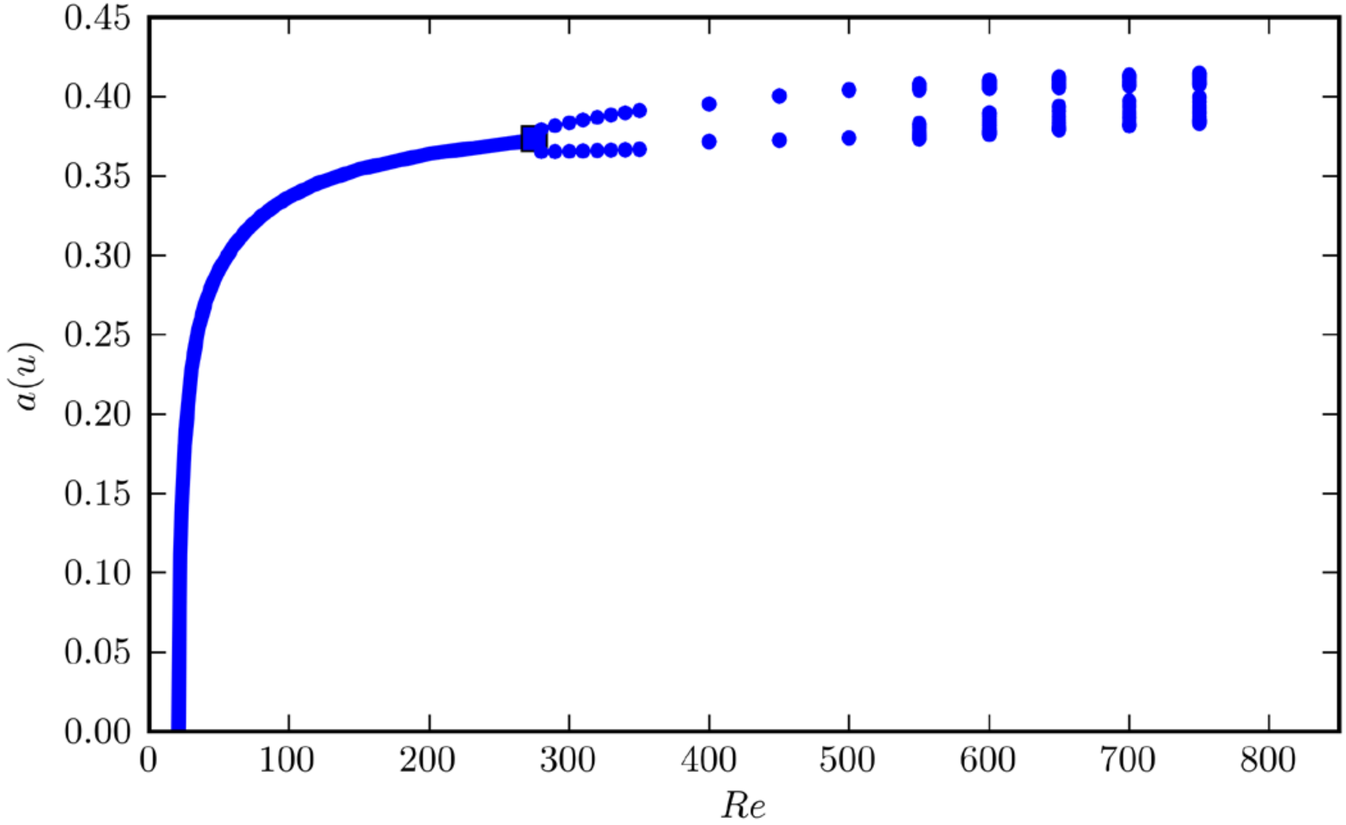
Parameters:

$$Re = \frac{U_0 d}{\nu}$$

$$Ro = \frac{2\Omega d}{U_0}$$

Fixed at 0.5

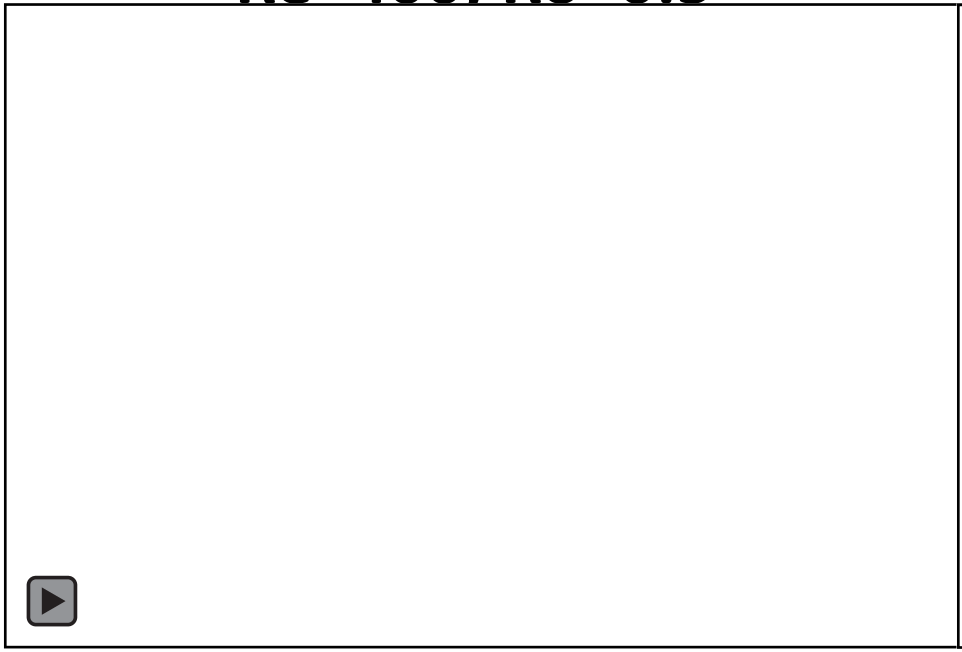
Rotating plane Couette flow (RPCF)



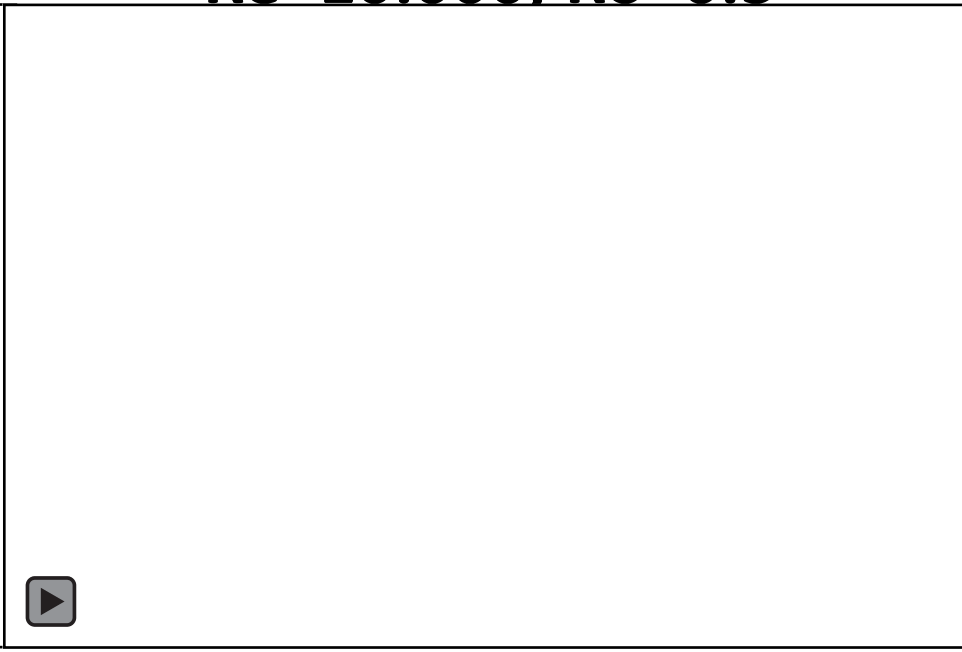
Rotating plane Couette flow (RPCF)

Temporal dynamics:

Re=400, Ro=0.5



Re=10.000, Ro=0.5



The quasilinear approximation

- $\vec{u}_1$ no variation in spanwise direction ($k_z = 0$)
- $\vec{u}_2$ with variation in spanwise direction ($k_z \neq 0$)

$$\Rightarrow \quad \vec{u} = \vec{u}_1 + \vec{u}_2$$

Projectors: $P_1\vec{u} = \vec{u}_1$

$P_2\vec{u} = \vec{u}_2$

Nonlinear Term:

$$(\vec{u} \cdot \nabla)\vec{u} = (\vec{u}_1 \cdot \nabla)\vec{u}_1 + (\vec{u}_1 \cdot \nabla)\vec{u}_2 + (\vec{u}_2 \cdot \nabla)\vec{u}_1 + (\vec{u}_2 \cdot \nabla)\vec{u}_2$$

=0 =0

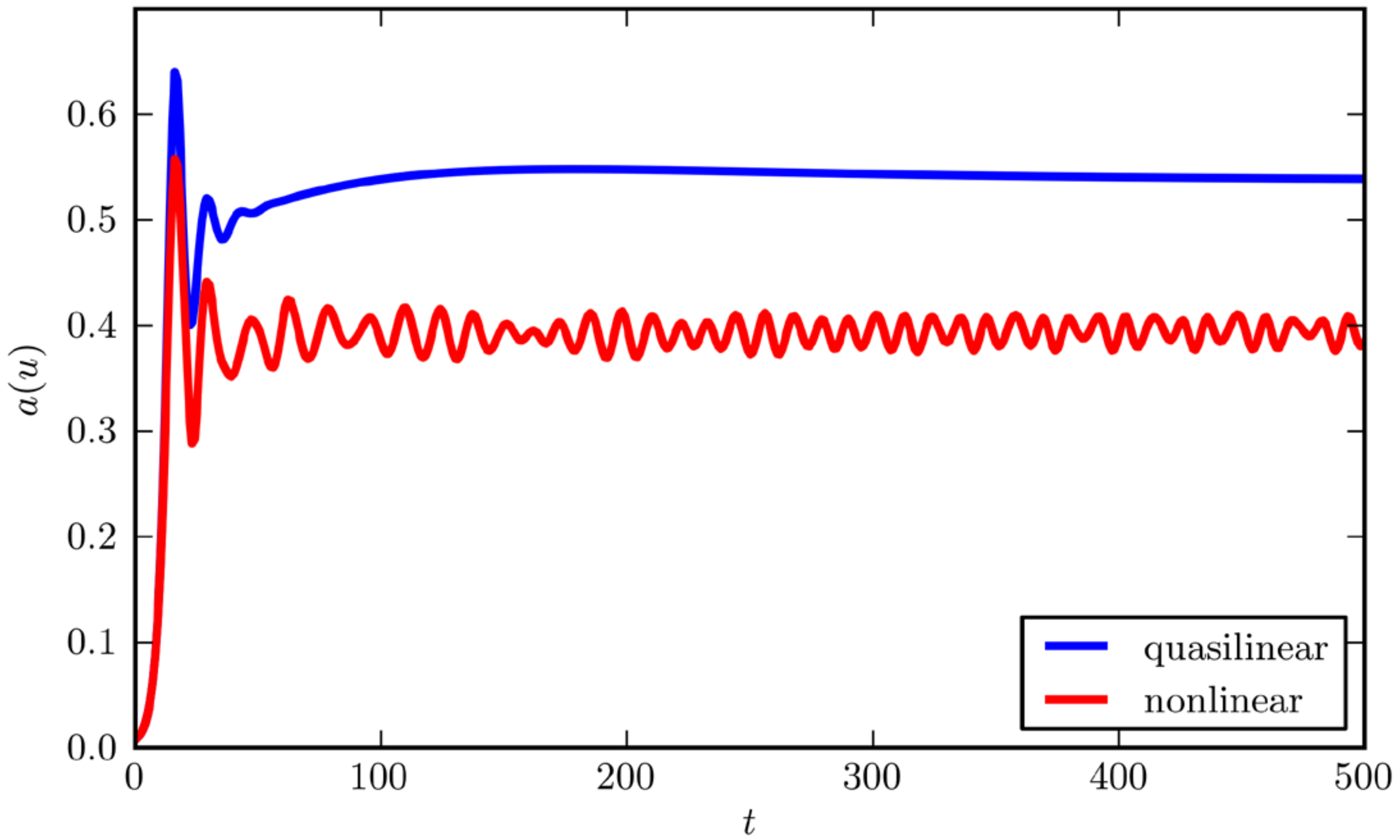
$$(\vec{u} \cdot \nabla)\vec{u} \approx (\vec{u}_2 \cdot \nabla)\vec{u}_1 + P_1(\vec{u}_2 \cdot \nabla)\vec{u}_2$$

QLA → Steve Tobias Talk

Omitted: $P_2(\vec{u}_2 \cdot \nabla)\vec{u}_2$

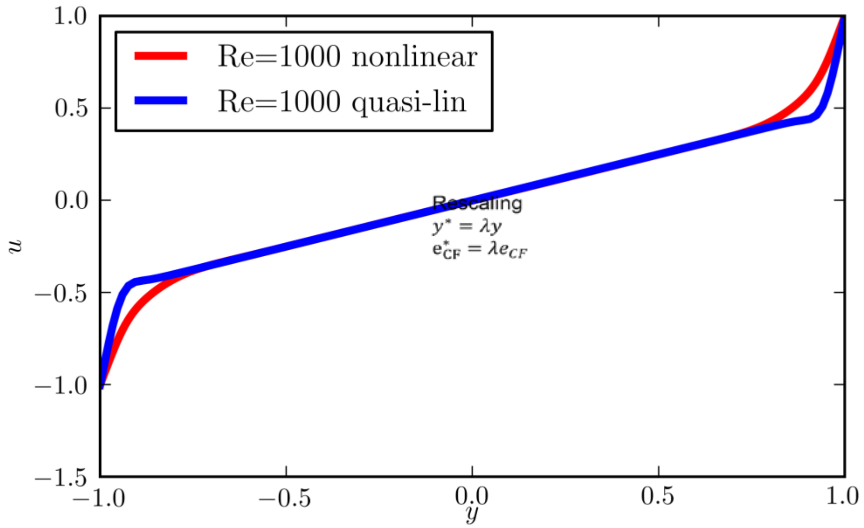
Dynamics in the quasilinear approximation

Re=800

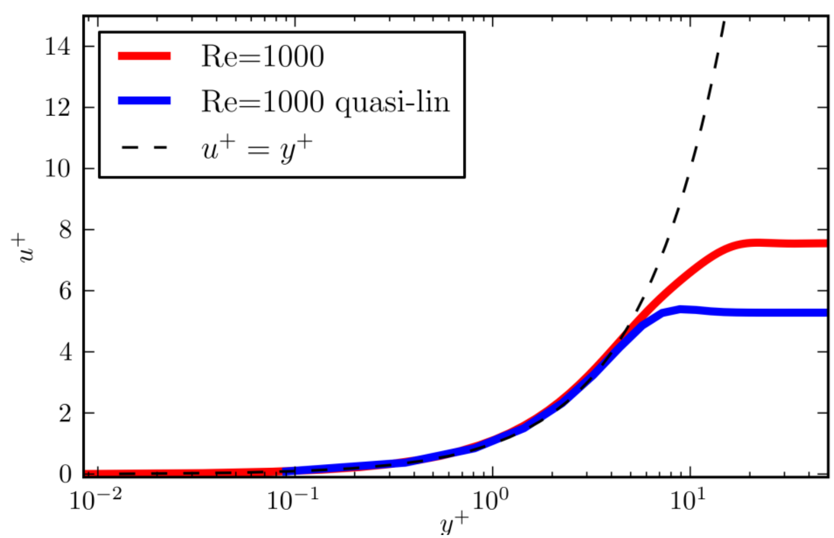


Much simpler dynamics! Attracting state is always stationary.

Comparison of the profiles

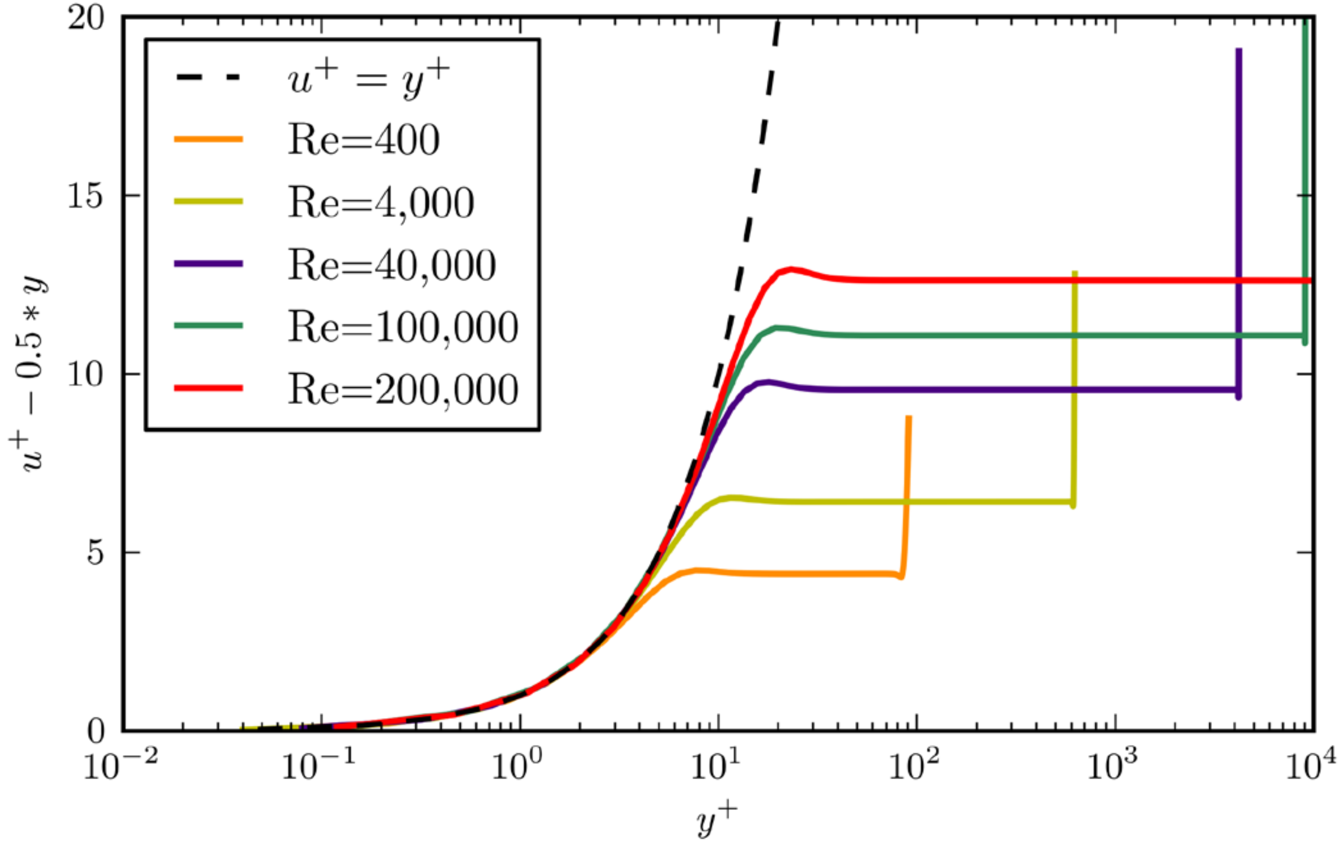


Wall units:



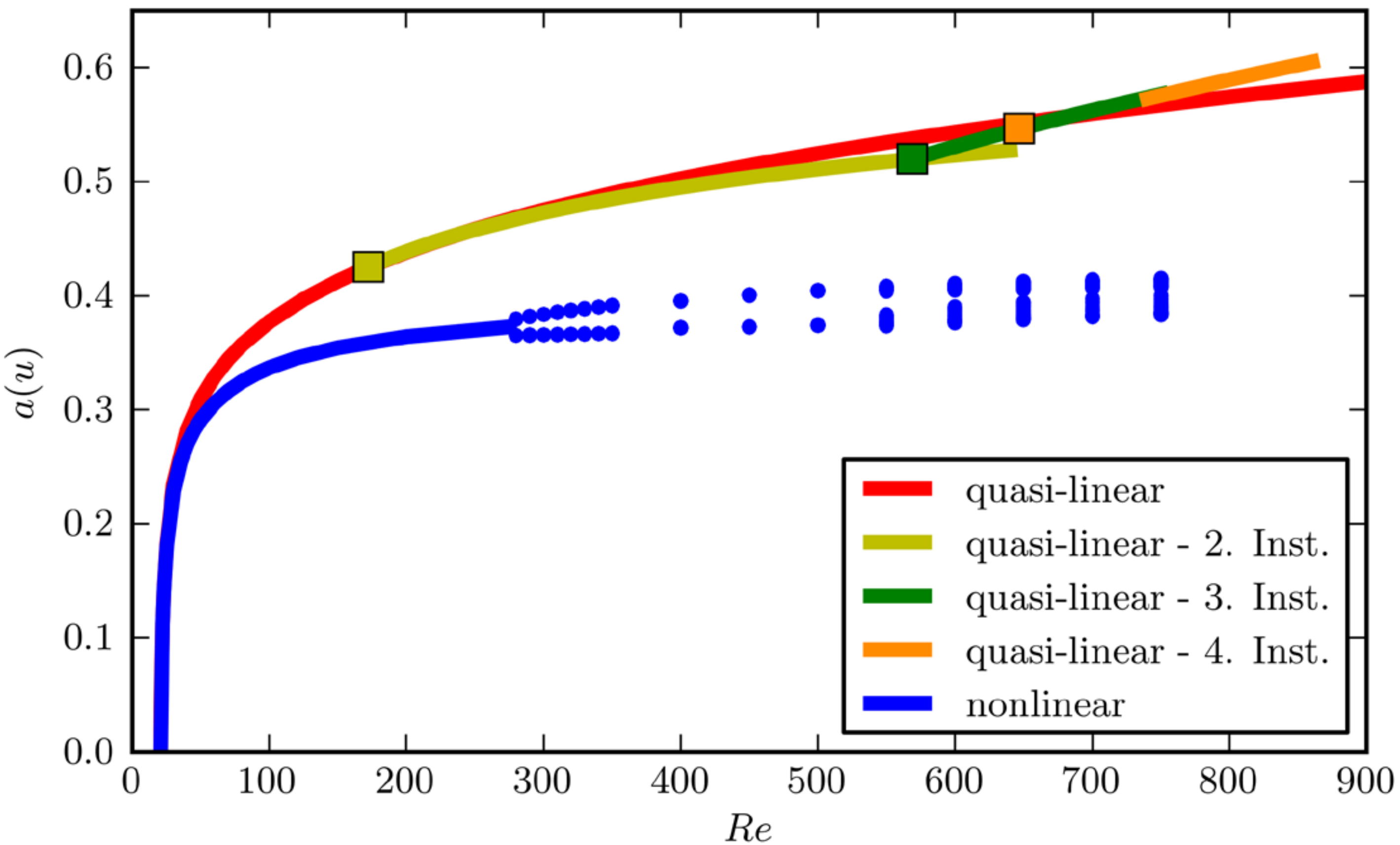
Profiles are qualitatively similar.

Velocity profiles for RPCF



Mechanism for the creation of the profiles?

Bifurcation diagram

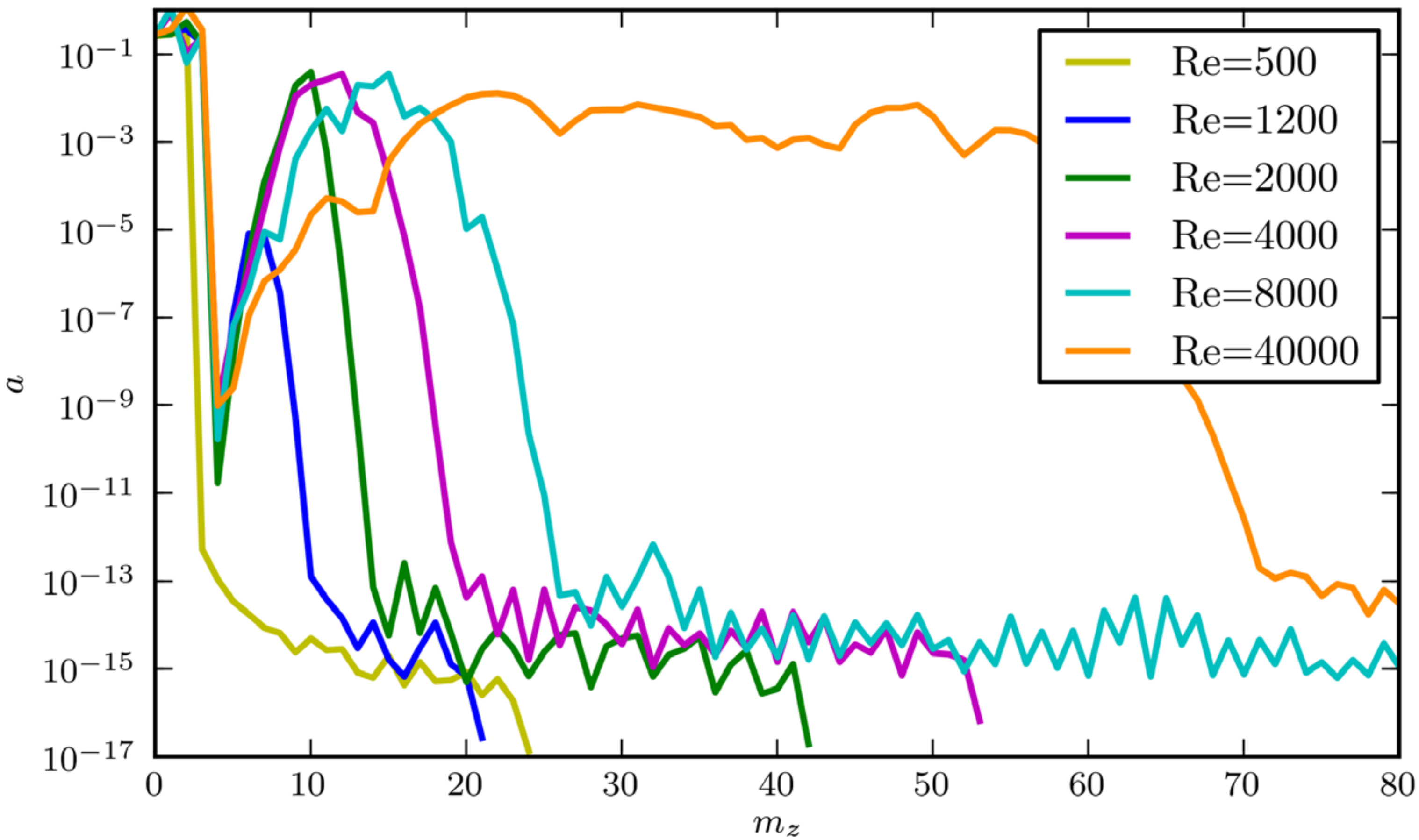


What happens at the bifurcation points?

$$\vec{u} = \sum_{m_z=0}^{M_z-1} \vec{\tilde{u}}_{m_z}(y) e^{2\pi i(\frac{k_z z}{L_z})} + c.c.$$

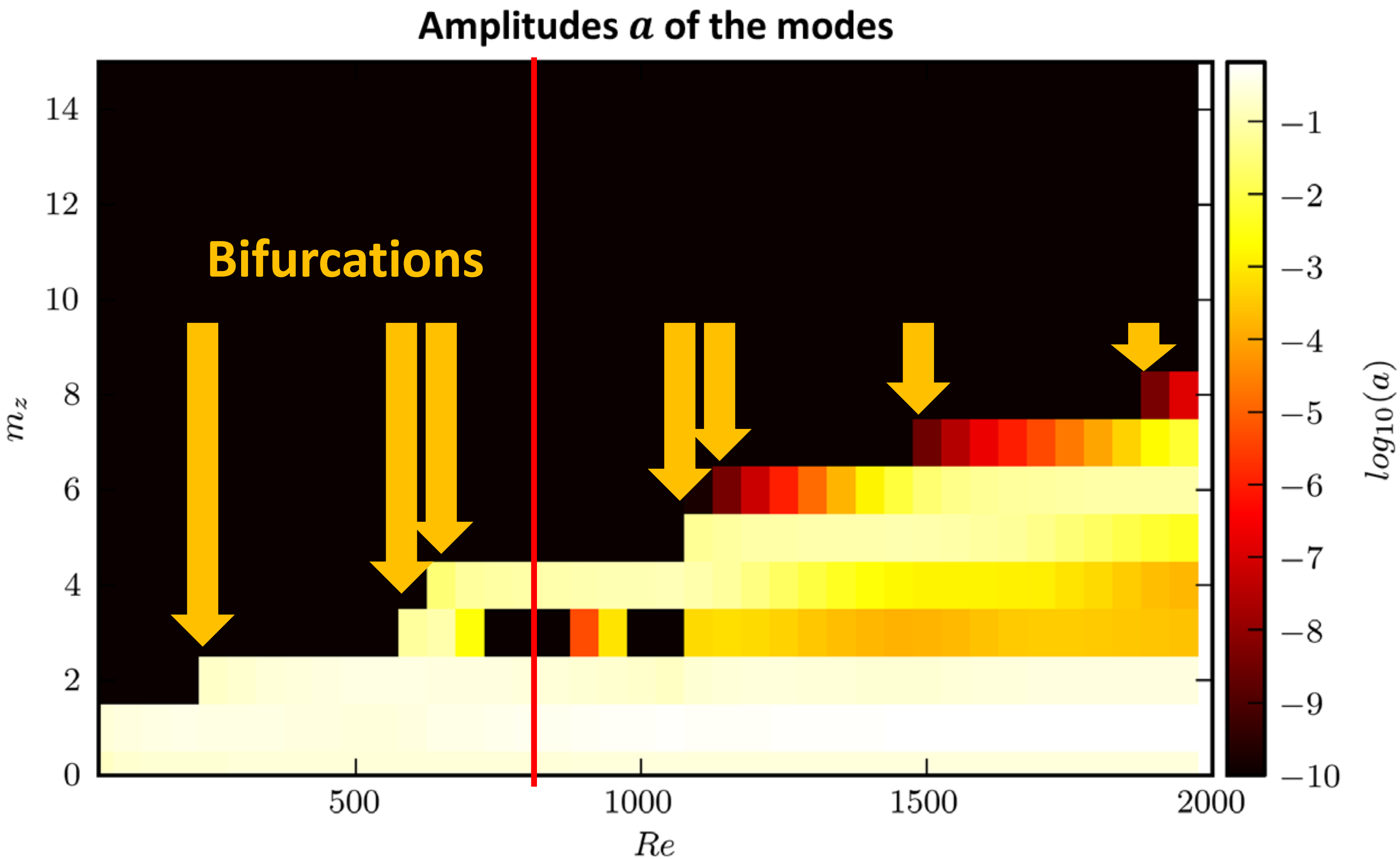
$$\vec{u}_{m_z} = \vec{\tilde{u}}_{m_z}(y) e^{2\pi i(\frac{k_z z}{L_z})} + c.c$$

$$a(\vec{u}_{m_z}) = \sqrt{\frac{1}{L_y\, L_z} \iint \vec{u}_{m_z}^2 dV}$$



Re=500
Re=1200
Re=2000
Re=4000
Re=8000
Re=40000
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{-13}
10^{-15}
10^{-17}
a
0
10
20
30
40
50
60
70
80
m_z

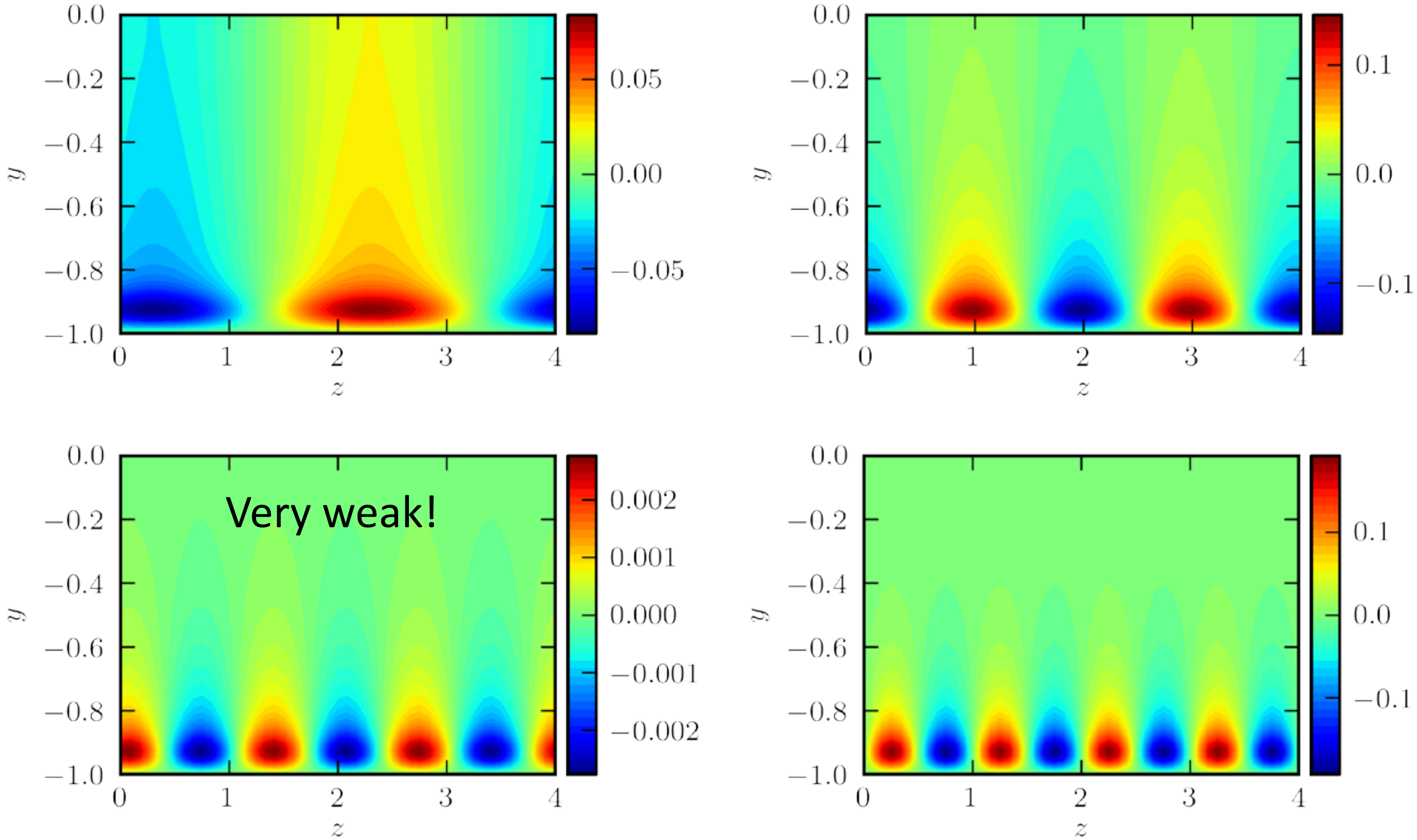
Bifurcation diagram



Bifurcations add modes.

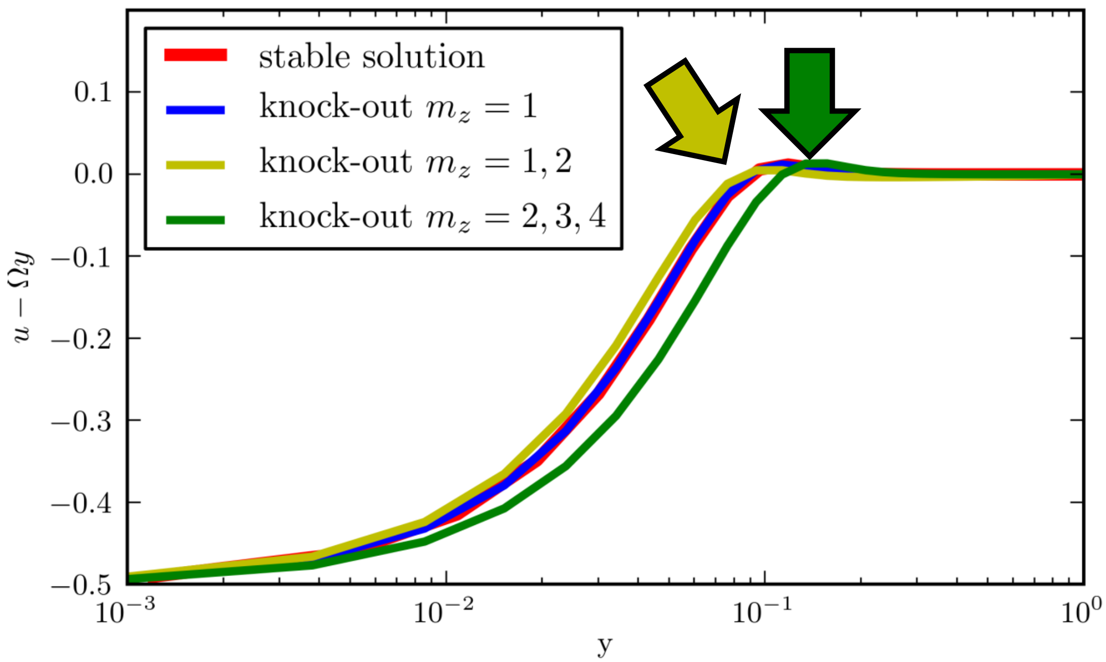
Contributions to the profile

Re=800 → Stable state has four active modes



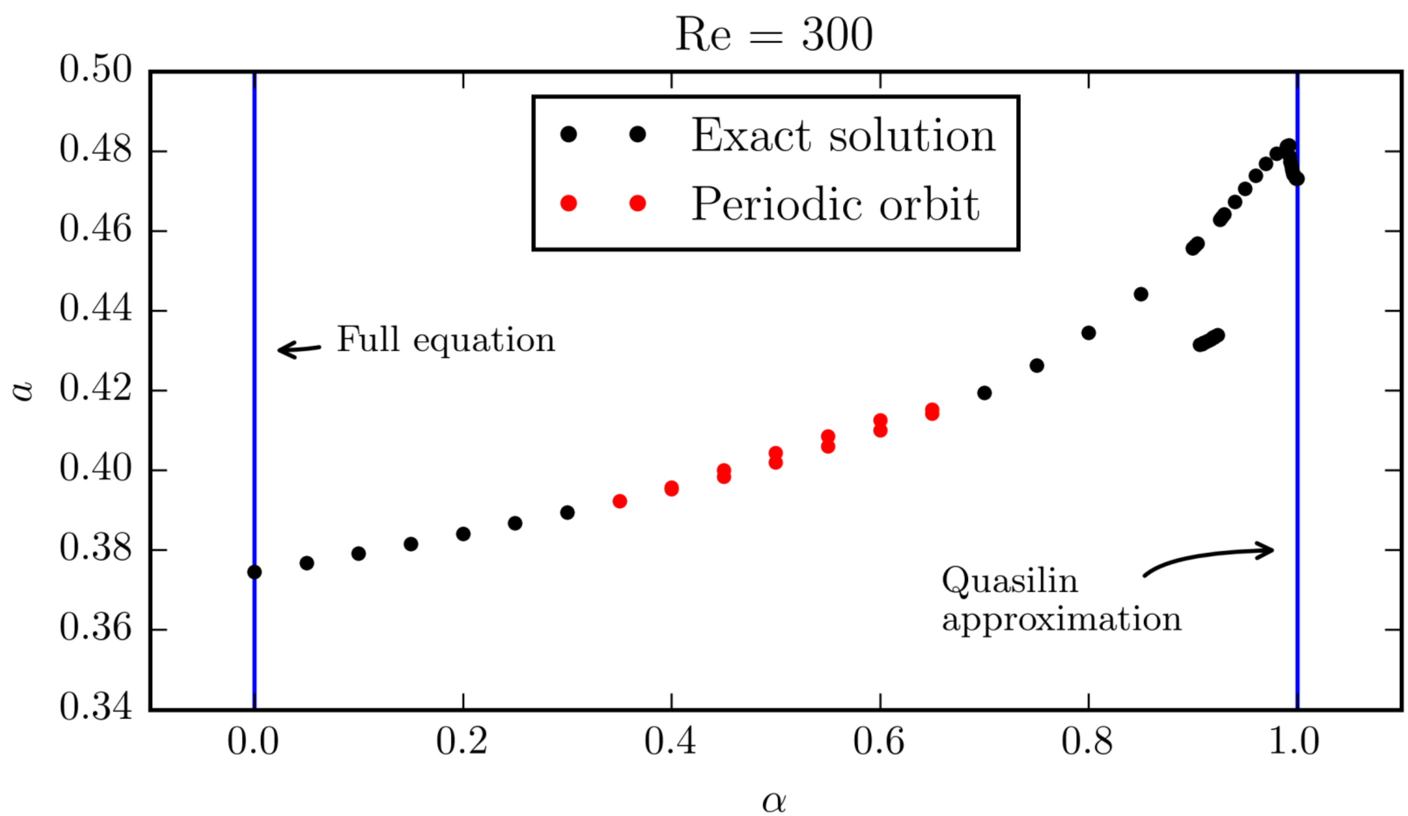
Each mode can be knocked out → Unstable equilibrium solutions

Contributions to the profile



- Low mode numbers → Linear profile in the bulk
- Higher mode numbers → Boundary layers

Continuation to the nonlinear system

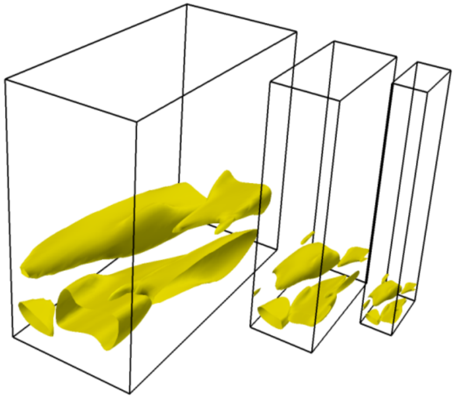


Continuation of the ECS to the ninlinear system is possible → Homotopy parameter α

Conclusions

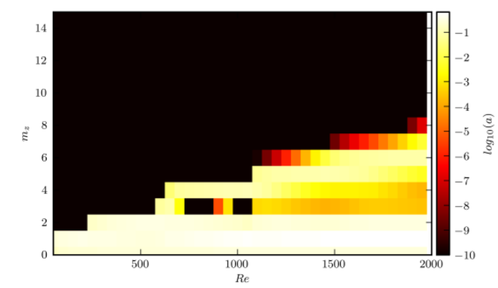
Nonlinear DNS:

- ECS with different scales in PCF
  - Wall-attached solutions
  - Localized solutions



Quasilinear DNS:

- Profile of Rot. PCF are qualitatively identical to the nonlinear case
- Formation of the profile can be understood by bifurcation analysis
  →Bifurcations add smaller scales
- Continuation to the nonlinear system is possible



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