Spiral chaos in a simple model

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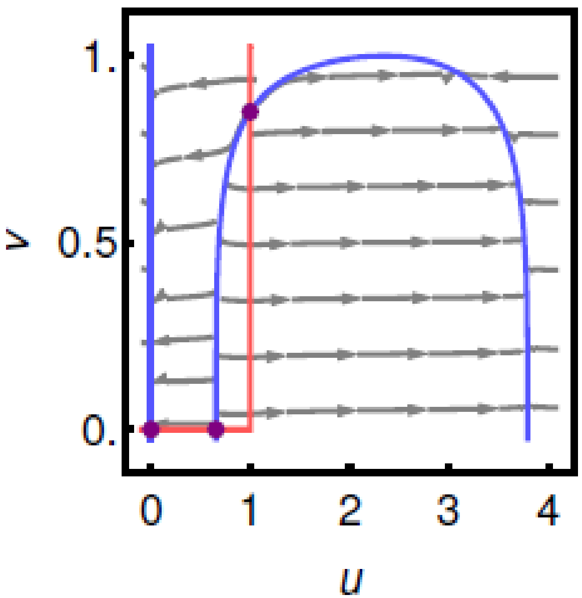
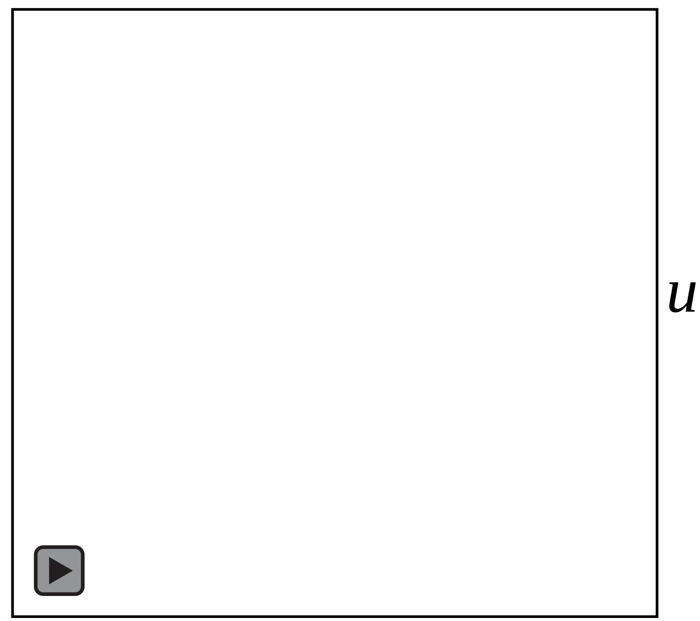
Reaction-diffusion system:

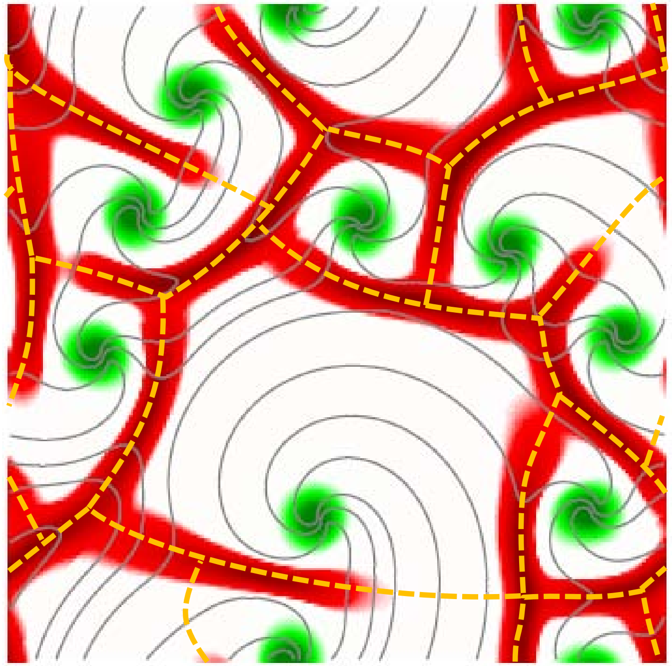
$$\partial_t u = D\nabla^2 u + f(u,v)$$
$$\partial_t v = g(u,v)$$

Karma (1994)

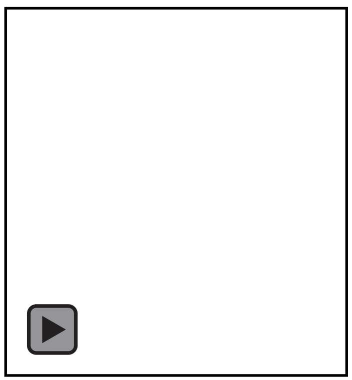




Dynamics on tiles



Local Euclidean symmetry



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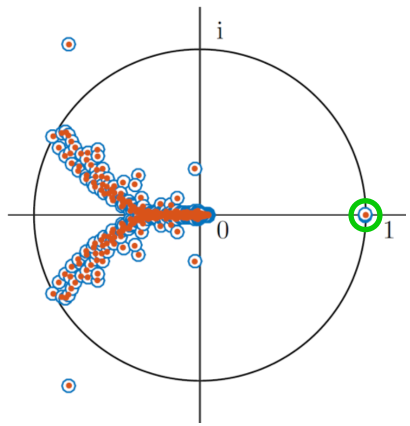
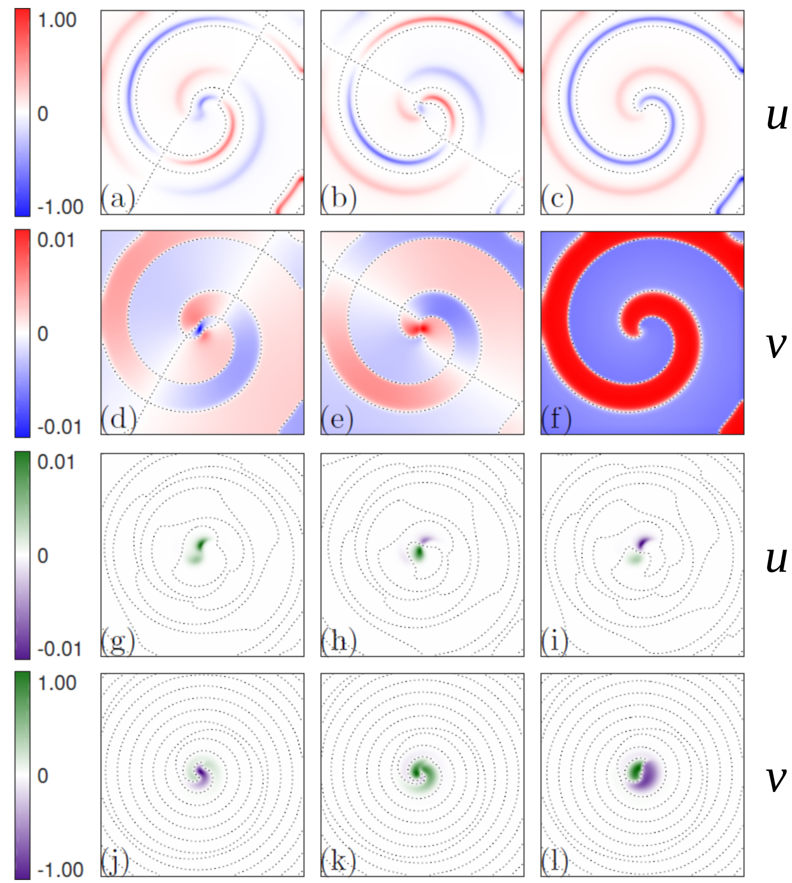


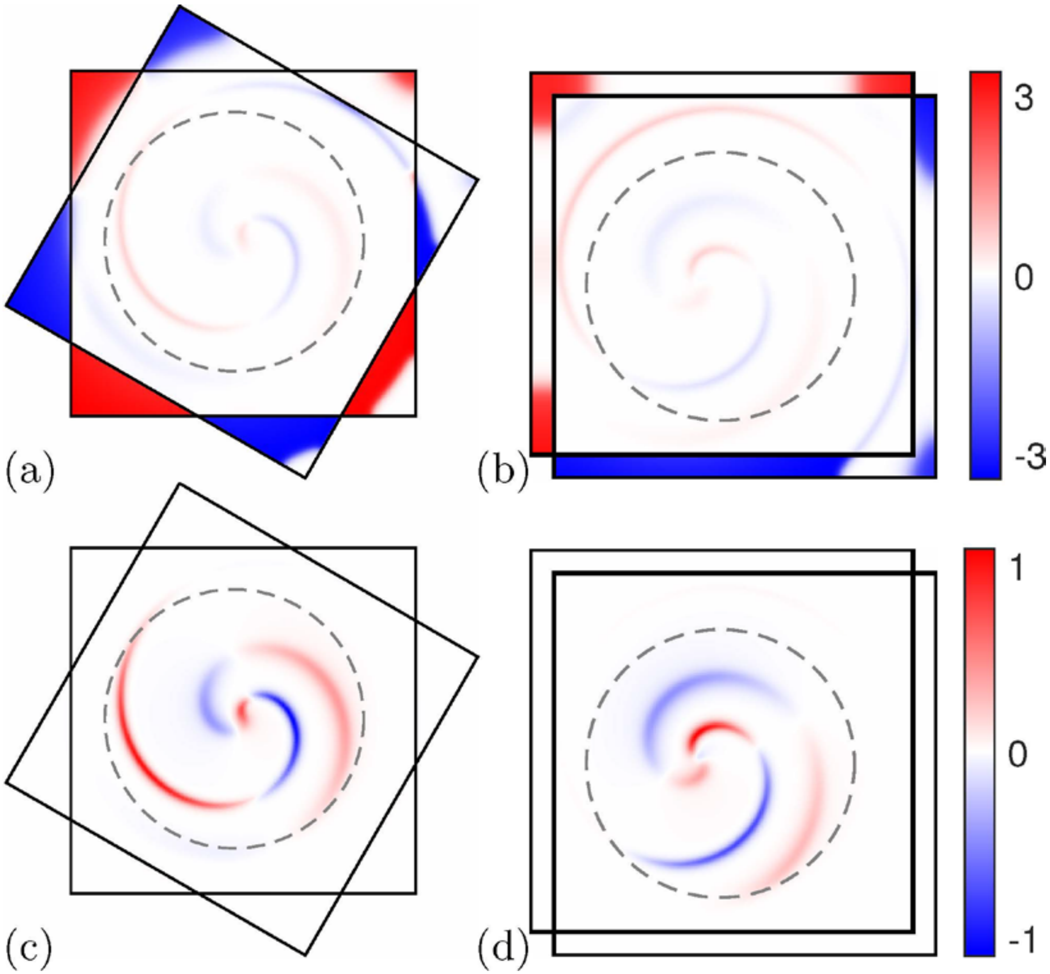
Floquet multipliers

Goldstone modes/right eigenfunctions

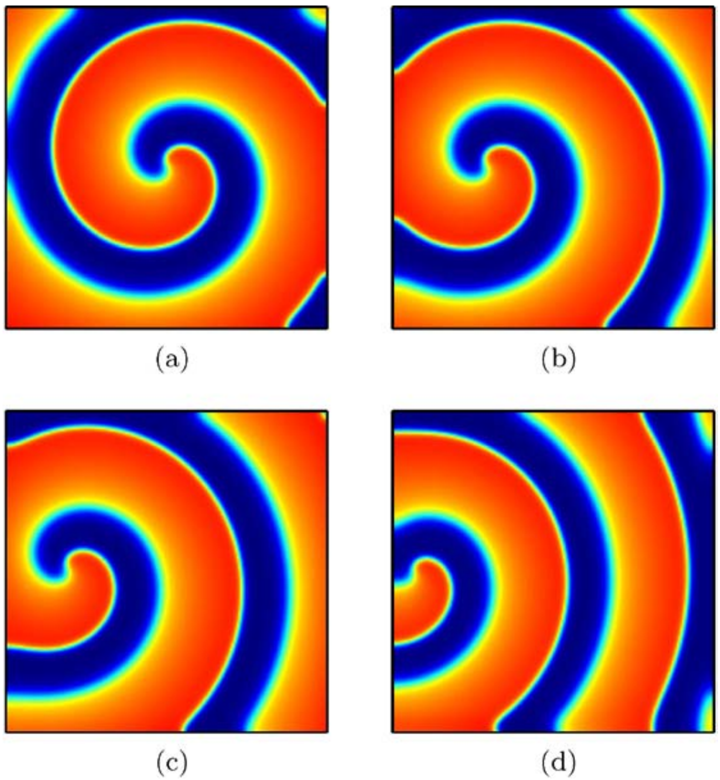


Response functions/left eigenfunctions

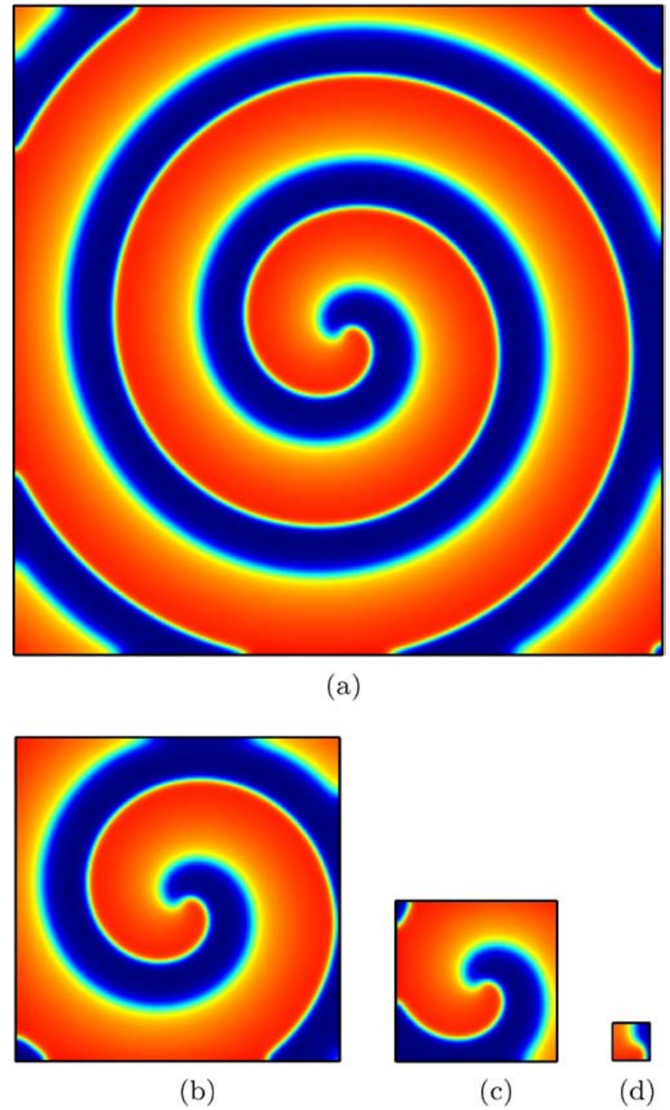
Local Euclidean symmetry



Generalized relative solutions

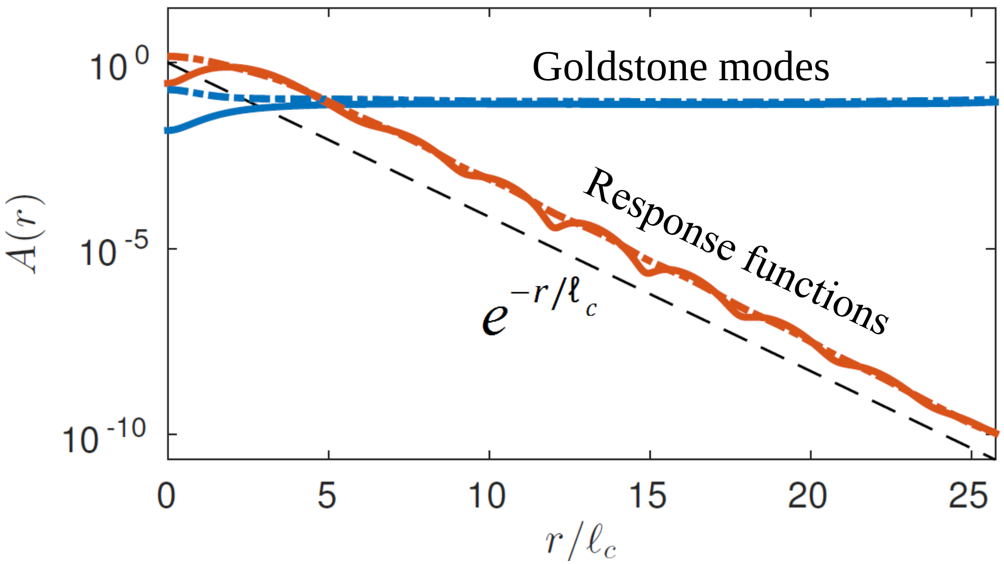


(a) (b) (c) (d)

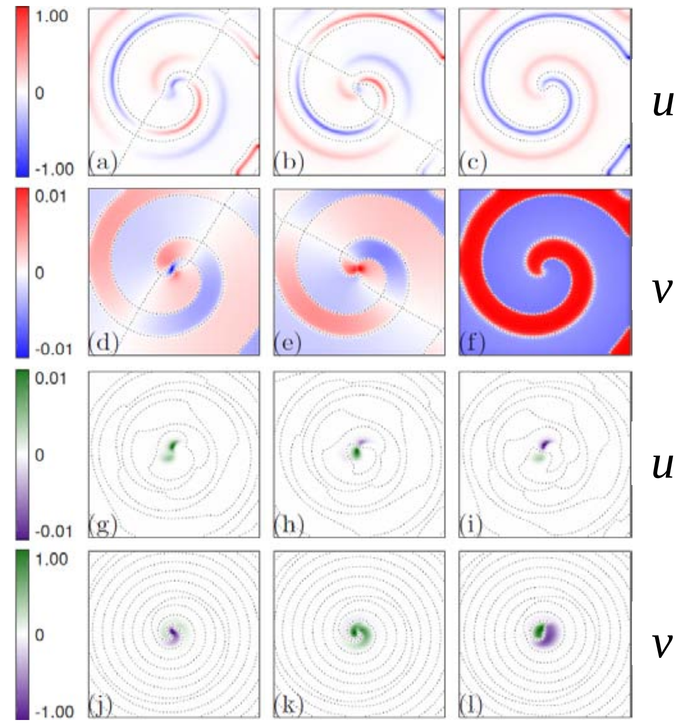


(a) (b) (c) (d)

Asymptotic freedom

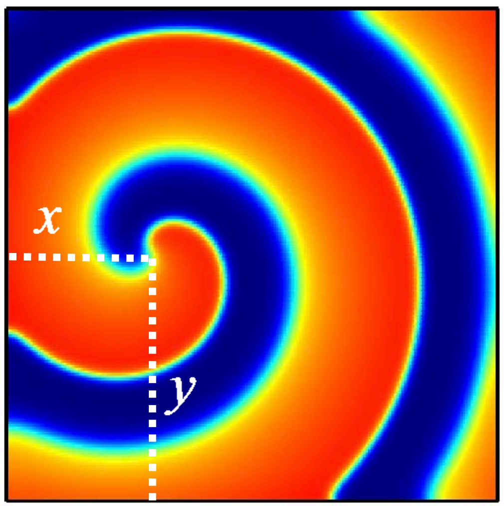


Goldstone modes



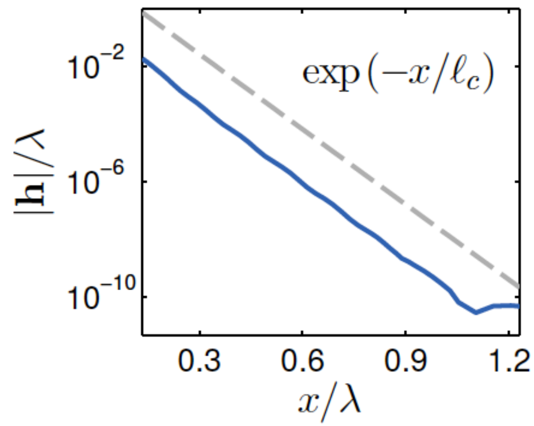
Response functions

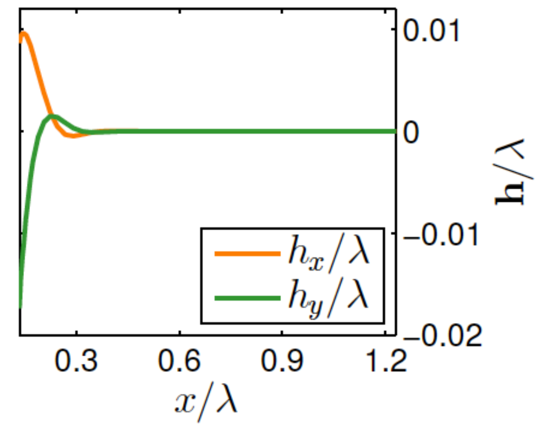
Interaction of cores with boundaries

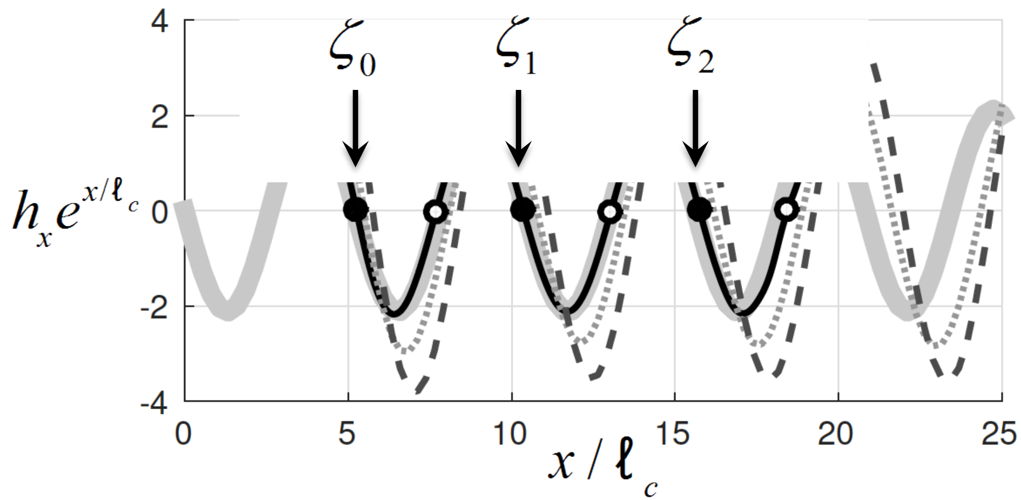


$$\mathbf{x}^{n+1} = \mathbf{x}^{n} + \mathbf{h}(\mathbf{x}^{n}),$$

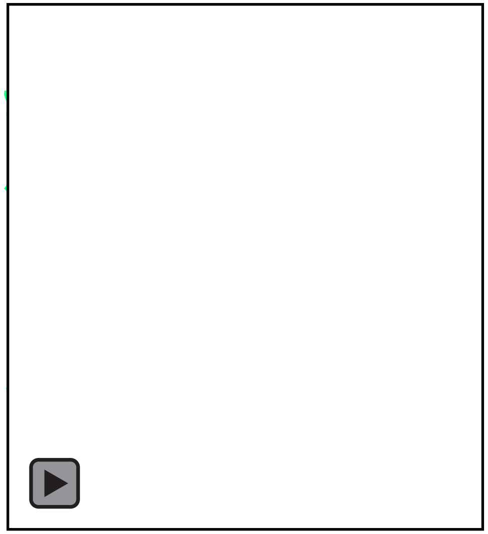
$$\mathbf{x}^{n} = \mathbf{x}(nT)$$

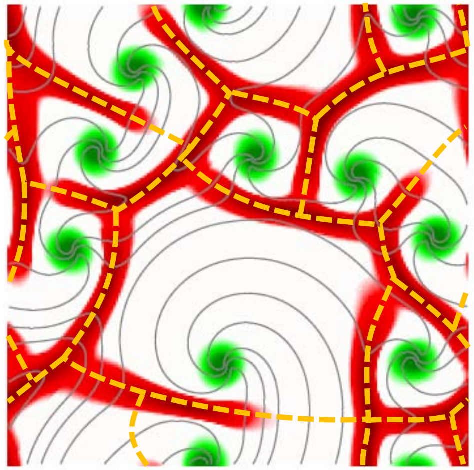


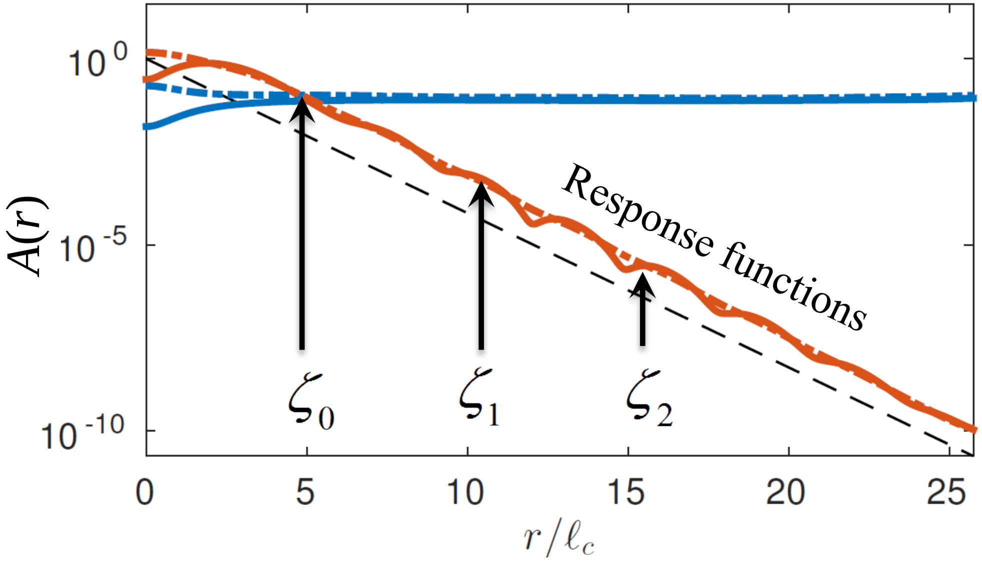




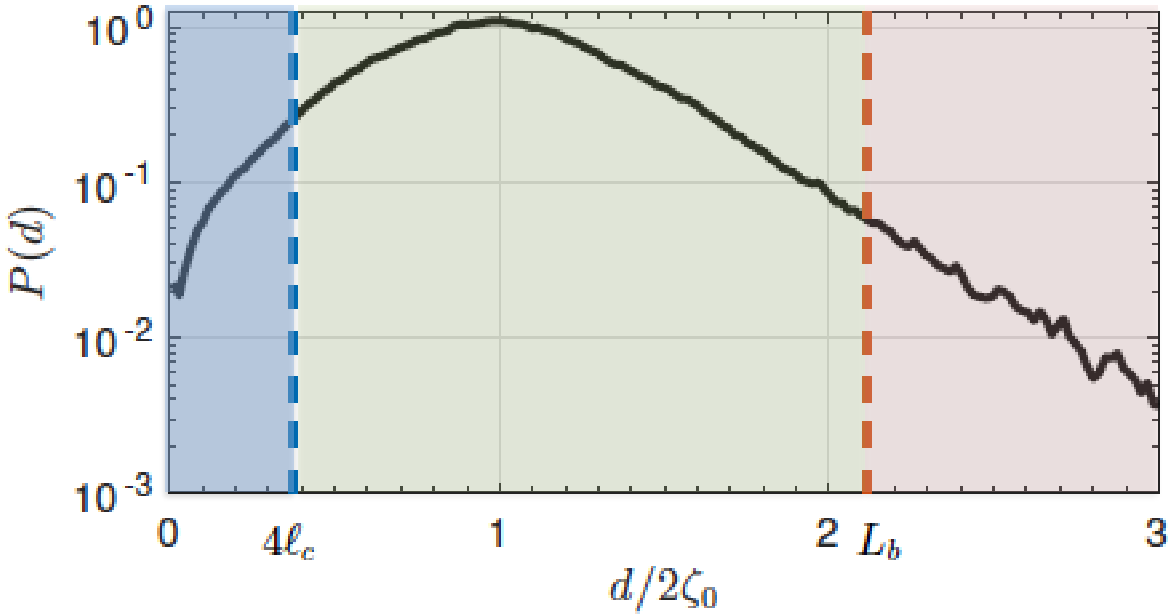
Core-core interaction







Core-core separation (& tile size)

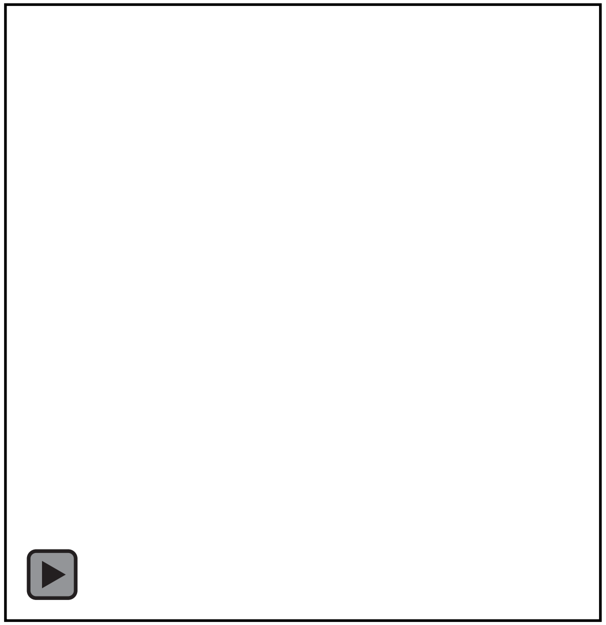


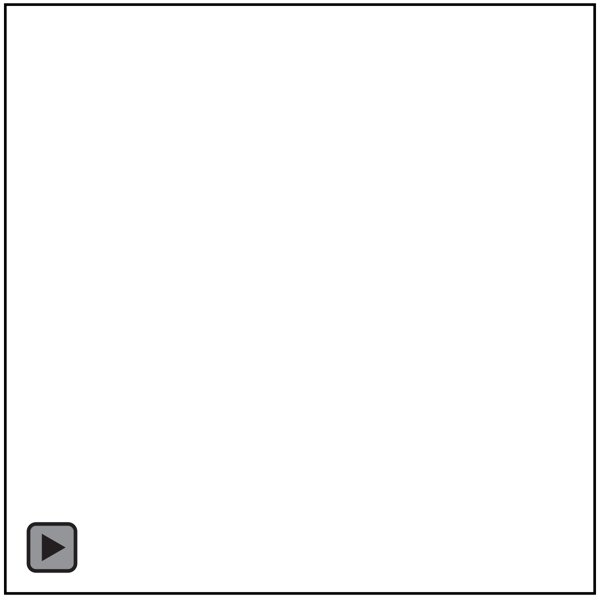
Virtual pairs

Core drift/ meander/ collapse

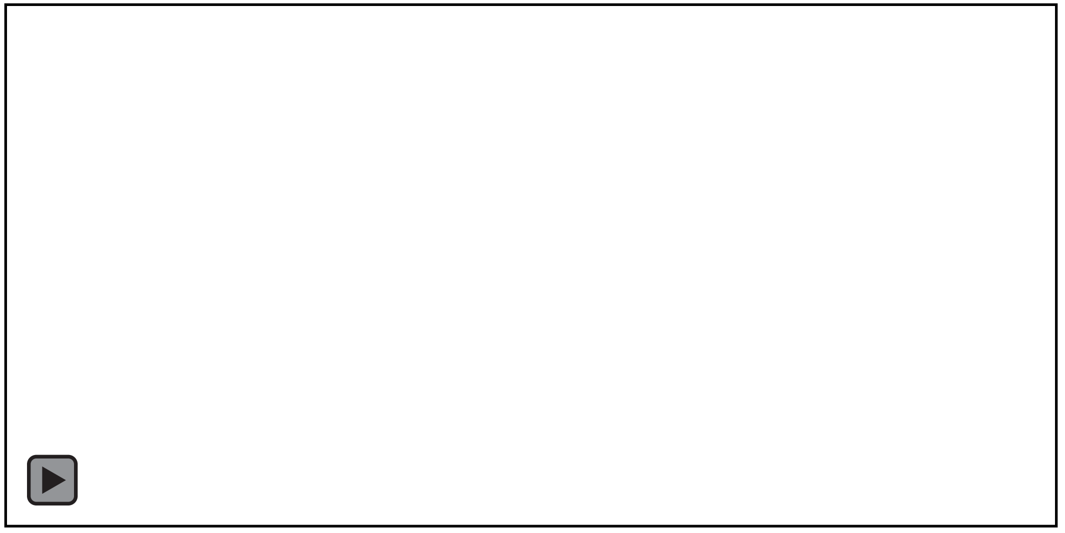
Break-up

Core meander

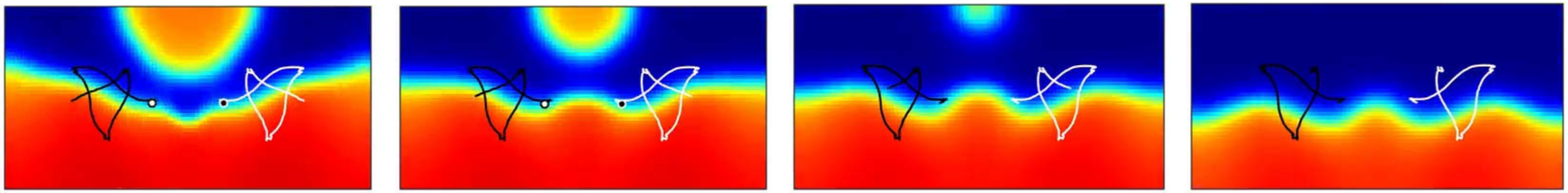


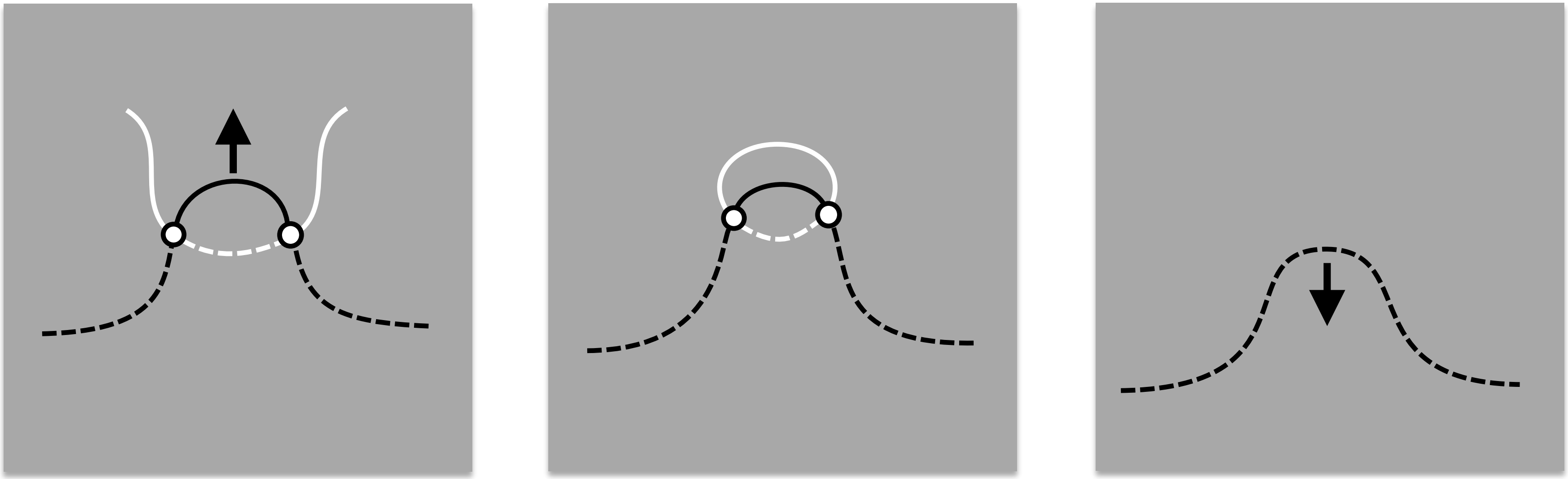


Wave collapse



Wave collapse





The mechanism of spiral chaos

- Slow dynamics (of tiles)
  - ✓ The tiles are generally of different size
  - ✓ The frequencies of spirals differ
  - ✓ The tiles boundaries drift (slowly)
  - ✓ Small (fast) spirals grow at the expense of big (slow) ones

- Fast dynamics (of spirals)
  - ✓ Large spirals ($L > L_b$) break up due to alternans instability
  - ✓ Small spirals ($L < 4l_c$) survive for less than one period and collapse (with a neighbor)
  - ✓ Medium size spirals ($4l_c < L < L_b$) interact with each other in nontrivial ways

Implications for fluid turbulence

- No *global* ECS on domains much larger than the relevant coherence length, no matter what the physics is
- Need to look for localized solutions that respect *local* Euclidean symmetries and their interactions
- Coherence length can be defined with the help of *adjoint* eigenfunctions (to-do for fluid dynamicists)
- Spatial *correlations* may decay exponentially even when solutions do not
- Does exponential decay of *velocity/energy* imply short spatial correlations? What about *pressure*?