Reductive Asymptotics for Recurrent Flows

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I. Mini-tutorial on reductive asymptotic methods for **PDEs**

II. Application to coupled uniform momentum zones and internal shear layers in turbulent wall flows

Reductive Asymptotic Methods for PDEs I.

- Extreme parameter regimes give rise to small parameters (ϵ) in governing PDEs
- Generally implies separation of spatiotemporal scales or of sizes of field variables
- Can be exploited by seeking asymptotic approximations:

$$f(x;\epsilon) \quad \sim \quad \sum_{n=0}^{N} \delta_n(\epsilon) f_n(x), \quad \text{where: } \delta_{n+1}/\delta_n \to 0 \text{ as } \epsilon \to 0 \text{ for each } n$$

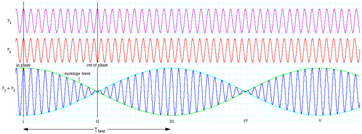
- Frequently, expansion not *uniformly* asymptotic (i.e. for all x)
- Two sources of non-uniformity, or singularity, include: (i) **infinite domains**, (ii) **change of type of PDE** (including reduction of order)
- Must suitably generalize asymptotic approximation:

$$f(x;\epsilon) \quad \sim \quad \sum_{n=0}^{N} \delta_n(\epsilon) F_n(x;\epsilon), \quad \text{where, e.g. } F_n(x;\epsilon) = x/\epsilon$$

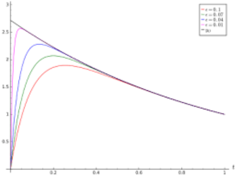
- Key challenge is to determine appropriate (re-)scalings through dominant balance arguments or by inspecting precise way in which 'regular' expansion breaks down

Reductive Asymptotic Methods for PDEs II: 2 Primary Methods

1. **Method of Multiple Scales**
    - Encompasses a variety of related methods: *Two-Timing, Averaging, Homogenization Theory, WKBJ Analysis, Slowly-Varying Waves*
    - Useful for problems with two or more disparate spatial and/or temporal scales that occur simultaneously over the domain



2. **Method of Matched Asymptotics**
    - Useful for problems with two or more disparate spatial (sometimes temporal) scales that occur in distinct regions



Outline

Part I:

1. Simple ODE example of method of **multiple scales**

2. Involved PDE example of method of **multiple scales** (pattern formation)

3. Simple ODE (BVP) example of **matched asymptotics**

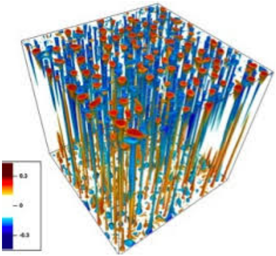
Part II:

4. Very involved PDE application of **matched asymptotics** to ECS at large Re_τ (including a discussion of VWI)

Possible Relevance of Asymptotic Methods to Recurrent Flows Activities

- Reduced PDEs for numerical simulations in otherwise inaccessible parameter regimes (cf. GFD, constrained turbulent flows including rapidly-rotating flows, Langmuir turbulence, . . .)
- ECS pattern formation theory (techniques *not* limited to weakly nonlinear states)
- Development of customized multiscale numerical methods
- Motivation for generalized quasilinear (GQL) approximation
- Asymptotic ECS (described in **Part II**)

Examples: Reduced PDEs for Externally-Constrained Anisotropic **Turbulent** Flows



- **Physically:** Strong external constraint induces anisotropic flow structures and reduces mode coupling in particular directions
- **Mathematically:** Strong external constraint introduces large undifferentiated terms into governing PDEs, e.g.

Rapidly-rotating convection:

$$D_t\mathbf{u} + \frac{1}{Ro}\hat{\mathbf{z}} \times \mathbf{u} = -\nabla p + B\theta\hat{\mathbf{z}} + \frac{1}{Re}\nabla^2\mathbf{u}$$

Langmuir turbulence:

$$D_t\mathbf{u} = -\nabla p + \frac{1}{La_t^2}\left[\mathbf{u}_s \times (\nabla \times \mathbf{u})\right] + \frac{1}{Re_*}\nabla^2\mathbf{u}$$