Spiral wave chaos: Tiling, local symmetries, and asymptotic freedom

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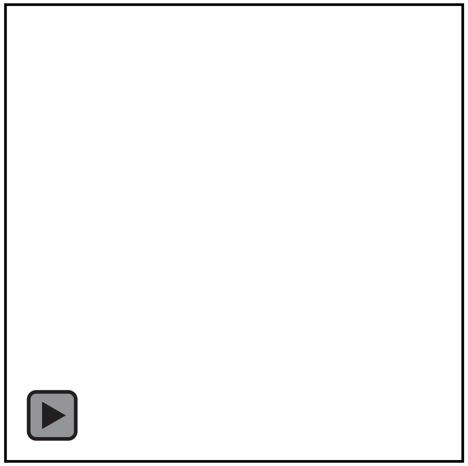
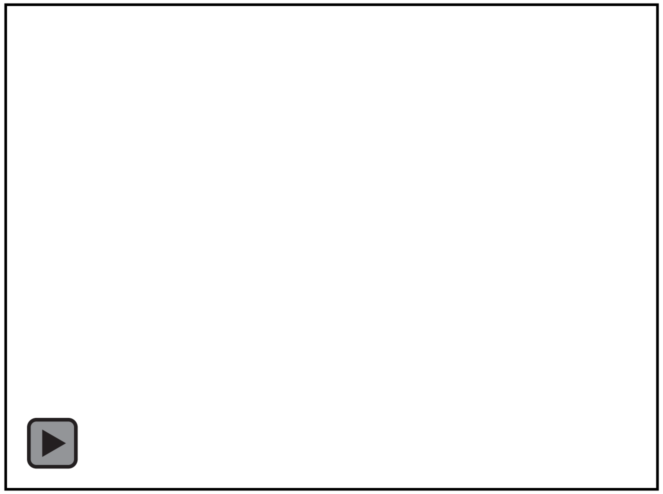
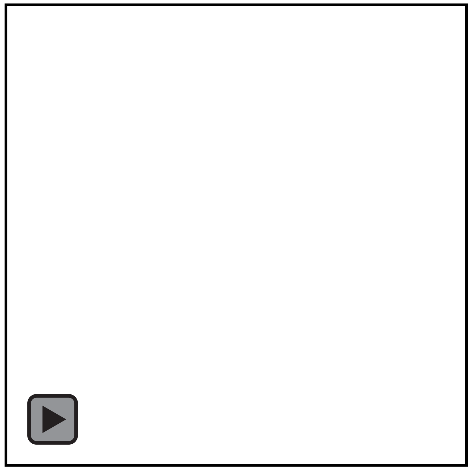
Spiral waves in excitable systems



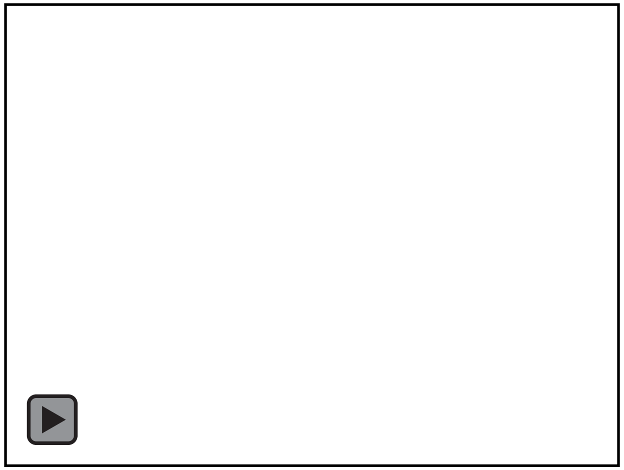
BZ reaction



Slime mold



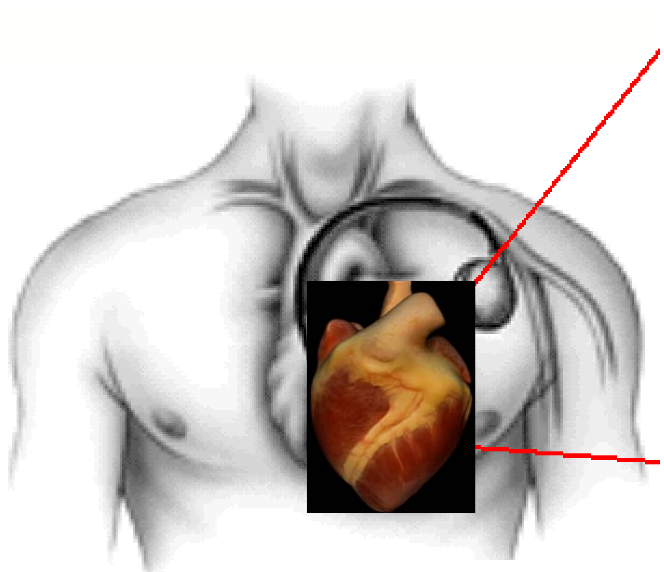
HL-1 cell culture

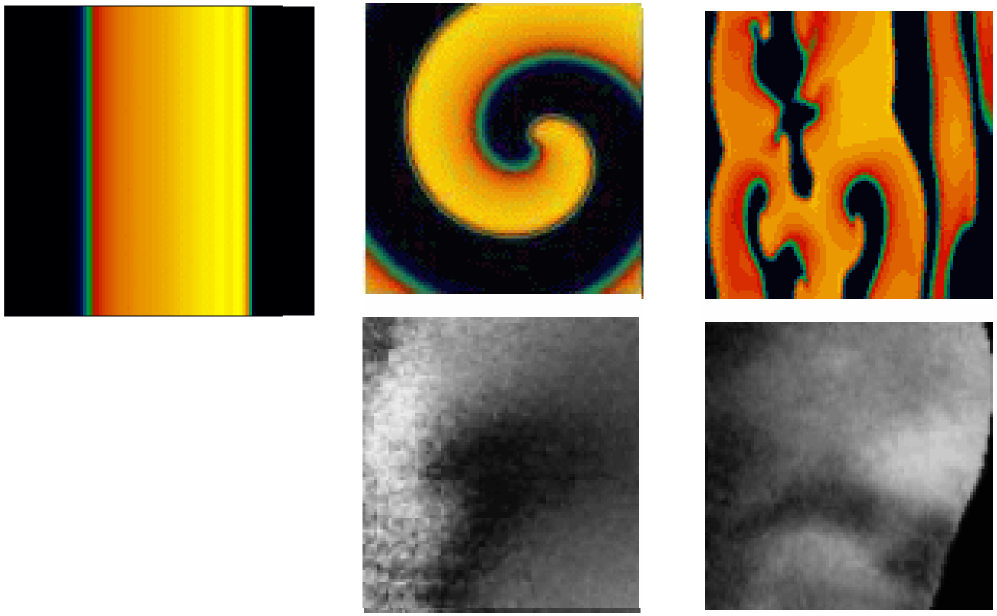


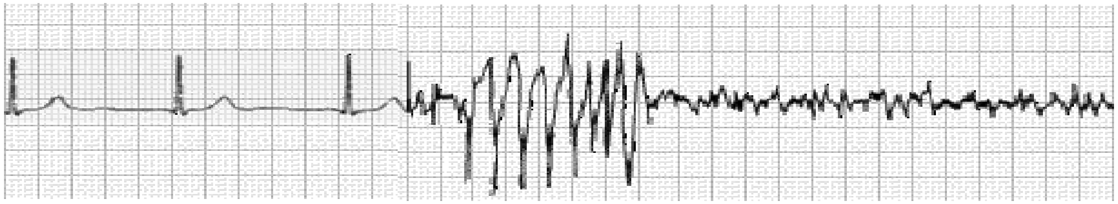
Human crowd

Fenton +
GT students

Cardiac rhythms and arrhythmias







Normal Rhythm

Ventricular Tachycardia

Ventricular Fibrillation

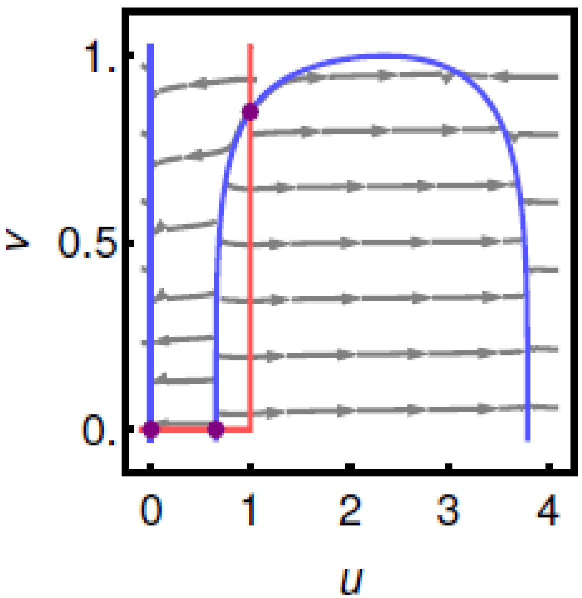
Fenton, Cherry
thevirtualheart.org

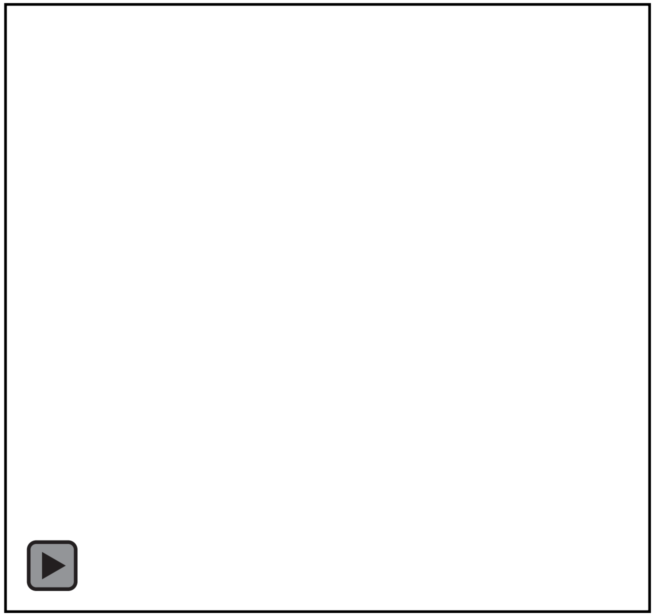
Spiral chaos in a simple model

Reaction-diffusion system:

$$\partial_t u = D\nabla^2 u + f(u,v)$$
$$\partial_t v = g(u,v)$$

Karma (1994)





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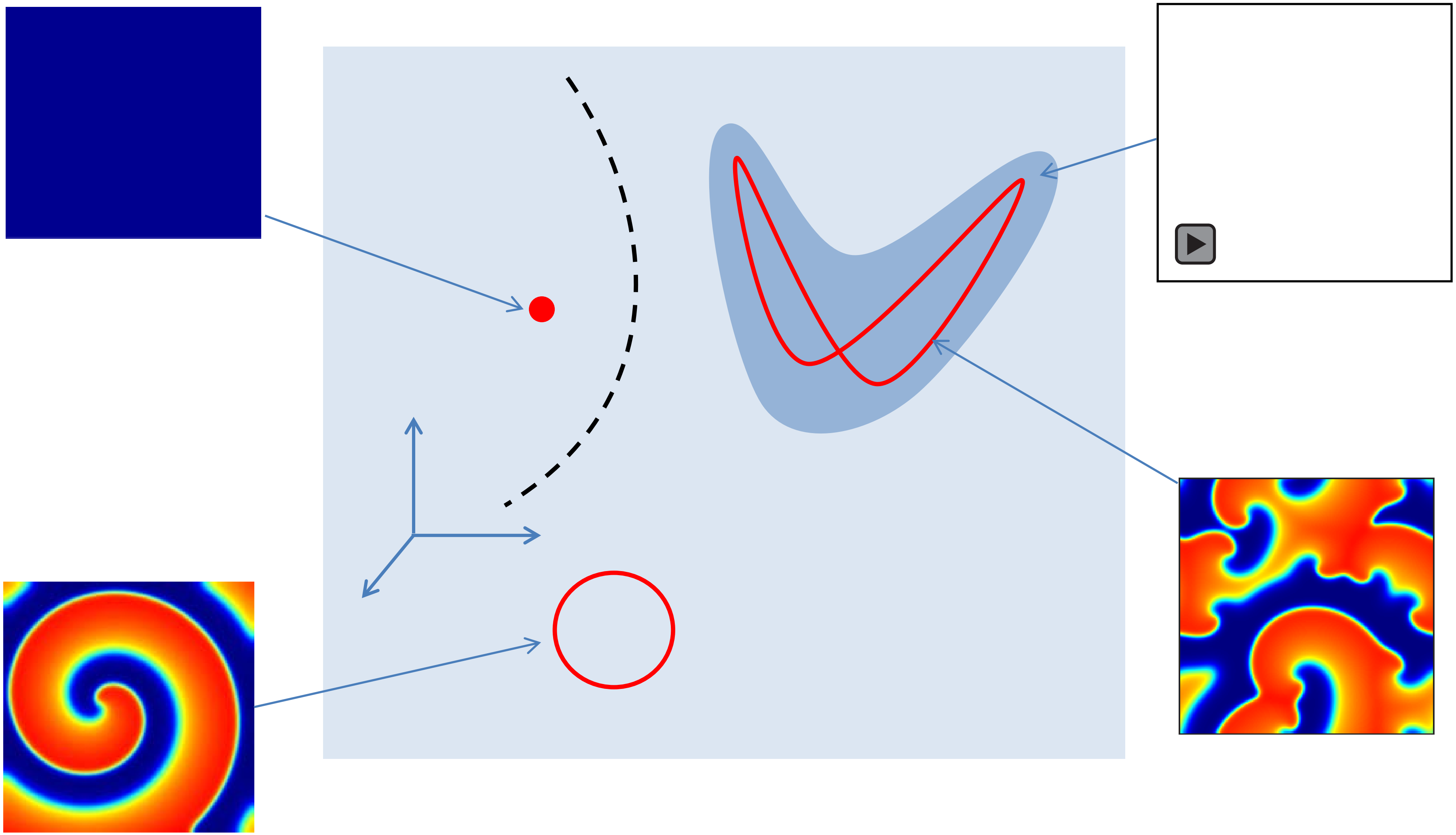
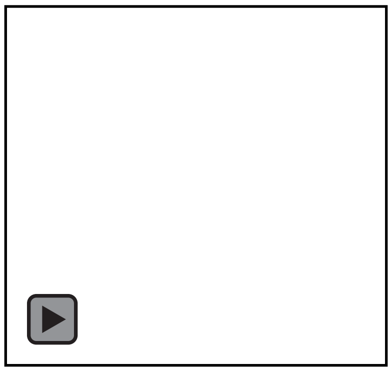
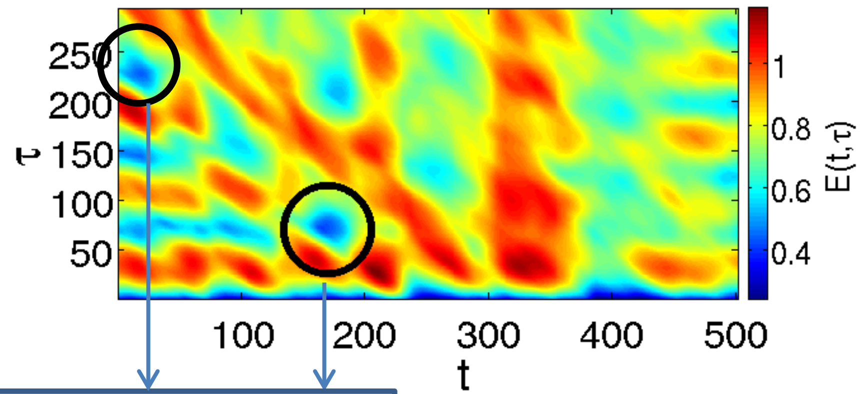
Nonchaotic solutions and ECS



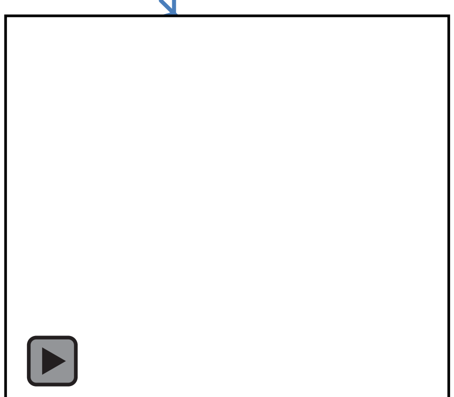
Computing ECS

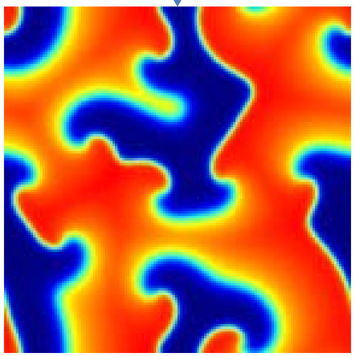


$$E(t,\tau) = \min_{g \in G} \left\| \mathbf{w} - g\mathbf{w}_\tau \right\| / \left\| \mathbf{w} \right\|$$

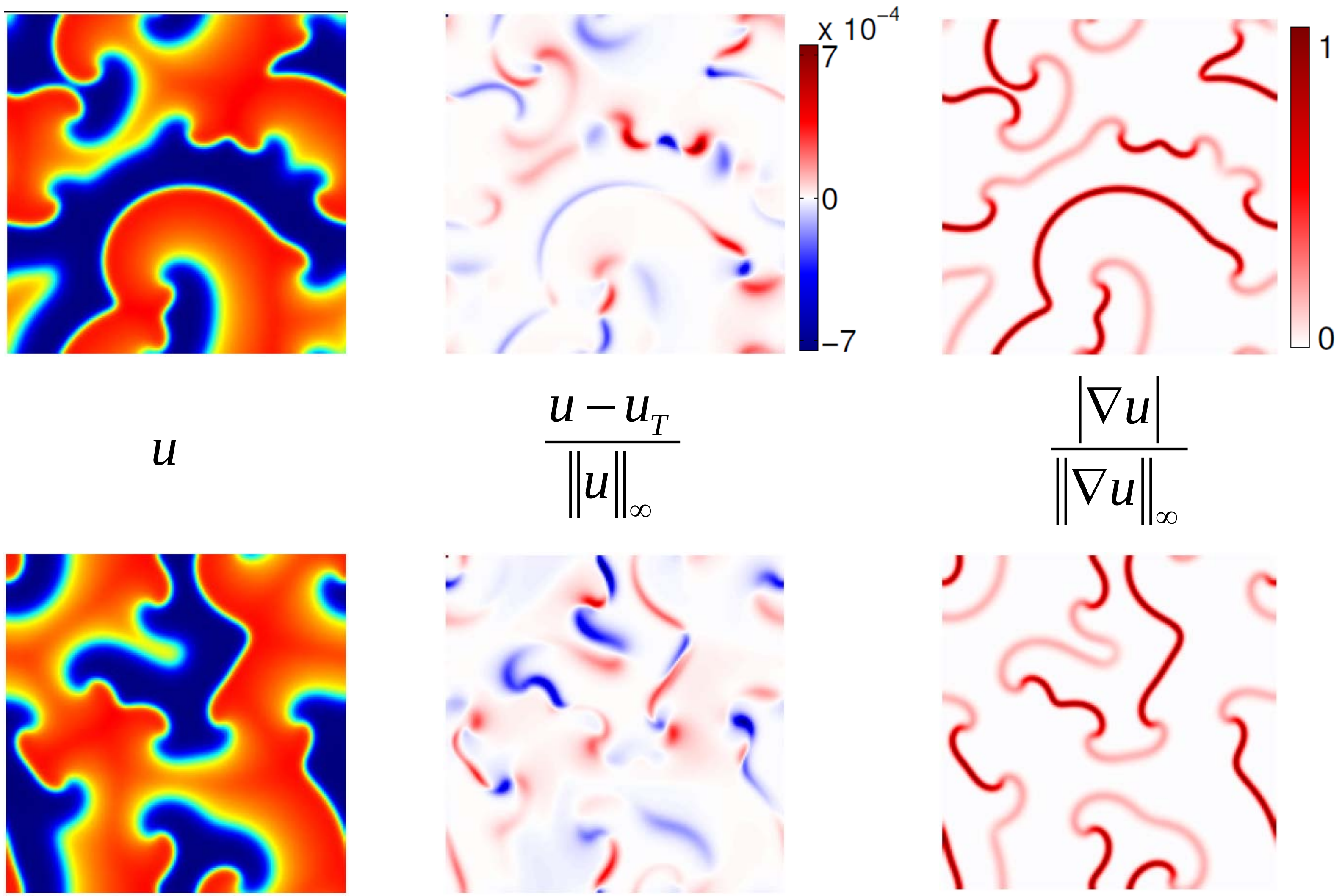


Newton-Krylov solver





Unstable periodic solutions?



Weakly nonlinear waves

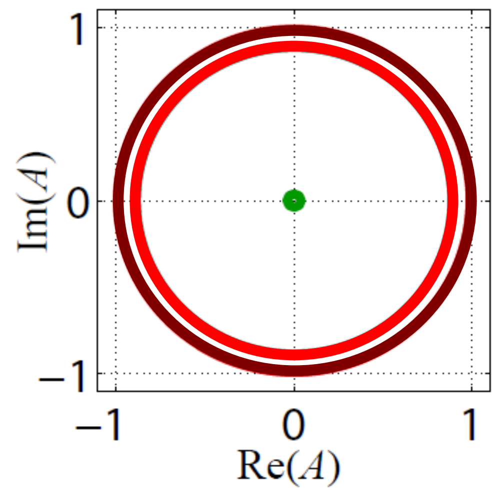
Complex Ginzburg-Landau Equation:

$$\partial_t A = A + (1+i\alpha)\nabla^2 A - (1+i\beta)|A|^2 A$$

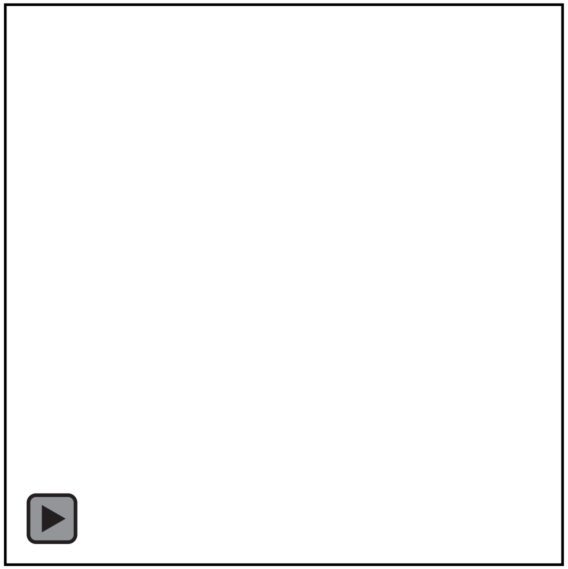
Tile the domain using amplitude ridges:

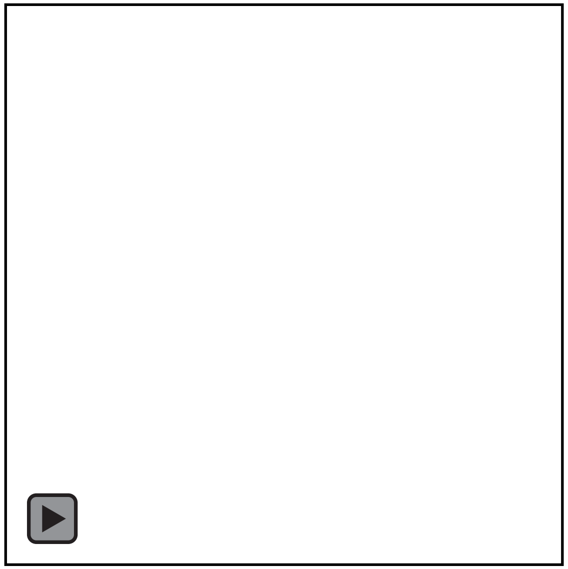
$$A = \rho e^{i\varphi}$$

Bohr, Huber, Ott (1996)



Re A





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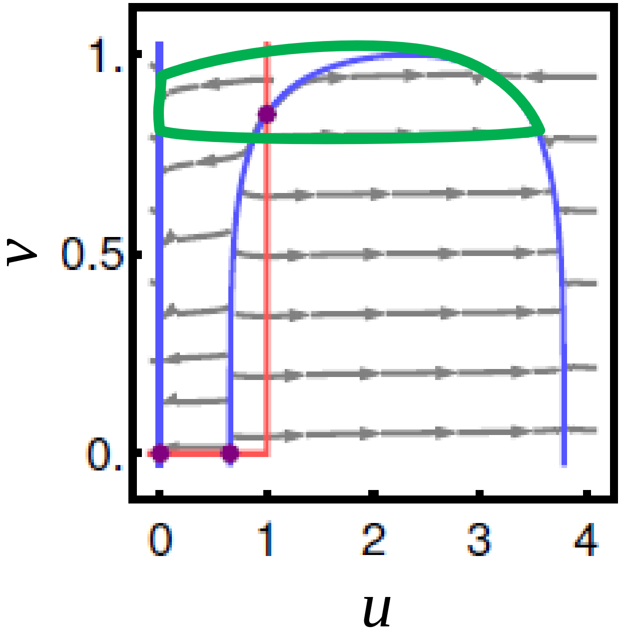
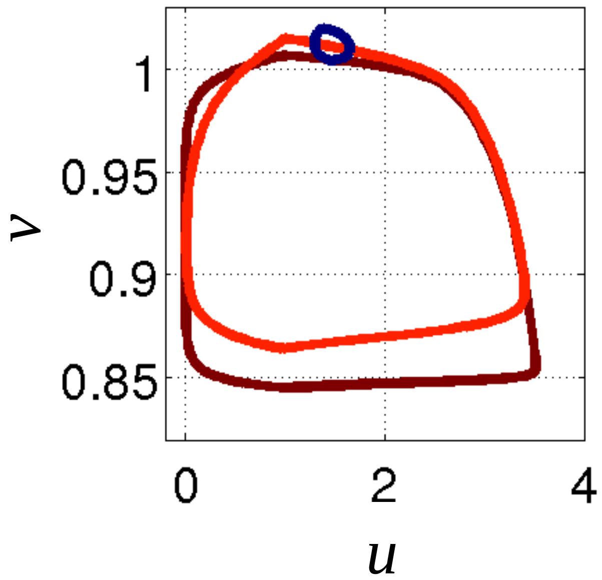
Strongly nonlinear waves





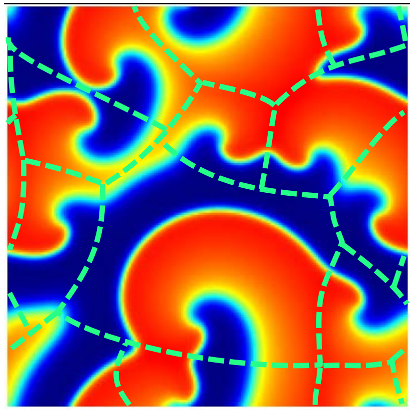
Use cycle area instead of amplitude:

$$I = \oint v\, du = \int_0^T v\dot{u}\, dt$$

and elapsed time from crossing a Poincare section instead of phase:

$$\theta = \int \omega\, dt, \quad \omega = \frac{2\pi}{T}$$

Tiling multispiral states



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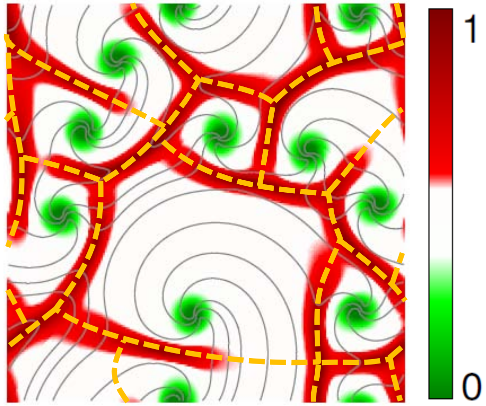

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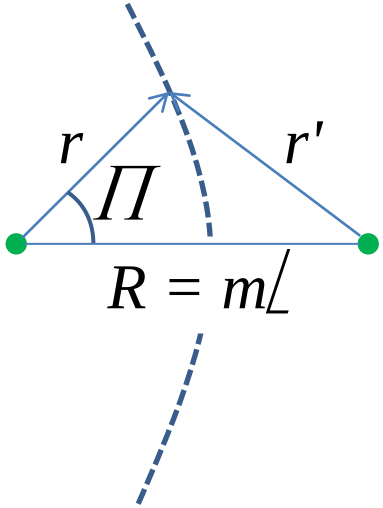
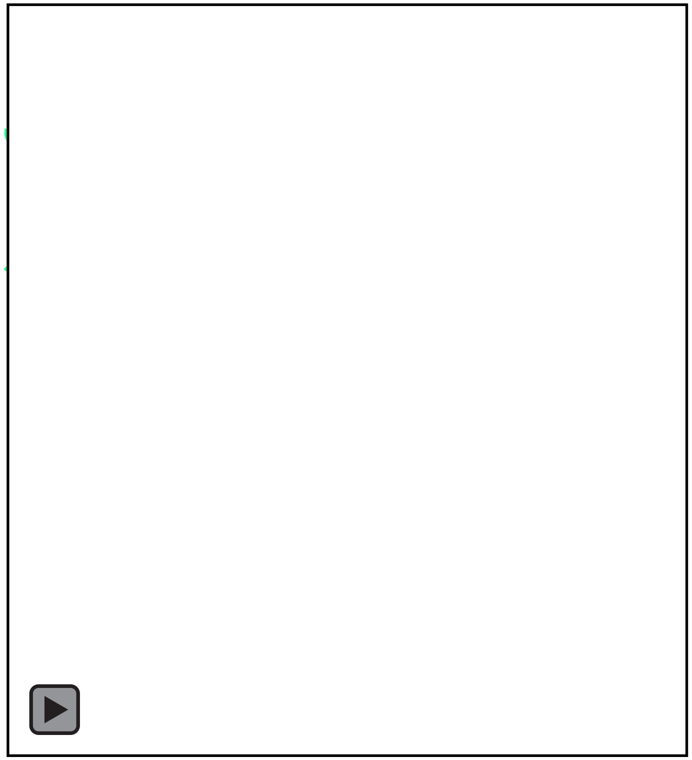


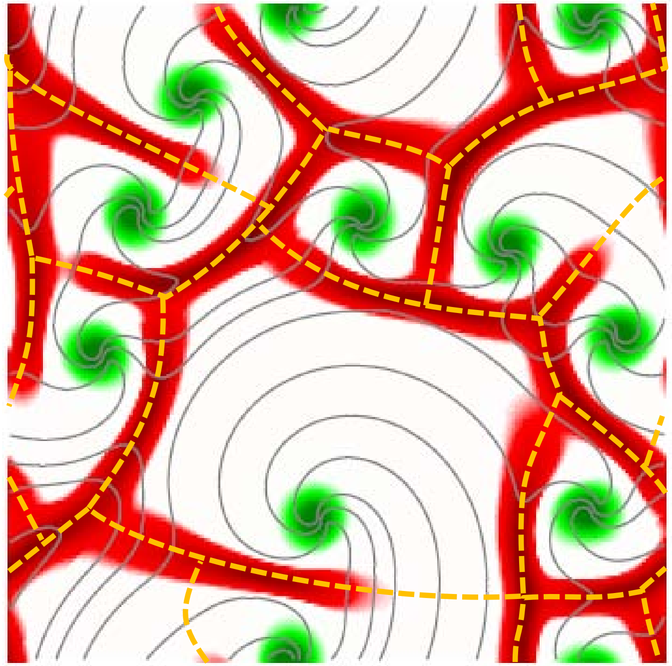
Can also describe tile boundaries analytically:

$$\frac{dr}{d\varphi} = \frac{-\sigma r'^2 - \sigma' r(R\cos\varphi - r) + 2\pi m r' r \sin\varphi}{\sigma' R^2 \sin\varphi + m\,(r'^2 - r'(r - R\cos\varphi))}$$

Luo, Zhang, Zhan (2009)

Boundary conditions





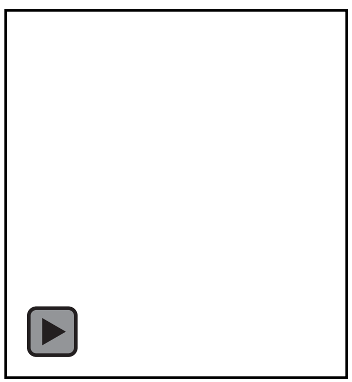
- Tiles are noncircular
- Neumann boundary conditions

Break-up

Drift

Collapse

Stability of spiral waves



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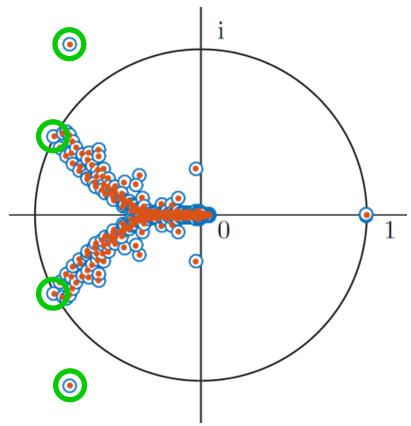
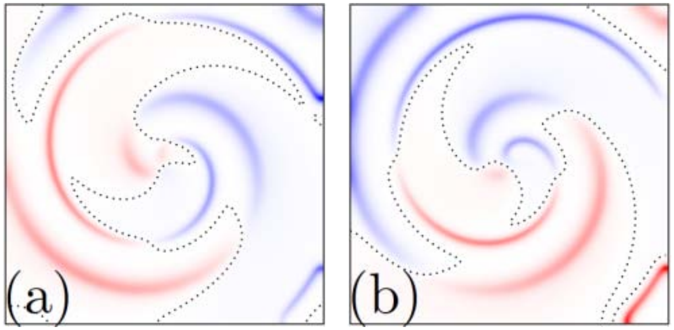
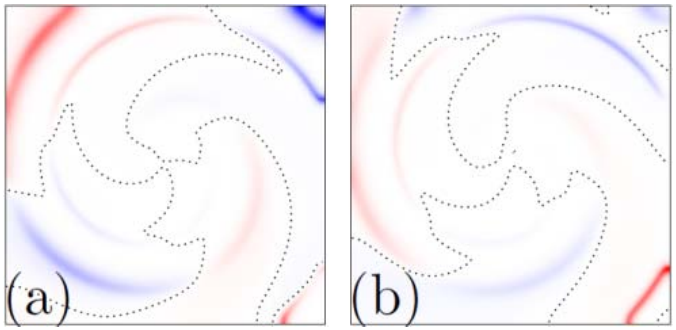


Floquet multipliers

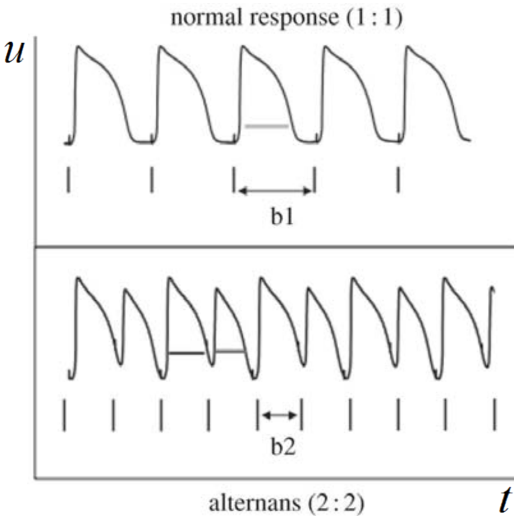


Absolute instability

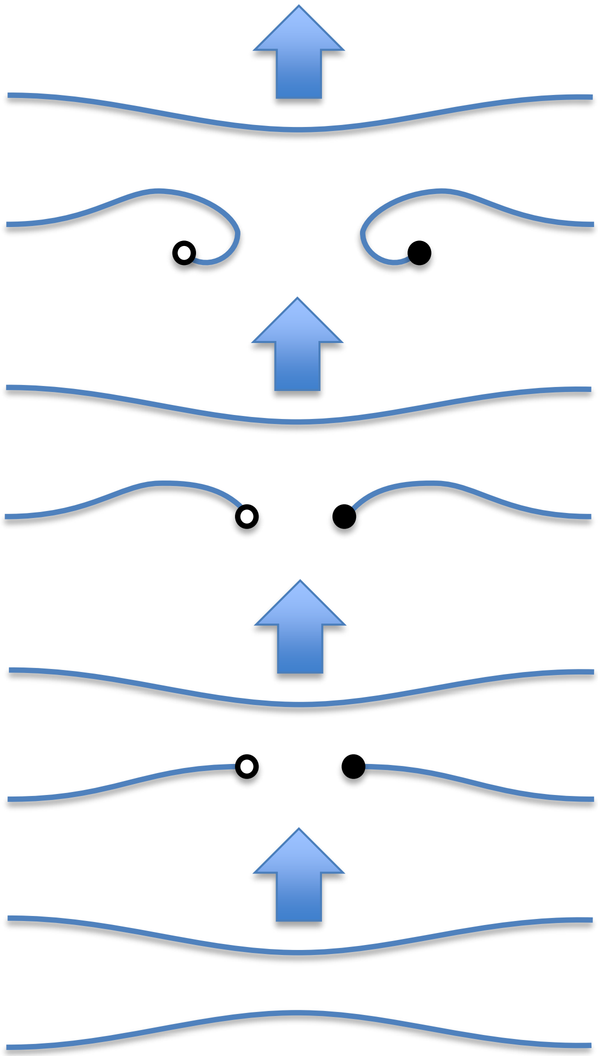


Convective instability

Alternans instability



Petrie, Zhao (2012)



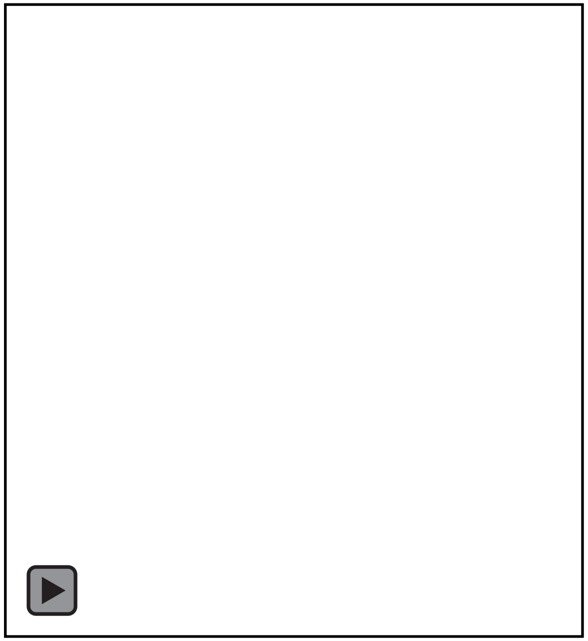
Break-up

Drift

Collapse

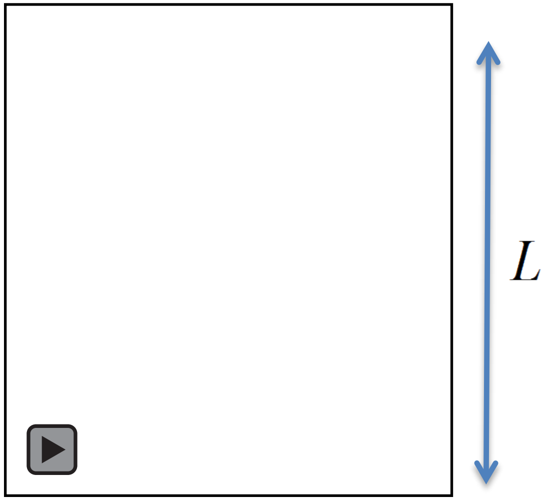
For some initial conditions…

Stroboscopic map:

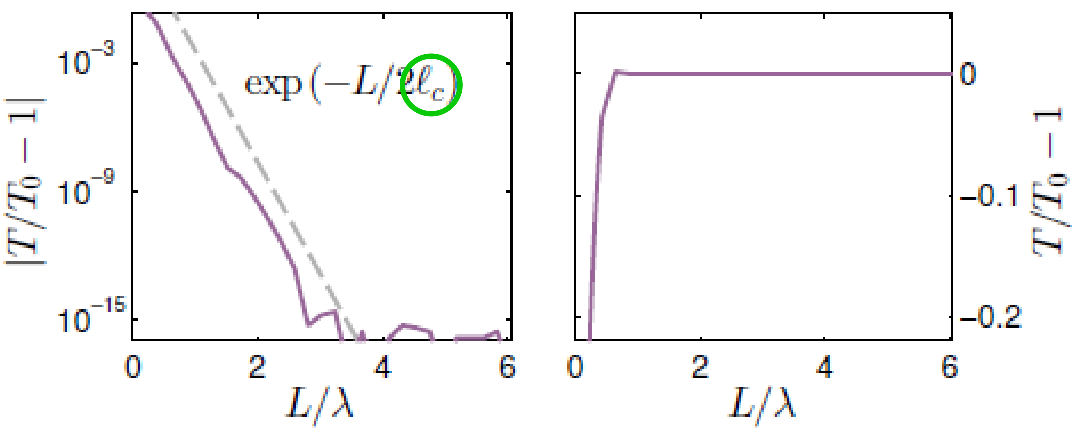


- Cores are drifting → tiles have to deform
- Can we understand this drift and deformation?

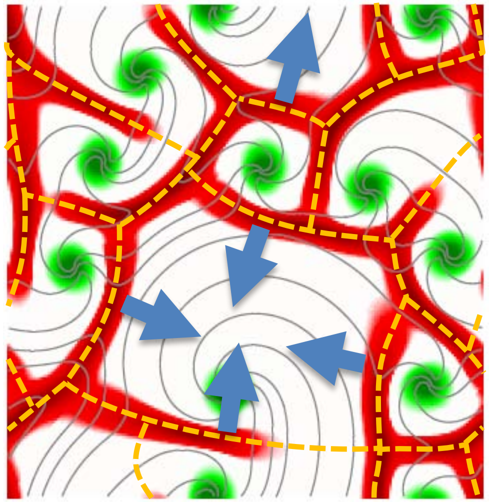
Dynamics of spirals on tiles



Period as a function of tile size



Dynamics of tiles



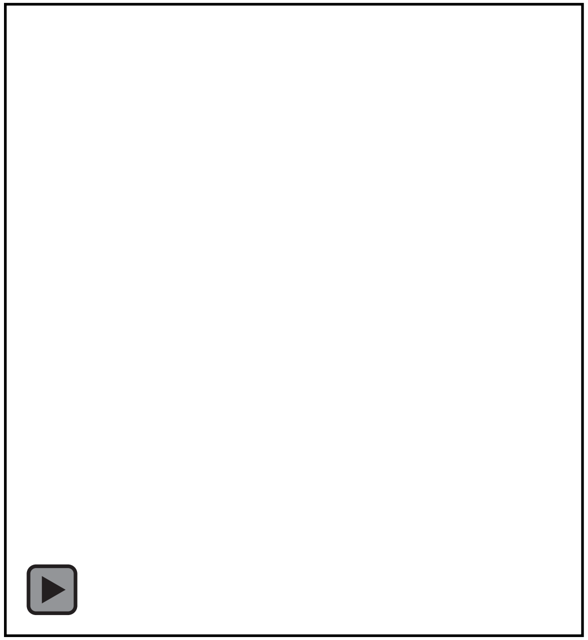
Motion of the boundary:

$$\mathbf{c} = (\omega_1 - \omega_2)\frac{\mathbf{k}_1 - \mathbf{k}_2}{|\mathbf{k}_1 - \mathbf{k}_2|^2}$$

Howard, Kopell (1977)

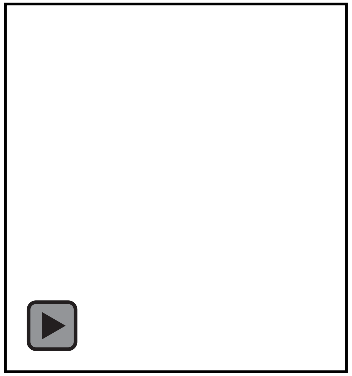
Dynamics of tiles (continued…)

Stroboscopic map:



- Why are some cores moving (and others are not)?
- Why is their motion so slow?
- What sets the distance between cores?

Local Euclidean symmetry



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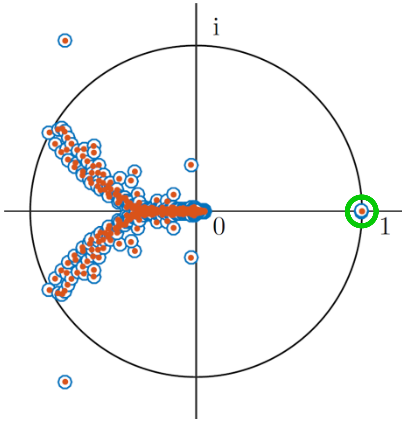
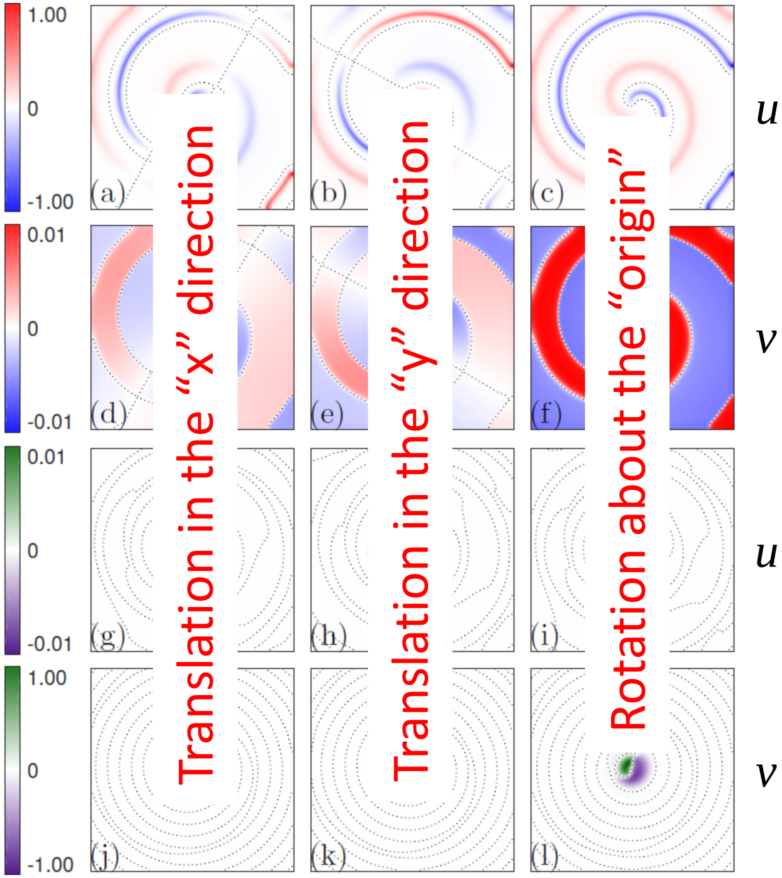


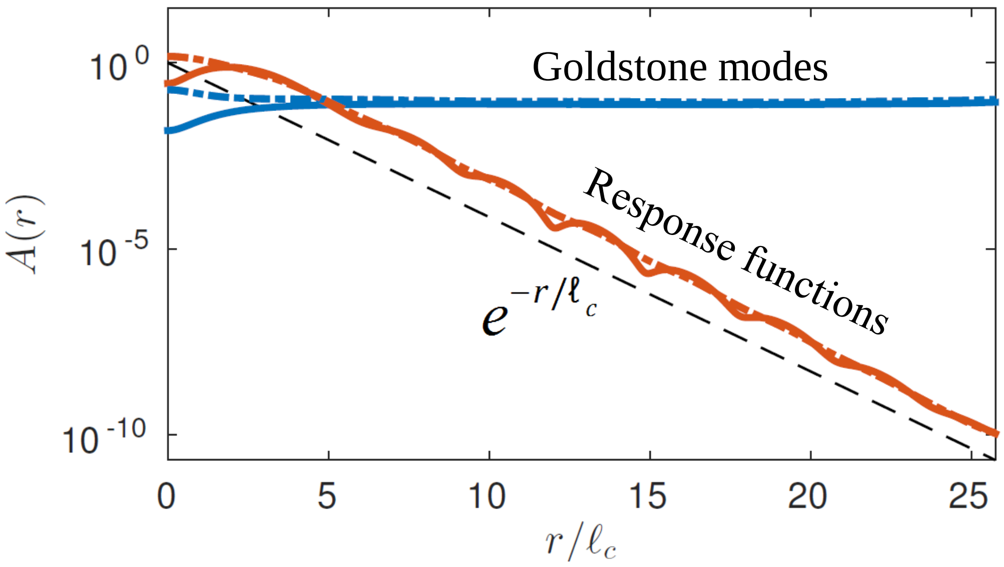
Floquet multipliers

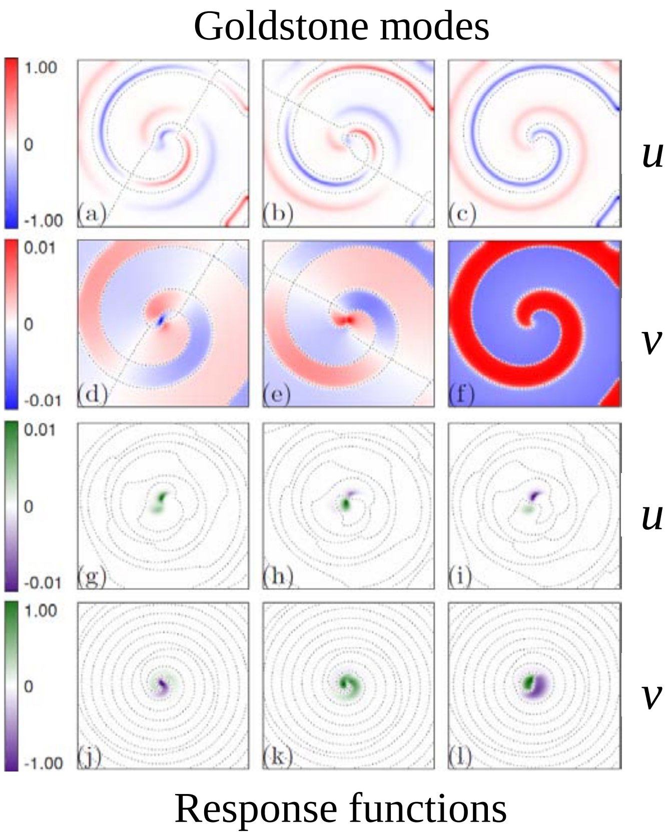
Goldstone modes/right eigenfunctions



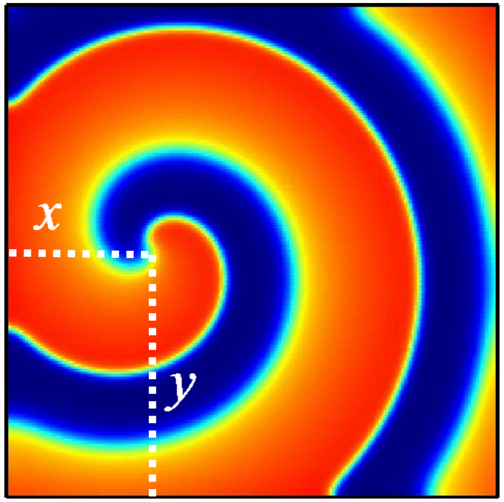
Response functions/left eigenfunctions

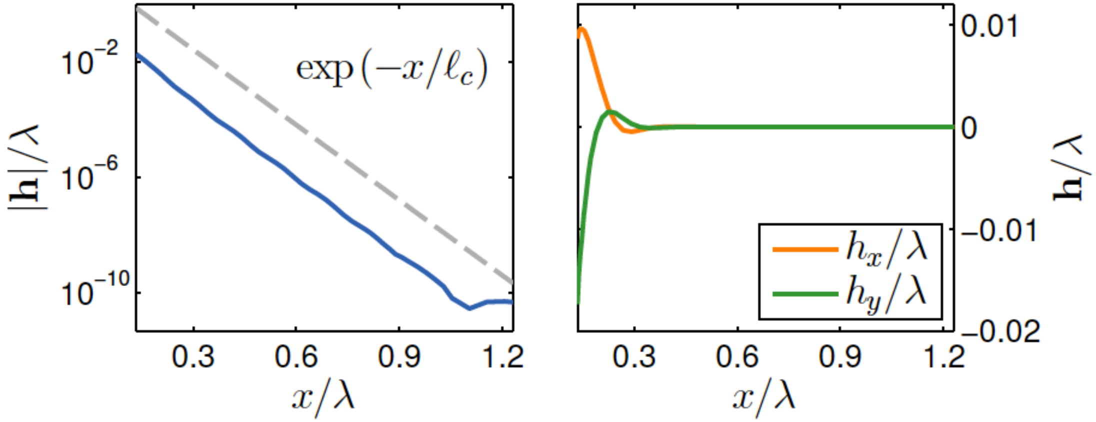
Asymptotic freedom



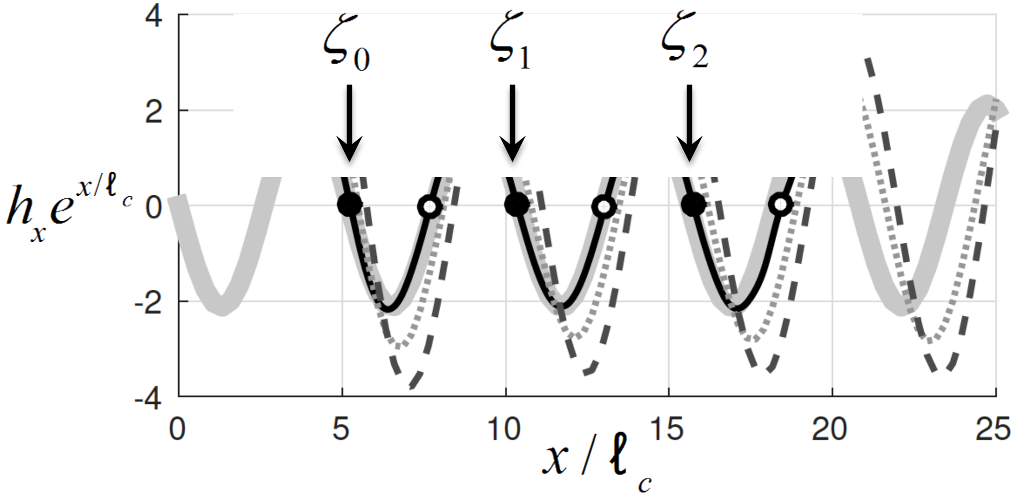


Interaction of cores with boundaries

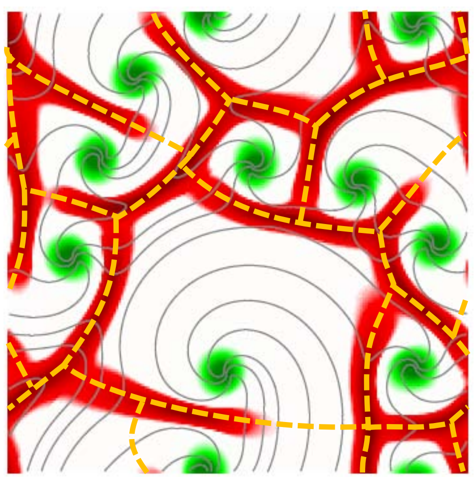


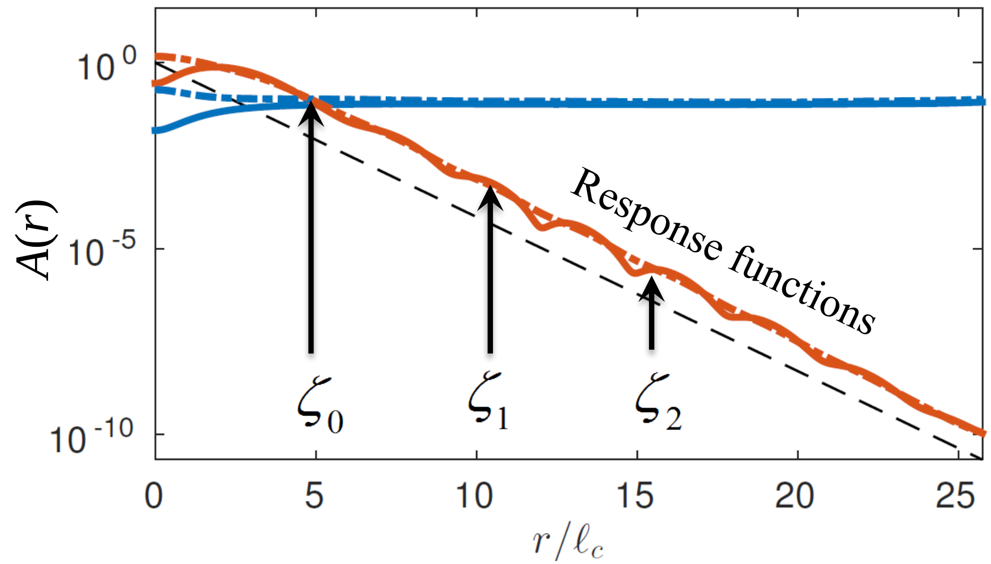


$$\mathbf{x}^{n+1} = \mathbf{x}^n + \mathbf{h}(\mathbf{x}^n),$$
$$\mathbf{x}^n = \mathbf{x}(nT)$$

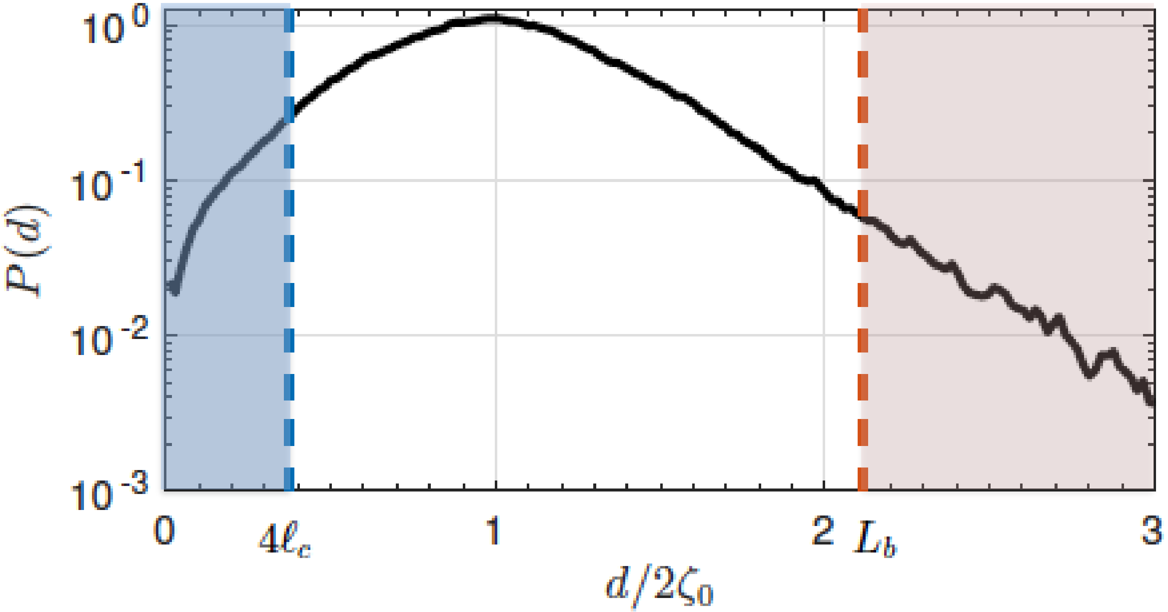


Core-core interaction





Core-core separation (& tile size)



No time-periodic solutions

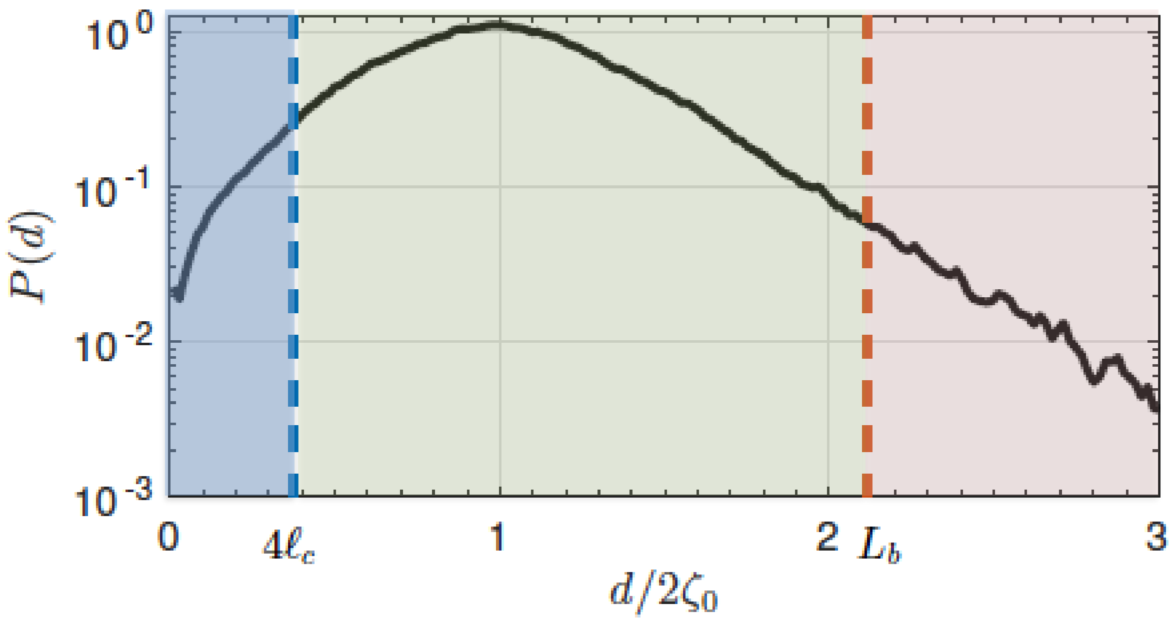
Solutions unstable (alternans)

Break-up

Drift

Collapse

Core-core separation (& tile size)

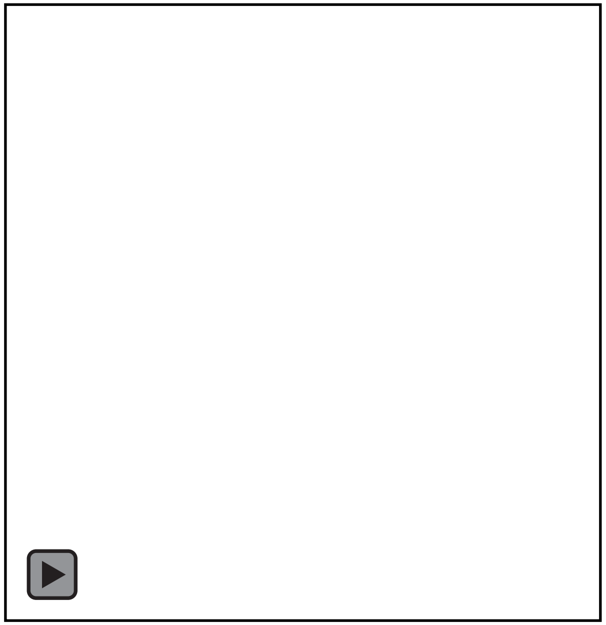


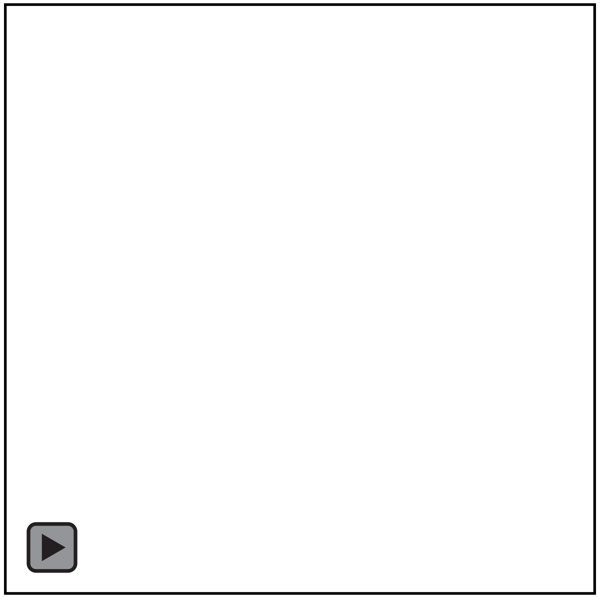
Virtual pairs

Core drift/ meander/ collapse

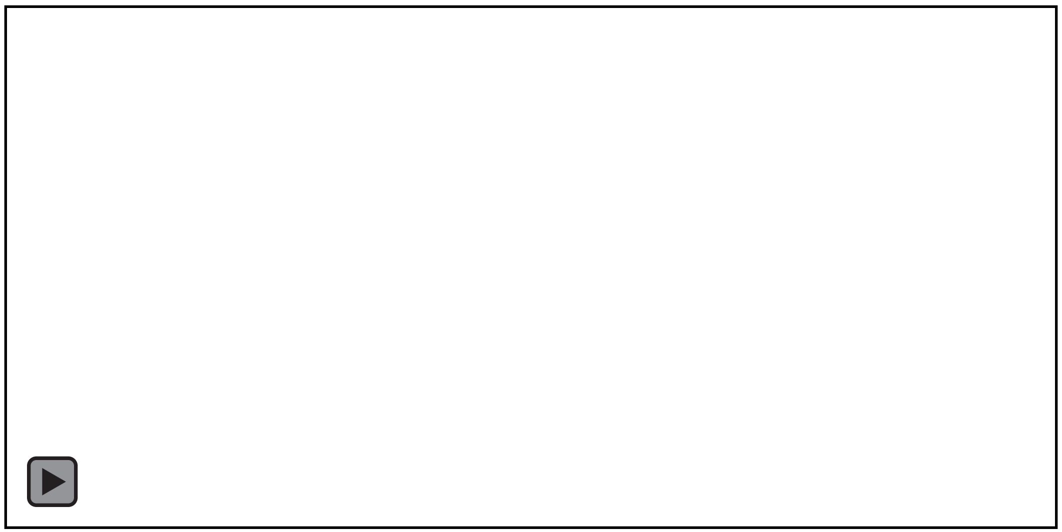
Break-up

Core meander

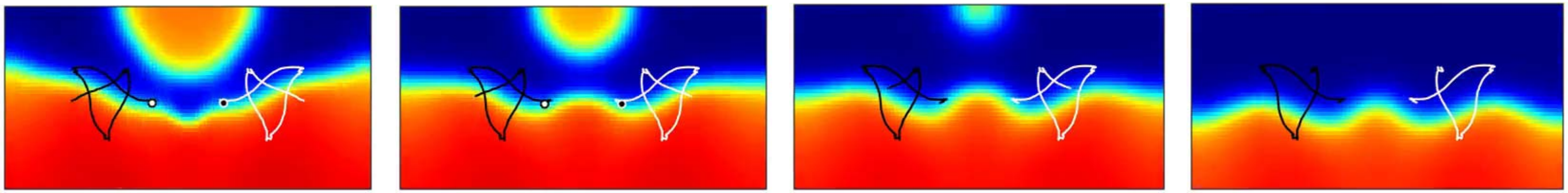


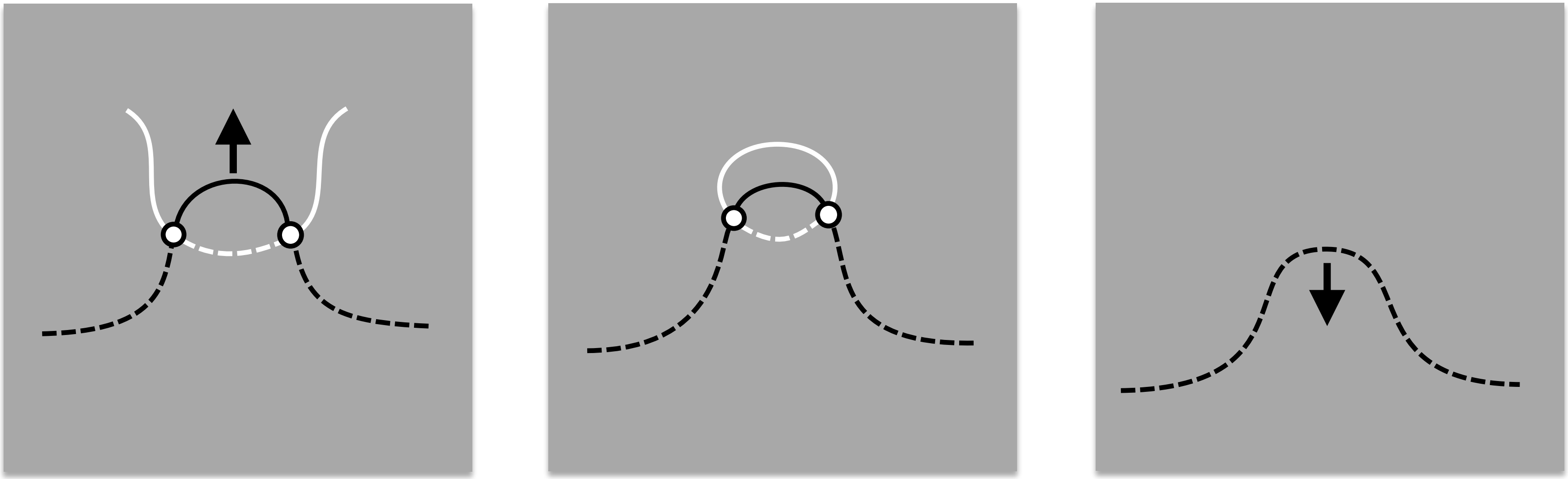


Wave collapse



Wave collapse





The mechanism of spiral chaos

- Slow dynamics (of tiles)
  - ✓ The tiles are generally of different size
  - ✓ The frequencies of spirals differ
  - ✓ The tiles boundaries drift (slowly)
  - ✓ Small (fast) spirals grow at the expense of big (slow) ones

- Fast dynamics (of spirals)
  - ✓ Large spirals ($L > L_b$) break up due to alternans instability
  - ✓ Small spirals ($L < 4l_c$) survive for less than one period and collapse (with a neighbor)
  - ✓ Medium size spirals ($4l_\mathrm{c} < L < L_b$) interact with each other in nontrivial ways

Implications for fluid turbulence

- No *global* ECS on domains much larger than the relevant coherence length, no matter what the physics is
- Need to look for localized solutions that respect *local* Euclidean symmetries and their interactions
- Coherence length can be defined with the help of *adjoint* eigenfunctions (to-do for fluid dynamicists)
- Spatial *correlations* may decay exponentially even when solutions do not
- Does exponential decay of *velocity/energy* imply short spatial correlations? What about *pressure*?