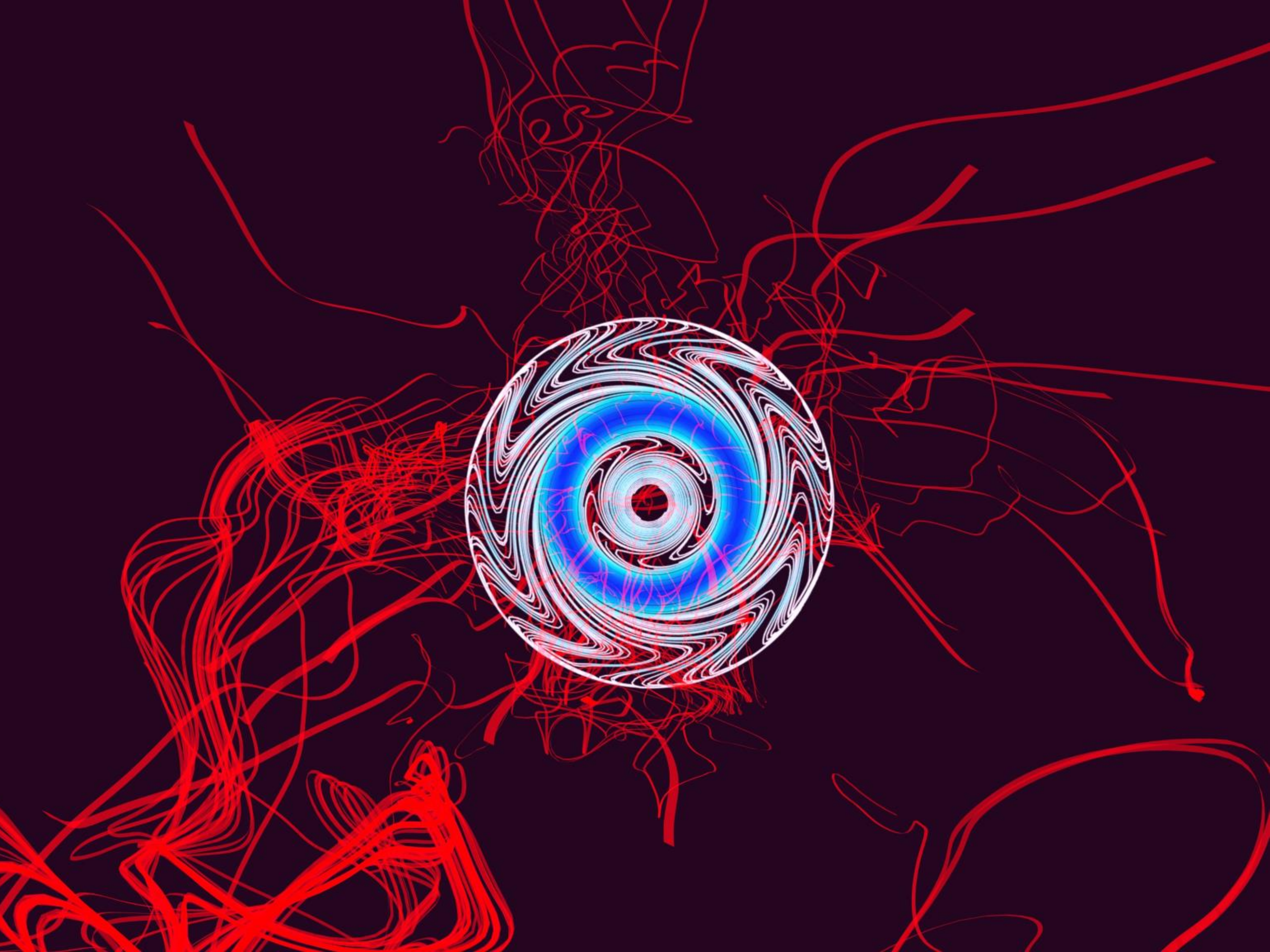
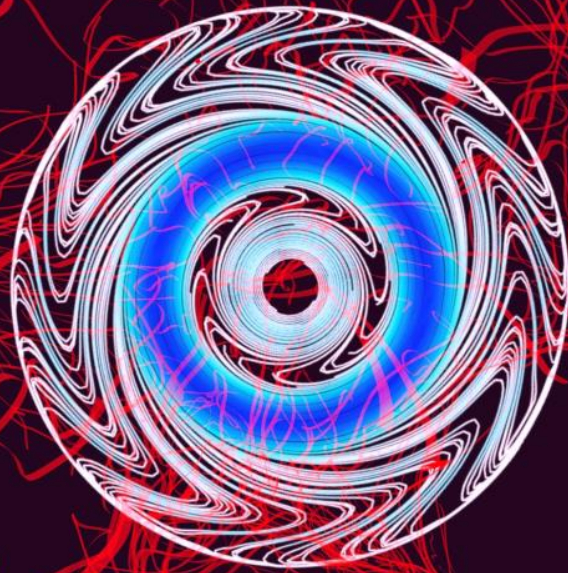


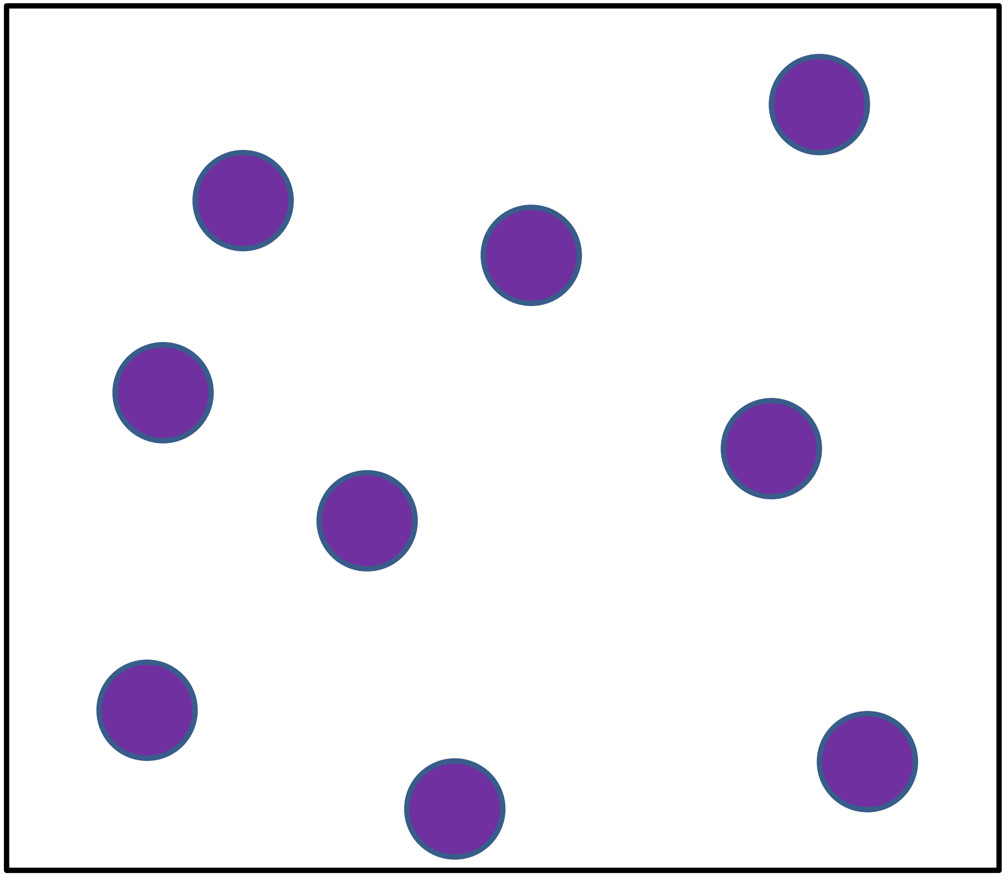
Statistical mechanics of the phase transition to turbulence: zonal flows, ecological collapse and extreme value statistics



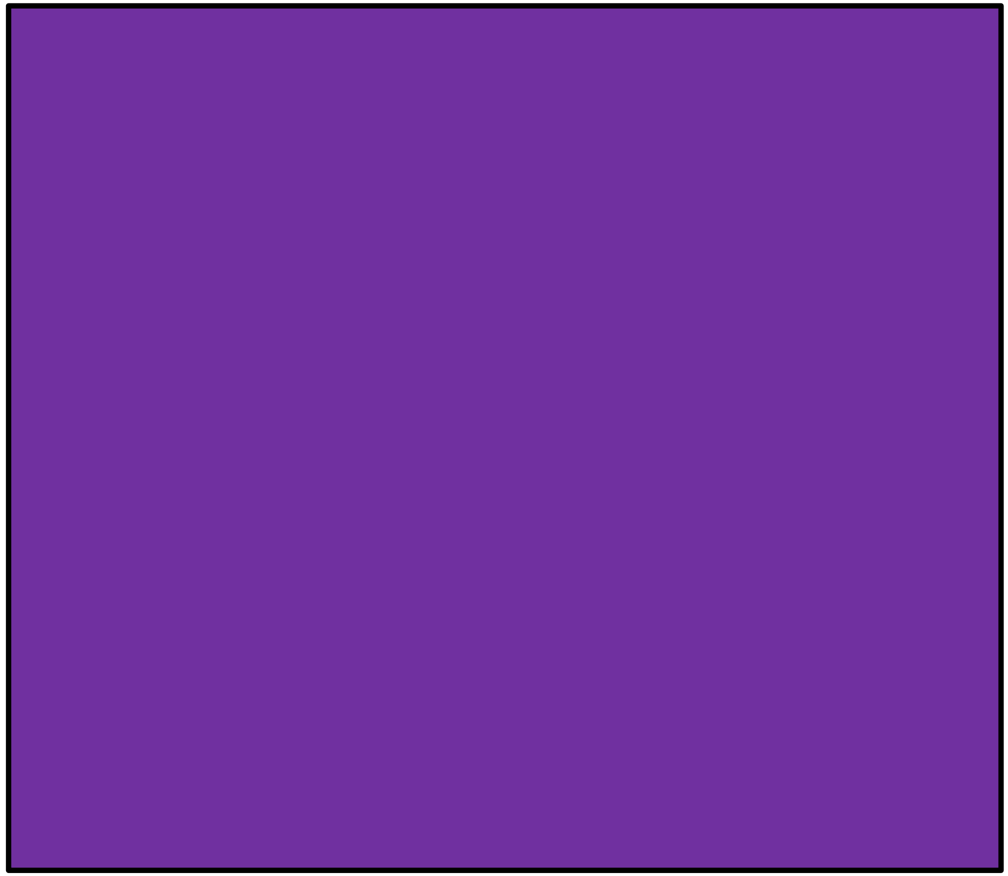
Hong-Yan Shih, Tsung-Lin Hsieh, Nigel Goldenfeld

~~Grudgingly~~ Partially supported by NSF-DMR-1044901

Nature Physics, March 2016: Advance Online Publication 15 Nov 2015



Deterministic classical mechanics of many particles in a box ➔ statistical mechanics



Deterministic classical mechanics of infinite number of particles in a box

= Navier-Stokes equations for a fluid

➔ statistical mechanics

$$\nabla \cdot \mathbf{u} = 0$$

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \mu \nabla^2 \mathbf{u}$$

Deterministic classical mechanics of infinite number of particles in a box

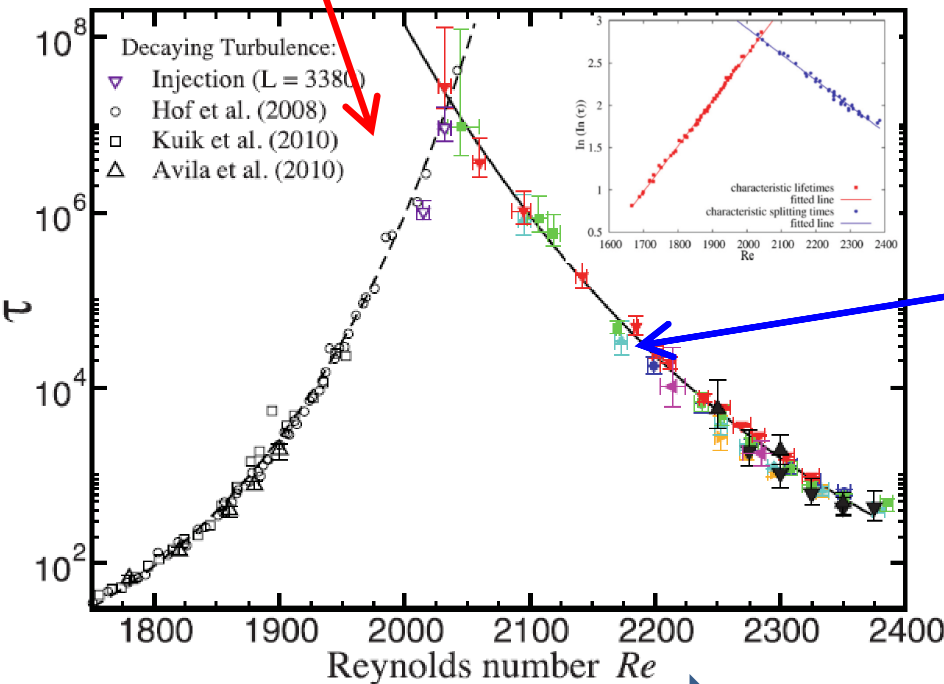
= Navier-Stokes equations for a fluid

➔ statistical mechanics

Fluid in a pipe near onset of turbulence

Turbulent puff lifetime

Log (lifetime)



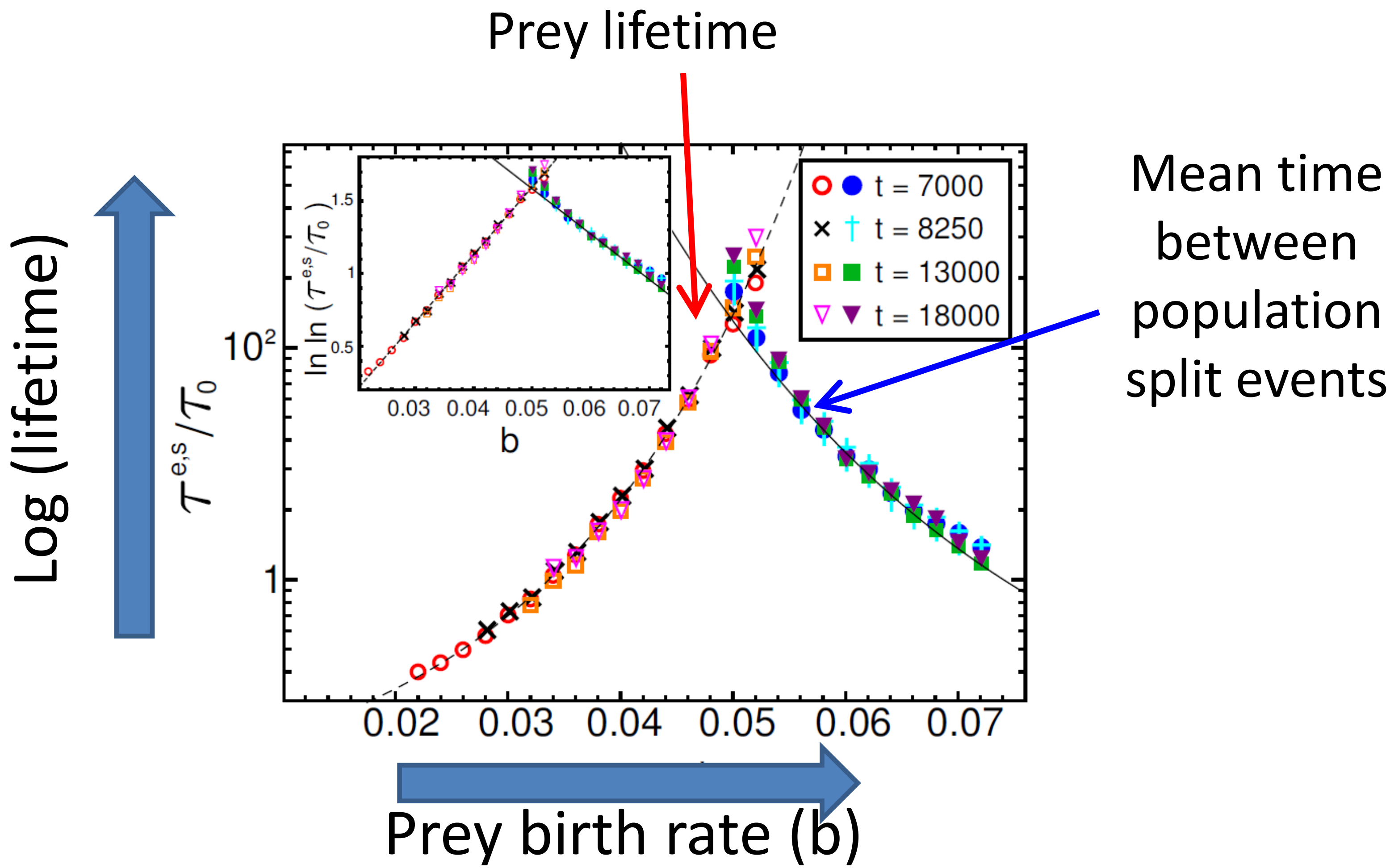
Mean time between puff split events

Reynolds number

Super-exponential scaling: $\frac{\tau}{\tau_0} \sim \exp(\exp \mathrm{Re})$

Avila *et al.*, *Science* **333**, 192 (2011)   Song *et al.*, *J. Stat. Mech.* 2014(2**)**, P020010

Predator-prey ecosystem in a pipe near extinction

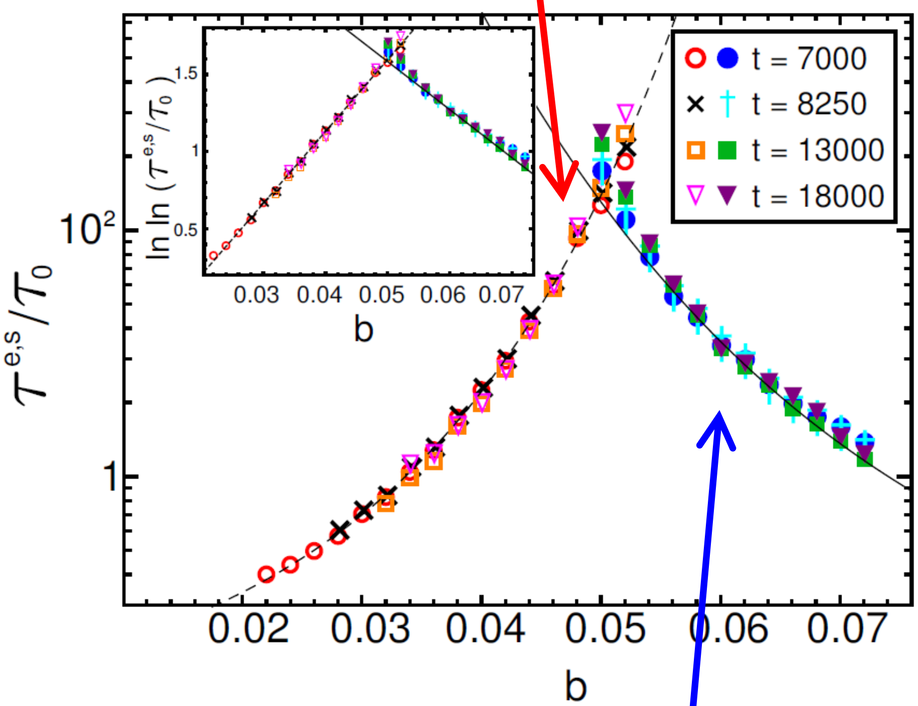


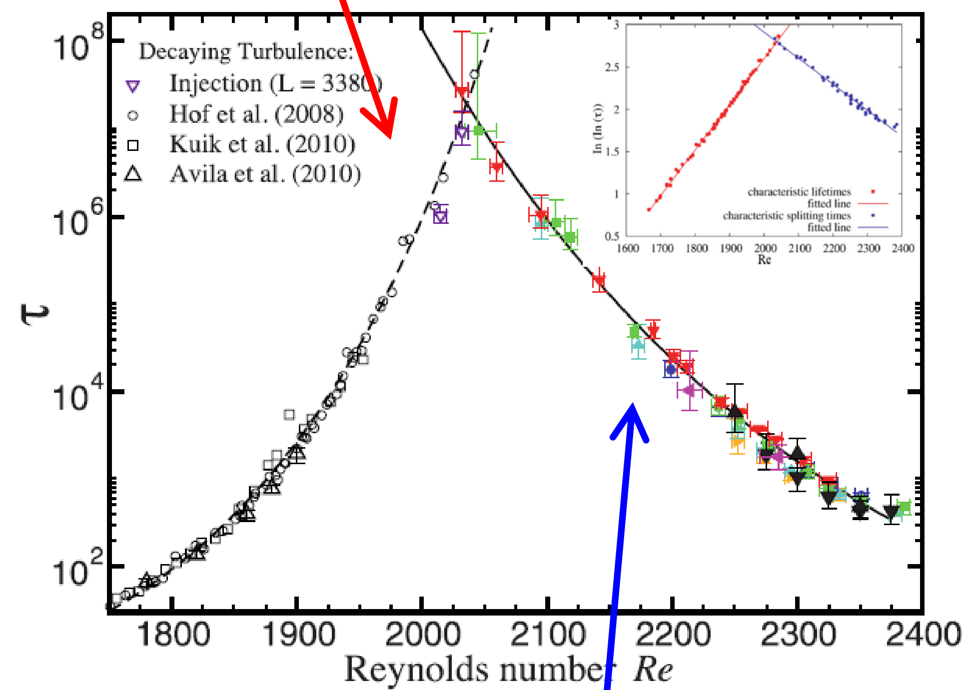
Super-exponential scaling: $\frac{\tau}{\tau_0} \sim \exp(\exp \mathrm{b})$

Predator-prey vs. transitional turbulence

Prey lifetime

Turbulent puff lifetime





Mean time between population split events

Mean time between puff split events

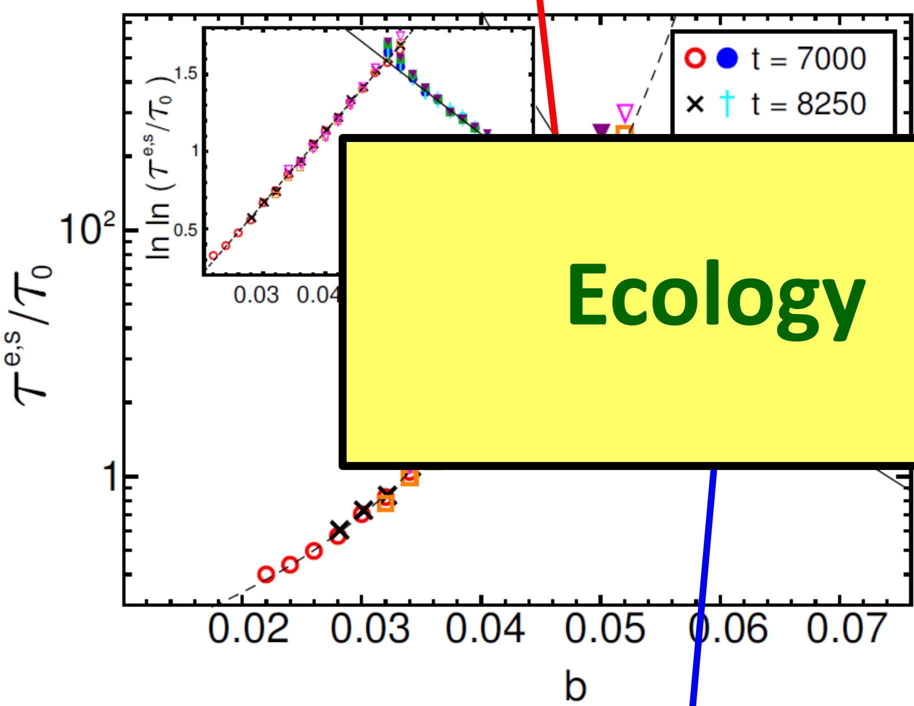
Avila *et al.*, *Science* **333**, 192 (2011)

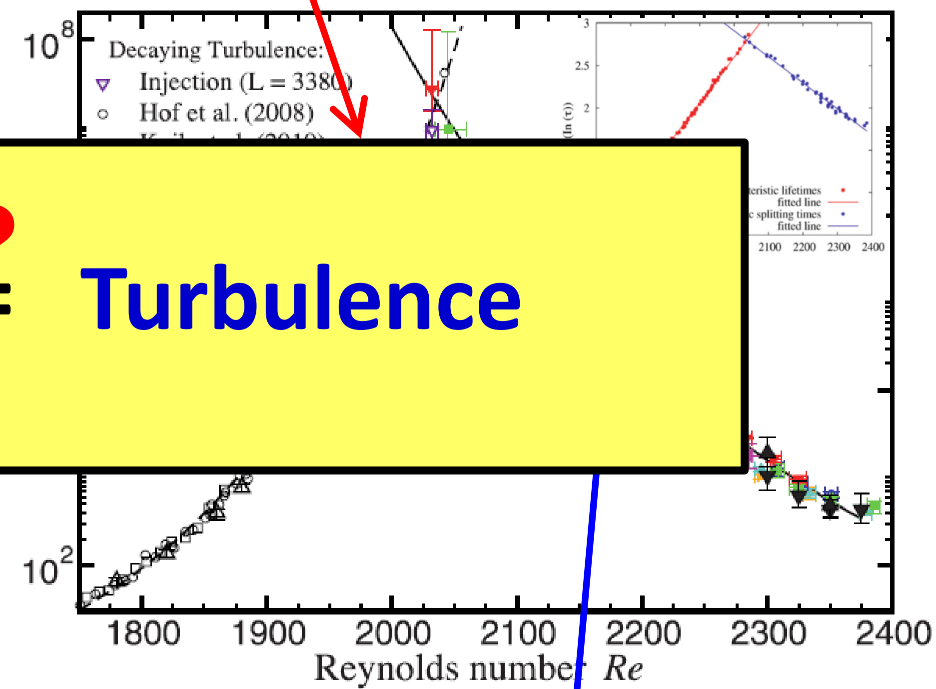
Song *et al.*, *J. Stat. Mech.* 2014(2**)**, P020010

Predator-prey vs. transitional turbulence

Prey lifetime

Turbulent puff lifetime





Ecology $\overset{?}{=}$ Turbulence

Mean time between population split events

Mean time between puff split events

Avila *et al.*, *Science* **333**, 192 (2011)

Song *et al.*, *J. Stat. Mech.* 2014(2**)**, P020010

Interpretation of super-exponential

Super-exponential scaling: $\frac{\tau}{\tau_0} \sim \exp(\exp \mathrm{Re})$

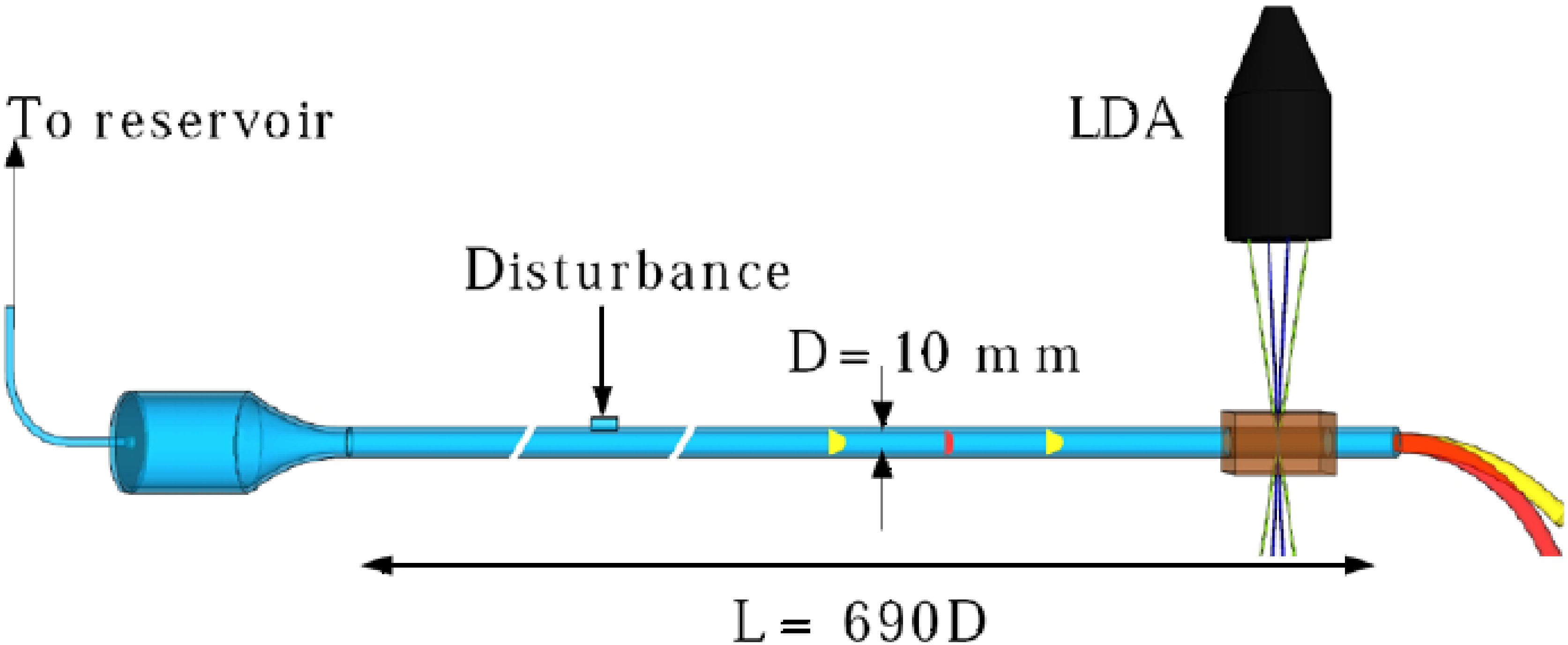
- Lifetime of turbulence is always finite
  - No critical Reynolds number for laminar-turbulence transition
- Naïve interpretation: turbulence is long-lived transient
  - Such phenomena seen in simplified models of spatio-temporal complex patterns

Crutchfield & Kaneko, PRL (1988)

Hof et al., Nature (2006)

Precision measurement of turbulent transition

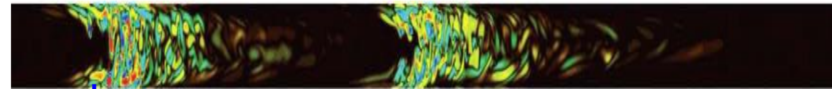
Q: will a puff survive to the end of the pipe?



Many repetitions ➔ survival probability P(Re, t)

Hof et al., *PRL* **101**, 214501 (2008)

Phase diagram of pipe flow



Single puff spontaneously decays

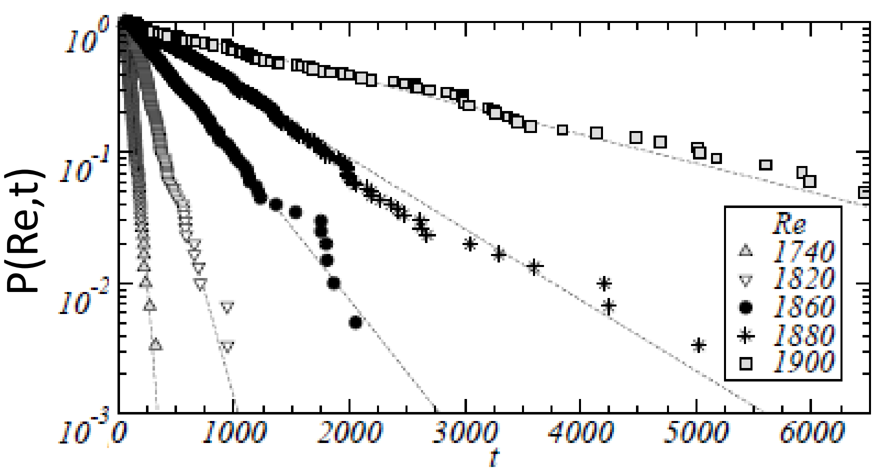
Splitting puffs

laminar	metastable puffs	spatiotemporal intermittency	expanding slugs

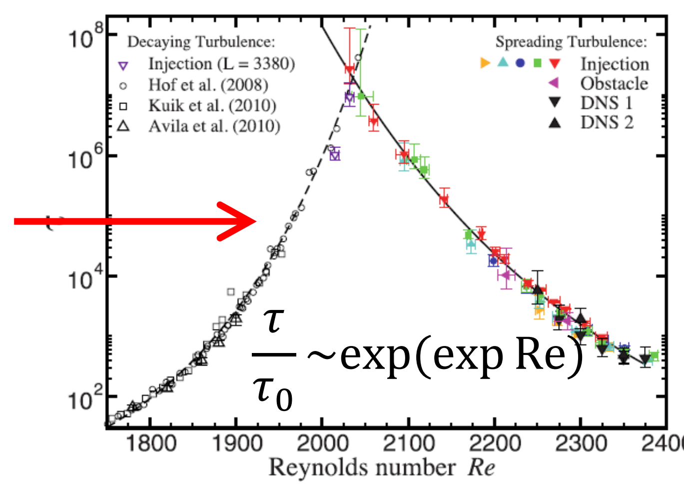
Re: 1775, 2100, 2500

Survival probability $P(\mathrm{Re}, t) = e^{-\frac{t-t_0}{\tau(\mathrm{Re})}}$

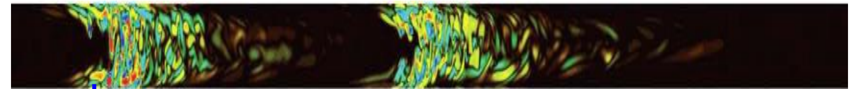
Avila *et al.*, *Science* **333**, 192 (2011)
Hof *et al.*, *PRL* **101**, 214501 (2008)



Avila *et al.*, (2009)



Phase diagram of pipe flow



Single puff spontaneously decays

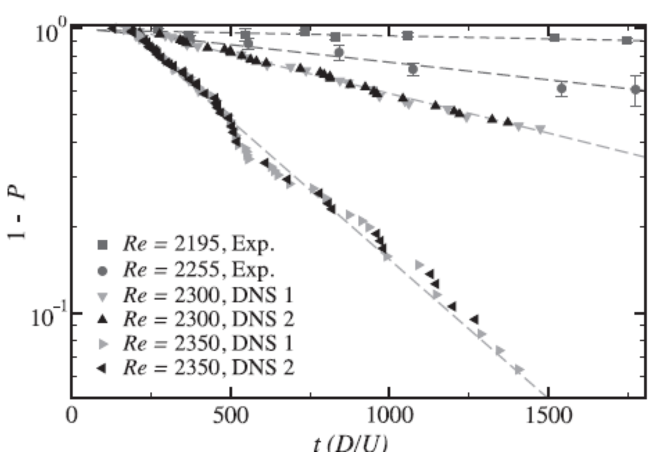
Splitting puffs

laminar	metastable puffs	spatiotemporal intermittency	expanding slugs

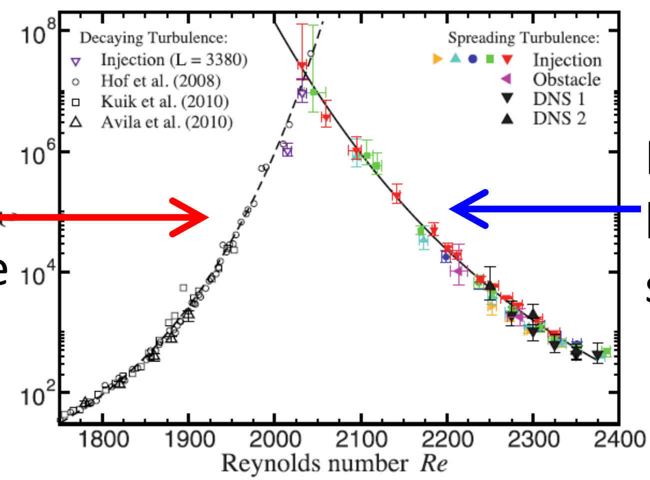
Re 1775 2100 2500

Survival probability $1 - P(\mathrm{Re}, t) = e^{-\frac{t - t_0}{\tau(\mathrm{Re})}}$

Avila *et al.*, *Science* **333**, 192 (2011)
Hof *et al.*, *PRL* **101**, 214501 (2008)

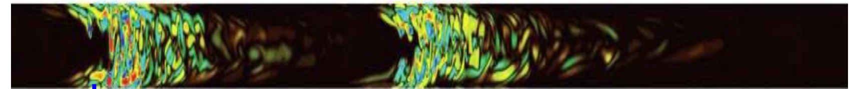


Puff lifetime



Mean time between split events

Phase diagram of pipe flow



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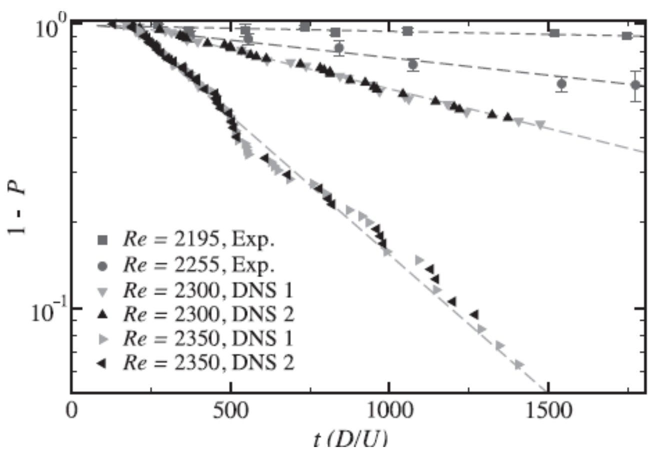
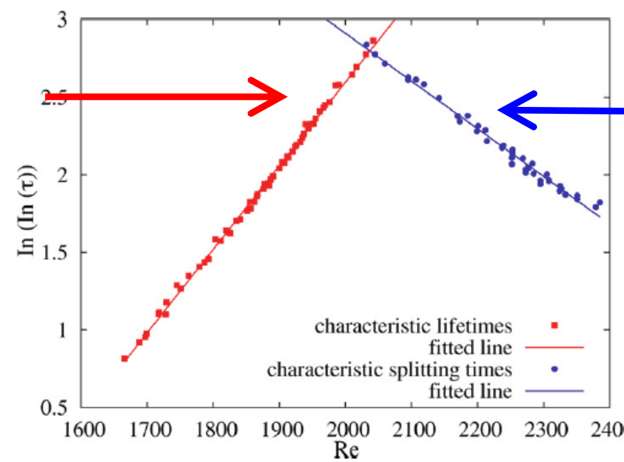
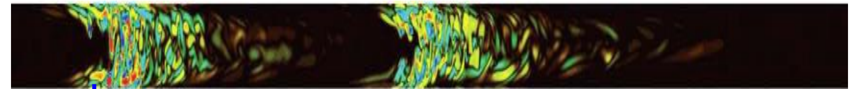


Puff lifetime



Mean time between split events

Phase diagram of pipe flow



Single puff spontaneously decays

Splitting puffs

laminar

metastable

spatiotemporal

expanding

Super-exponential scaling:

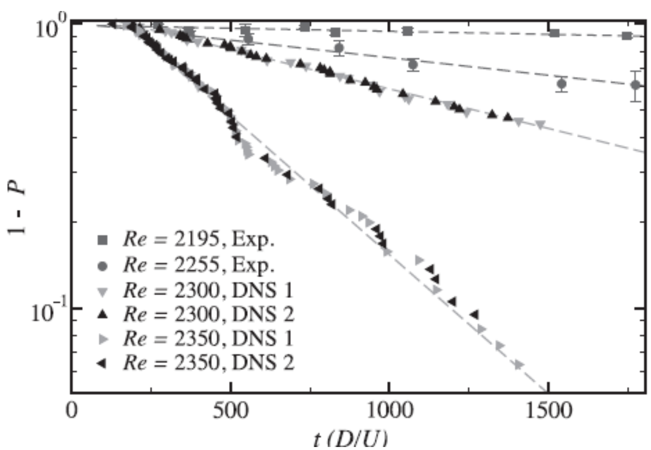
$$\frac{\tau}{\tau_0} \sim \exp(\exp \mathrm{Re})$$

Su

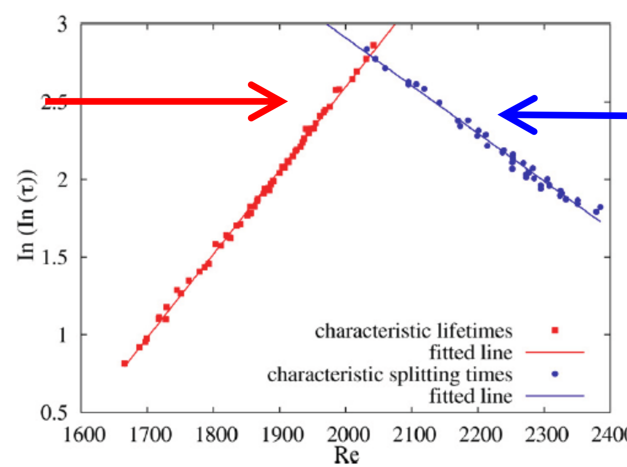
1)

3)

Song *et al.*, *J. Stat. Mech.* 2014(2), P020010

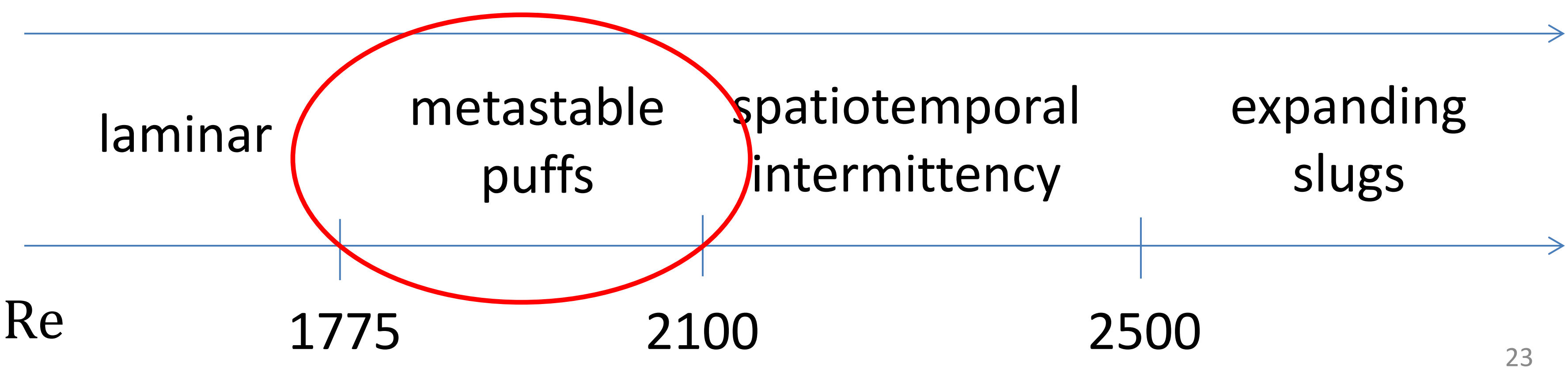


Puff lifetime



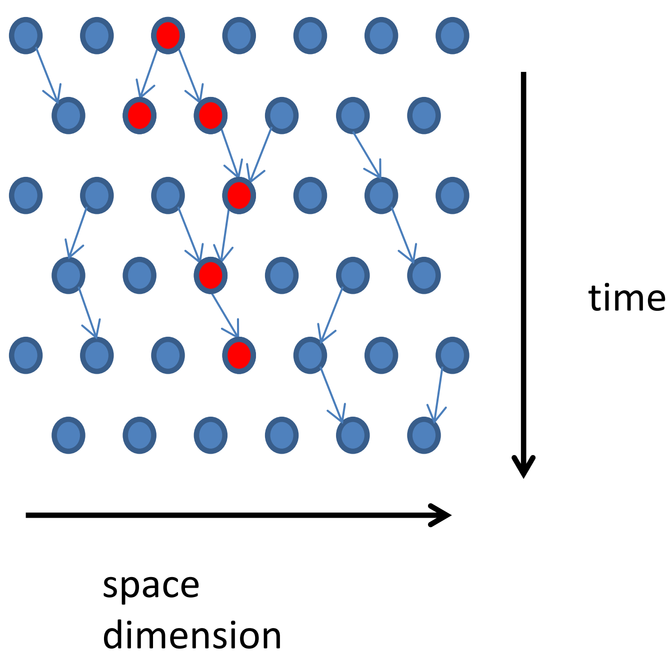
Mean time between split events

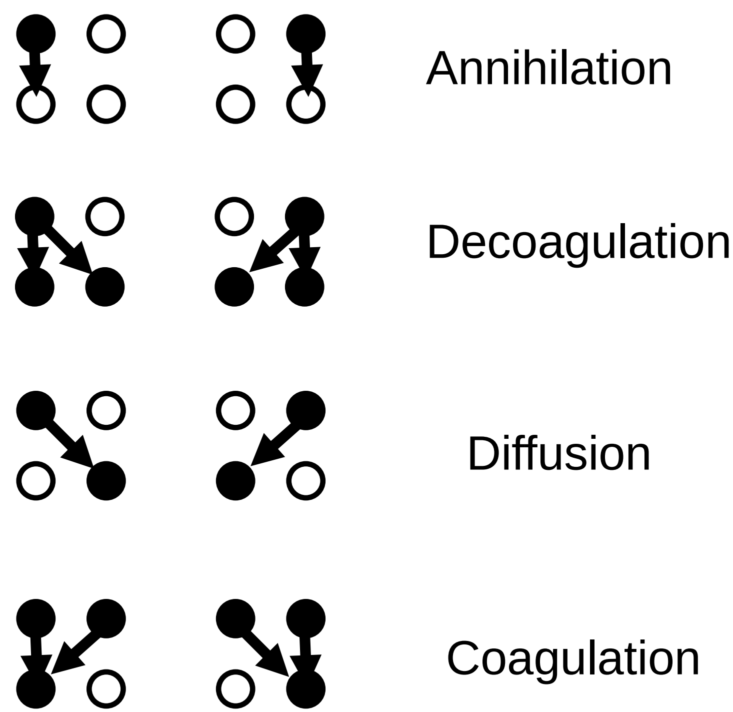
MODEL FOR METASTABLE TURBULENT PUFFS



DP & the laminar-turbulent transition

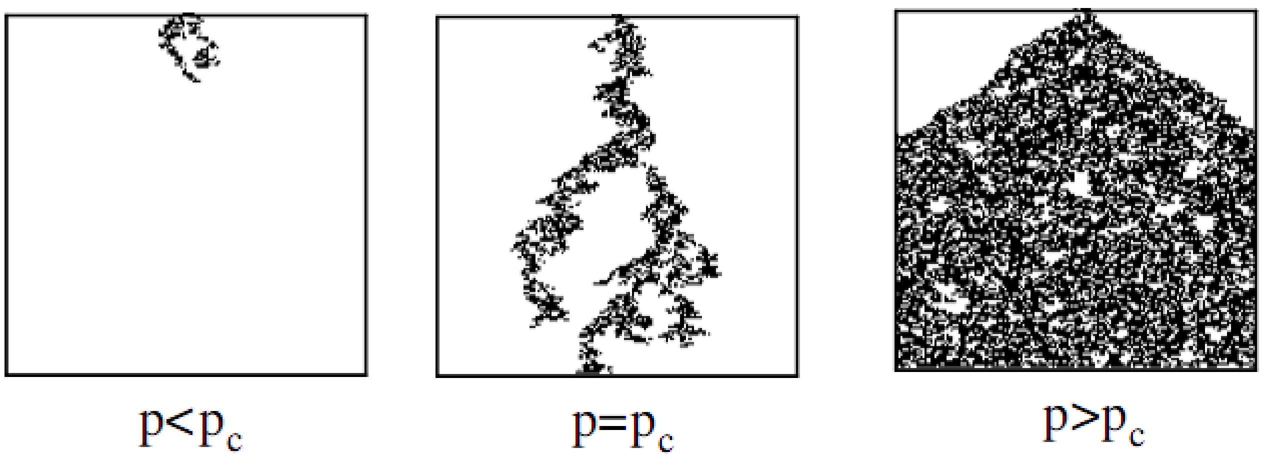
- Turbulent regions can spontaneously relaminarize (go into an absorbing state).
- They can also contaminate their neighbourhood with turbulence. (Pomeau 1986)





Directed Percolation Transition

- A continuous phase transition occurs at p_c.



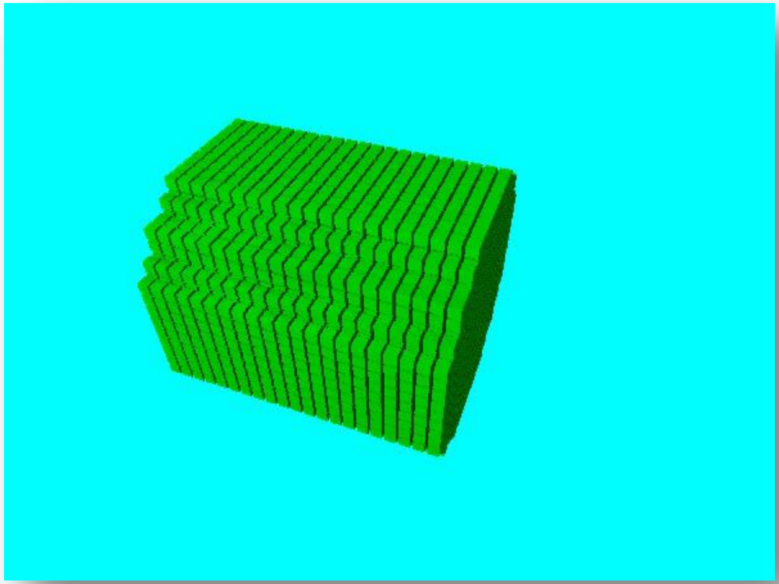
Hinrichsen (Adv. in Physics 2000)

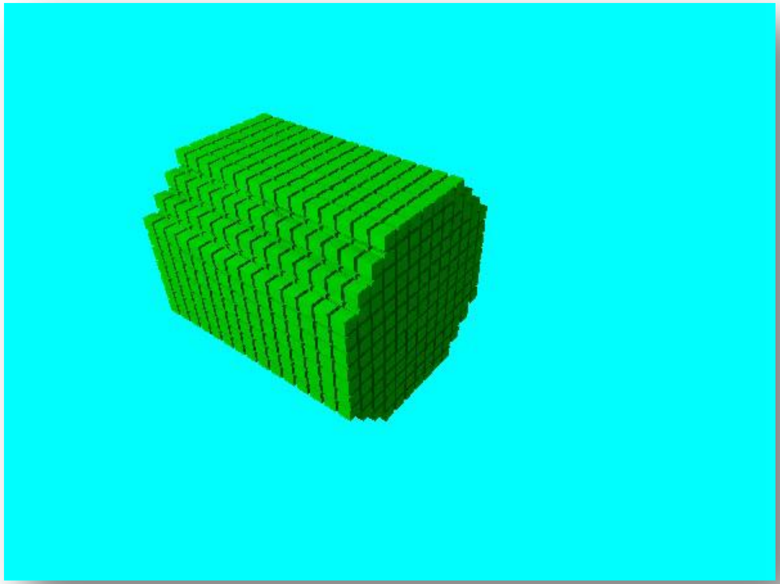
- Phase transition characterized by universal exponents:

$$\rho \sim (p - p_c)^{\beta} \qquad \xi_{\perp} \sim (p - p_c)^{-\nu_{\perp}} \qquad \xi_{\parallel} \sim (p - p_c)^{-\nu_{\parallel}}$$

to DP models

DP in 3 + 1 dimensions in pipe





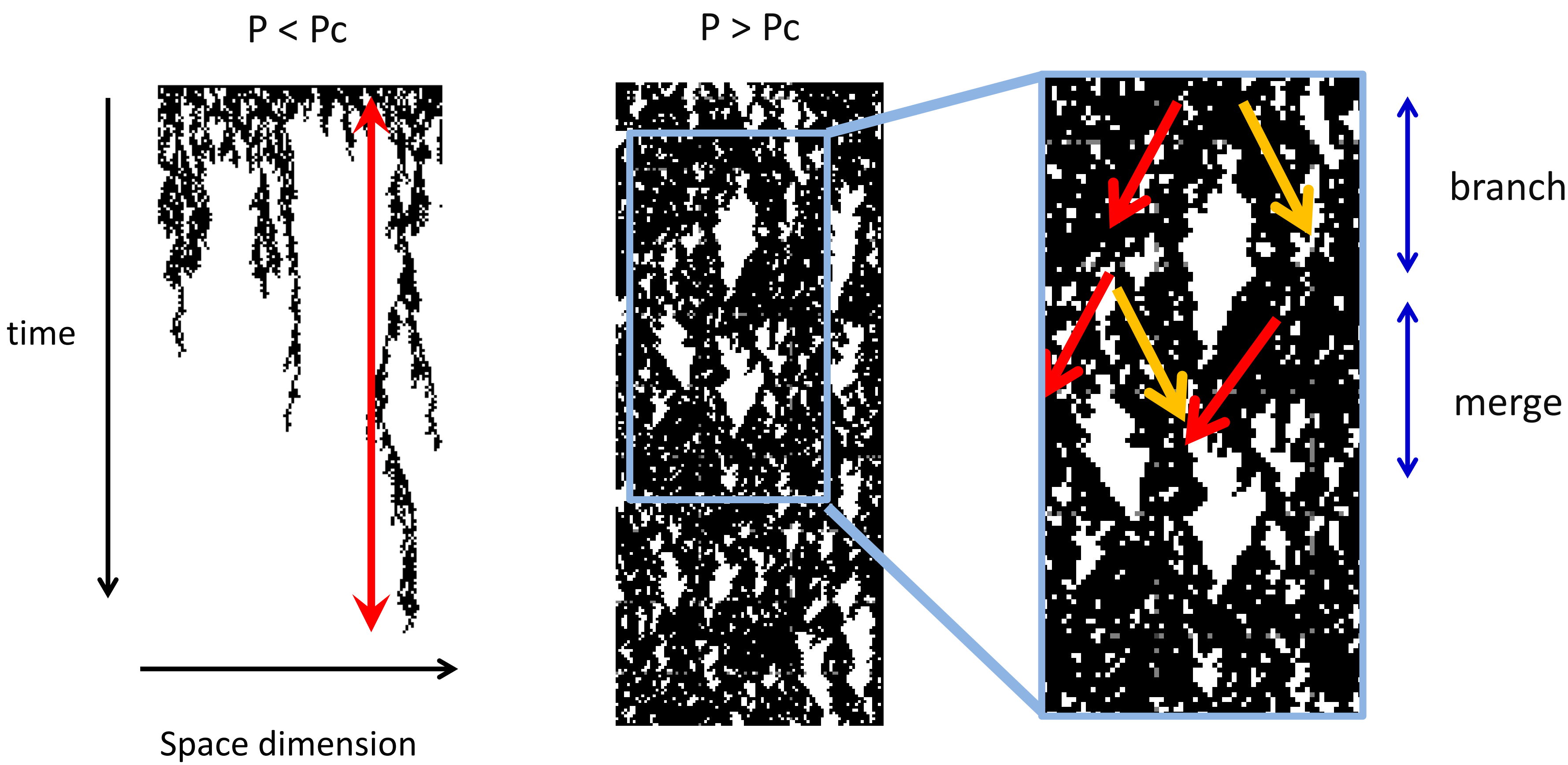
Puff decay

Slug spreading

Sipos and Goldenfeld, *PRE* **84**, 035304(R) (2011)

1+1 Directed Percolation

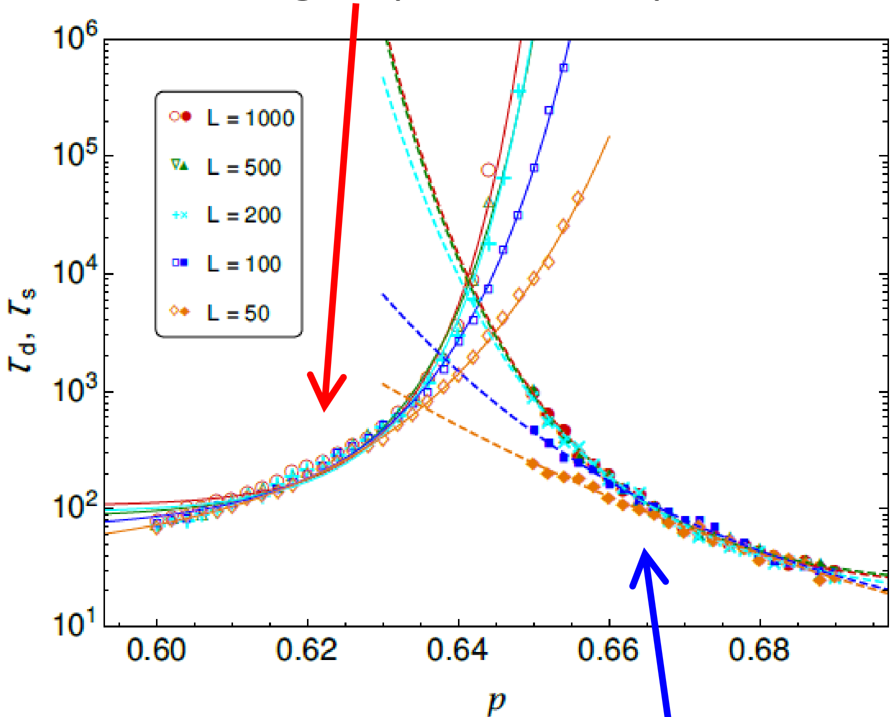
- Turbulent puff lifetime → longest percolation path
- Turbulent puff splitting time → longest length of empty site



Directed percolation vs. transitional turbulence

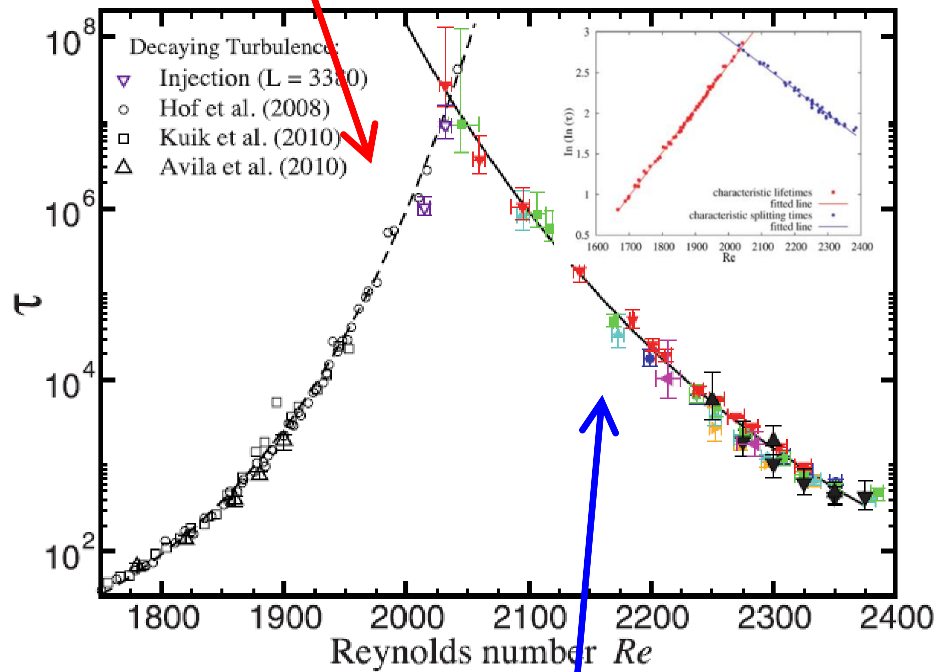
Survival probability $P(\mathrm{Re}, t) = e^{-\frac{t - t_0}{\tau(\mathrm{Re})}}$

Longest percolation path



Longest length of empty site

Turbulent puff lifetime



Mean time between puff split events

Avila *et al.*, (2011)   Song *et al.*, (2014)

Super-exponential scaling and extreme statistics

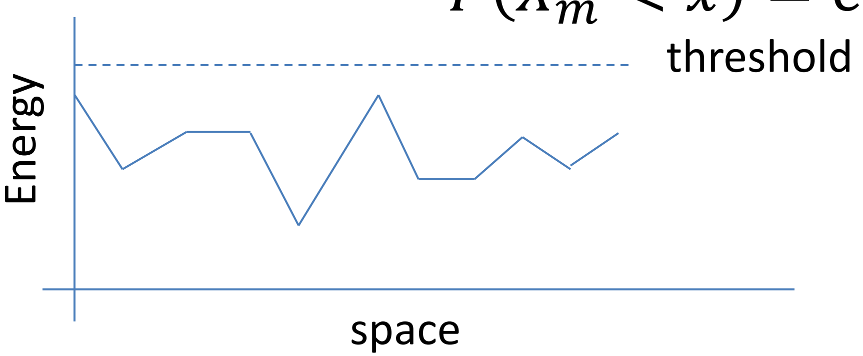
- Consider identical and independently distributed random variables X_i whose distribution decays sufficiently fast at infinity
- Their mean $\bar{X} \propto \sum_i X_i$ is normally distributed (Central limit theorem).

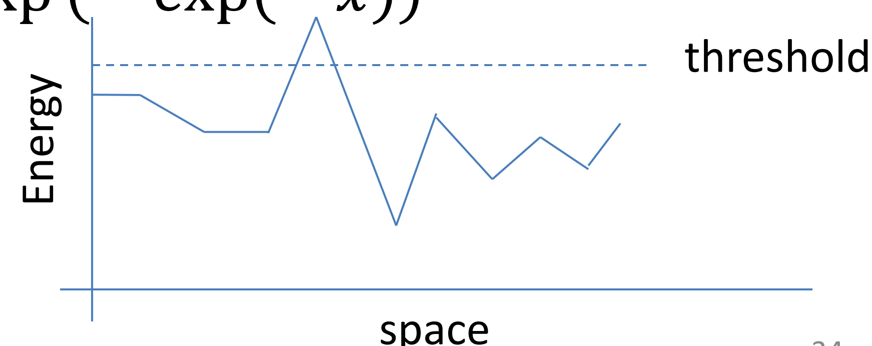
more on FT

- Their maximum $X_m \propto \max X_i$ is distributed according to the Fisher-Tippett type I distribution:

more on FT 2

$$P(X_m < x) = \exp\left(-\exp(-x)\right)$$





Goldenfeld, Gioia, Guttenberg (2010)

Mean lifetime has extreme value distribution

- Probability of puff decay per unit time = probability largest turbulent energy fluctuation exceeds a Re-dependent threshold

- Taylor expand near critical Re.

there is a probability p that the puff will be suppressed within each time interval τ_0. Then, the lifetime statistics will be Poisson. The probability P that turbulence persists to a time t after becoming established at a time t_0 is $P=(1-p)^M$, where the number of intervals is $M=(t-t_0)/\tau_0$. Therefore

$$\ln(P) = M\ln(1-p) = \frac{1}{\tau_0}(t-t_0)\ln(1-p), \tag{1}$$

and so it follows that $\tau_0/\tau=-\ln(1-p)$. Since $1 \gg p > 0$, we can estimate $\ln(1-p)=-p$ and therefore express the lifetime in the form

$$\tau = \tau_0/p, \tag{2}$$

where p depends on Re.

$$F(X) \equiv \int_{-\infty}^{X} P_M(x)dx = \exp\{-\exp[-(X-\mu)/\beta]\}.$$

Thus, $p=F(B_c)$, where B_c is the threshold.

$$\tau = \tau_0 \exp[\exp\{-[B_c^0 + B_c^1(\mathrm{Re}-\mathrm{Re}_0) + O((\mathrm{Re}-\mathrm{Re}_0)^2)]\}]$$

Goldenfeld, Gioia, Guttenberg (2010)

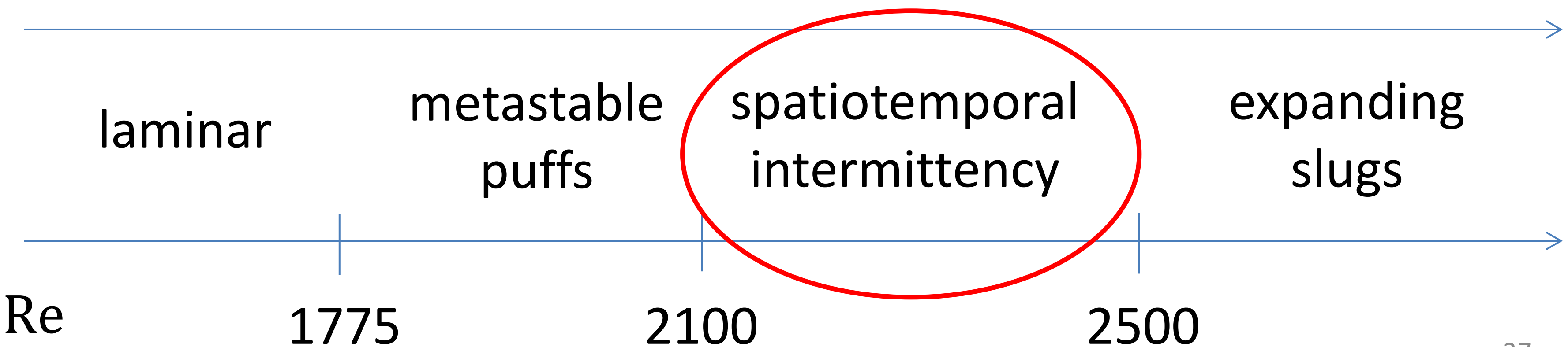
Super-exponential scaling and extreme statistics

- Active state persists until the most long-lived percolating "strands" decay.
  - extreme value statistics
- Why do we not observe the power law divergence of lifetime of DP near transition?
- Close to transition, transverse correlation length diverges, so initial seeds are not independent
  - Crossover to single seed behaviour
  - Asymptotically will see the power law behavior in principle

$$\xi_\perp \sim (p - p_c)^{-\nu_\perp}$$

MODEL FOR SPATIOTEMPORAL INTERMITTENCY

Very complex behavior and we need to understand precisely what happens at the transition, and where the DP universality class comes from.



Logic of modeling phase transitions

Magnets

Electronic structure

↓

Ising model

↓

Landau theory

↓

RG universality class

Logic of modeling phase transitions

Magnets	Turbulence
Electronic structure	Kinetic theory
↓	↓
Ising model	Navier-Stokes eqn
↓	↓
Landau theory	**?**
↓	↓
RG universality class	**?**

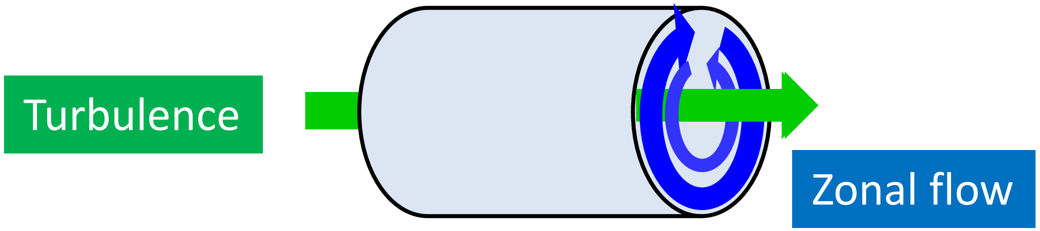
Logic of modeling phase transitions

Magnets		Turbulence
Electronic structure		Kinetic theory
↓		↓
Ising model		Navier-Stokes eqn
↓		↓
Landau theory	⟷	**?**
↓		↓
RG universality class	⟷	**?**

Identification of collective modes at the laminar-turbulent transition

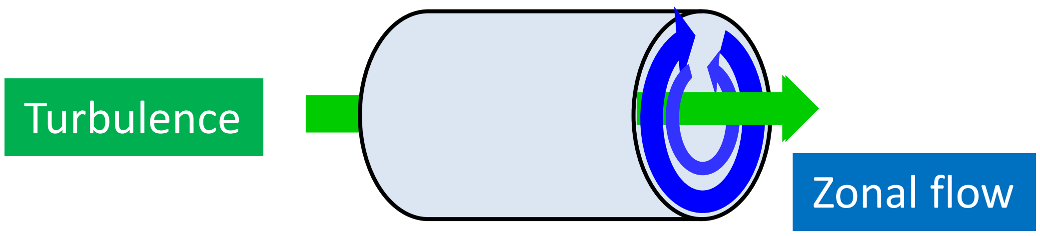
To avoid technical approximations,
we use DNS of Navier-Stokes

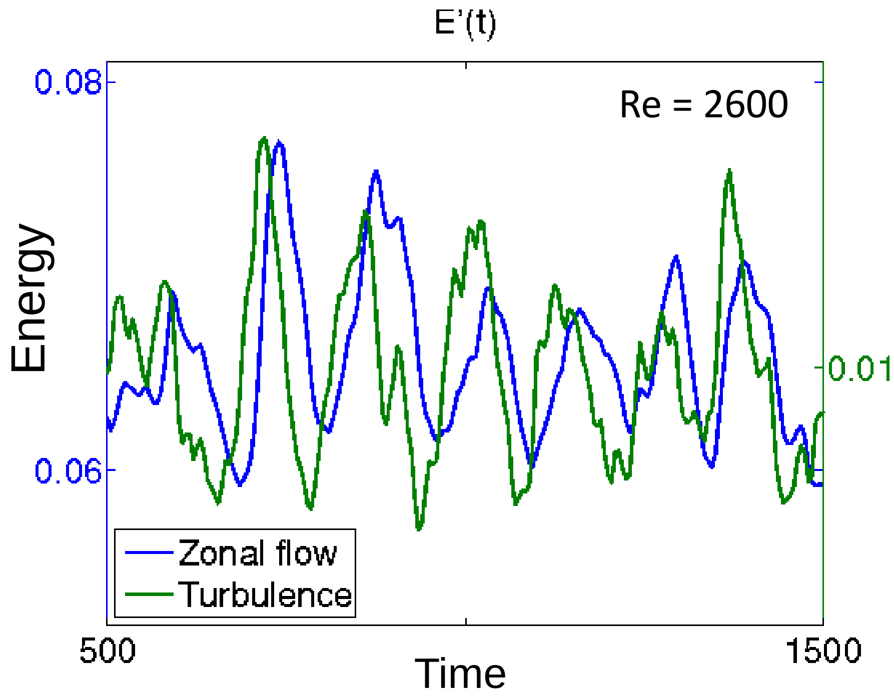
Observation of predator-prey oscillations in numerical simulation of pipe flow

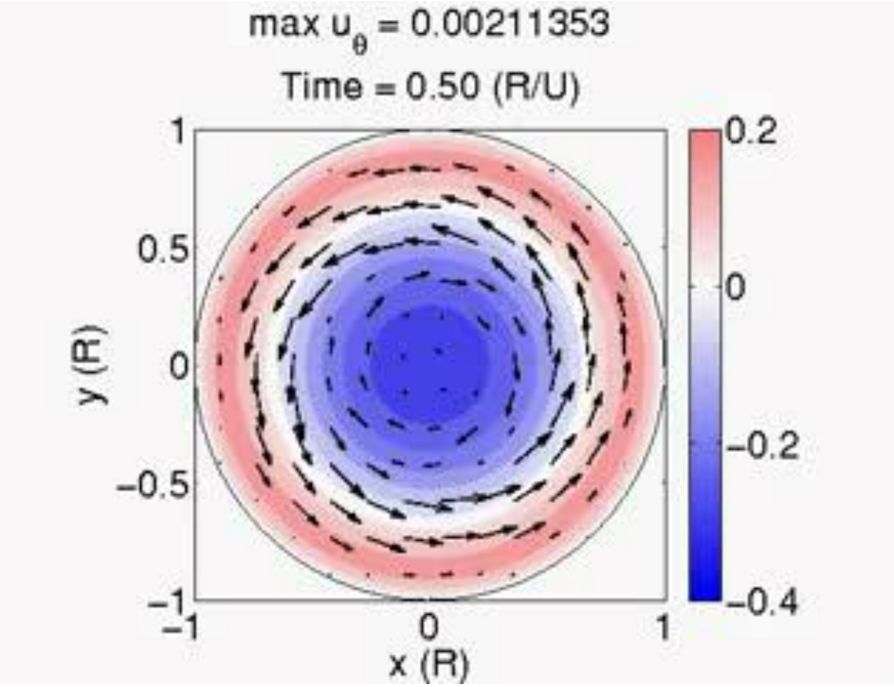


Simulation based on the open source code by Ashley Willis: openpipeflow.org

Predator-prey oscillations in pipe flow







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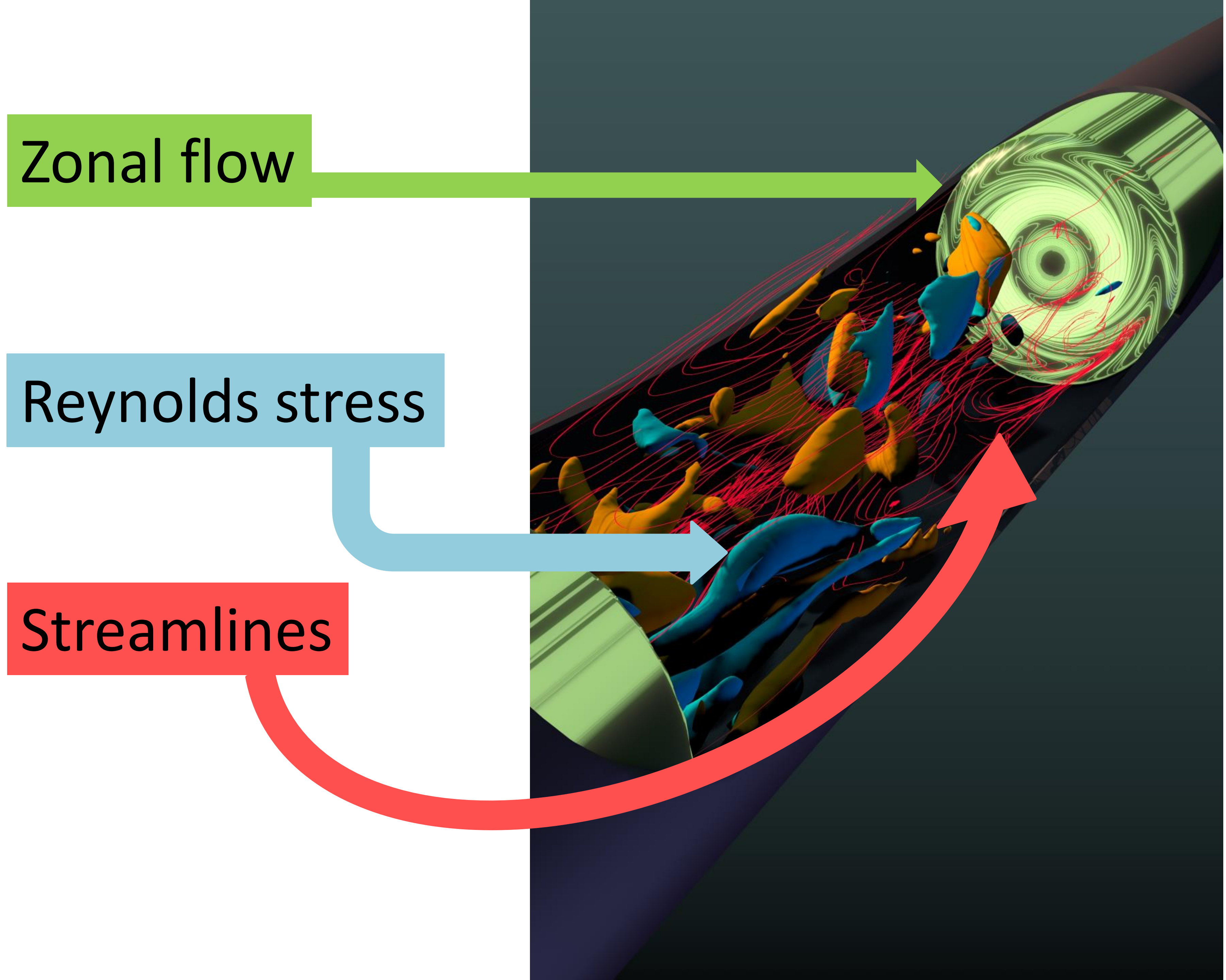
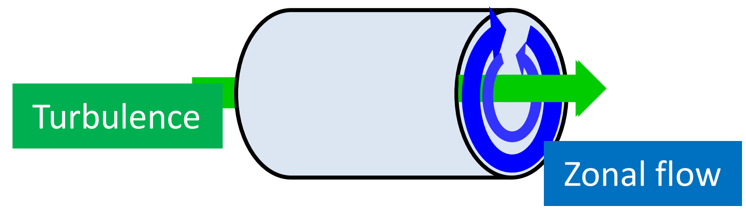

Zonal flow
Reynolds stress
Streamlines

What drives the zonal flow?

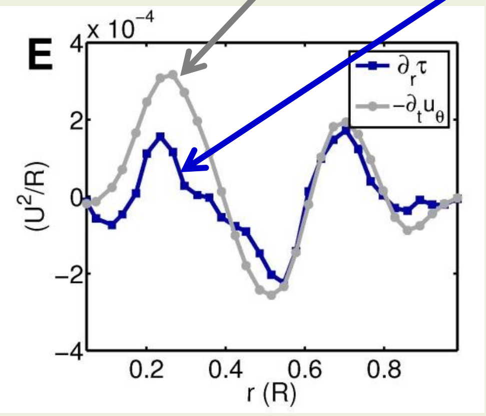
- **Interaction in two fluid model**
  - Turbulence, small-scale (k>0)
  - Zonal flow, large-scale (k=0,m=0): induced by turbulence and creates shear to suppress turbulence

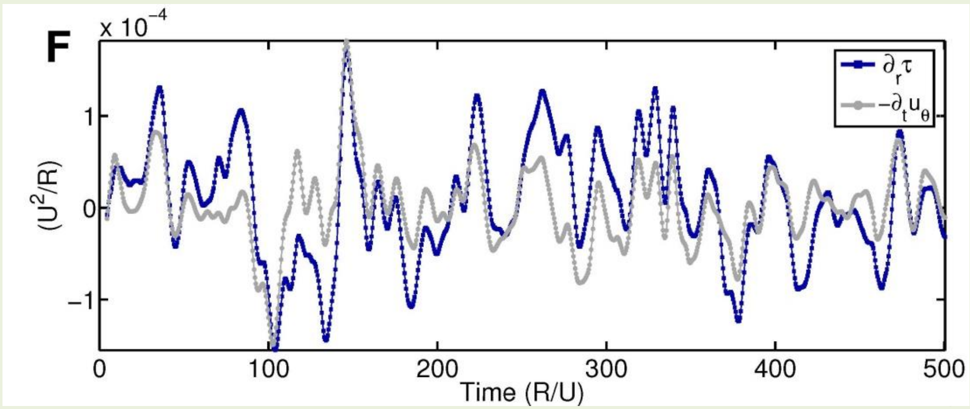


1) Anisotropy of turbulence creates Reynolds stress which generates the mean velocity in azimuthal direction

$$\partial_t \langle v_\theta \rangle = -\partial_r \langle (\widetilde{v}_\theta \cdot \widetilde{v}_r) \rangle - \mu \langle v_\theta \rangle$$

2) Mean azimuthal velocity decreases the anisotropy of turbulence and thus suppress turbulence





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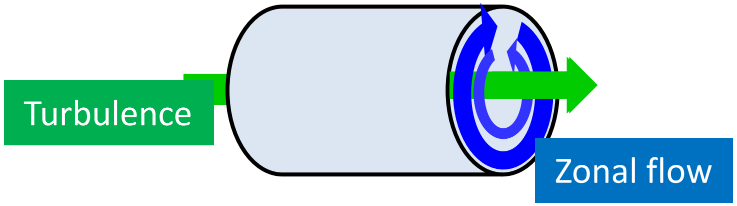

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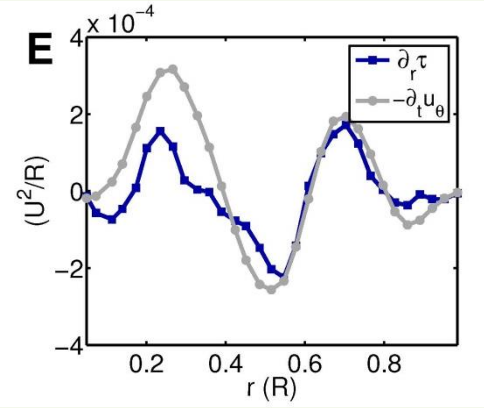

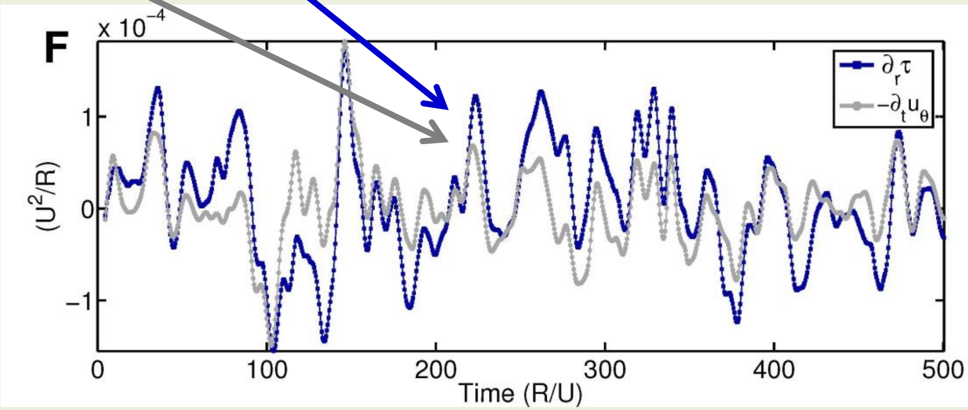

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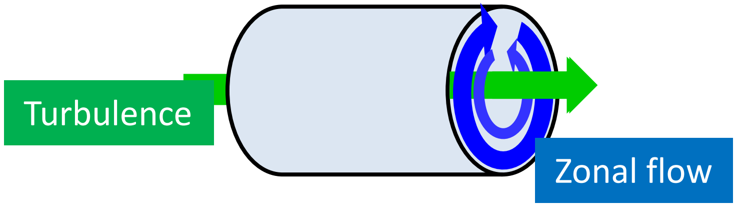

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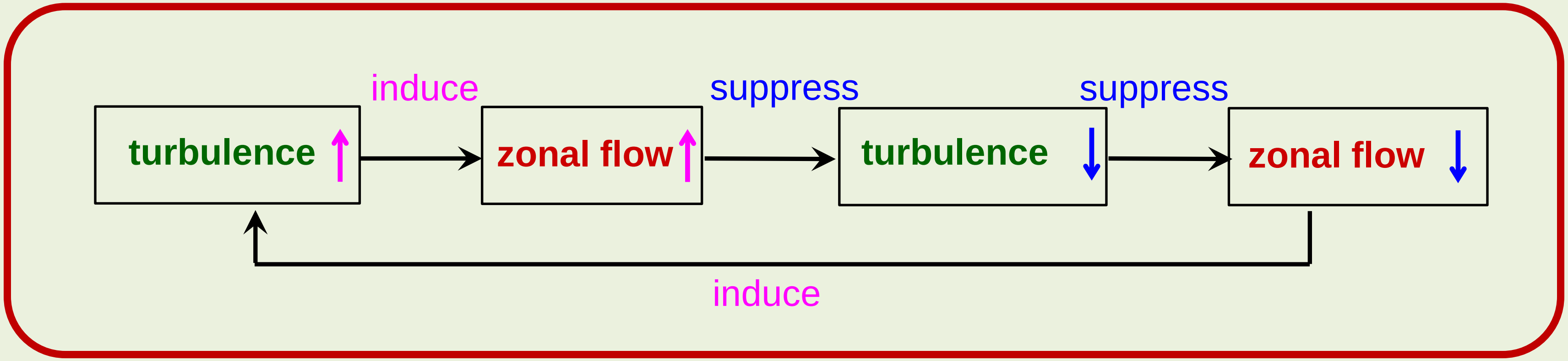
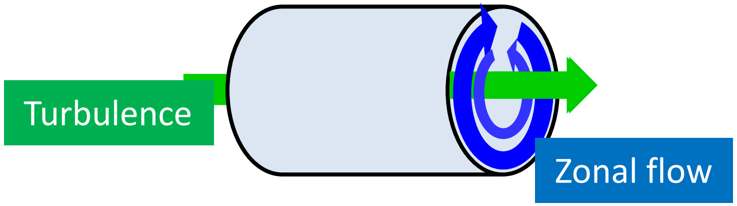


What drives the zonal flow?

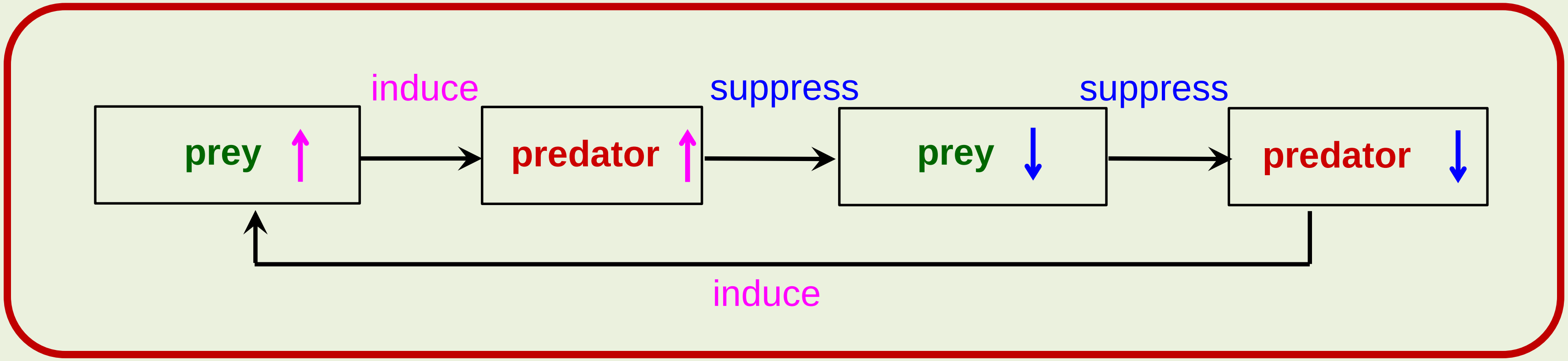


- **Interaction in two fluid model**
  - Turbulence, small-scale (k>0)
  - Zonal flow, large-scale (k=0,m=0): induced by turbulence and creates shear to suppress turbulence

1) Anisotropy of turbulence creates Reynolds stress which generates the mean velocity in azimuthal direction

$$\partial_t \langle v_\theta \rangle = -\partial_r \langle (\widetilde{v}_\theta \cdot \widetilde{v}_r) \rangle - \mu \langle v_\theta \rangle$$

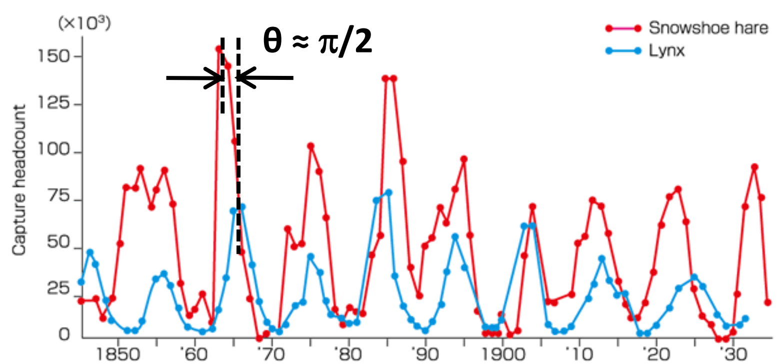
2) Mean azimuthal velocity decreases the anisotropy of turbulence and thus suppress turbulence



Normal population cycles in a predator-prey system

Resource → Prey → Predator

π/2 phase shift between prey and predator population



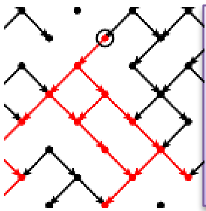
Persistent oscillations
+
Fluctuations

https://interstices.info/jcms/n_49876/des-especes-en-nombre

Q. What is the universality class of the transition to turbulence?

Tentative answer: directed percolation … but why?

Strategy: transitional turbulence to directed percolation



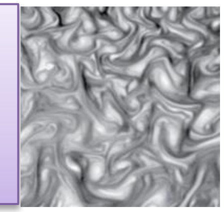
(Wikimedia Commons)

Directed Percolation

?

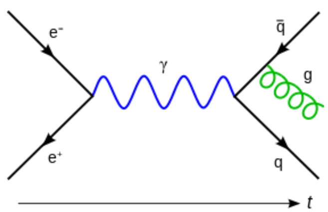
(Classical) Turbulence

(Boffetta and Ecke, 2012)

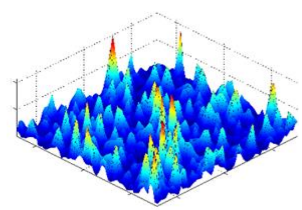


Reggeon field theory
(Janssen, 1981)

Field Theory



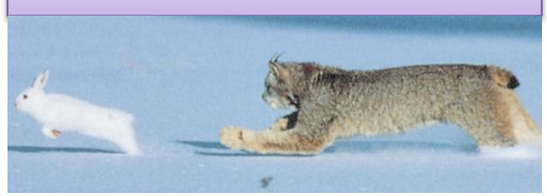
(Wikimedia Commons)



Two-fluid model

Extinction transition
(Mobilia et al., 2007)

Predator-Prey



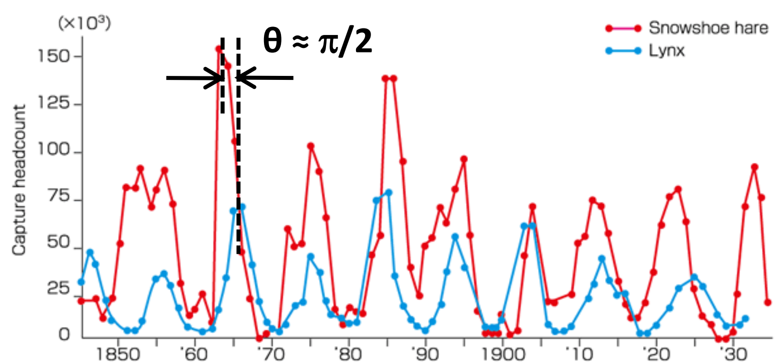
(Pearson Education, Inc., 2009)

Introduction to stochastic predator-prey systems

Normal population cycles in a predator-prey system

Resource → Prey → Predator

π/2 phase shift between prey and predator population

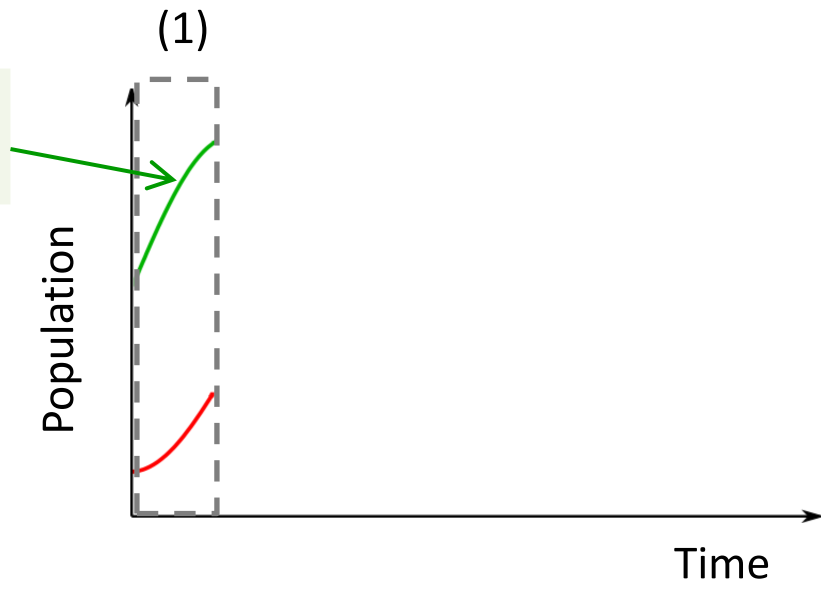


Persistent oscillations
+
Fluctuations

https://interstices.info/jcms/n_49876/des-especes-en-nombre

Cartoon picture for normal cycles ($\pi/2$ phase shift)

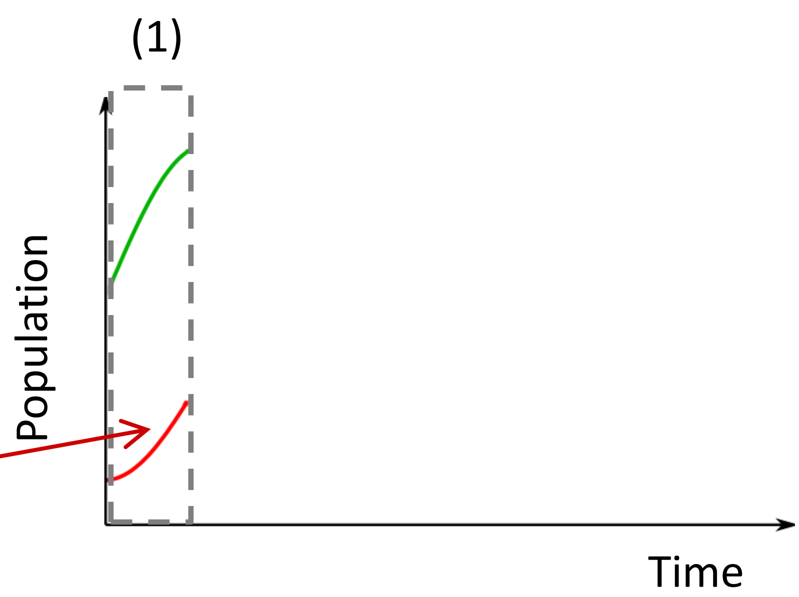
Prey consumes resource and grows



Cartoon picture for normal cycles (π/2 phase shift)

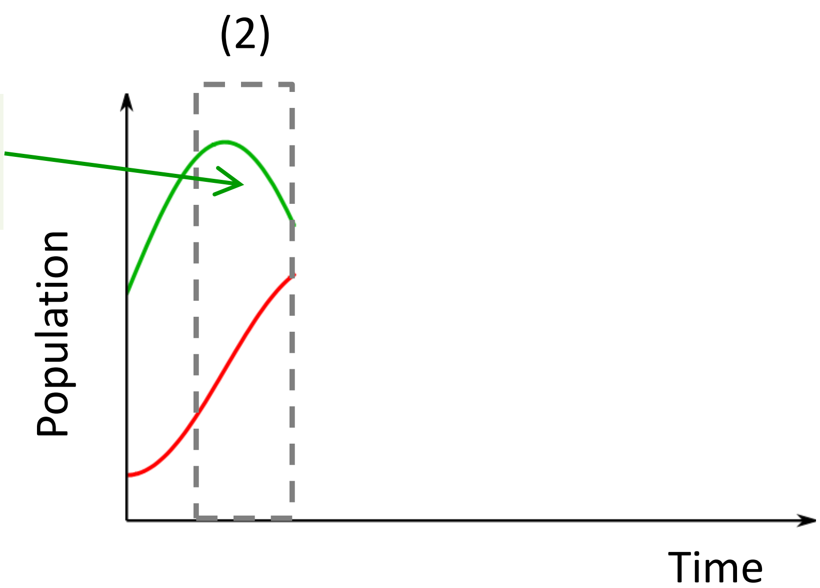
Prey consumes resource and grows

Predator eats prey and grows

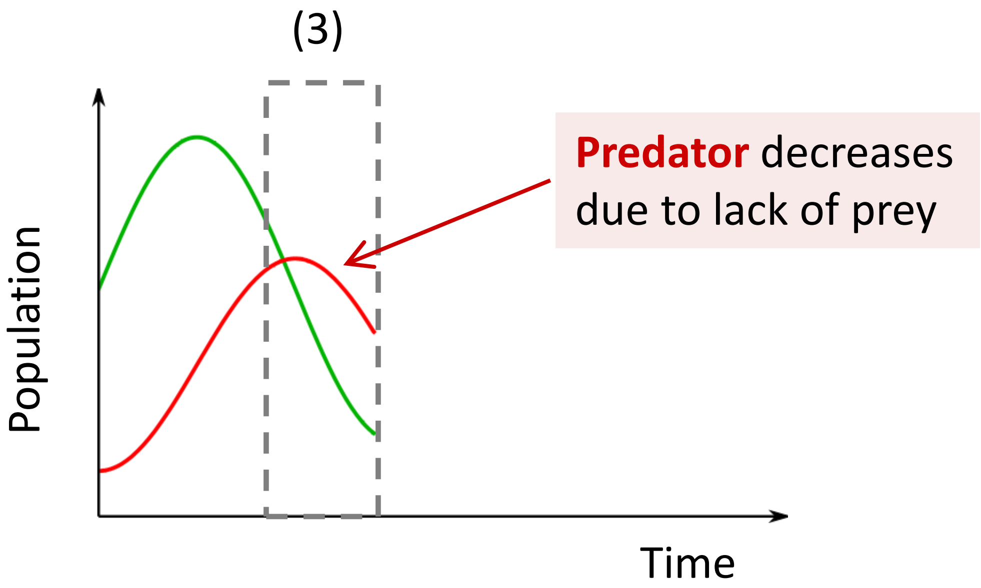


Cartoon picture for normal cycles (π/2 phase shift)

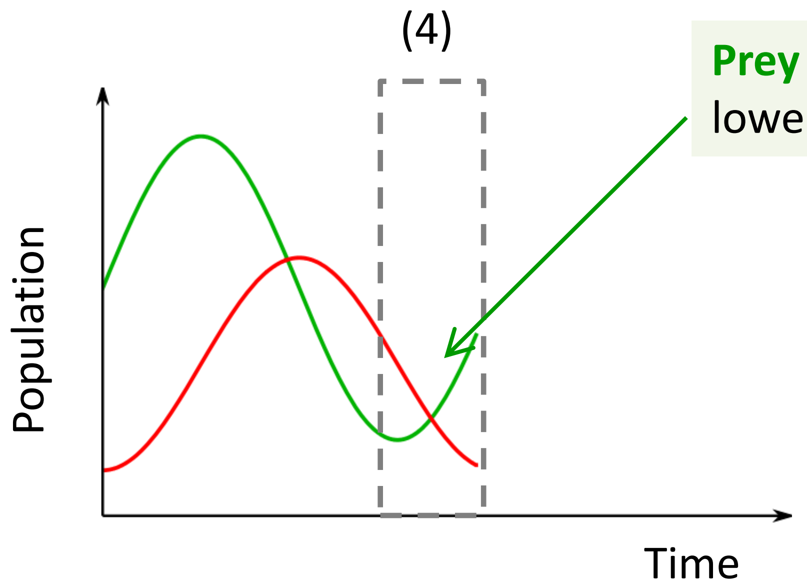
Prey decreases due to predation by predator



Cartoon picture for normal cycles (π/2 phase shift)

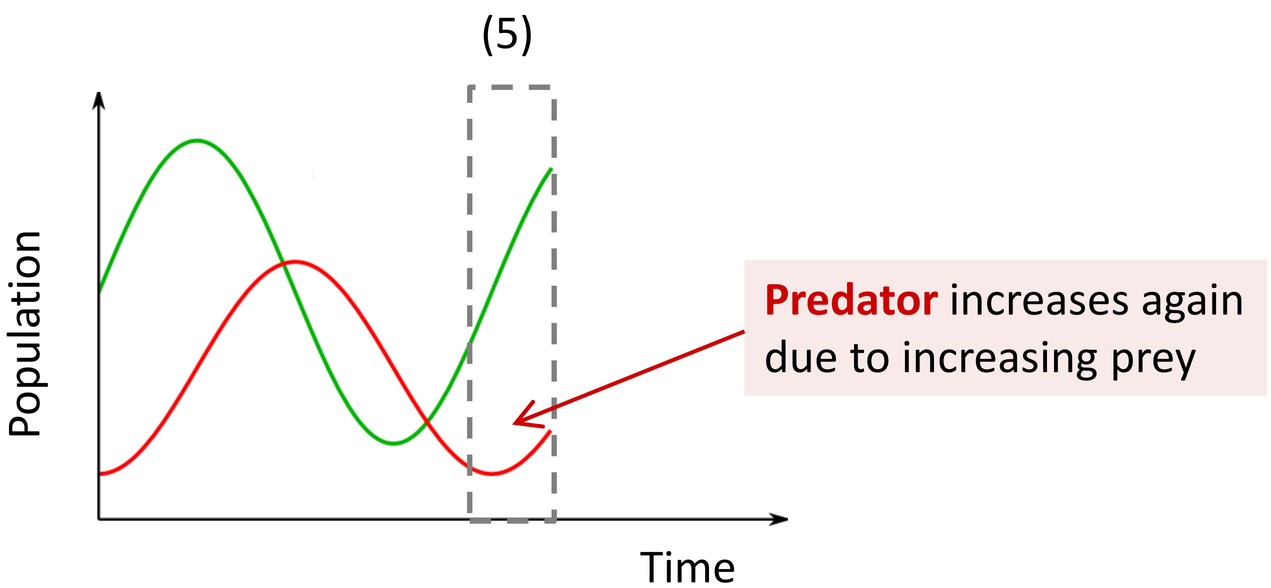


Cartoon picture for normal cycles (π/2 phase shift)

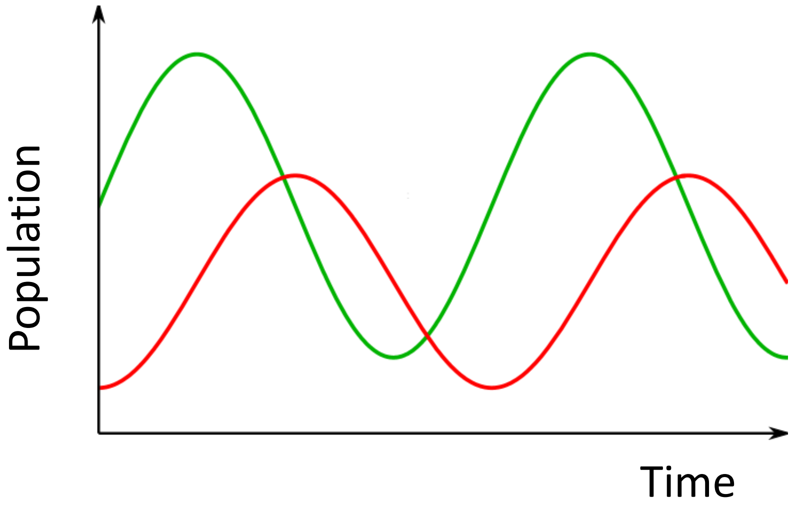


Prey increases because of lower predation pressure

Cartoon picture for normal cycles (π/2 phase shift)

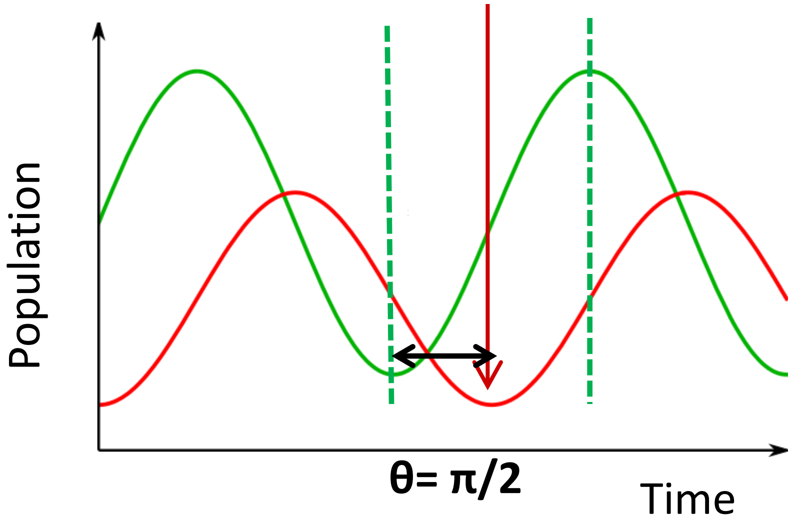


Cartoon picture for normal cycles (π/2 phase shift)



Cartoon picture for normal cycles (π/2 phase shift)

Predator can only start to grow after **prey** grows and before **prey** declines

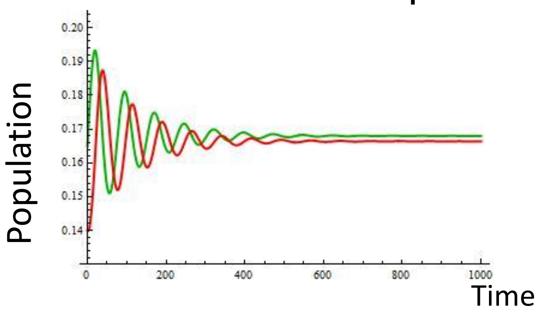


Phase shift is a quarter period

Models for predator-prey ecosystem

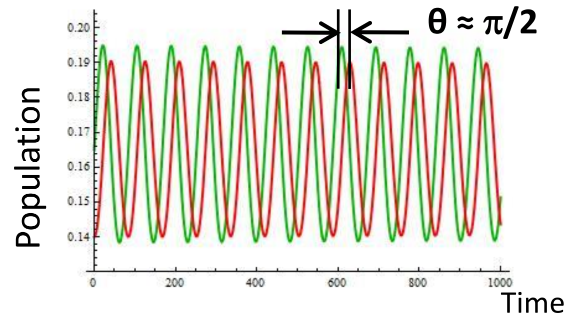
- **Deterministic models**

Lotka-Volterra equations



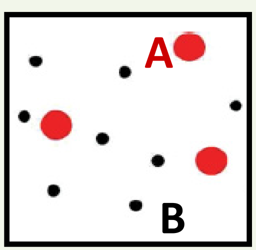
No persistent oscillations

Satiation model (Holling type II function)

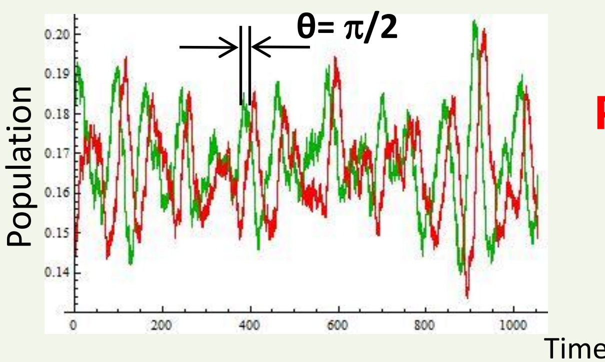


No fluctuations

- **Stochastic individual level model**
  fluctuations in number → demographic stochasticity that induces **quasi-cycles**



$$A \to \emptyset$$
$$A + B \to A + A$$
$$B \to B + B$$



Persistent oscillations + Fluctuations

McKane & Newman. *PRL* **94**, 218102 (2005).

Master equation as a quantum field theory

- Individuals in a population are quantized, so use annihilation and creation operators to count them and describe their interactions
  - When adding a new individual to the system, there is only one to chose
  - When removing an individual from the system there are many to chose
- Result: even classical identical particles obey commutation relations familiar from quantum field theory

Doi 1976; Grassberger & Scheunert 1980; Cardy & Sugar 1980; Mikhalov 1981; Goldenfeld 1982, 1984; Peliti 1985

Individual-level stochastic model of predator-prey dynamics

$$A \xrightarrow{d_1} E$$ **Predators**

$$B \xrightarrow{d_2} E$$ **Prey**

$$BE \xrightarrow{b} BB$$

$$AB \xrightarrow{p_1} AA$$

$$AB \xrightarrow{p_2} AE$$

$$\begin{aligned}
\partial_t P(m,n) &= d_1(-nP(m,n) + (n+1)P(m,n+1)) \\
&+ c(-n^2 P(m,n) + (n+1)^2 P(m,n+1)) \\
&+ b_1(-nP(m,n) + (n-1)P(m,n-1)) \\
&+ p_1(-mnP(m,n) + (n+1)mP(m,n+1)) \\
&+ p_2(-mnP(m,n) + (m-1)(n+1)P(m-1,n+1)) \\
&+ d_2(-mP(m,n) + (m+1)P(m+1,n))
\end{aligned}$$

Master equation as a quantum field theory

- Individuals in a population are quantized, so use annihilation and creation operators to count them and describe their interactions
- Time evolution given by Liouville equation

$$|\psi\rangle = \sum P(m,n)|m,n\rangle$$

$$\partial_t|\psi\rangle = -\hat{H}(a,\hat{a},b,\hat{b})|\psi\rangle$$

$$a|m,n\rangle = m|m-1,n\rangle$$

$$\hat{a}|m,n\rangle = |m+1,n\rangle$$

$$[a,\hat{a}] = 1$$

$$b|m,n\rangle = n|m,n-1\rangle$$

$$\hat{b}|m,n\rangle = |m,n+1\rangle$$

$$\left[b,\hat{b}\right] = 1$$

$$\hat{H} = b_1(\hat{b}b - \hat{b}^2 b) + d_1(\hat{b}b - b) + \frac{c}{V}(\hat{b}^2 b^2 - \hat{b}b^2)$$
$$+ \frac{p_1}{V}(\hat{a}a\hat{b}b - \hat{a}ab) + \frac{p_2}{V}(\hat{a}a\hat{b}b - \hat{a}^2 ab)$$
$$+ d_2(\hat{a}a - a)$$

Doi 1976; Grassberger & Scheunert 1980; Cardy & Sugar 1980; Mikhalov 1981; Goldenfeld 1982, 1984; Peliti 1985

Resonance from demographic noise

- Expand the number of predators and prey about average values in $\sqrt{N}$ expansion

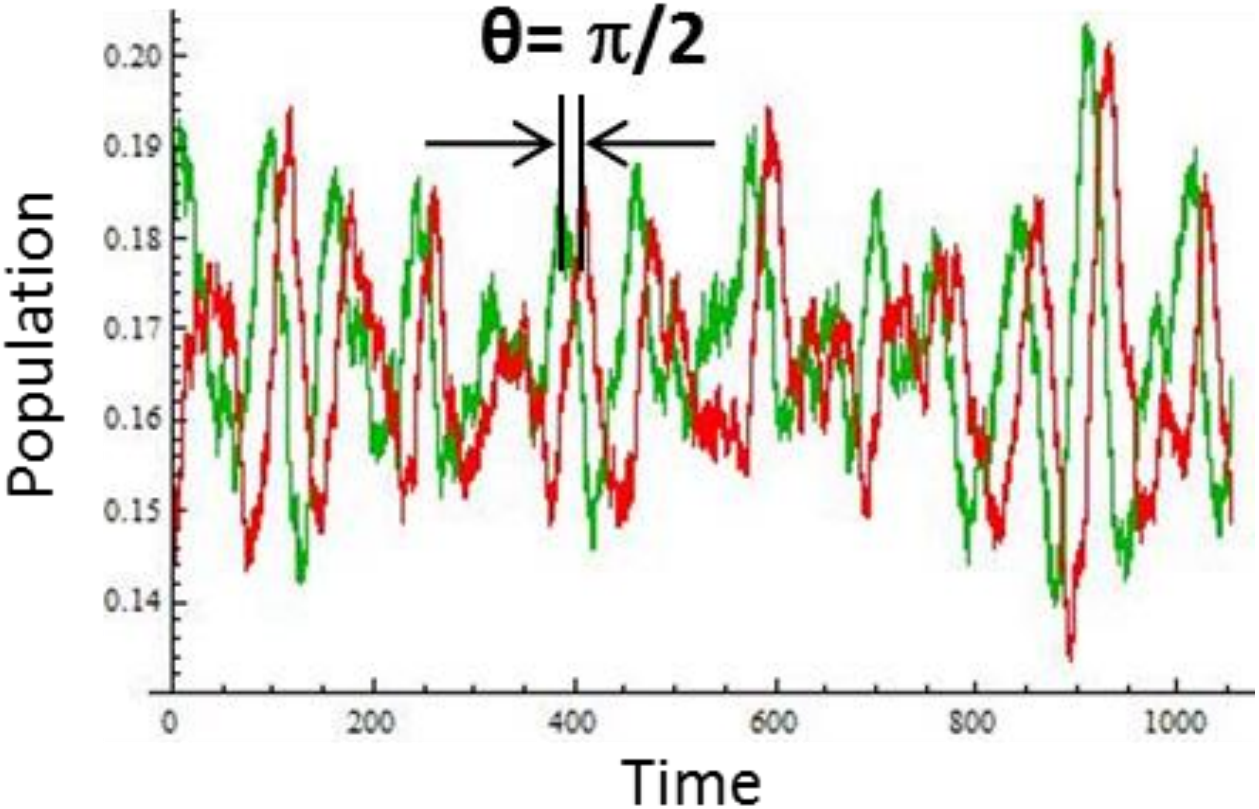
$$n/N = f_1 + x/\sqrt{N}$$

$$m/N = f_2 + y/\sqrt{N}$$

- Resulting equation is a linear stochastic equation in x, y with Langevin noise and power spectrum, sharply peaked about an internally-generated natural frequency

$$P(\omega) = \frac{\alpha + \beta\omega^2}{[(\omega^2 - \Omega_0^2)^2 + \Gamma^2\omega^2]}$$

Quasi-cycles



Extinction/decay statistics for stochastic predator-prey systems

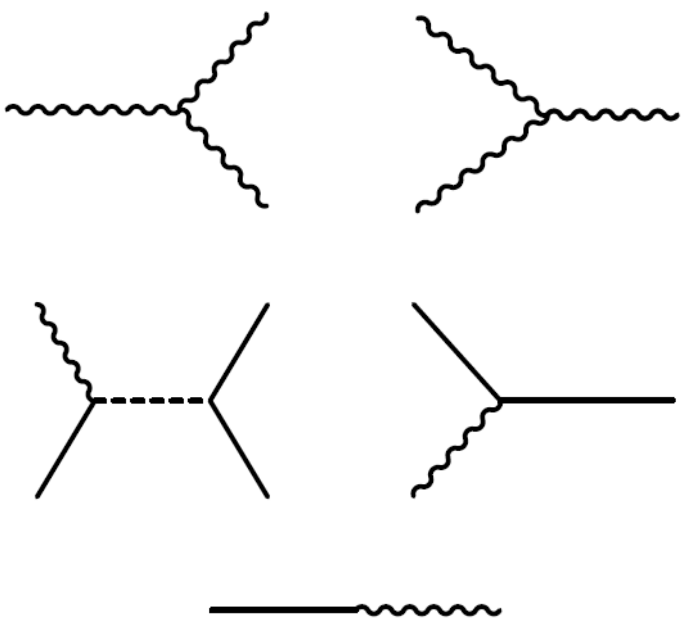
Derivation of predator-prey equations

Predator/Zonal flow

Prey/Turbulence

Zonal flow-turbulence

Predator-prey



$$B + E \xrightarrow{b} B + B$$

$$B + B \xrightarrow{c} B + E$$

$$A + B \xrightarrow{p} A + A$$

$$A + B \xrightarrow{p'} A + E$$

$$B \xrightarrow{m} A$$

$$A \xrightarrow{d_A} E \quad B \xrightarrow{d_B} E$$

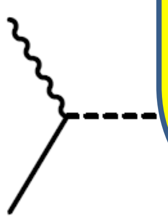
Derivation of predator-prey equations

nce

Zor

This phenomenological derivation of predator-prey dynamics is valid near critical point – it's Landau theory.

Can we motivate the predator-prey dynamics from mean field considerations?

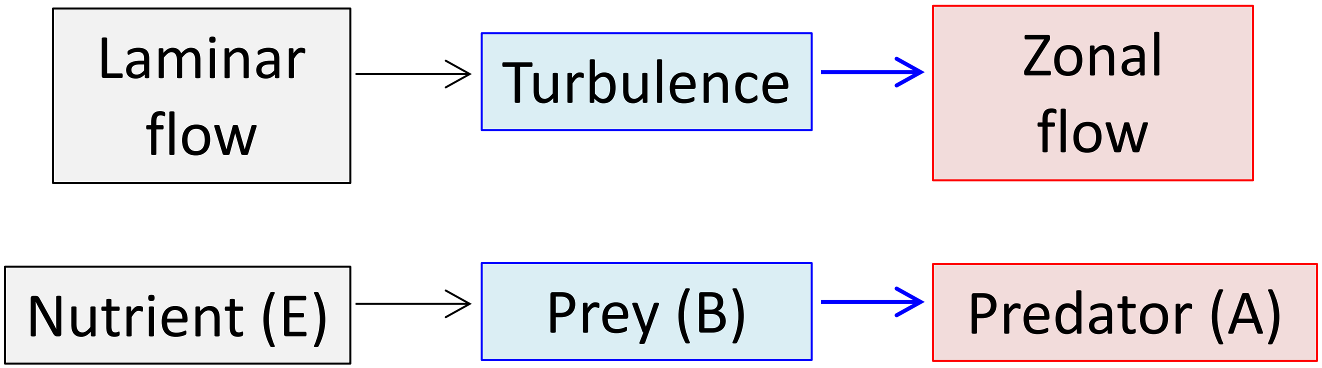


E

$$B \xrightarrow{m} A$$

$$A \xrightarrow{d_A} E \quad B \xrightarrow{d_B} E$$

Ecology model for turbulence



$$B + E \xrightarrow{b} B + B$$

$$A + B \xrightarrow{p} A + A$$

$$A \xrightarrow{d_A} E \quad B \xrightarrow{d_B} E$$

$$B \xrightarrow{m} A$$

mean-field rate equation:

$$\frac{dA}{dt} = pAB - d_A A + mB$$

$$\frac{dB}{dt} = b(1 - A - B)B - pAB - d_B B - mB$$

Heuristic derivation of predator-prey equation

Energy of turbulent fluctuations

$$E = \int |\tilde{v}|^2 dV$$

Shear at zonal flow

$$\Omega \equiv \partial \langle \overline{u}_\theta(r) \rangle / \partial r$$

Energy of sheared zonal flow

$$U \equiv \Omega^2$$

Turbulence dynamics

$$\frac{dE}{dt} = \gamma_0 E - \alpha_1 E^2 - \alpha_2 EU$$

Primary instability ($\gamma_0 E$)

Eddy interaction ($\alpha_1 E^2$)

Suppression by zonal flow ($\alpha_2 EU$)

Diamond et al. PRL (1994)

Heuristic derivation of predator-prey equation

Reynolds momentum balance equation

$$\frac{\partial \langle \overline{u}_\theta(r) \rangle}{\partial t} = -\frac{\partial \langle \tilde{v}_r \tilde{v}_\theta \rangle}{\partial r} - \mu \langle \overline{u}_\theta(r) \rangle$$

damping

Radial derivative

$$\partial_t \Omega = -\partial_r^2 \langle \tilde{v}_r \tilde{v}_\theta \rangle - \mu \Omega$$

$$\propto E$$

$$\propto +\Omega + O(\Omega^2)$$

Zonal flow dynamics

$$\boxed{\partial_t U = \alpha_3 E U - 2\mu U}$$

Diamond et al. PRL (1994)

Heuristic derivation of predator-prey equation

Turbulence dynamics

$$\frac{dE}{dt} = \gamma_0 E - \alpha_1 E^2 - \alpha_2 EU$$

Zonal flow dynamics

$$\partial_t U = \alpha_3 EU - 2\mu U$$

Prey dynamics

$$\dot{B} = bB(1 - B/\kappa) - pAB$$

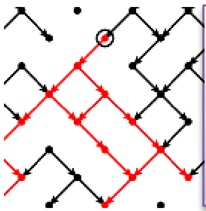
Predator dynamics

$$\dot{A} = pAB - dA$$

Universality class of the transition

Strategy: transitional turbulence to directed percolation

(Boffetta and Ecke, 2012)

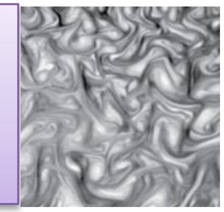


(Wikimedia Commons)

Directed Percolation

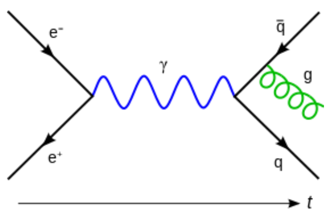
?

(Classical) Turbulence



Reggeon field theory
(Janssen, 1981)

Field Theory



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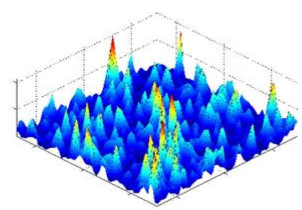
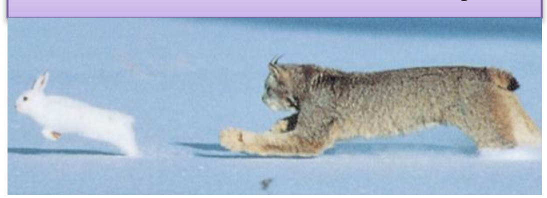


Two-fluid model

Extinction transition
(Mobilia et al., 2007)

Predator-Prey



(Pearson Education, Inc., 2009)

Universality class of predator-prey system near extinction

Basic individual processes in predator (A) and prey (B) system:

Death $B_i \xrightarrow{d_B} E_i$ $A_i \xrightarrow{d_A} E_i$ $B_i \xrightarrow{m} A_i$

Birth $B_i + E_j \xrightarrow[\langle ij \rangle]{b} B_i + B_j$ $A_i + B_j \xrightarrow[\langle ij \rangle]{p} A_i + A_j$

Diffusion $B_i + E_j \xrightarrow[\langle ij \rangle]{D} E_i + B_j$ $A_i + E_j \xrightarrow[\langle ij \rangle]{D} E_i + A_j$

Carrying capacity $B_i + B_j \xrightarrow[\langle ij \rangle]{c} B_i + E_j$

Universality class of predator-prey system near extinction

Near the transition to prey extinction, the prey (B) population is very small and no predator (A) can survive; A ~ 0.

Death $B_i \xrightarrow{d_B} E_i$ $\quad A_i \xrightarrow{d_A} E_i$ $\quad B_i \xrightarrow{m} A_i$

Birth $B_i + E_j \xrightarrow[\langle ij \rangle]{b} B_i + B_j$ $\quad A_i + B_j \xrightarrow[\langle ij \rangle]{p} A_i + A_j$

Diffusion $B_i + E_j \xrightarrow[\langle ij \rangle]{D} E_i + B_j$ $\quad A_i + E_j \xrightarrow[\langle ij \rangle]{D} E_i + A_j$

Carrying capacity $B_i + B_j \xrightarrow[\langle ij \rangle]{c} B_i + E_j$

Universality class of predator-prey system near extinction

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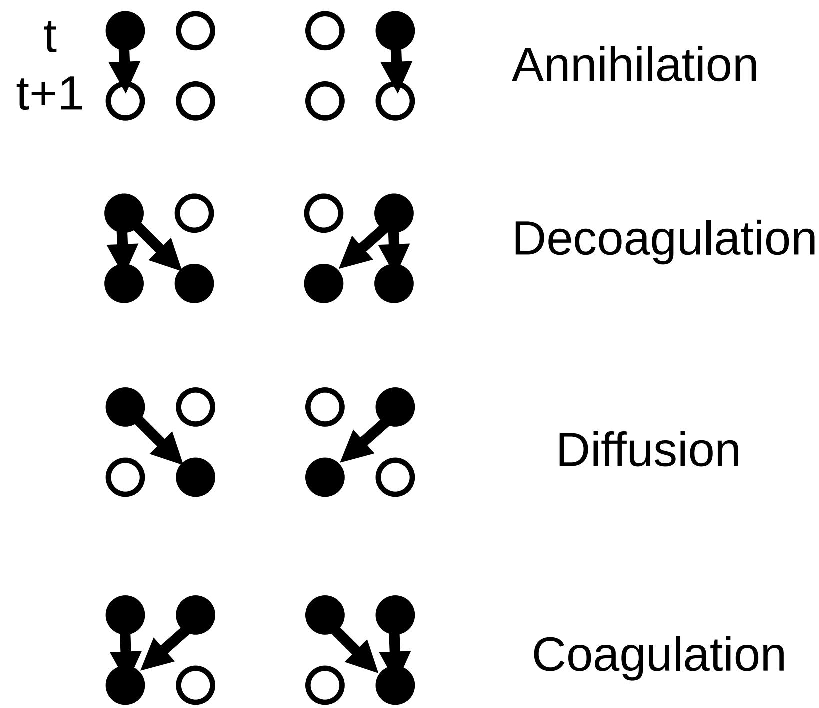
Carrying capacity $$B_i + B_j \xrightarrow[\langle ij \rangle]{c} B_i + E_j$$



Universality class of predator-prey system near extinction

Near the transition to prey extinction, the prey (B) population is very small and no predator (A) can survive; A ~ 0.

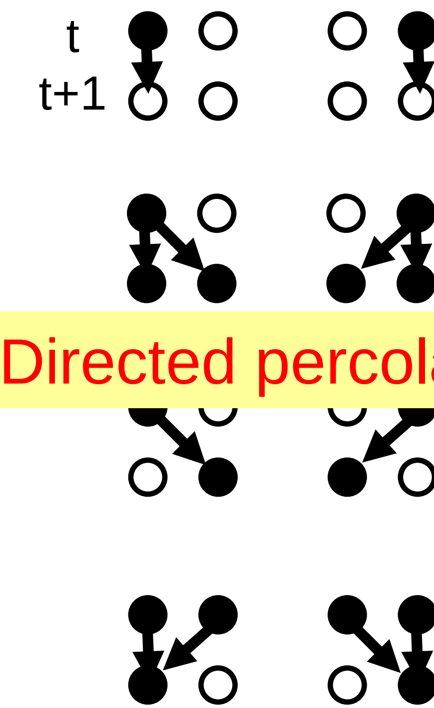
Death $B_i \xrightarrow{d_B} E_i$ — Annihilation

Birth $B_i + E_j \xrightarrow[\langle ij \rangle]{b} B_i + B_j$ — Decoagulation

Predator-prey = Directed percolation

Diffusion $B_i + E_j \xrightarrow[\langle ij \rangle]{D} E_i + B_j$ — Diffusion

Carrying capacity $B_i + B_j \xrightarrow[\langle ij \rangle]{c} B_i + E_j$ — Coagulation



Field theory for predator-prey model

- Near extinction model reduces to simpler system

$$
\begin{aligned}
A &\to \emptyset && \text{with rate } \mu, \\
A + B &\to A + A && \text{with rate } \lambda', \\
B &\to B + B && \text{with rate } \sigma.
\end{aligned}
$$

- Express as Hamiltonian

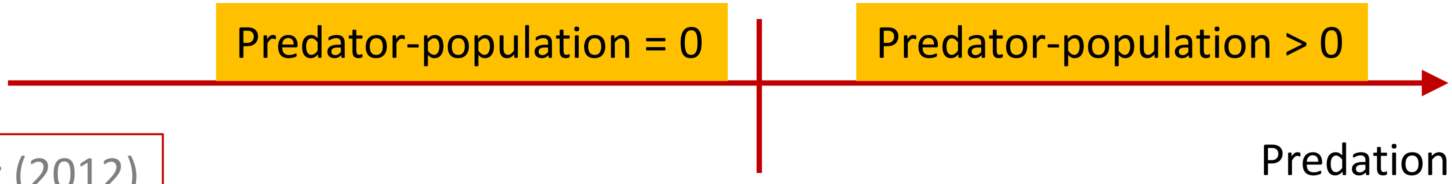
$$H_{\text{reac}} = -\sum [\mu(1 - a_i^\dagger)a_i + \sigma(b_i^\dagger - 1)\, b_i^\dagger\, b_i + \lambda'(a_i^\dagger - b_i^\dagger)a_i^\dagger\, a_i\, b_i]$$

- Map into a coherent state path integral

$$S[\hat{a}, \hat{b}; a, b] = \int d^d x \int dt \Bigg[\hat{a}\left(\frac{\partial}{\partial t} - D_A \nabla^2\right) a + \hat{b}\left(\frac{\partial}{\partial t} - D_B \nabla^2\right) b$$

$$+ \mu(\hat{a} - 1)\, a - \sigma(\hat{b} - 1)\hat{b}\, b\, e^{-a_0^d \hat{b} b} + \nu(\hat{b} - 1)\,\hat{b}\, b^2 - \lambda(\hat{a} - \hat{b})\hat{a}\, a\, b \Bigg]$$

- Phase diagram



See Tauber (2012)

Extinction in predator-prey systems

- This field theory can be reduced to

$$A \to \emptyset$$
$$A + B \to A + A$$
$$B \to B + B$$

$$S'_{\infty}[\widetilde{\psi}, \psi] = \int \mathrm{d}^d x \int \mathrm{d}t \left[\widetilde{\psi} \left(\frac{\partial}{\partial t} + D_A (r_A - \nabla^2) \right) \psi - u\, \widetilde{\psi} (\widetilde{\psi} - \psi) \psi + \tau \widetilde{\psi}^2 \psi^2 \right]$$

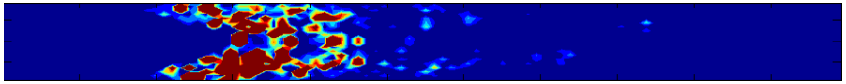
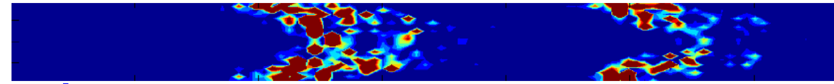
- Reggeon field theory ↔ Extinction transition in predator-prey model (Mobilia et al (2007)
- Reggeon field theory ↔ DP universality class: non-equilibrium critical dynamics with absorbing state

- **Summary: ecological model of transitional turbulence predicts the DP universality class**

Puff splitting in ecology model

Driven by emerging traveling waves
of populations

Pipe flow turbulence

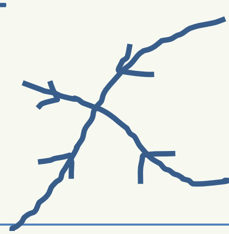
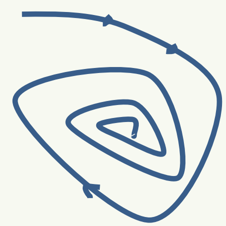



laminar | metastable puffs | spatiotemporal intermittency | expanding slugs

Re | 1775 | 2050 | 2500

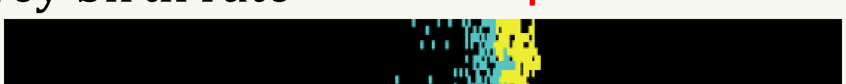
Predator-prey model

linear stability of mean-field solutions

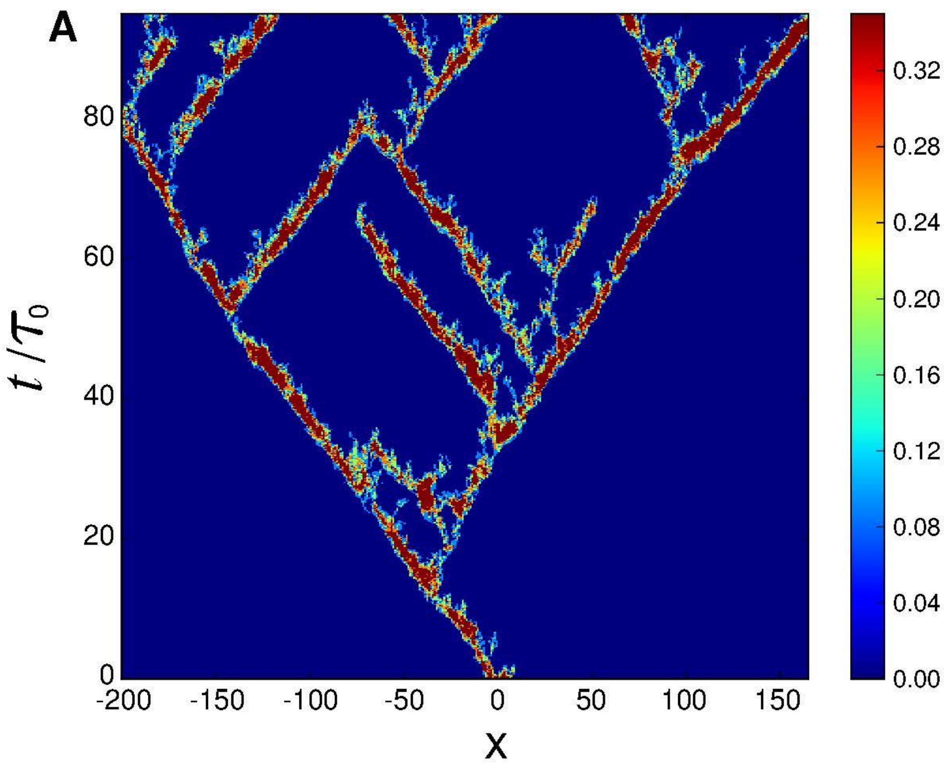



nutrient only | metastable population | traveling fronts | expanding population

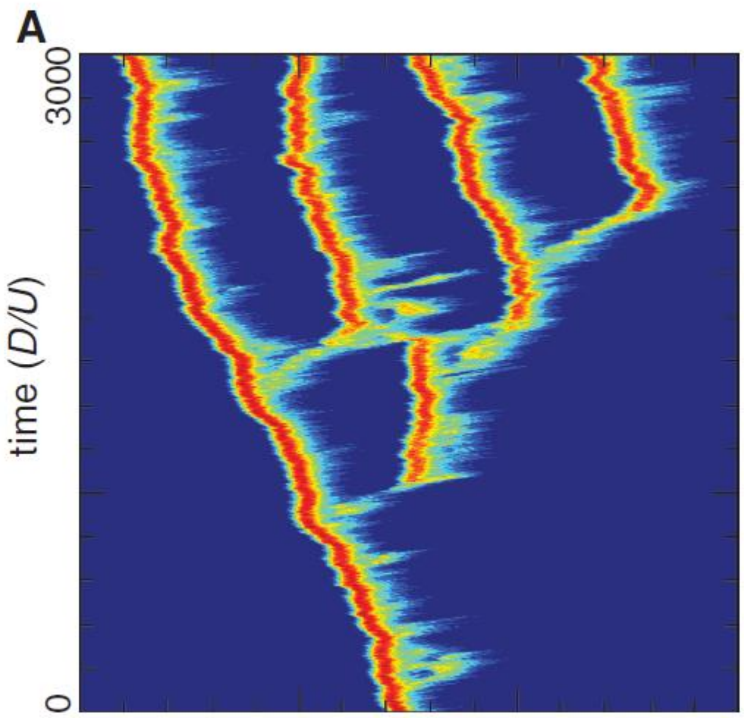
prey birth rate




Puff splitting in predator-prey systems



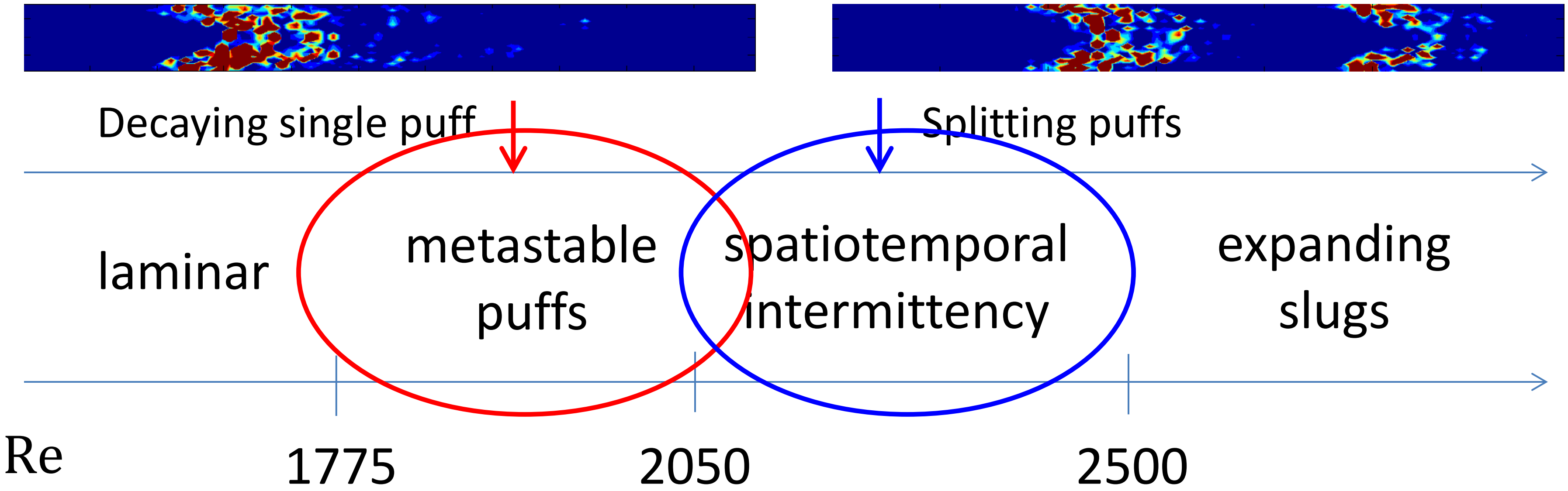
Puff-splitting in predator-prey ecosystem in a pipe geometry



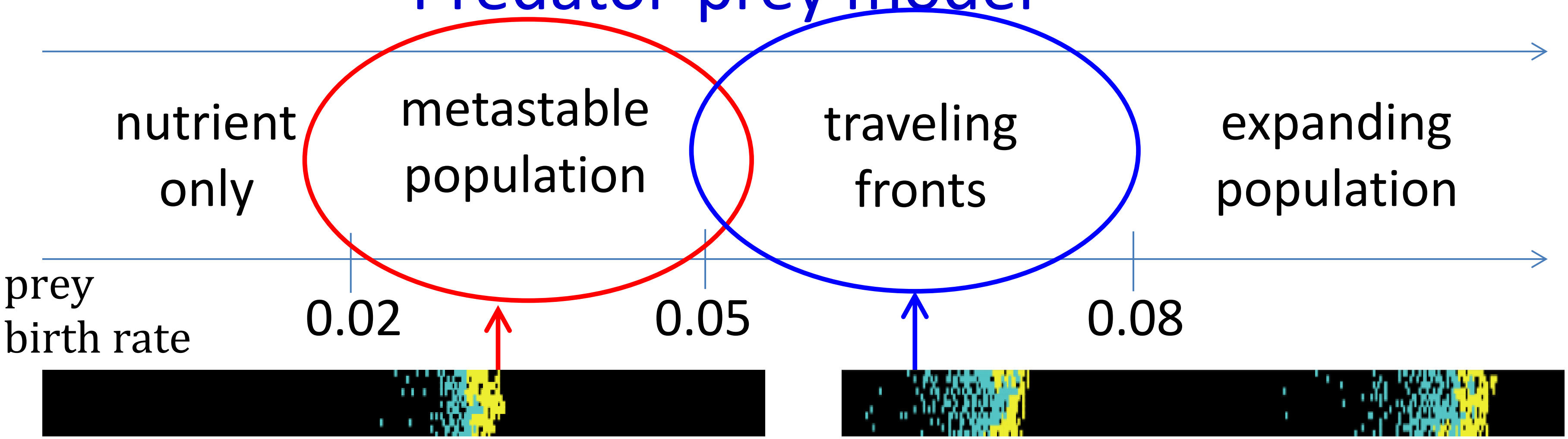
Puff-splitting in pipe turbulence

Avila et al., Science (2011)

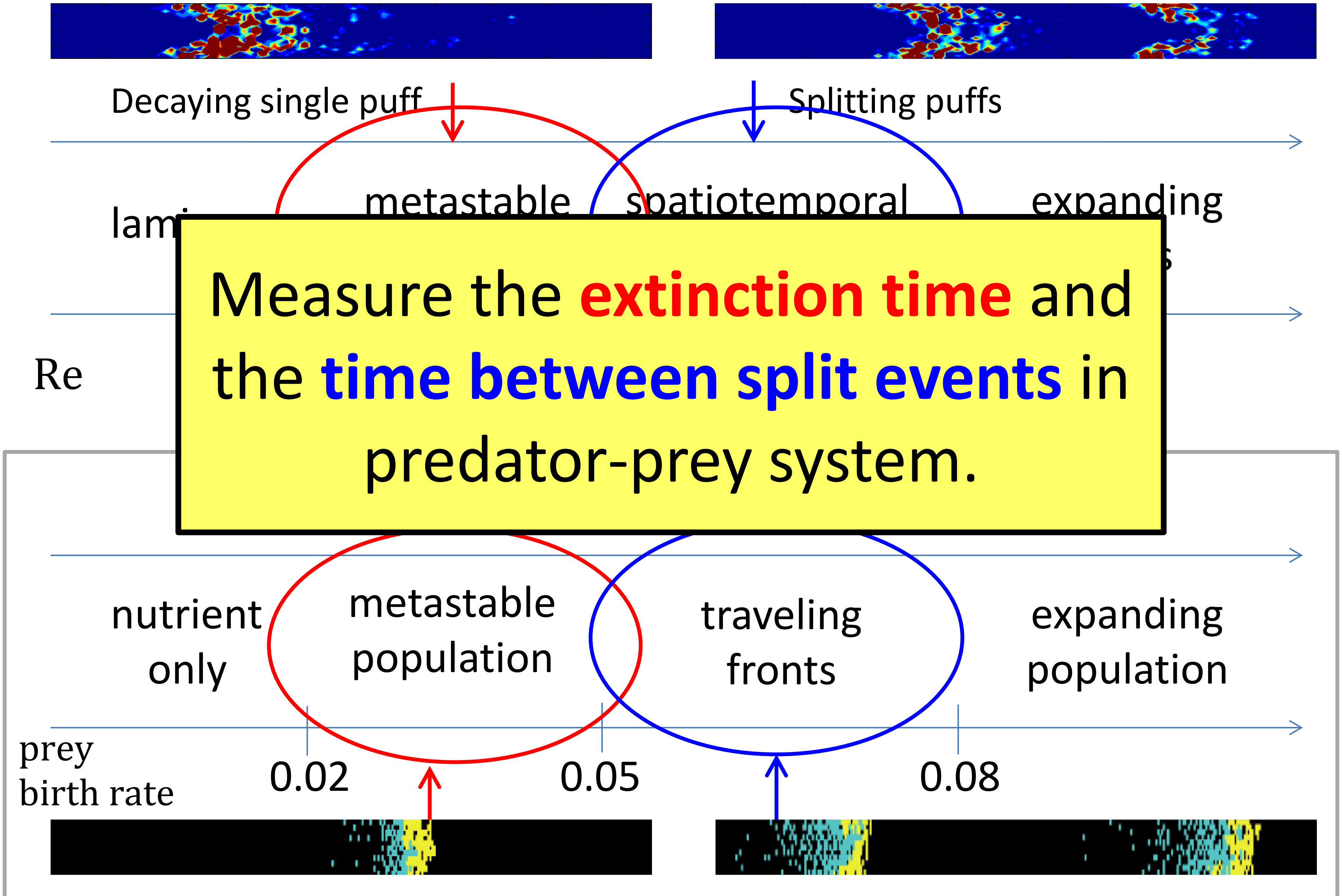
Pipe flow turbulence



Predator-prey model



Pipe flow turbulence

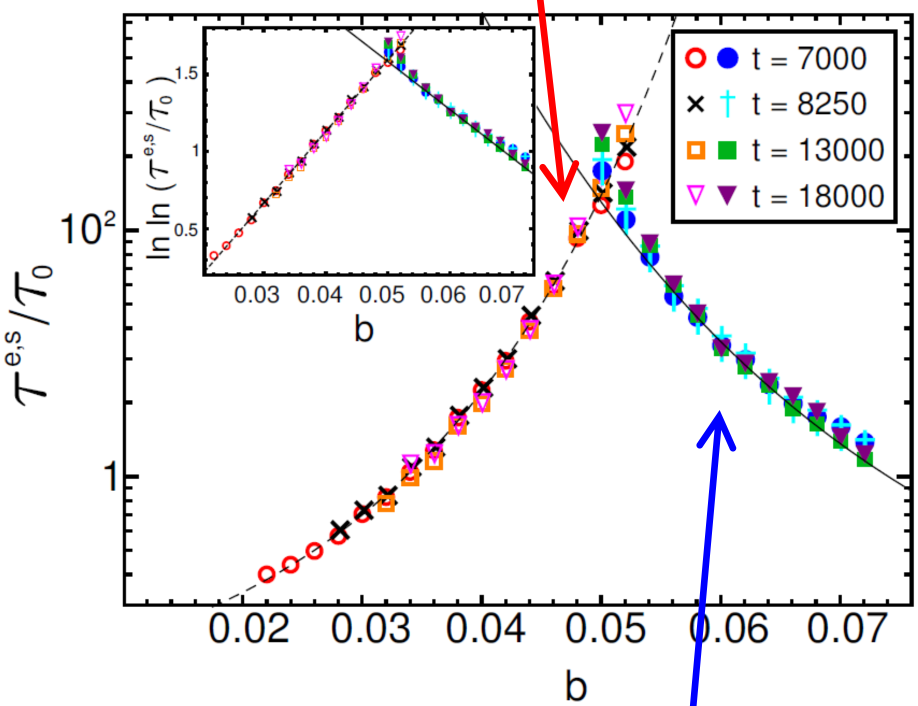


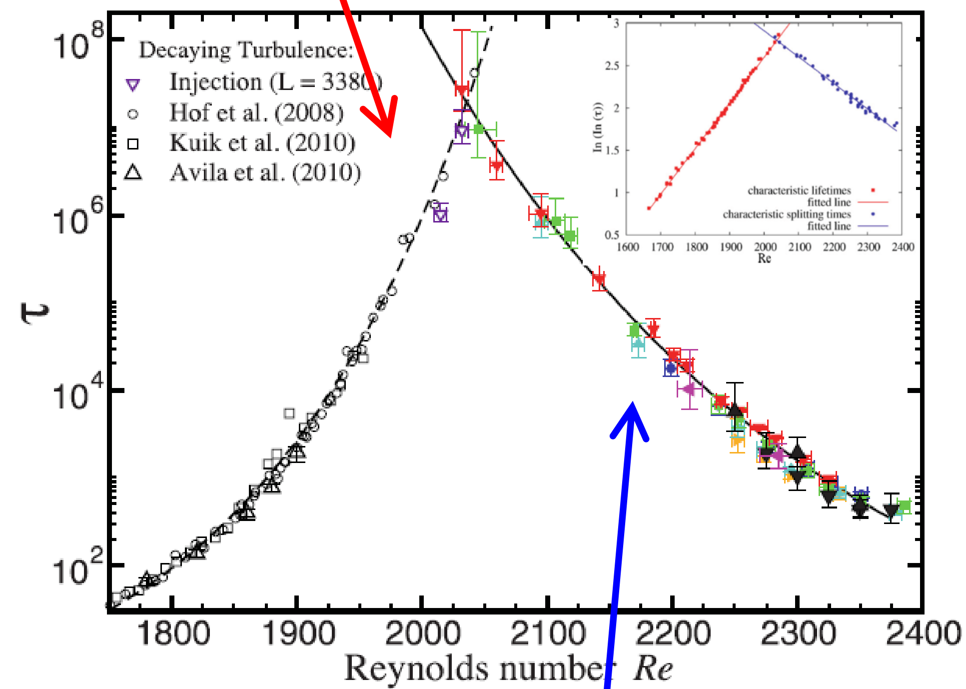
Measure the **extinction time** and the **time between split events** in predator-prey system.

Predator-prey vs. transitional turbulence

Prey lifetime

Turbulent puff lifetime





Mean time between population split events

Mean time between puff split events

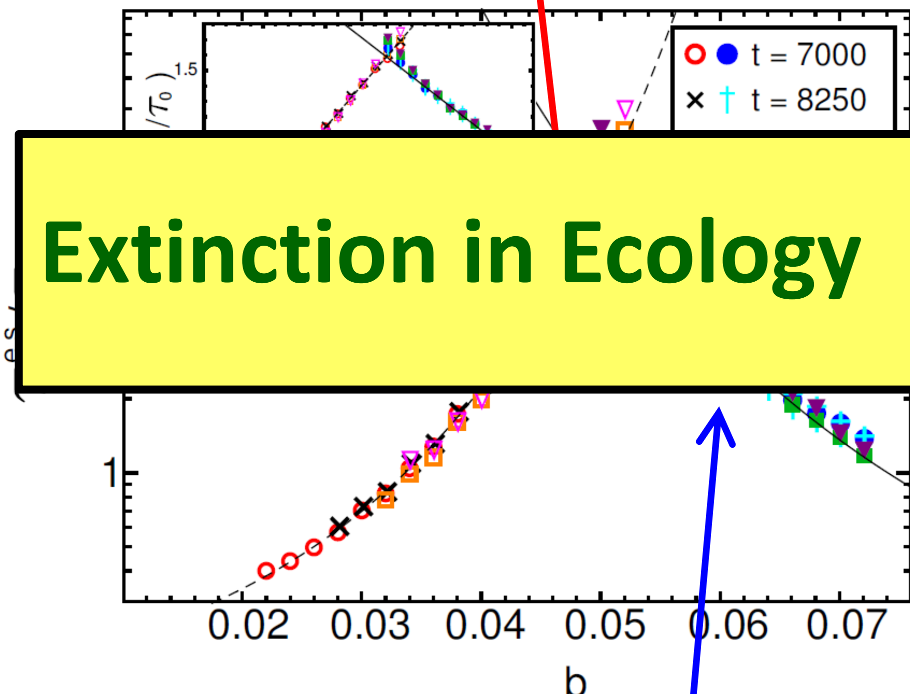
Avila *et al.*, *Science* **333**, 192 (2011)

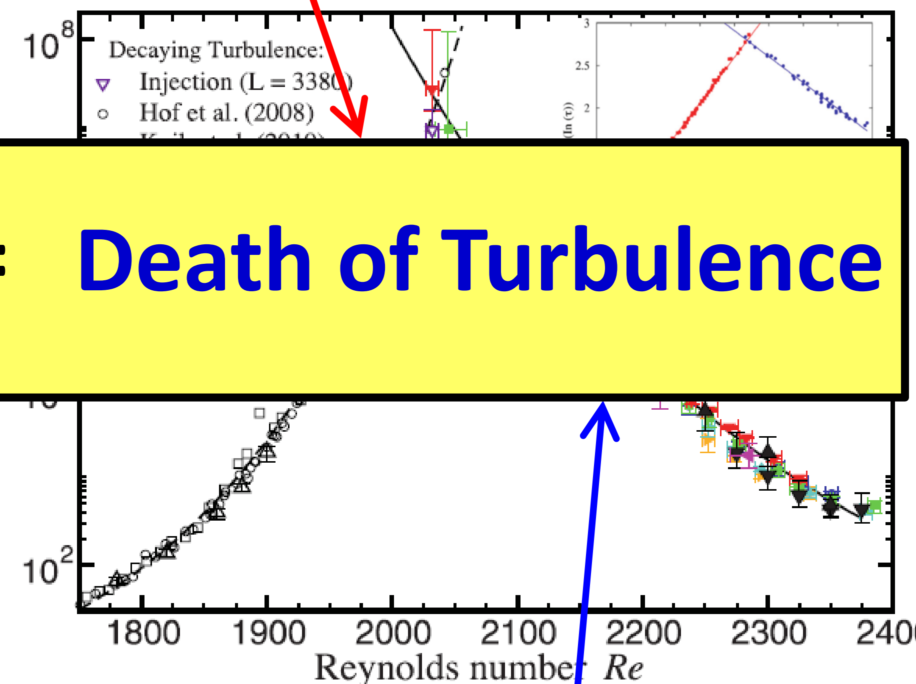
Song *et al.*, *J. Stat. Mech.* 2014(2**)**, P020010

Predator-prey vs. transitional turbulence

Prey lifetime

Turbulent puff lifetime





Extinction in Ecology = Death of Turbulence

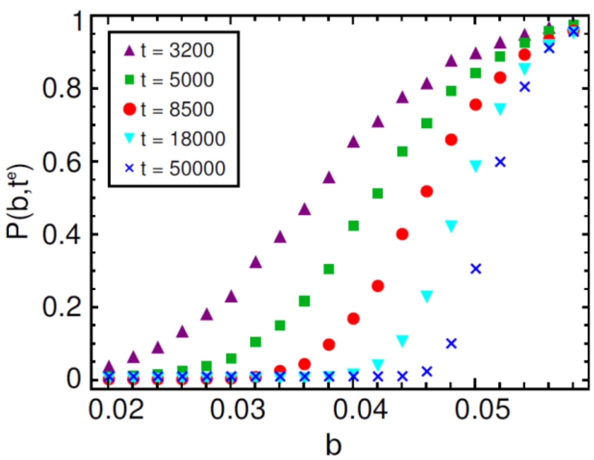
Mean time between population split events

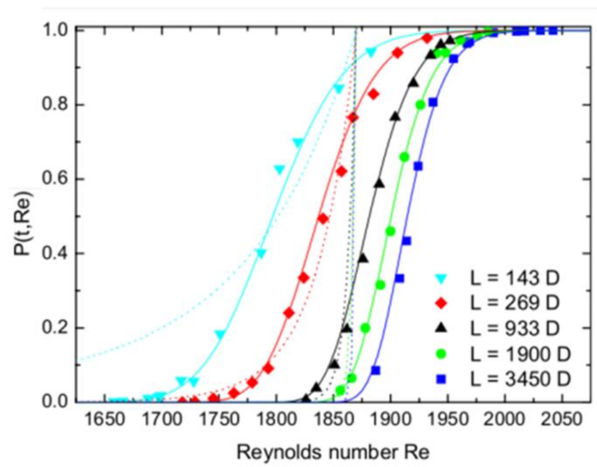
Mean time between puff split events

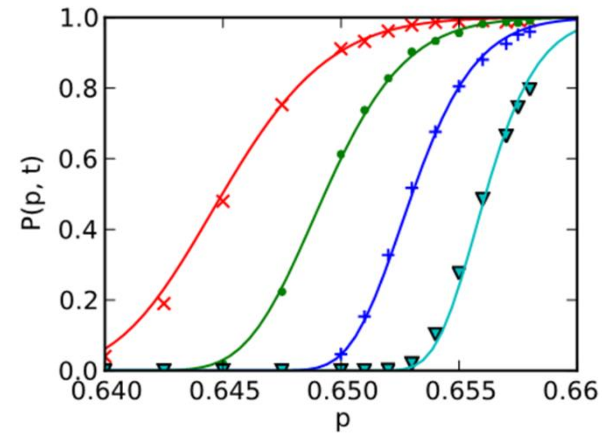
Avila *et al.*, *Science* **333**, 192 (2011)

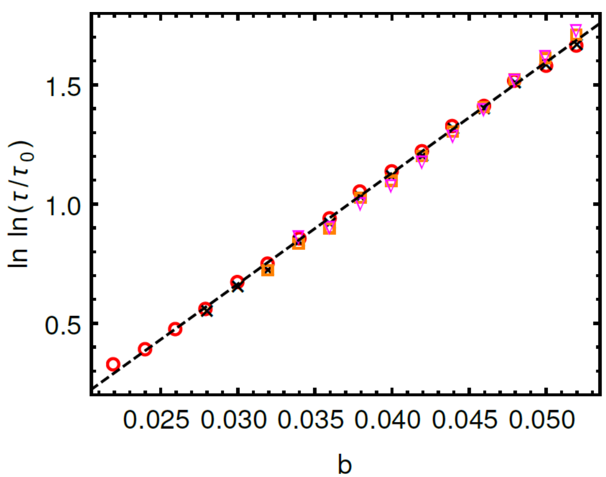
Song *et al.*, *J. Stat. Mech.* 2014(2), P020010

Ecology = turbulence = DP

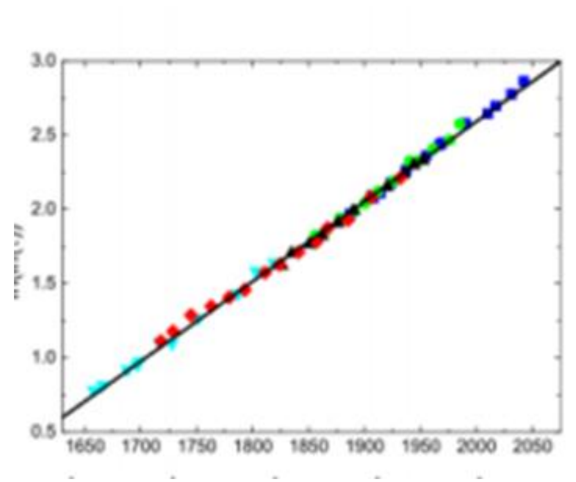




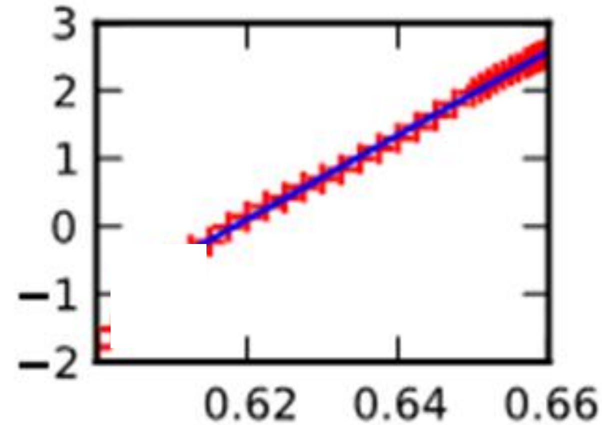




Shih, Hsieh, NG (2015)

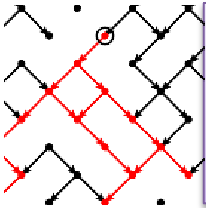


Hof et al., *PRL* **101**, 214501 (2008)



Sipos and Goldenfeld, *PRE* **84**, 035304(R) (2011)

Summary: universality class of transitional turbulence

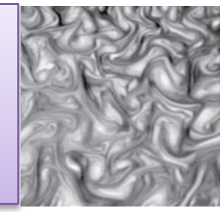


(Wikimedia Commons)

Directed Percolation

(Classical) Turbulence

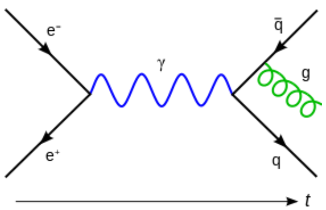
(Boffetta and Ecke, 2012)



Reggeon field theory (Janssen, 1981)

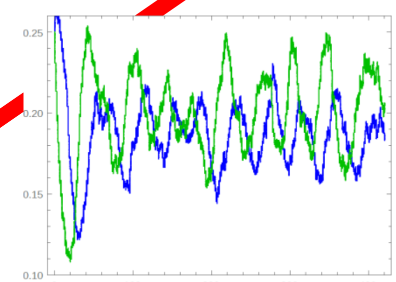
Direct Numerical Simulations of Navier-Stokes

Field Theory



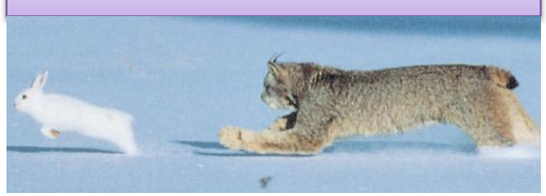
(Wikimedia Commons)

Two-fluid model



Extinction transition (Mobilia et al., 2007)

Predator-Prey



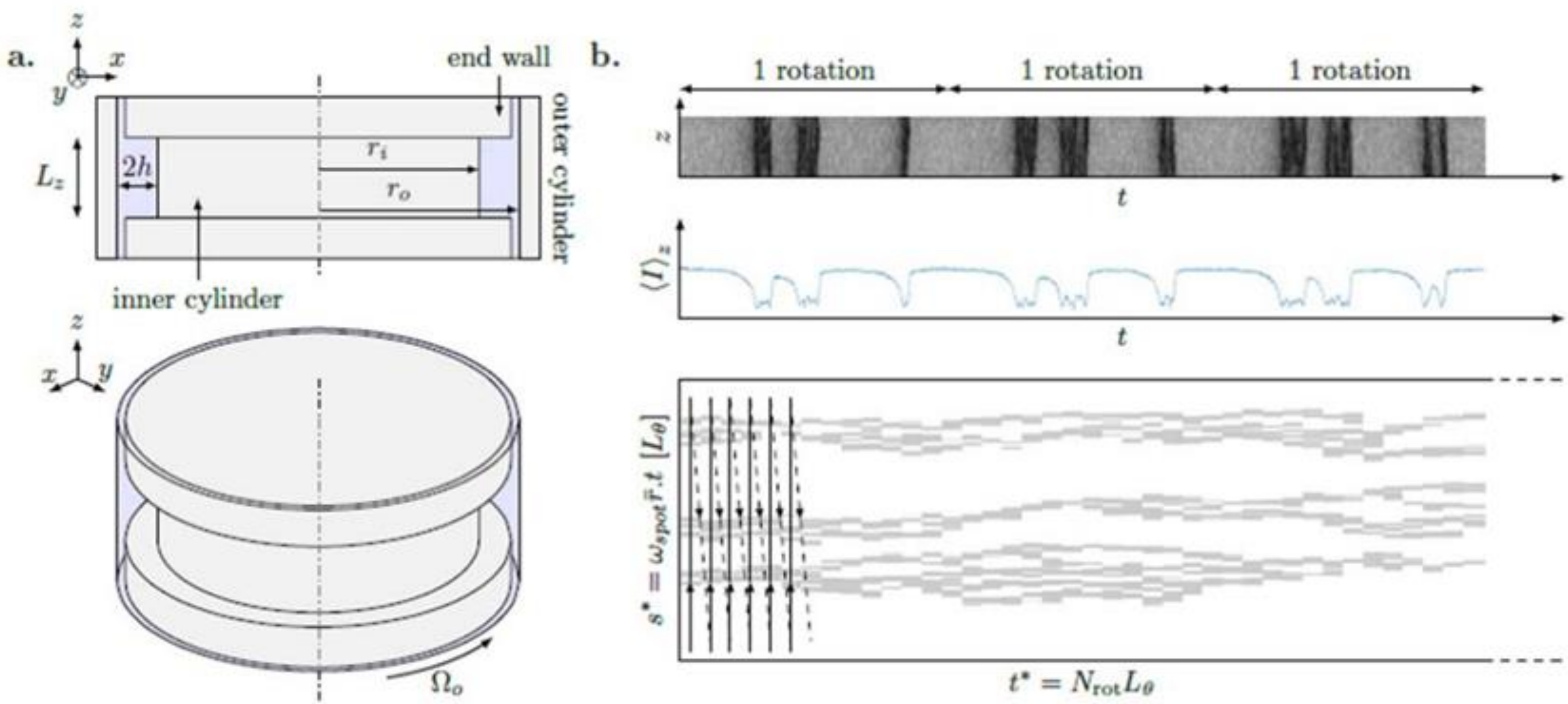
(Pearson Education, Inc., 2009)

Precise measurements of DP

Turbulence and directed percolation

Fluid between concentric cylinders, outer one rotating

Turbulent patches

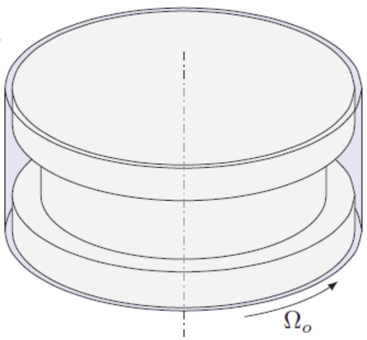


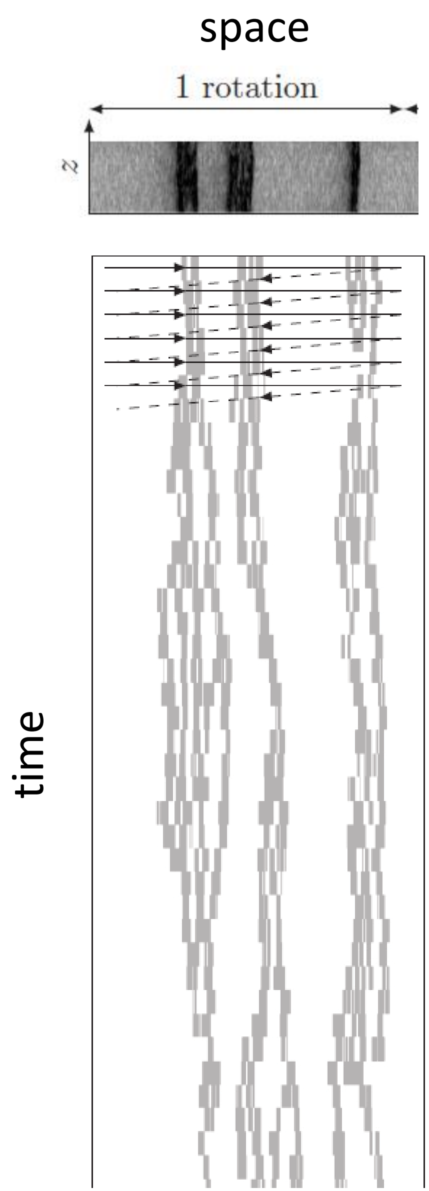
Position of turbulent patches changes in time

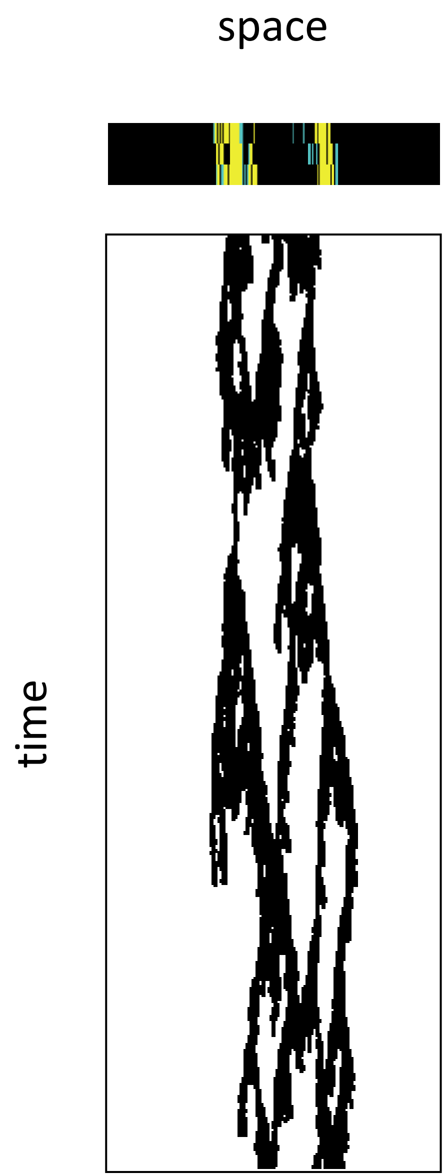
Lemoult et al., Nature Physics (2016)

Turbulence and directed percolation

Couette

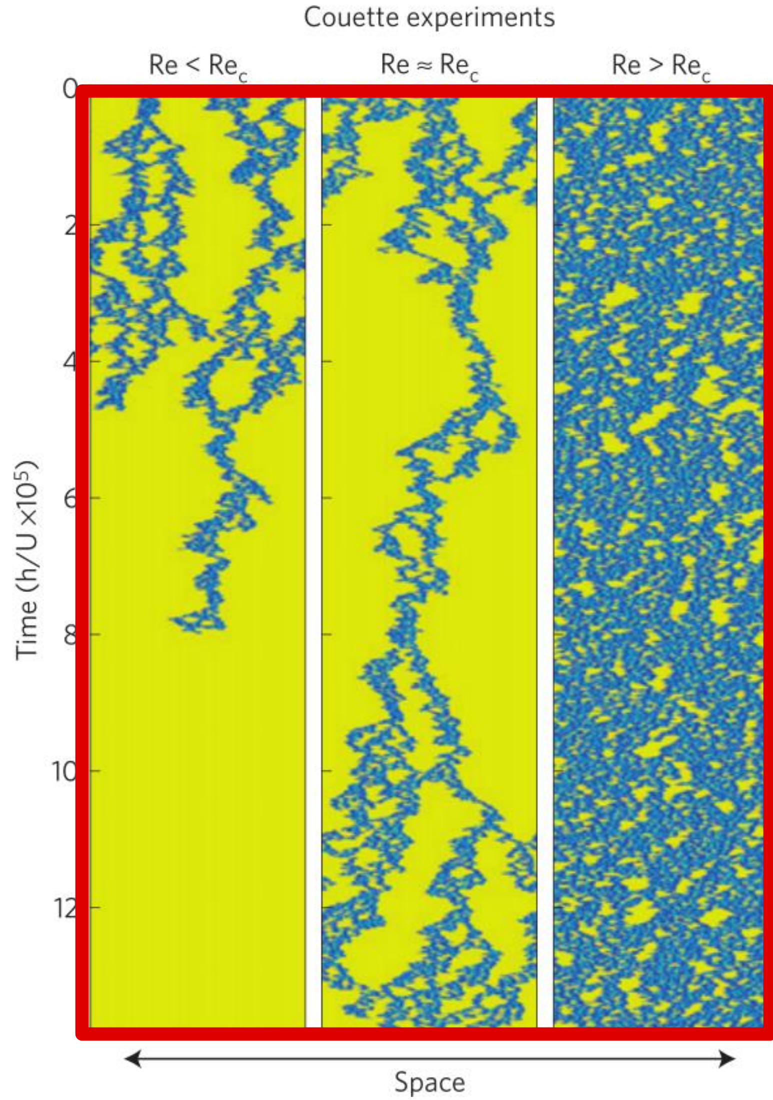






Ecology

Turbulence and directed percolation



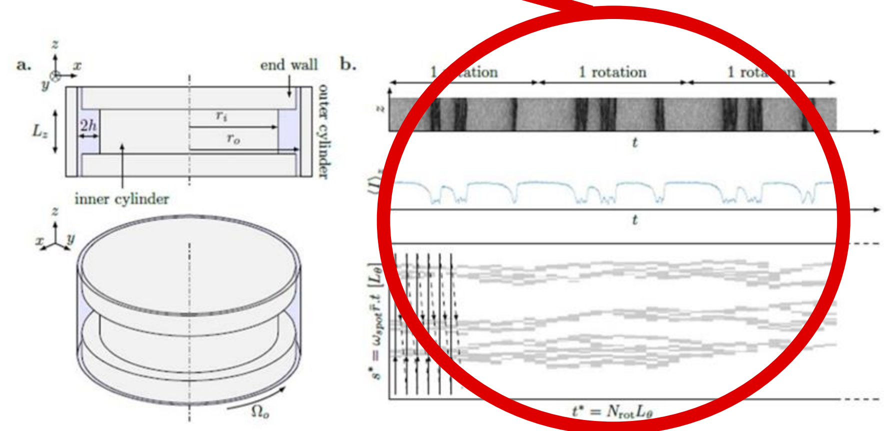


Figure S1: **a** Schematic of experimental set up (not to scale) see text [illegible] details. **b** Image of turbulent spots and the conversion to a intensity time series (see text for details).

Lemoult et al., Nature Physics (2016)

DP in large aspect ratio Taylor-Couette

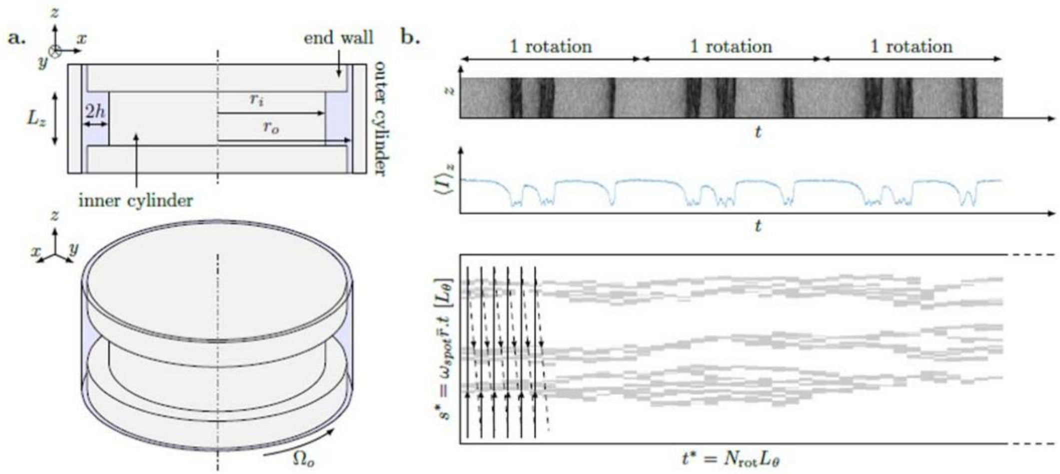
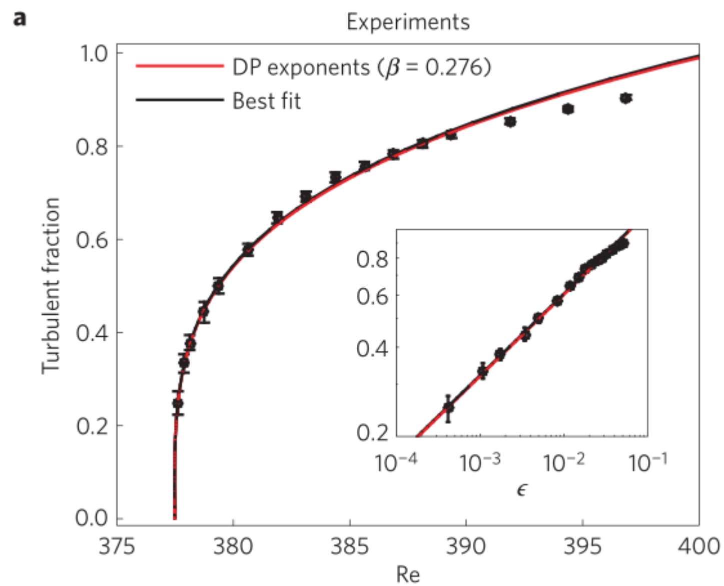
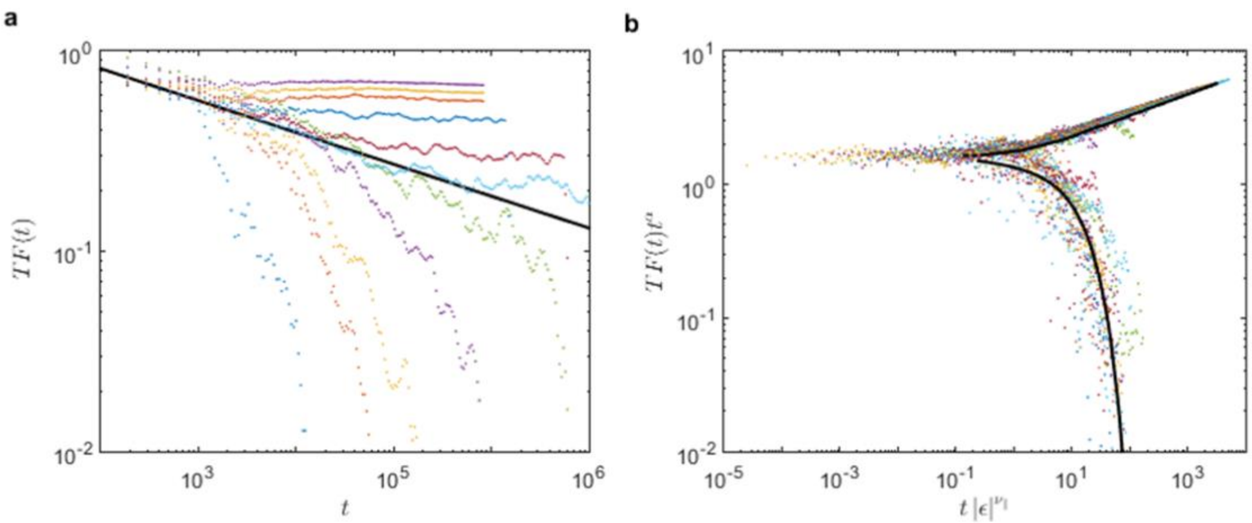


Figure S1: **a** Schematic of experimental set up (not to scale) see text for details. **b** Image of turbulent spots and the conversion to a intensity time series (see text for details).





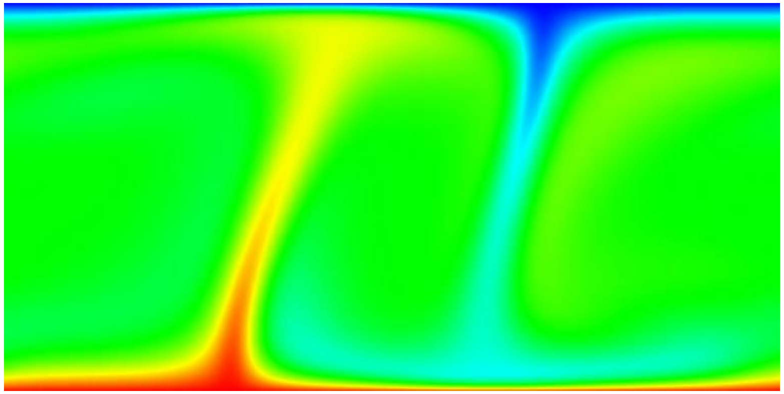
Dynamic scaling of turbulent fraction following a critical quench from $Re > Re_c$

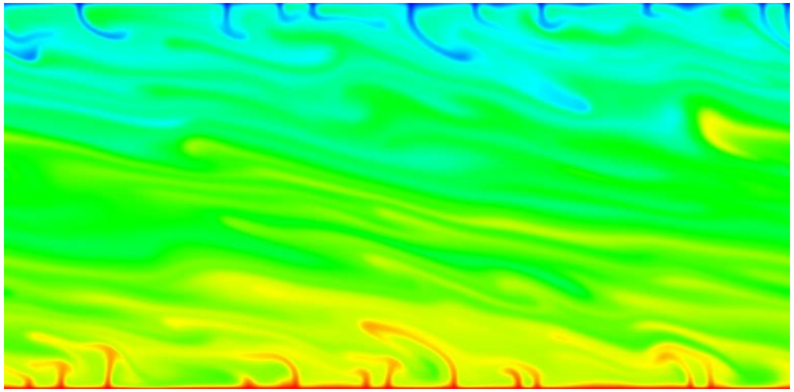
$$TF(t) \simeq t^{-\alpha} f(\varepsilon t^{1/\nu_\parallel})$$

Lemoult et al., Nature Physics (2016)

But Nigel, is this
the transition
to turbulence or
a transition
to turbulence?

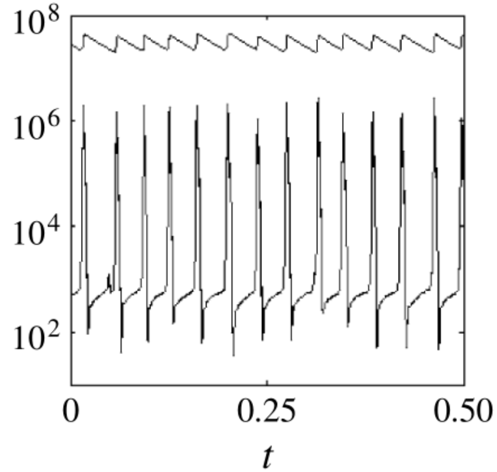
Predator-prey oscillations in convection



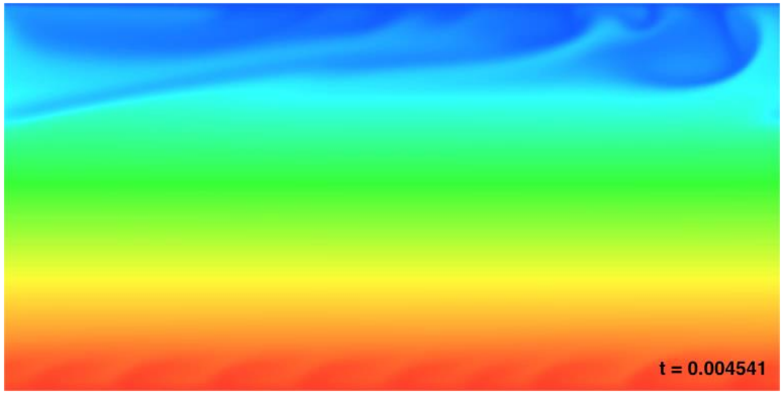


Pr=10 Ra=2 x 10^5 **Sustained shearing convection** Pr=10 Ra=2 x 10^8

E_x = Horizontal component of KE



E_z = Vertical component of KE



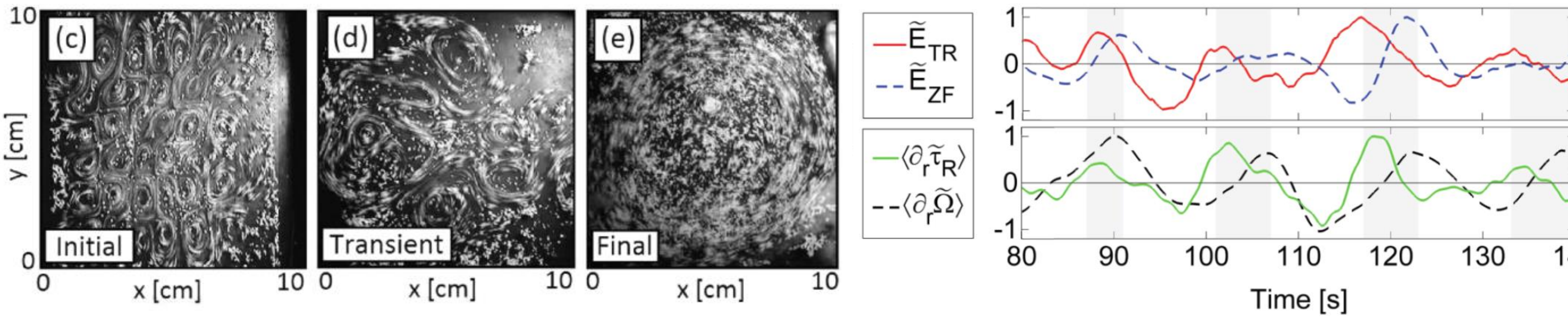
Pr = 1 Ra=2 x 10^8 **Bursty shearing convection**

Energy in zonal flow and vertical plumes shows predator-prey oscillations

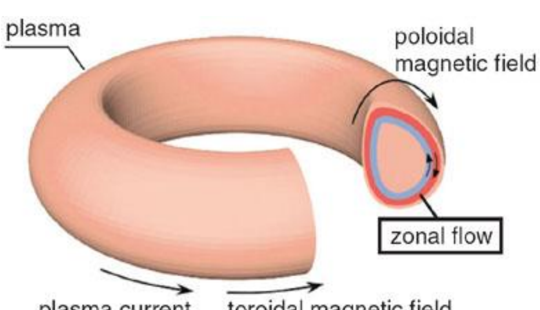
D. Goluskin et al. JFM (2014)

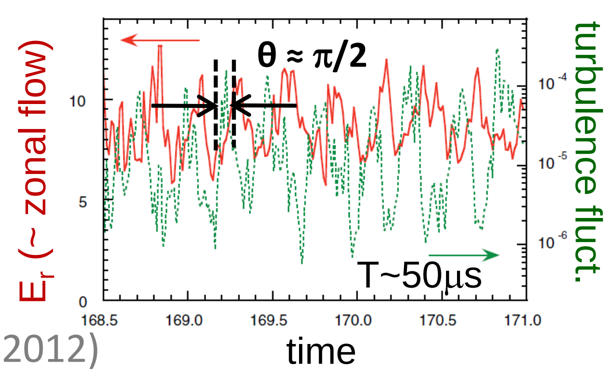
Universal predator-prey behavior in transitional turbulence

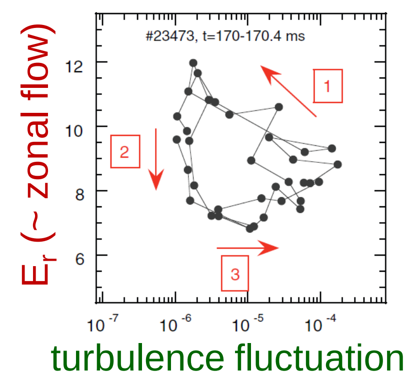
- Experimental observations
  - L-H mode transition in fusion plasmas in tokamak
  - 2D magnetized electroconvection



Bardoczi et al. Phys. Rev E (2012)







Estrada et al. EPL (2012)

Transition to turbulence in Taylor-Couette flow

PHYSICAL REVIEW E **81**, 025301(R) (2010)

Transient turbulence in Taylor-Couette flow

Daniel Borrero-Echeverry and Michael F. Schatz
Center for Nonlinear Science and School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332-0430, USA

Randall Tagg
Department of Physics, University of Colorado, Denver, Colorado 80217-3364, USA
(Received 4 May 2009; revised manuscript received 2 December 2009; published 19 February 2010)

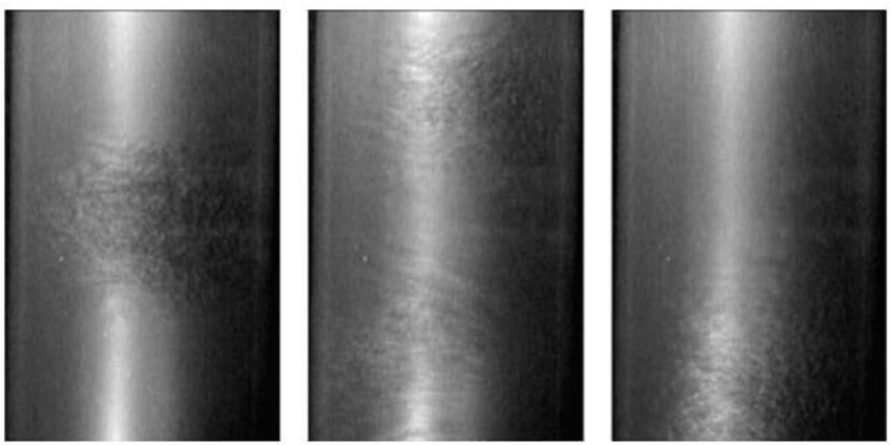
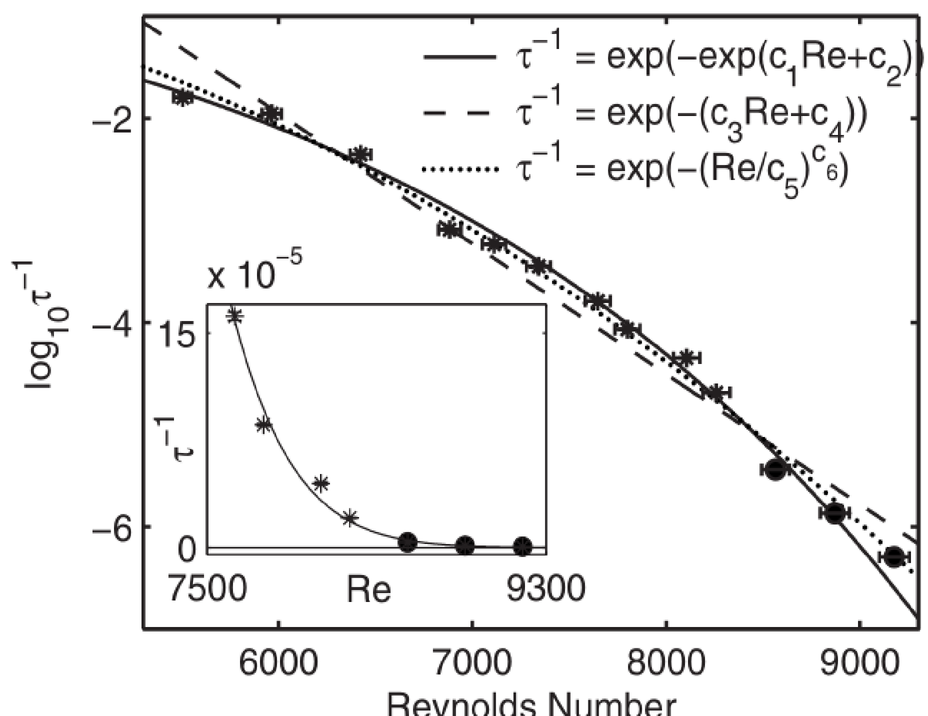


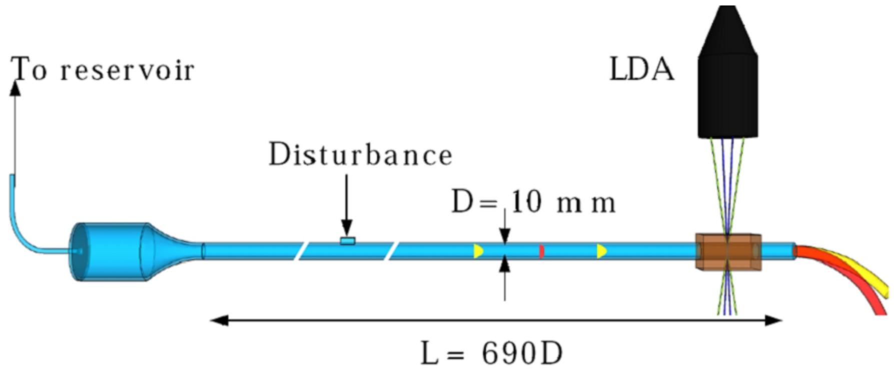
FIG. 1. Photographs of turbulent patches in TCF at Re=7500 with only the outer cylinder rotating. In this regime, turbulent patches coexist with the laminar flow and evolve in space and time. For all Re studied, these patches decay away in a probabilistic manner with a characteristic time scale τ dependent on Re. The photographs show a 25 cm high region of the flow.



Is there a well-defined sharp transition in turbulence?

If so, why do we see superexponential scaling rather than asymptotic criticality like in directed percolation?

Superexponential vs critical



Superexponential but no critical Re or critical exponents

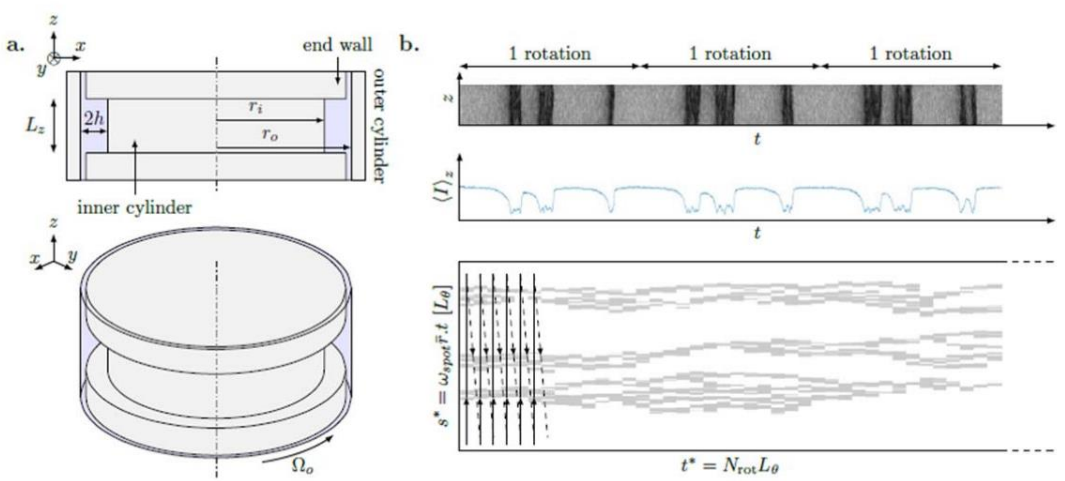


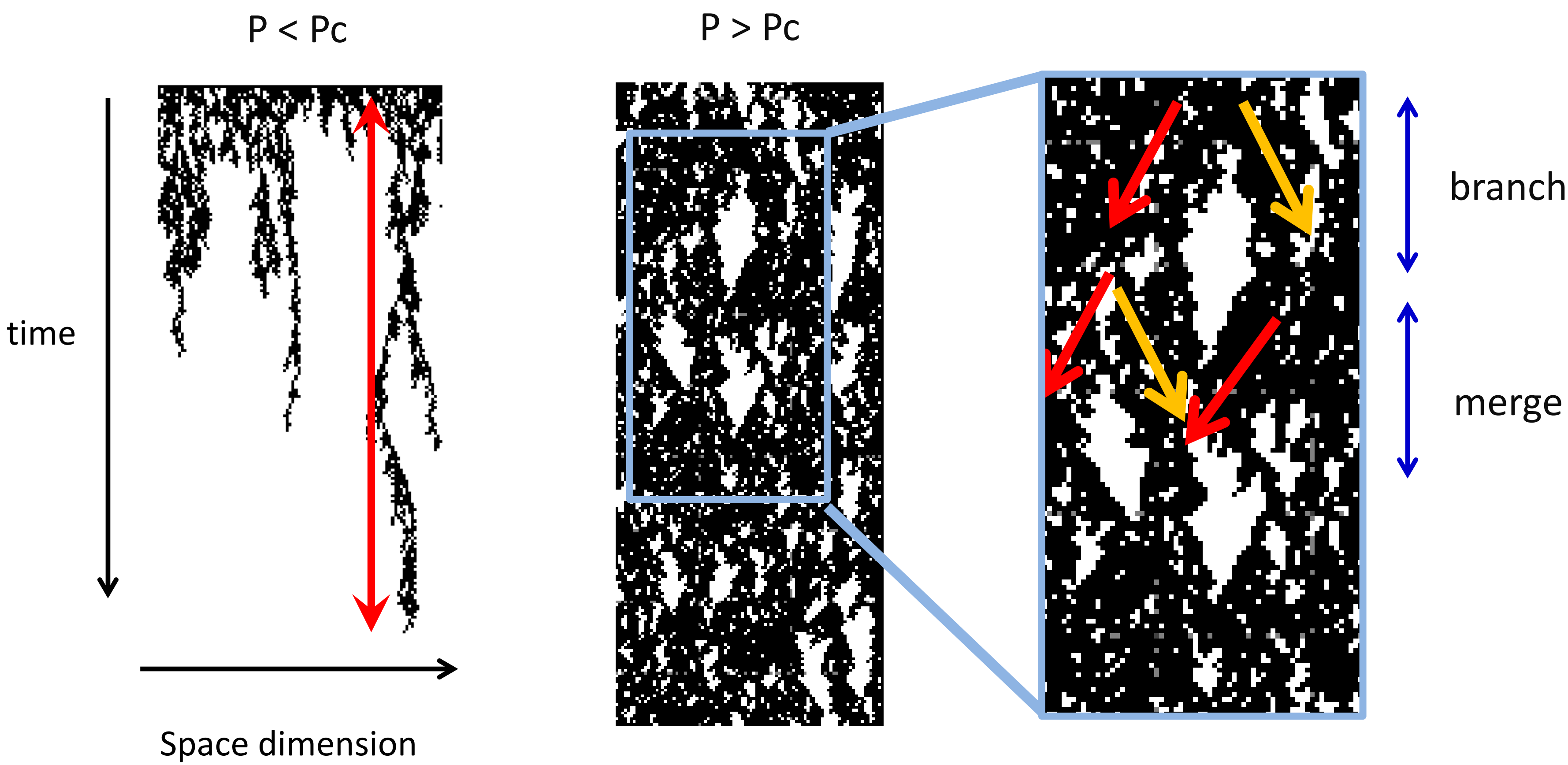
Figure S1: **a** Schematic of experimental set up (not to scale) see text for details. **b** Image of turbulent spots and the conversion to a intensity time series (see text for details).

Critical Re, critical exponents and scaling functions but no superexponential

Lemoult et al., Nature Physics (2016)   Hof *et al.*, *PRL* **101**, 214501 (2008)   Sano & Tamai, Nature Physics (2016)

Survival probability in 1+1 DP

- Turbulent puff lifetime → longest percolation path
- Turbulent puff splitting time → longest length of empty site



Super-exponential scaling from extreme value statistics

- Calculate survival probability from extreme value statistics

$$P(t_{\mathrm{d}} > t) = 1 - P(t_{\mathrm{d}} < t) = 1 - e^{-e^{-(t-\mu_{\mathrm{d}})/\beta_{\mathrm{d}}}}$$

- Define decay rate by fitting survival probability with exponential

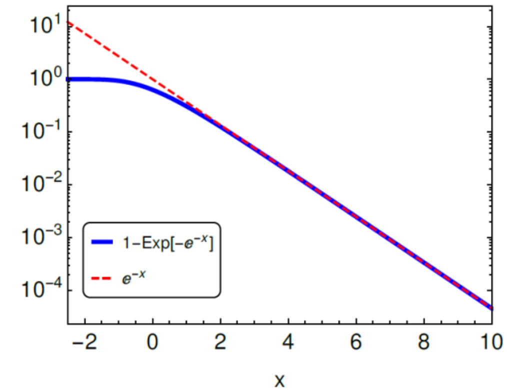
$$P(t_{\mathrm{d}} > t) \equiv e^{-(t-t_0)/\tau_{\mathrm{d}}}$$

$$\Longrightarrow \quad e^{-(t-t_0)/\tau_{\mathrm{d}}} = 1 - e^{-e^{-(t-\mu_{\mathrm{d}})\beta_{\mathrm{d}}}}$$

$$t_1 \equiv t - t_0 \quad t_2 \equiv t - \mu_{\mathrm{d}}(p)$$

$$\Longrightarrow \quad -t_1/\tau_{\mathrm{d}} = \ln(1 - e^{-e^{-t_2/\beta_{\mathrm{d}}}}) \sim -e^{-e^{-t_2/\beta_{\mathrm{d}}}}$$

$$\Longrightarrow \quad \boxed{\frac{\tau_{\mathrm{d}}(p)}{t_1} = e^{e^{bt_2 p - at_2 + \mathcal{O}(p^2)}}}$$



Decay rate τ scales super-exponentially due to extreme value statistics

Coexistence of super-exponential and power law

- **Decay rate $\boldsymbol{\tau}$** from survival probability scales super-exponentially:

$$P(\mathrm{p},t) = e^{-\frac{t-t_0}{\boldsymbol{\tau}(\mathrm{p})}} \quad \Longrightarrow \quad \boldsymbol{\tau} \sim \exp(\exp \mathrm{p})$$

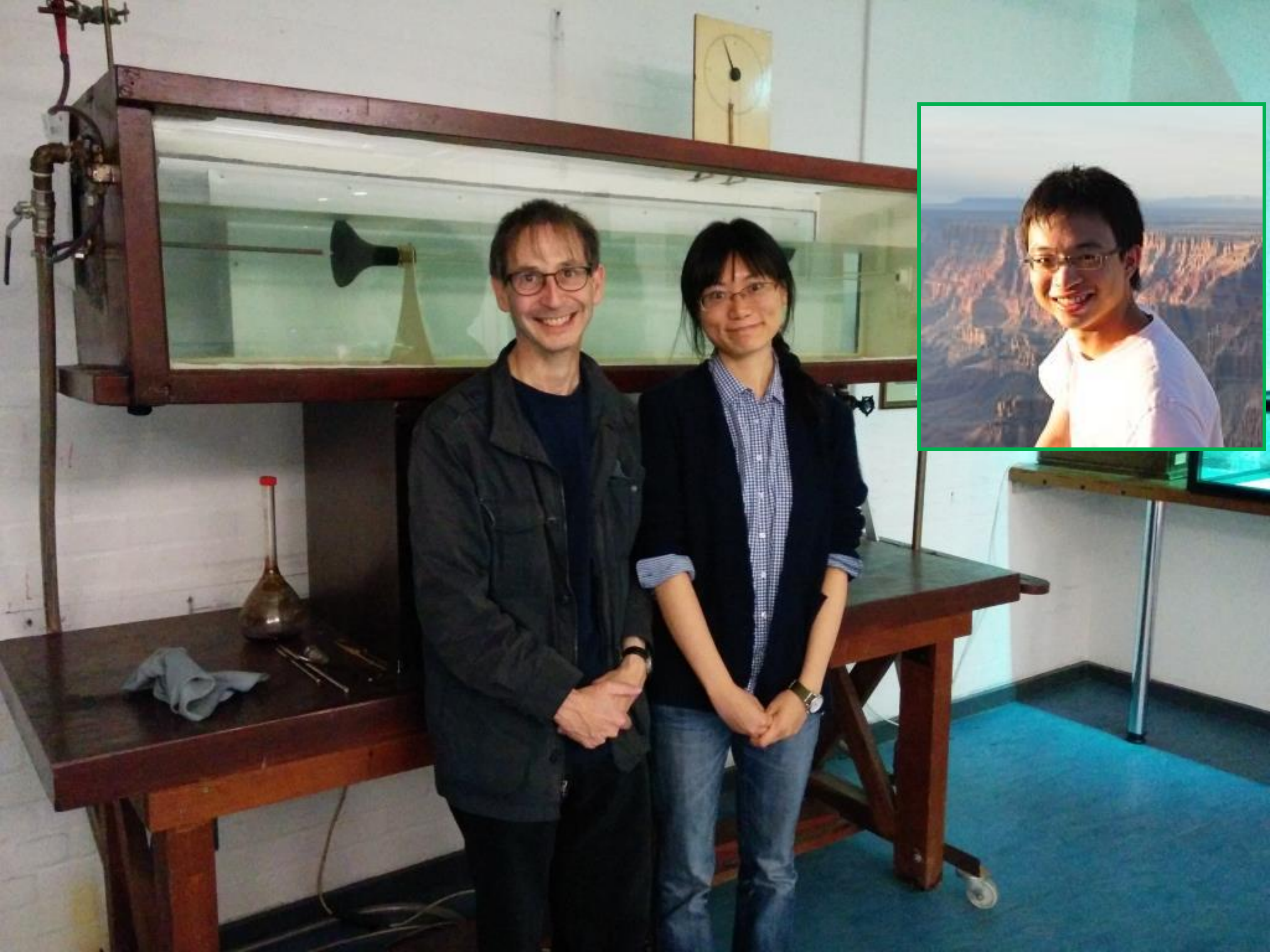
- **Mean lifetime $\langle \boldsymbol{t_{max}} \rangle$** scales in power law:

$$\langle t_{\max} \rangle \sim \xi_{||}(p) \sim |p - p_c|^{-\nu_{||}}$$

Summary

- Transitional turbulence is governed by predator-prey interactions leading to directed percolation
  - Turbulence is the prey
  - Zonal (azimuthal) flow is the predator
  - Heuristic derivation of zonal flow-Reynolds stress interaction in mean field theory
- Is there a sharp well-defined transition even though the measured lifetime scales super-exponentially?
  - Yes. The survival probability lifetime reflects extreme value statistics (longest-lived path or turbulent patch)
  - The mean value of the lifetime scales with the DP critical exponents

Superexponential scaling and power-law critical scaling coexist



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- Maksim Sipos and Nigel Goldenfeld. Directed percolation describes lifetime and growth of turbulent puffs and slugs. *Phys. Rev. E Rapid Communications* **84**, 035305 (4 pages) (2011)
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QUASI-CYCLES AND FLUCTUATION-INDUCED PREDATOR-PREY OSCILLATIONS

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- T. Butler and Nigel Goldenfeld. Fluctuation-driven Turing patterns. *Phys. Rev. E* **84**, 011112 (12 pages) (2011)
- Hong-Yan Shih and Nigel Goldenfeld. Path-integral calculation for the emergence of rapid evolution from demographic stochasticity. *Phys. Rev. E Rapid Communications* **90**, 050702 (R) (7 pages) (2014)

Turbulence is a life force. It is opportunity.
Let's love turbulence and use it for change.

Lucky Numbers 34, 15, 28, 4, 19, 20