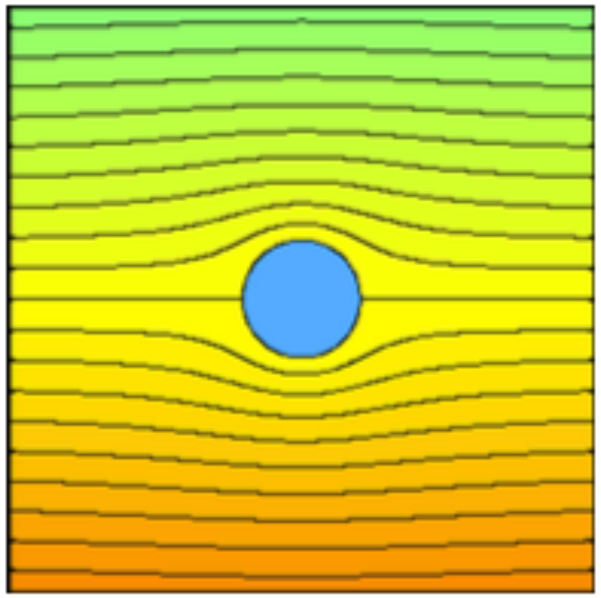
Mean flows and frequency prediction

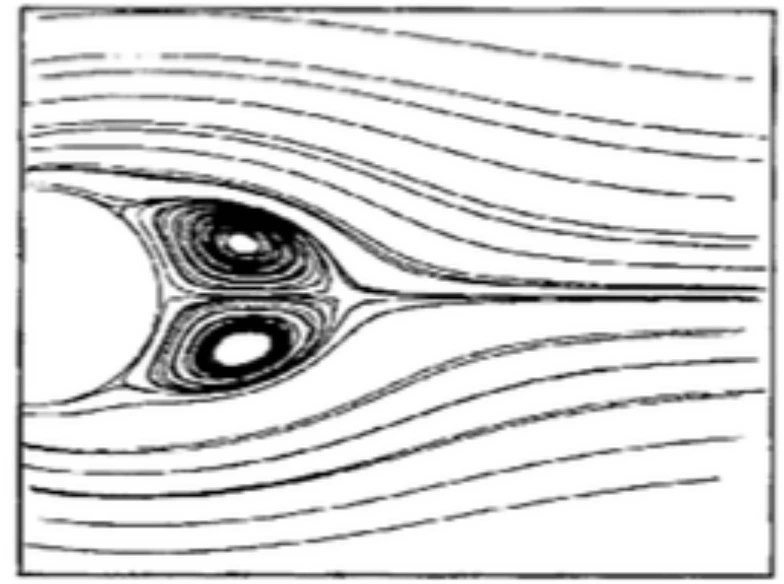
Sam Turton (Cambridge, Part III —> MIT)
Yacine Bengana (PMMH-CNRS-ESPCI)
Nicolas Périnet (Univ. of Santiago)
Laurette Tuckerman (PMMH-CNRS-ESPCI)
Dwight Barkley (University of Warwick)

Cylinder wake

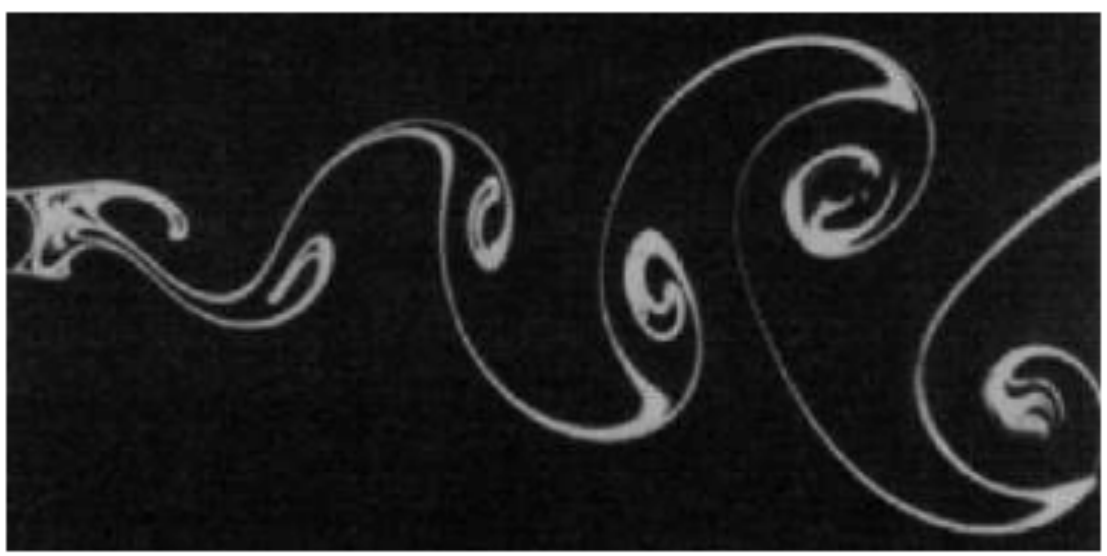
Ideal flow



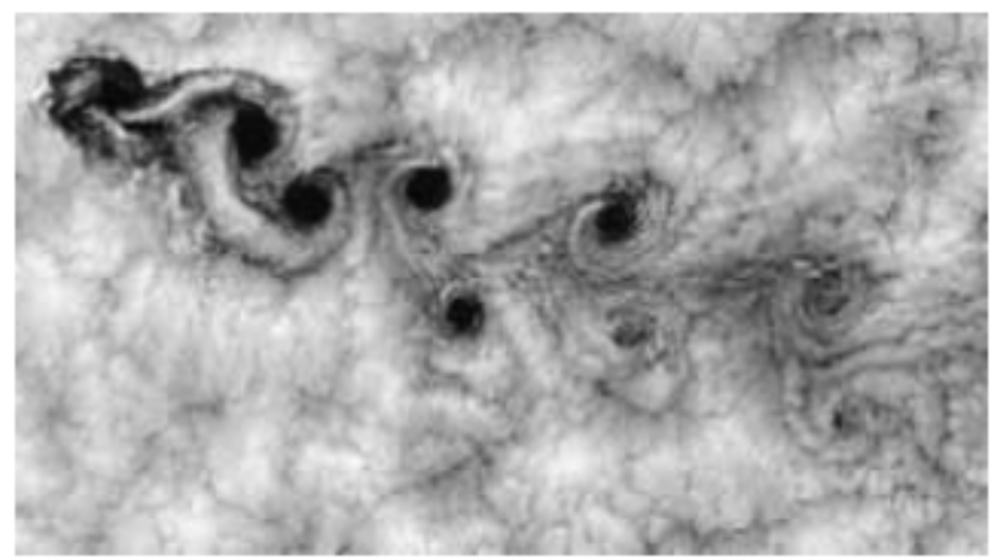
with downstream recirculation zone



Von Kármán vortex street $(\mathrm{Re} \geq 46)$

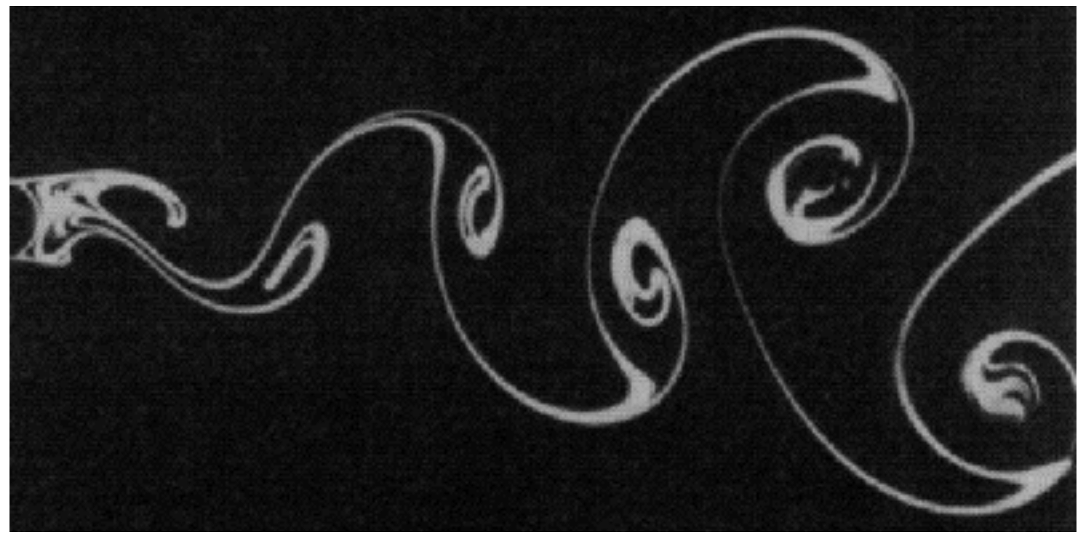


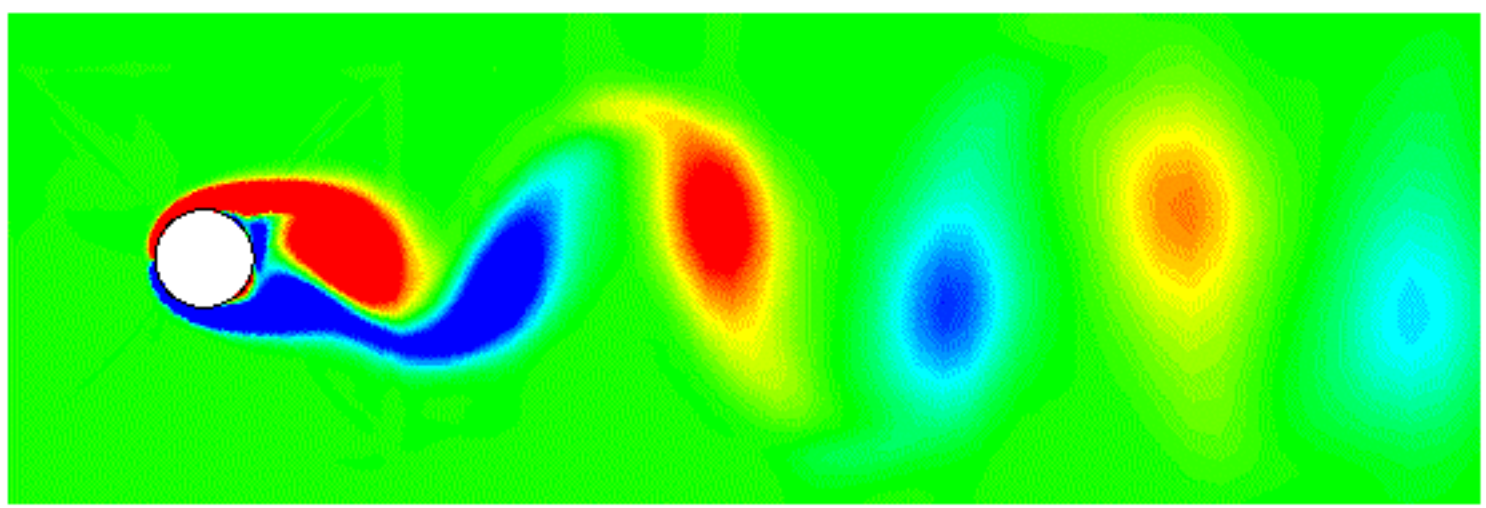
Laboratory experiment (Taneda, 1982)



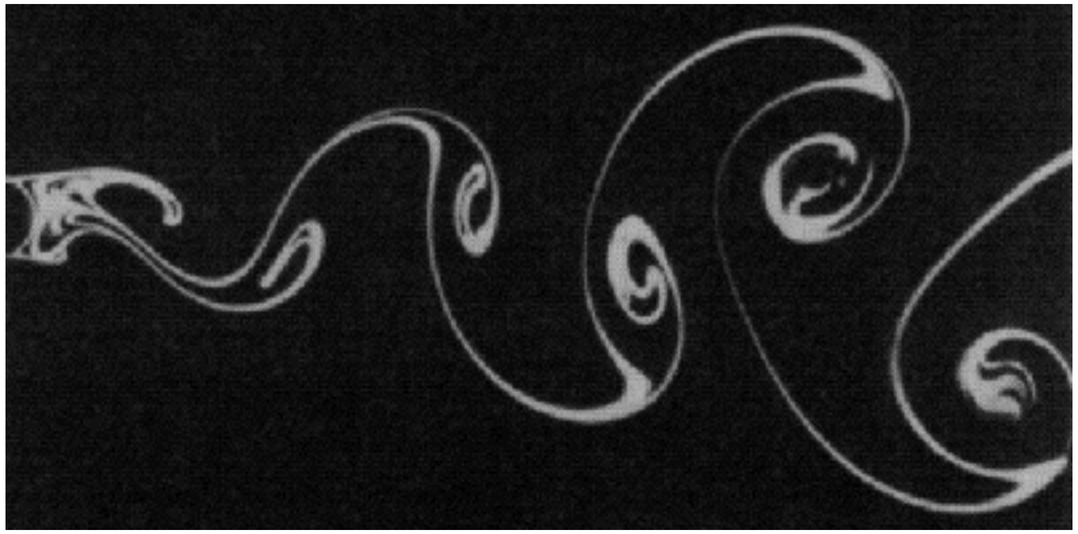
Off Chilean coast past Juan Fernandez islands

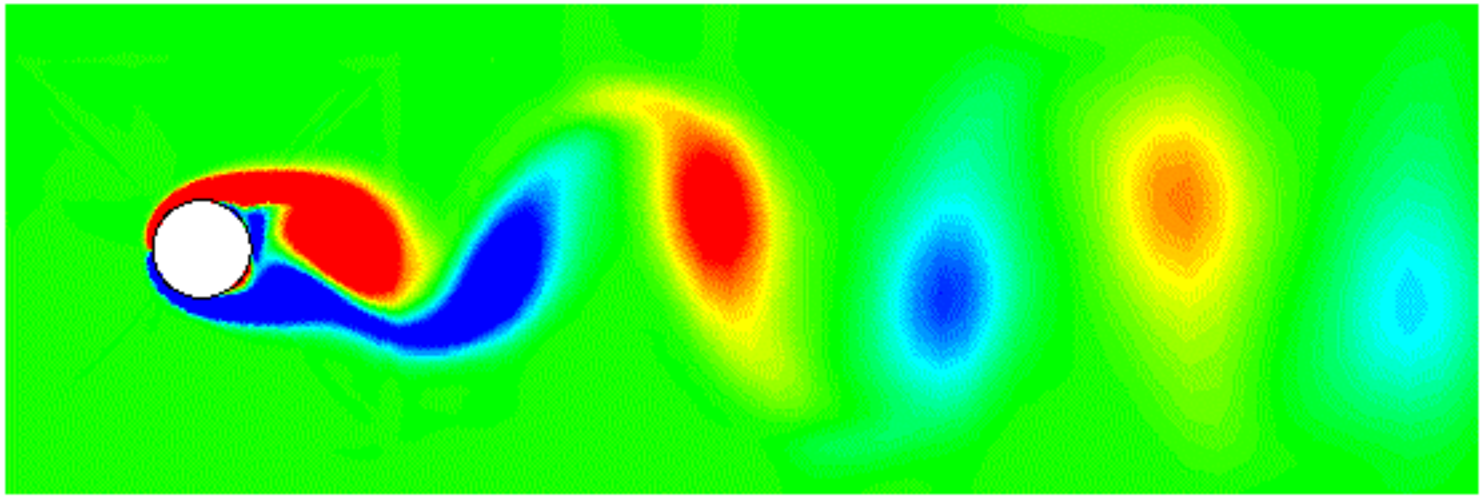
Vortex-shedding frequency of cylinder wake





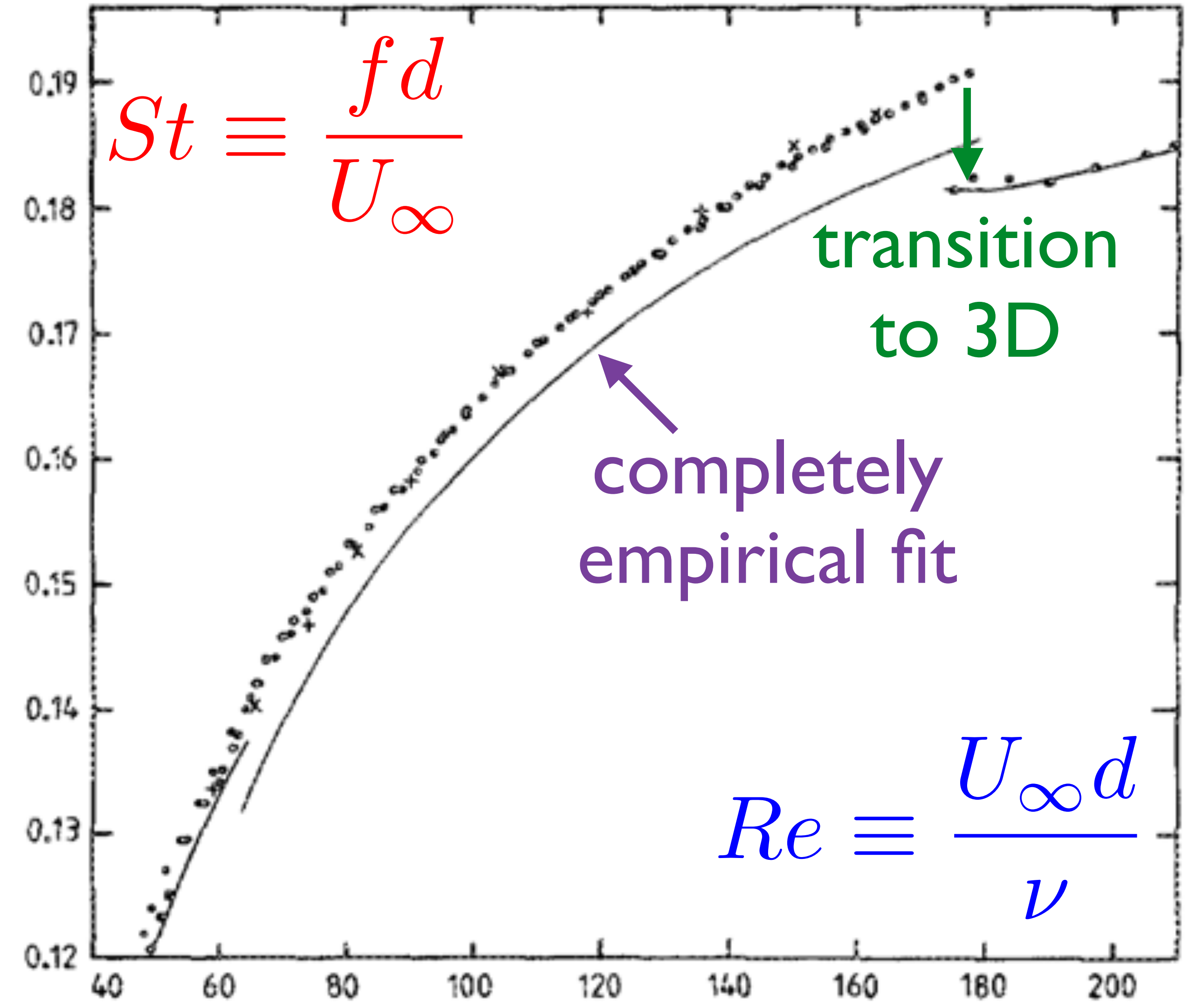
Vortex-shedding frequency of cylinder wake





Defining a universal and continuous Strouhal–Reynolds number relationship for the laminar vortex shedding of a circular cylinder

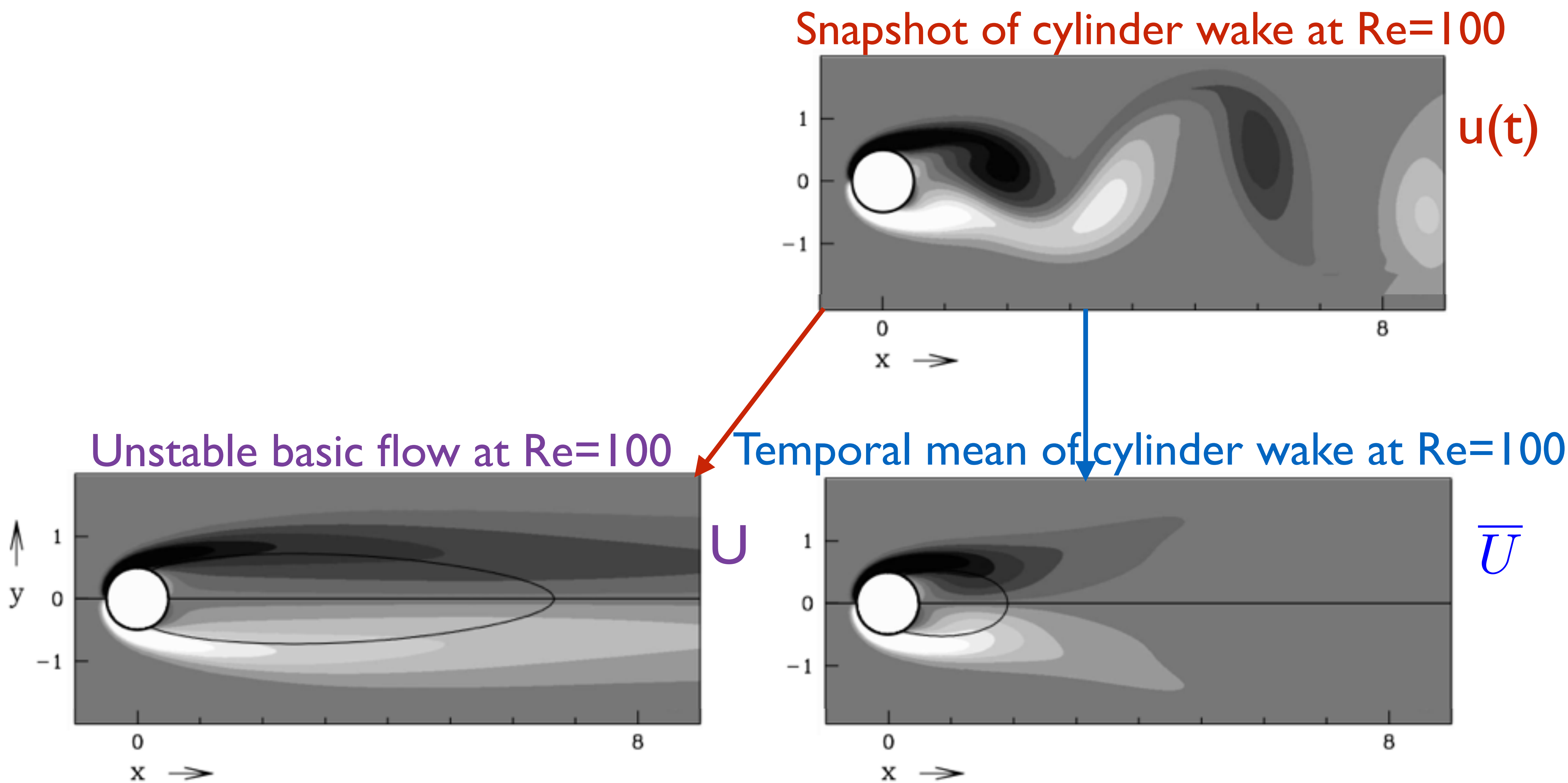
C. H. K. Williamson Physics of Fluids **31**, 2742 (1988)



Seek prediction from equations/physics

Linear analysis of the cylinder wake mean flow

D. Barkley(*) *Europhys. Lett.*, **75** (5), pp. 750–756 (2006)



Basic flow

$$0 = -(U \cdot \nabla)U - \nabla P + \frac{1}{Re}\nabla^2 U$$

Temporally periodic wake flow

$$\partial_t u = -(u \cdot \nabla)u - \nabla p + \frac{1}{Re}\nabla^2 u$$

Temporal mean

$$\overline{U} \equiv \frac{1}{T}\int_0^T u(t)dt$$

Linearise about steady base flow

$$\partial_t u = -(U \cdot \nabla)u - (u \cdot \nabla)U - \nabla p + \frac{1}{Re}\nabla^2 u$$

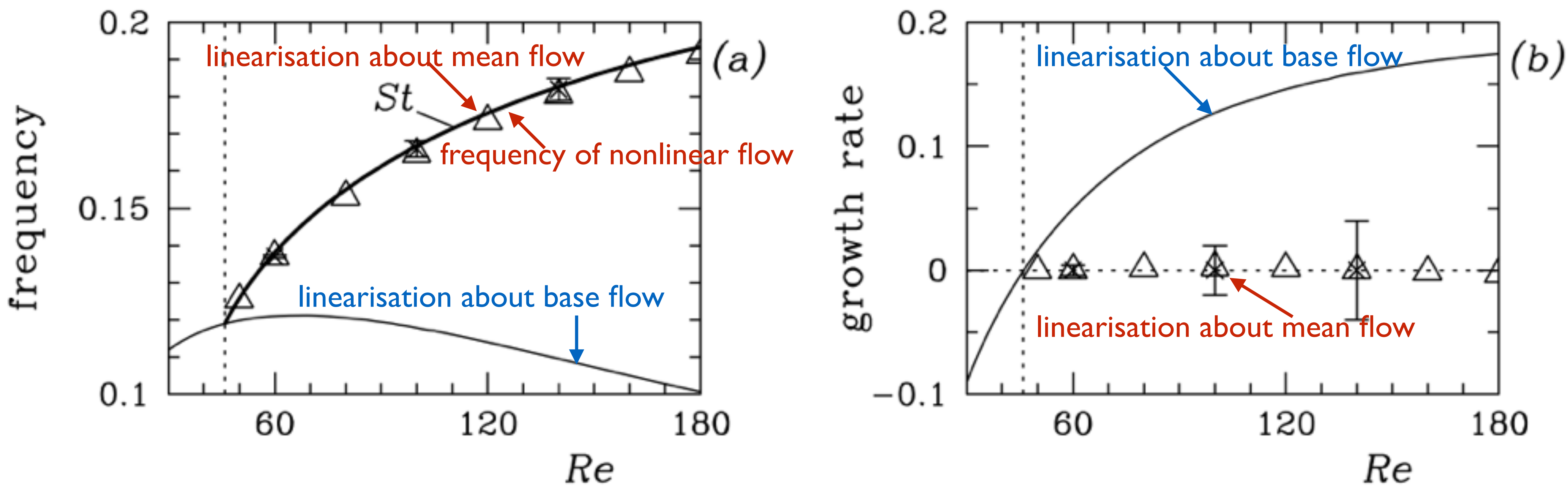
Linearise about temporal mean

$$\partial_t u = -(\overline{U} \cdot \nabla)u - (u \cdot \nabla)\overline{U} - \nabla p + \frac{1}{Re}\nabla^2 u$$

Strange and unjustified procedure, but quite successful !

Linear analysis of the cylinder wake mean flow

D. BARKLEY(*) *Europhys. Lett.*, **75** (5), pp. 750–756 (2006)



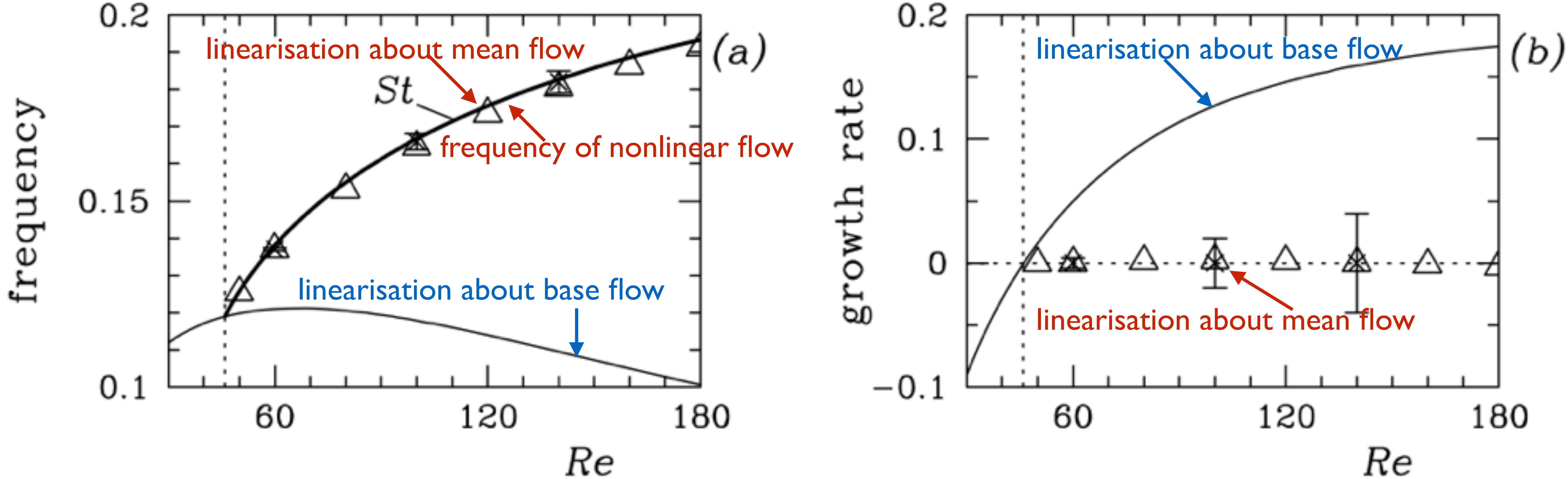
Mean flow eigenvalue has **RZIF** property:

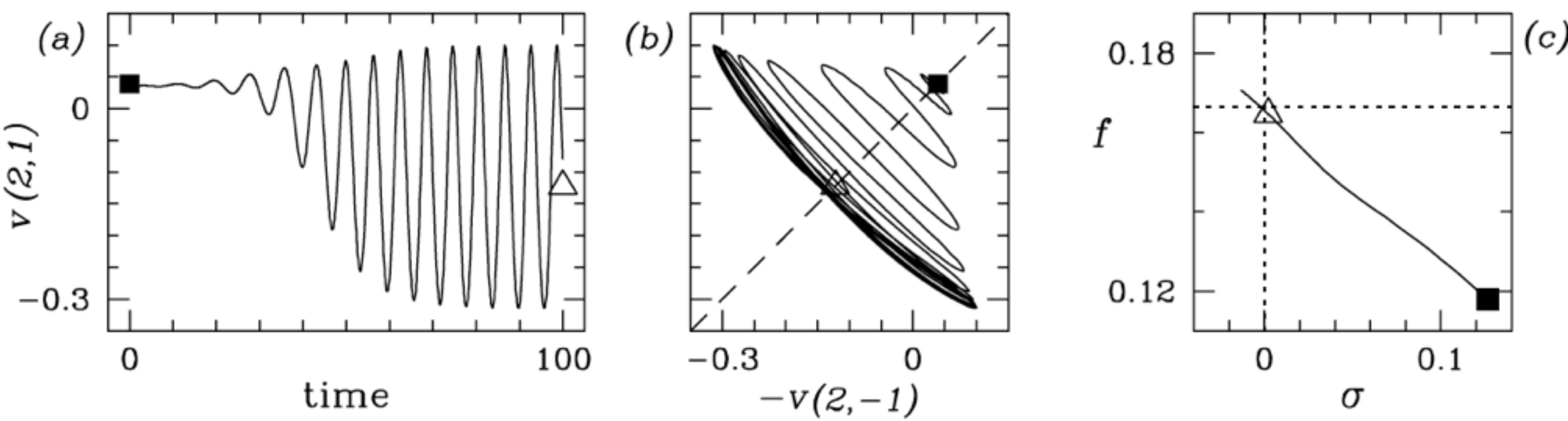
Real part is near **Zero**.

Imaginary part is near exact nonlinear **Frequency**.

Linear analysis of the cylinder wake mean flow

D. Barkley(*) *Europhys. Lett.*, **75** (5), pp. 750–756 (2006)





Malkus theory: Temporal mean of turbulent flow should be marginally stable

Outline of a theory of turbulent shear flow

W. V. R. Malkus

Journal of Fluid Mechanics / Volume 1 / Issue 05 / November 1956, pp 521 - 539

1956 Cambridge University Press

VOLUME 79, NUMBER 20 PHYSICAL REVIEW LETTERS 17 NOVEMBER 1997

Strongly Nonlinear Effect in Unstable Wakes

B. J. A. Zielinska,[2,1] S. Goujon-Durand,[1,3] J. Dušek,[4] and J. E. Wesfreid[1]
[1]*Ecole Supérieure de Physique et Chimie Industrielles de Paris (ESPCI), PMMH-URA CNRS No. 857,*

J. Fluid Mech. (2002), *vol.* 458, *pp.* 407–417. © 2002 Cambridge University Press
DOI: 10.1017/S0022112002008054 Printed in the United Kingdom

On the frequency selection of finite-amplitude vortex shedding in the cylinder wake

By BENOÎT PIER

Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Silver Street, Cambridge CB3 9EW, UK

J. Fluid Mech. (2003), *vol.* 497, *pp.* 335–363. © 2003 Cambridge University Press
DOI: 10.1017/S0022112003006694 Printed in the United Kingdom

A hierarchy of low-dimensional models for the transient and post-transient cylinder wake

By BERND R. NOACK[1]†, KONSTANTIN AFANASIEV[2], MAREK MORZYŃSKI[3], GILEAD TADMOR[4] AND FRANK THIELE[1]

J. Fluid Mech. (2007), vol. 593, pp. 333–358.

doi:10.1017/S0022112007008907 Printed in the United Kingdom

Global stability of base and mean flows: a general approach and its applications to cylinder and open cavity flows

DENIS SIPP AND ANTON LEBEDEV
ONERA, 8 rue des Vertugadins, 92190 Meudon, France

INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN FLUIDS
Int. J. Numer. Meth. Fluids 2008; **58**:111–118
Published online 20 December 2007 in Wiley InterScience (www.interscience.wiley.com). DOI: 10.1002/fld.1714

SHORT COMMUNICATION

Global linear stability analysis of time-averaged flows

Sanjay Mittal*,†

PRL **113,** 084501 (2014) PHYSICAL REVIEW LETTERS week ending 22 AUGUST 2014

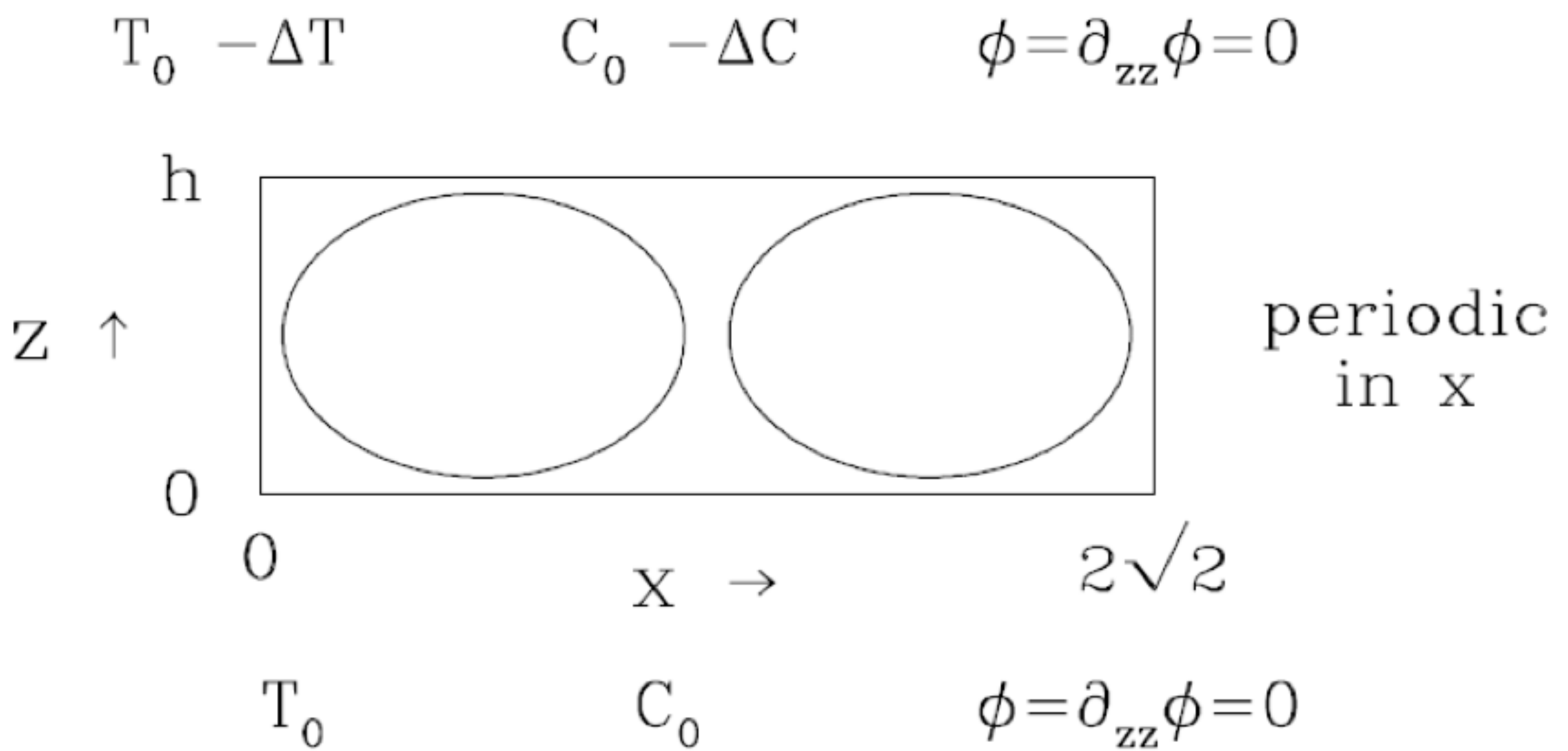
Self-Consistent Mean Flow Description of the Nonlinear Saturation of the Vortex Shedding in the Cylinder Wake

Vladislav Mantič-Lugo,* Cristóbal Arratia,† and François Gallaire
Laboratory of Fluid Mechanics and Instabilities, École Polytechnique Fédérale de Lausanne, EPFL-STI-IGM-LFMI, Lausanne CH-1015, Switzerland
(Received 19 December 2013; published 20 August 2014)

When is RZIF satisfied ?

Why is RZIF satisfied?

Simple Model: 2D Thermosolutal Problem



Vertical thermal and solutal gradients imposed at $z = 0, 1$

Boundary conditions: free-slip at $z = 0, 1$; periodic in x with length $2\sqrt{2}$

Streamfunction $\mathbf{U} = \nabla \times \phi(x, z)\mathbf{e}_y$

Density: $\rho(T, C) = \rho_0 + \rho_T(T - T_0) + \rho_C(C - C_0)$

Diffusivities: κ_T (thermal), κ_C (solutal), ν (momentum)

Conductive solution:

$$T = T_0 - z\Delta T/h, \qquad C = C_0 - z\Delta C/h, \qquad \mathrm{U} = \nabla \times \phi \mathrm{e}_y = 0$$

Four nondimensional parameters:

Fix: Lewis number $L \equiv \frac{\kappa_C}{\kappa_T} \ll 1$ Prandtl number $P \equiv \frac{\nu}{\kappa_T} \gg 1$.

Vary: Rayleigh number $R \equiv \frac{g\rho_T \Delta T h^3}{\nu \kappa_T}$ Separation ratio $S \equiv \frac{\rho_C \Delta C}{\rho_T \Delta T}$

Subtract conductive solution and nondimensionalize.

Governing Equations:

$$\begin{aligned}
\partial_t \tilde{T} &= \partial_x \tilde{\phi} + \mathrm{e_y} \cdot (\nabla \tilde{\phi} \times \nabla \tilde{T}) + \nabla^2 \tilde{T} \\
\partial_t \tilde{C} &= \partial_x \tilde{\phi} + \mathrm{e_y} \cdot (\nabla \tilde{\phi} \times \nabla \tilde{C}) + L \nabla^2 \tilde{C} \\
\partial_t \nabla^2 \tilde{\phi} &= PR\partial_x(\tilde{T} + S\tilde{C}) + \mathrm{e_y} \cdot (\nabla \tilde{\phi} \times \nabla \nabla^2 \tilde{\phi}) + P\nabla^4 \tilde{\phi}
\end{aligned}$$

Linear Analysis:

$$\left\{\begin{array}{c}\tilde{T}\\ \tilde{C}\\ \tilde{\phi}\end{array}\right\}(x,z,t) = \left\{\begin{array}{c}T\cos(kx)\\ C\cos(kx)\\ \phi\sin(kx)\end{array}\right\}\sin(\pi z)e^{(k^2+\pi^2)\sigma t}$$

Nonlinear interaction of these eigenmodes of the basic state

$$\nabla\phi\times\nabla\nabla^2\phi = \nabla\phi\times\nabla(-k^2-\pi^2)\phi = 0$$

$$\nabla\phi\times\nabla T = \phi T\frac{k\pi}{2}\sin(2\pi z)$$

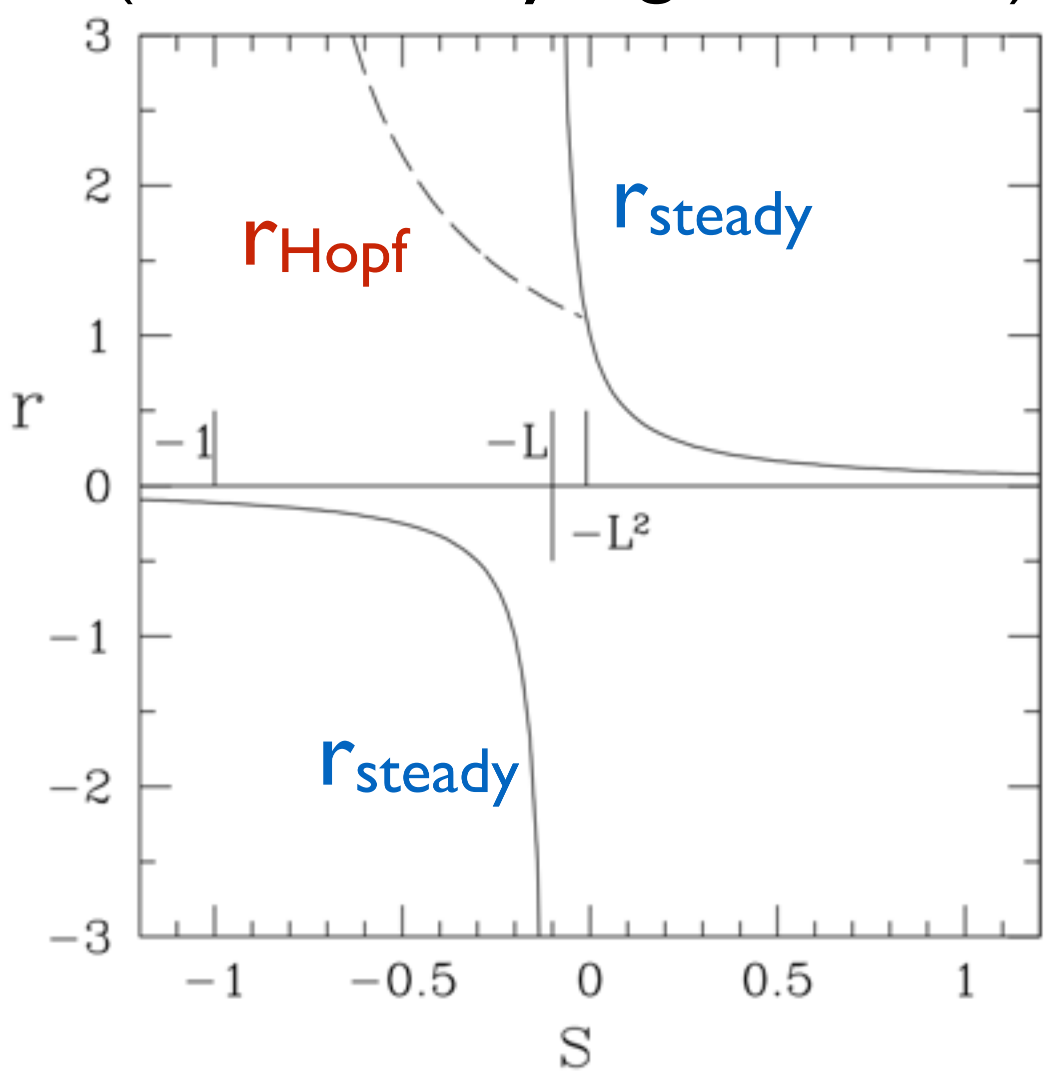
$$\nabla\phi\times\nabla C = \phi C\frac{k\pi}{2}\sin(2\pi z)$$

At lowest order, mean "flow" has $u = 0, T \neq 0, C \neq 0$

Hopf bifurcation to standing or traveling waves if separation ratio $S=Ra_C/Ra_T<0$

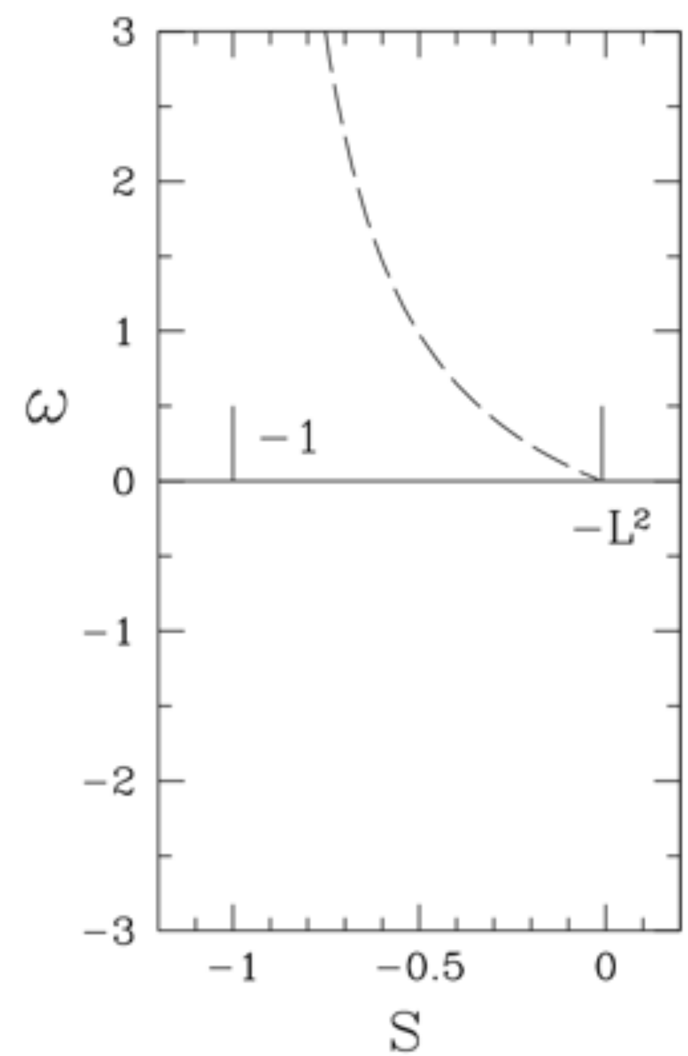
Temperature and concentration gradients are in opposite directions

convection threshold
(reduced Rayleigh number)



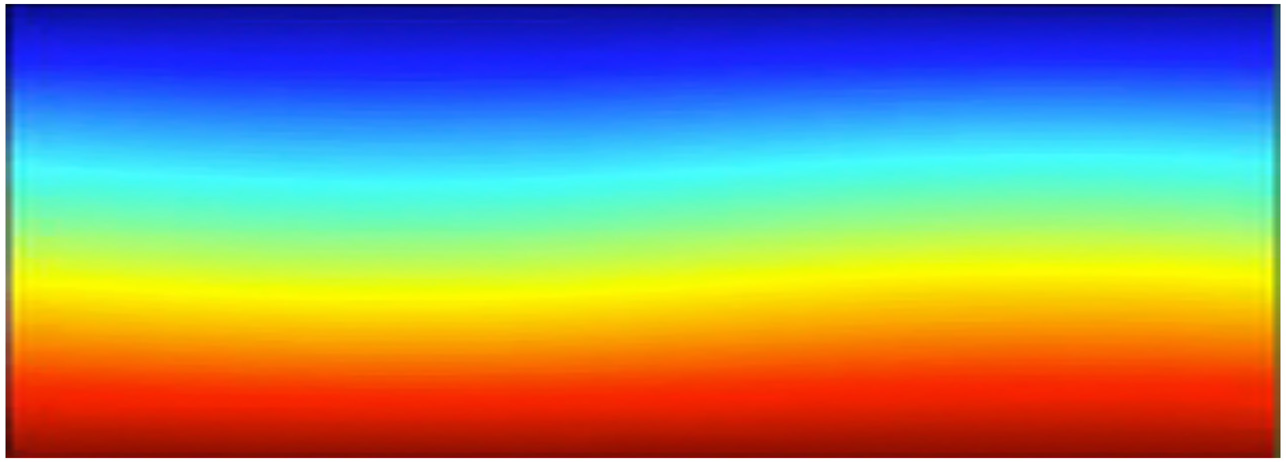
Separation ratio

Hopf frequency

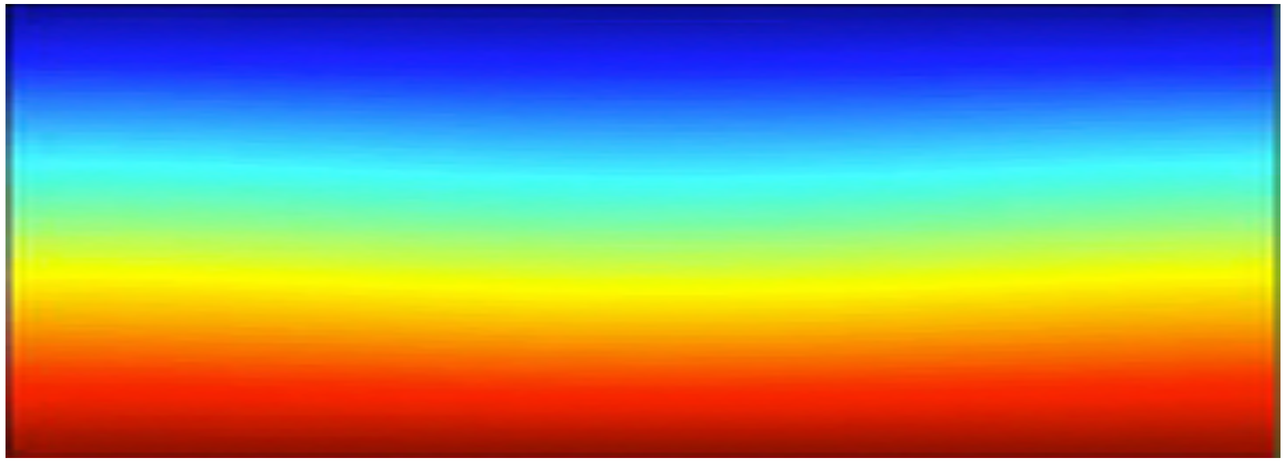


Separation ratio

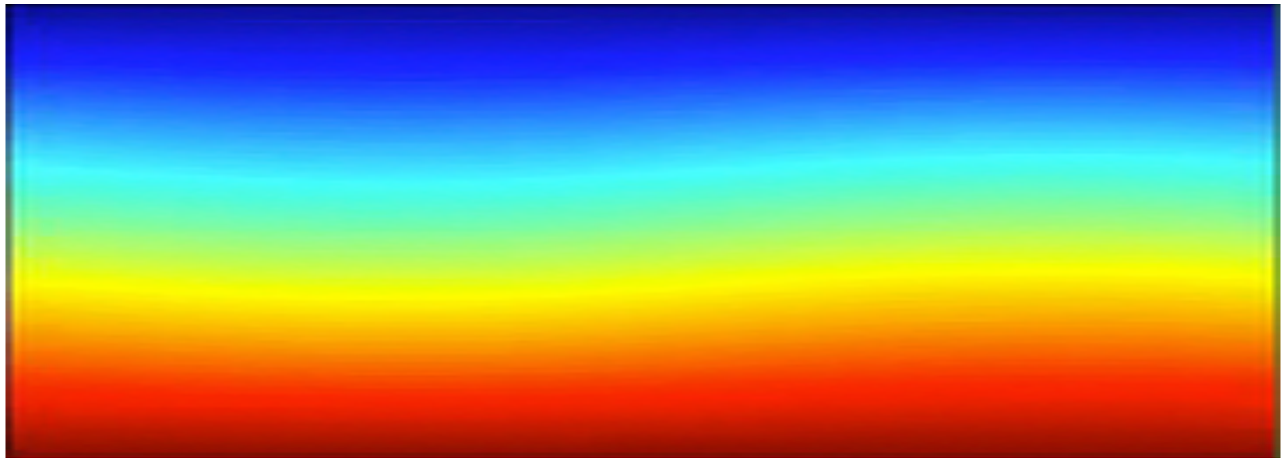
Traveling wave



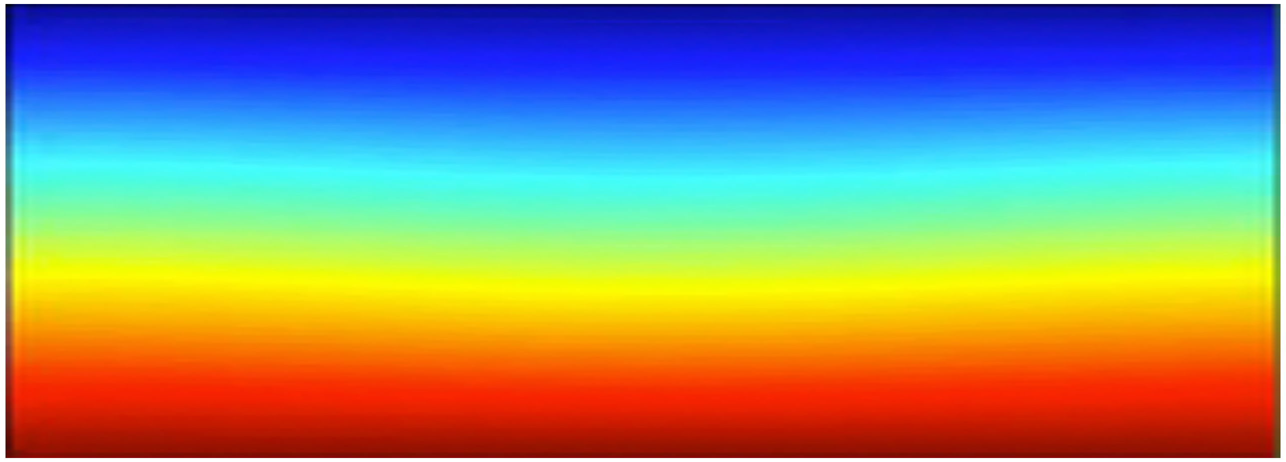
Standing wave



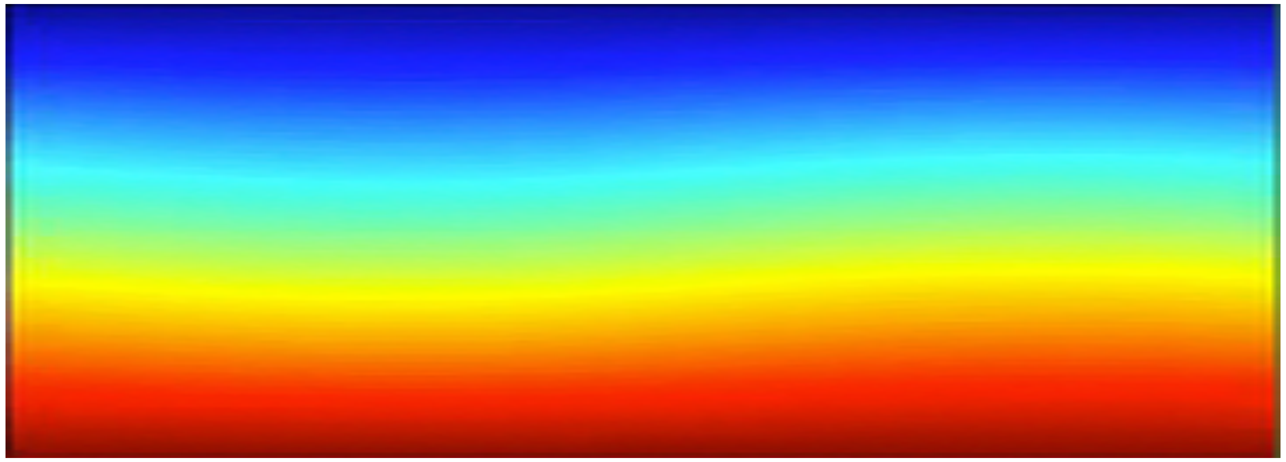
Traveling wave



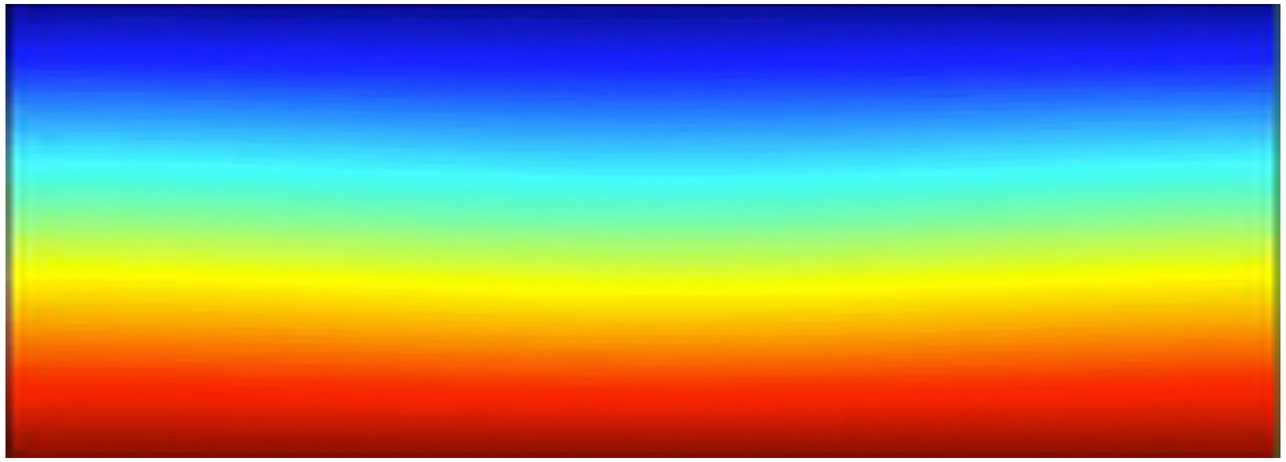
Standing wave



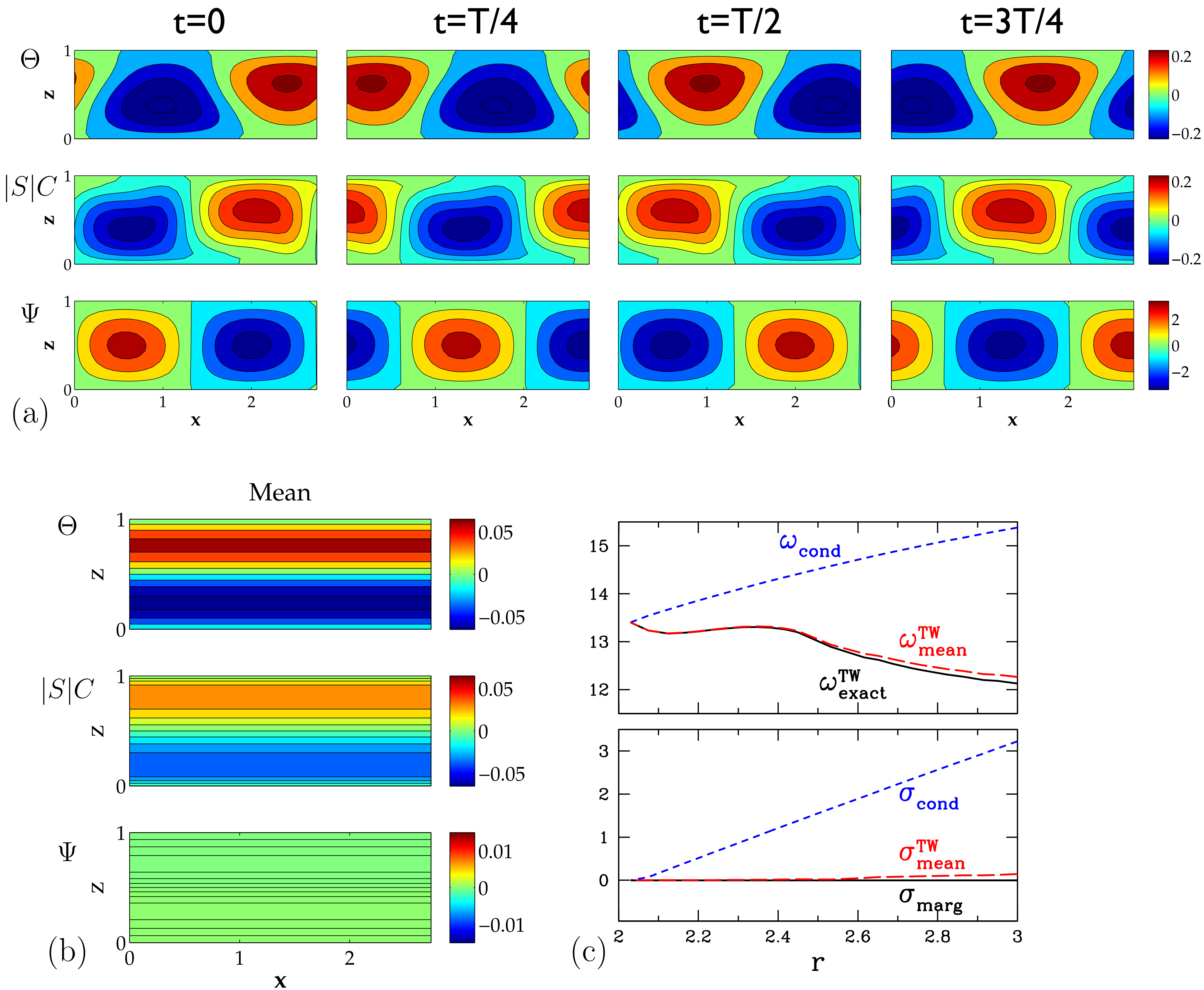
Traveling wave



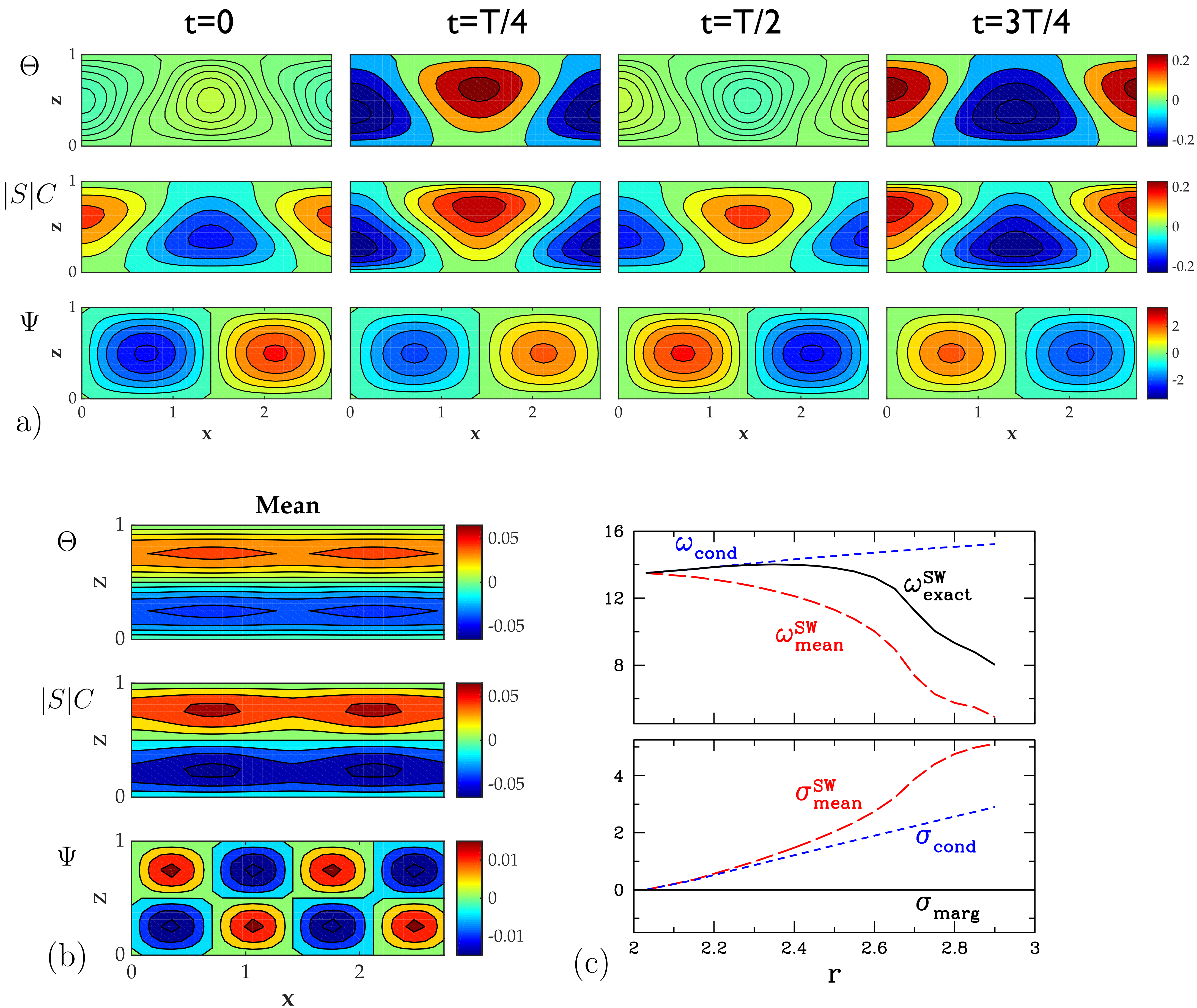
Standing wave



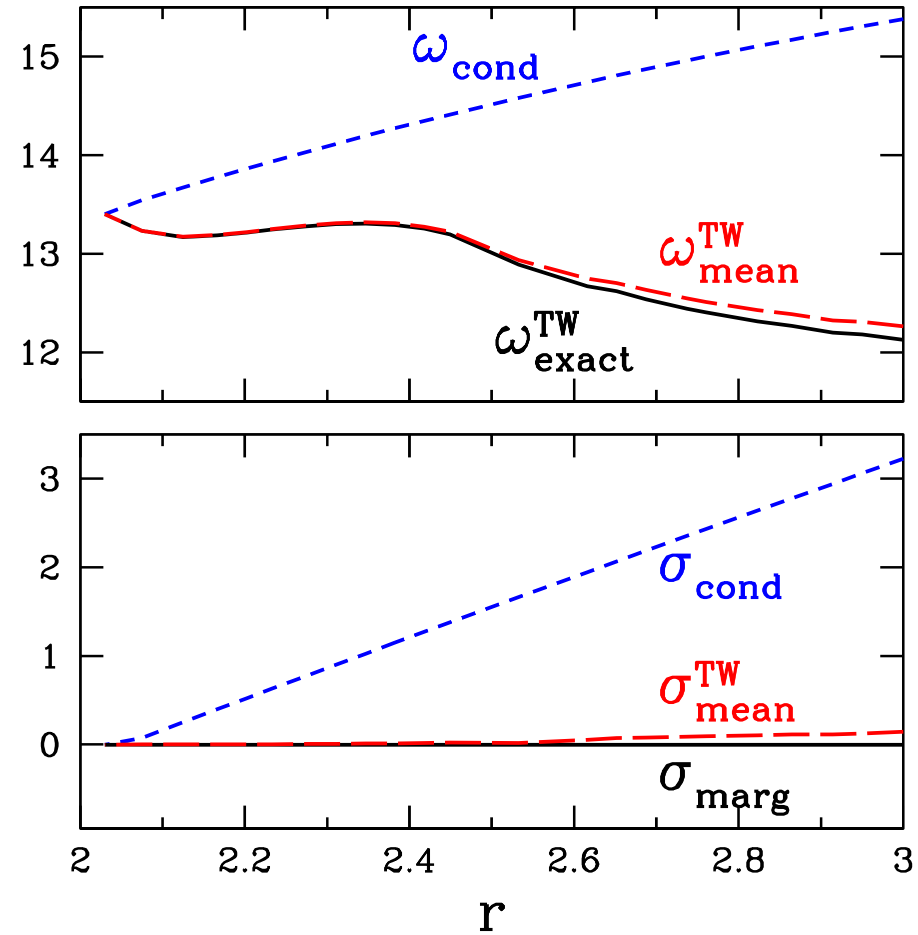
Traveling waves



Standing waves



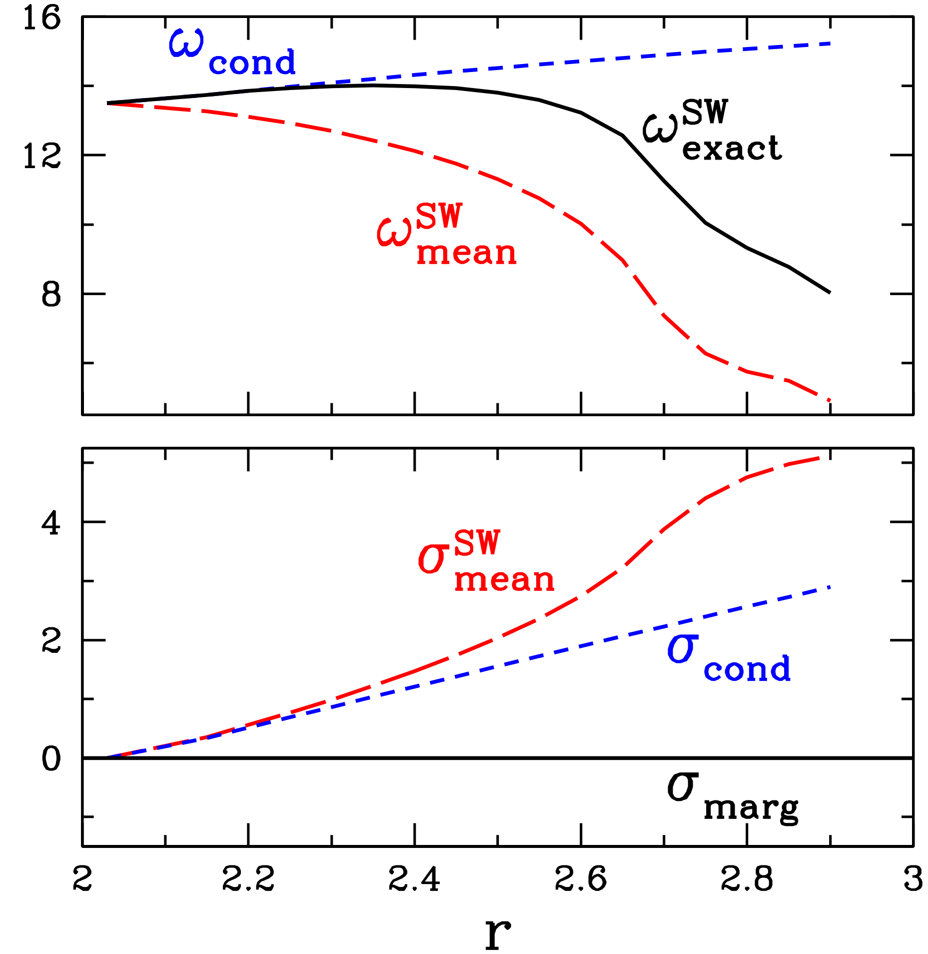
Traveling waves



mean = exact

RZIF

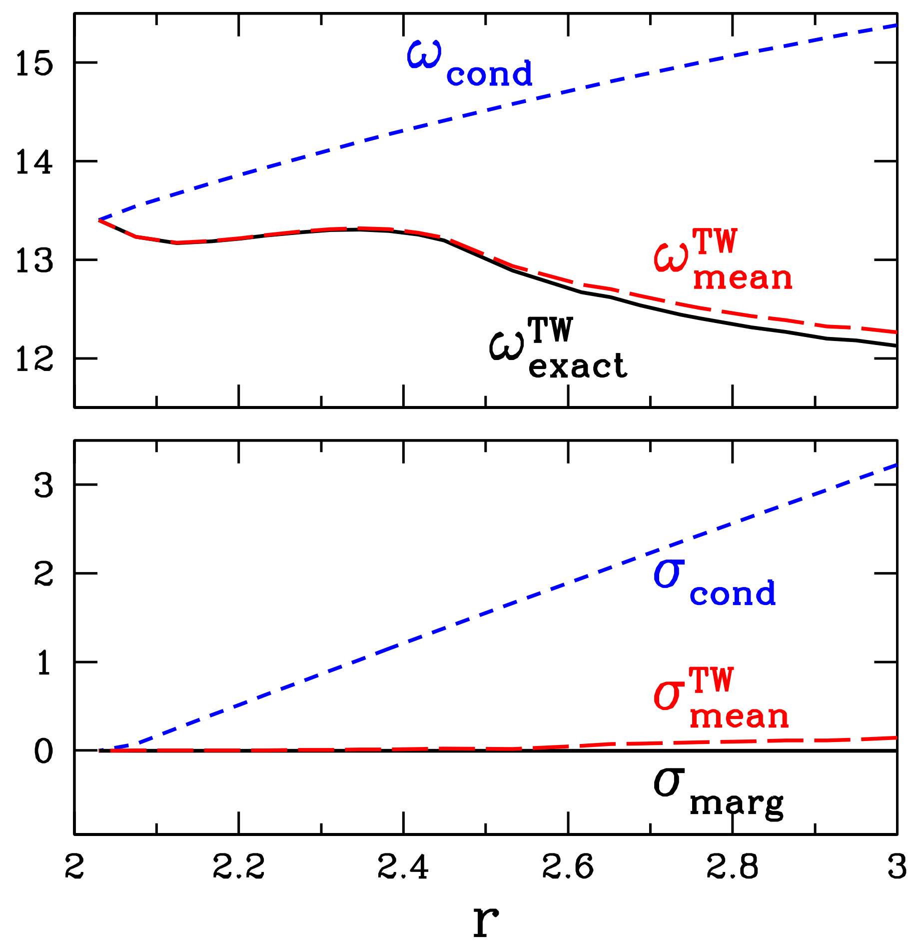
Standing waves



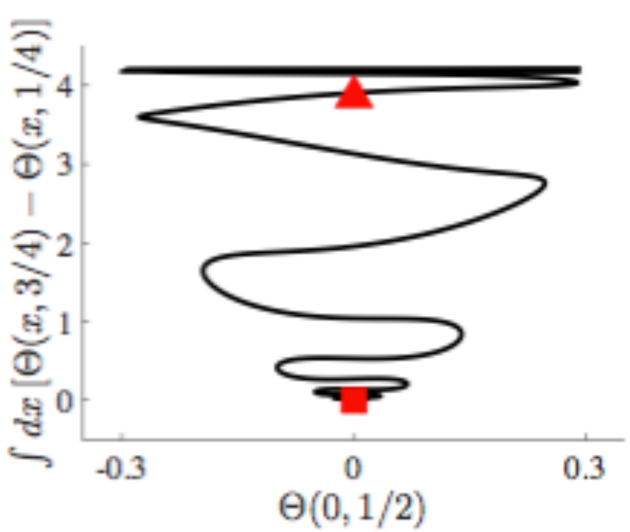
mean ≠ exact

RZIF

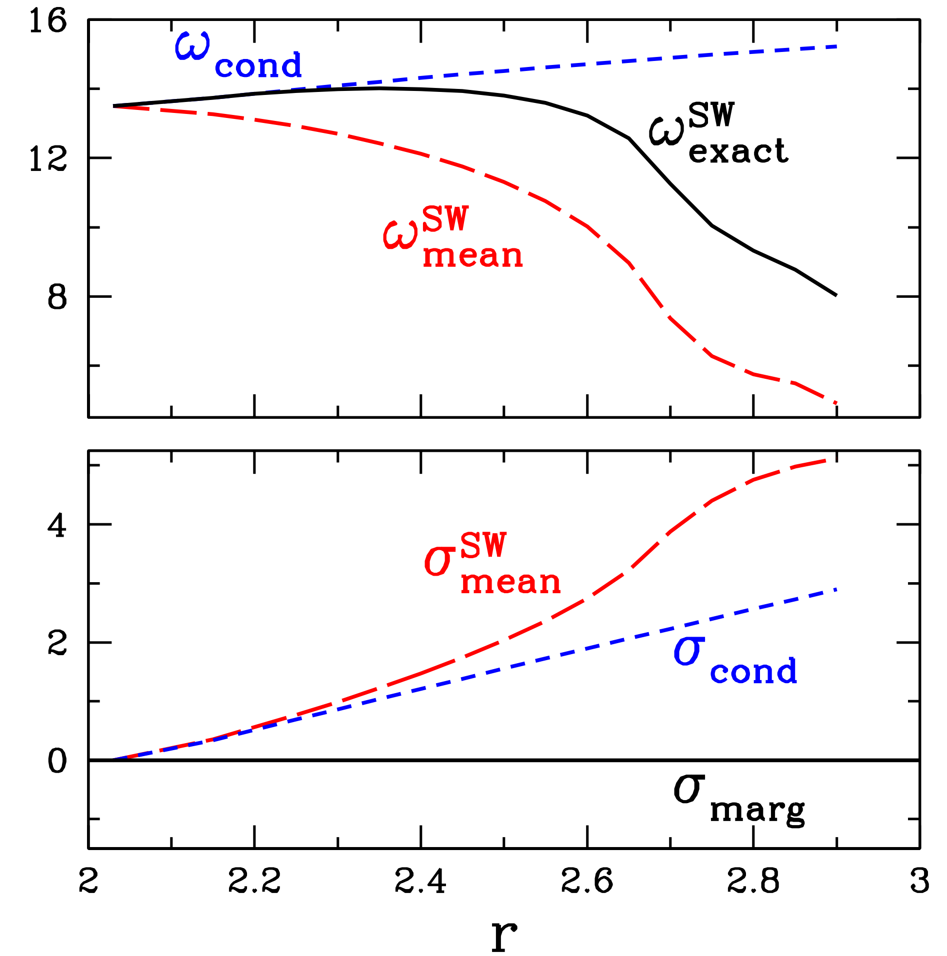
Traveling waves



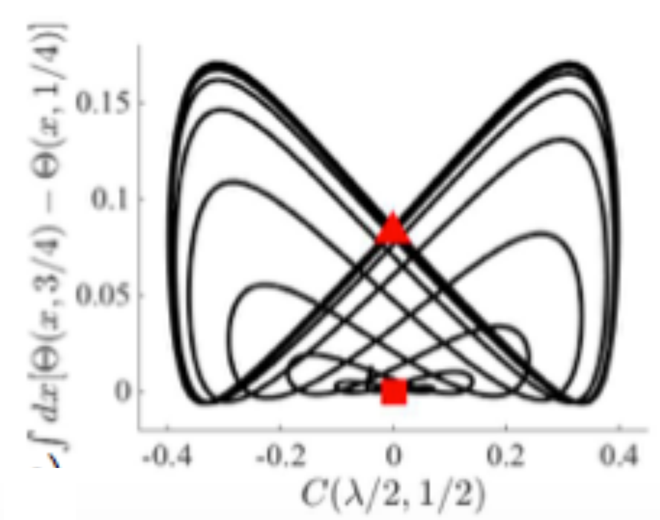
mean = exact



Standing waves



mean ≠ exact



Evolution equation:

$$\partial_t \mathbf{U} = \mathcal{L}\mathbf{U} + \mathcal{N}(\mathbf{U}, \mathbf{U})$$

Temporal Fourier decomposition:

$$\mathbf{U} = \overline{\mathbf{U}} + \sum_{n \neq 0} \mathbf{u}_n e^{in\omega t}$$

Substitute into evolution equation

Component 0:

$$0 = \mathcal{L}\overline{\mathbf{U}} + \mathcal{N}(\overline{\mathbf{U}}, \overline{\mathbf{U}}) + \sum_{m \neq 0} \mathcal{N}(\mathbf{u}_m, \mathbf{u}_{-m})$$

Component 1:

$$i\omega \mathbf{u}_1 = \underbrace{\mathcal{L}\mathbf{u}_1 + \mathcal{N}(\overline{\mathbf{U}}, \mathbf{u}_1) + \mathcal{N}(\mathbf{u}_1, \overline{\mathbf{U}})}_{\mathcal{L}_{\overline{\mathbf{U}}}\mathbf{u}_1} + \underbrace{\mathcal{N}(\mathbf{u}_2, \mathbf{u}_{-1}) + \mathcal{N}(\mathbf{u}_{-1}, \mathbf{u}_2) + \ldots}_{\text{small?}}$$

Evolution equation:

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$$\mathcal{N}_1$$

$\mathcal{N}(u_1, u_1)$ feeds u_2 $\mathcal{N}(u_1, u_{-1})$ feeds u_0

$$\mathcal{N}_1 \equiv \mathcal{N}(u_2, u_{-1}) + \mathcal{N}(u_{-1}, u_2) + \ldots \text{ feeds } u_1$$

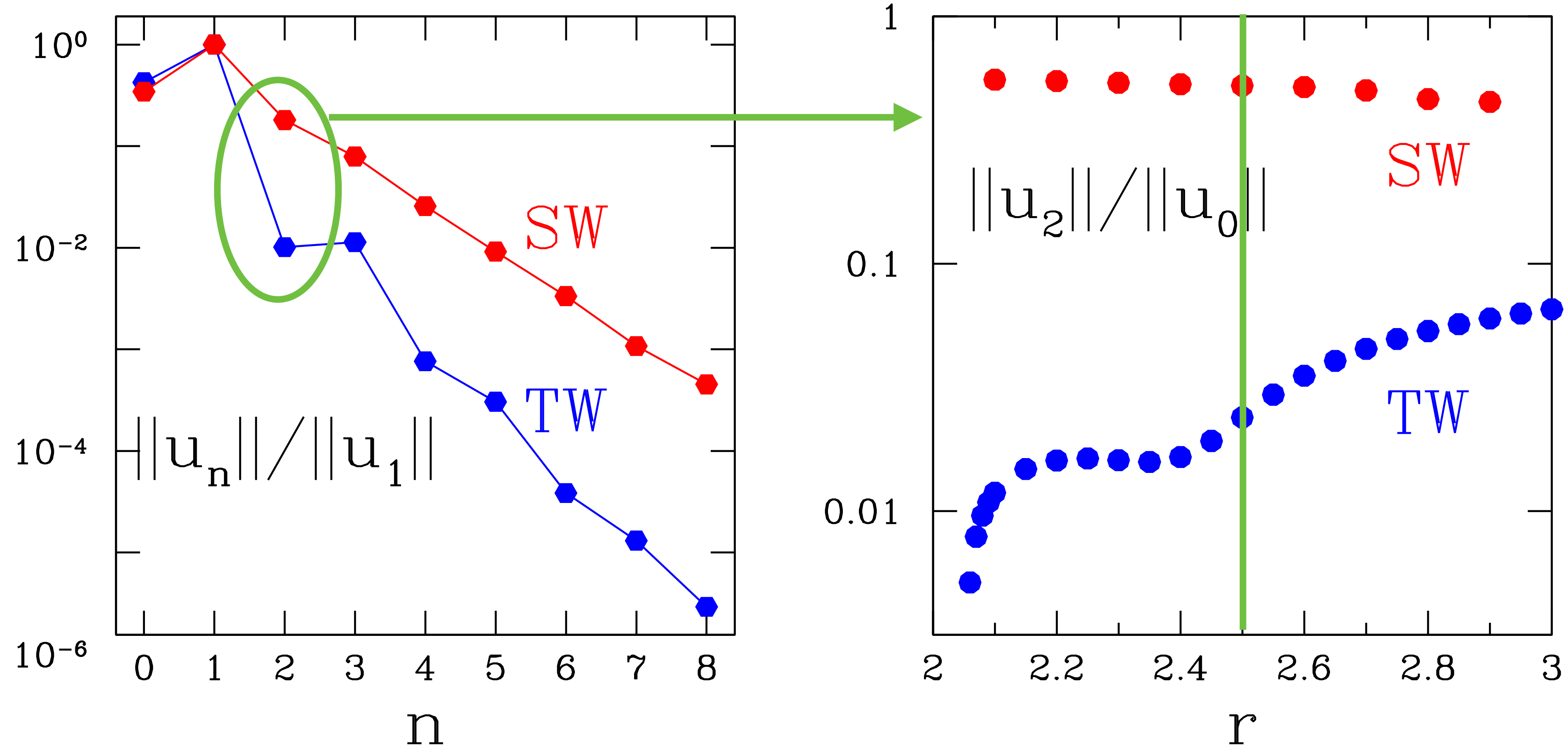
$$||u_n|| \sim \epsilon^{|n|} \implies \mathcal{N}_1 \ll (i\omega - \mathcal{L}_{\bar{U}})u_1$$

Mean flow eigenvalue has **RZIF** property:
Real part is near **Zero**.
Imaginary part is near exact nonlinear **Frequency**.

Do **TW** generally have **highly peaked** temporal spectra?
Do **SW** generally have **broad** temporal spectra?

Yes!

temporal spectrum for r=2.5



And **WHY** do **TW** have **highly peaked** temporal spectra?

And **WHY** do **SW** have **broad** temporal spectra?

Traveling waves

(N. Périnet)

$$\psi = \psi_1 \sin(kx - \omega t) \sin \pi z$$

$$T = T_1 \ \sin(kx - \omega t + \Delta) \ \sin \pi z$$

$$\nabla\psi \times \nabla T = \frac{\pi k}{2} \psi_1 T_1 \boxed{\sin \Delta} \sin 2\pi z$$

mean flow generated by phase difference Δ between ψ and T

no generation of second temporal harmonic

Standing waves

$$\psi = \psi_1 \ \sin kx \ \sin \omega t \ \sin \pi z$$

$$T = T_1 \cos kx \ \cos(\omega t + \Delta) \sin \pi z$$

$$\nabla\psi \times \nabla T = \frac{\pi k}{4} \psi_1 T_1 \left[\boxed{\sin \Delta} + \boxed{\sin(2\omega t + \Delta)}\right] \sin 2\pi z$$

mean flow generated by phase difference Δ between ψ and T

generation of **second** temporal harmonic

Does this argument hold for other cases ?

$\mathbf{u}_1$ from temporal Fourier decomposition of DNS:

$$i\omega\mathbf{u}_1 = \mathcal{L}_{\overline{\mathbf{U}}}\mathbf{u}_1 + \mathcal{N}_1$$

$\mathbf{u}_1$ as eigenvector:

$$(\sigma + i\omega)\mathbf{u}_1 = \mathcal{L}_{\overline{\mathbf{U}}}\mathbf{u}_1$$

Back to purely hydrodynamic flow, e.g. cylinder wake

Traveling waves

$$\psi = |\psi_1| e^{i(kx - \omega t)} \sin \pi z + \text{c.c.}$$

$$\omega = -\nabla^2 \psi = (k^2 + \pi^2)\psi$$

$$\nabla \psi \times \nabla \omega = 0$$

no generation of mean flow

Standing waves

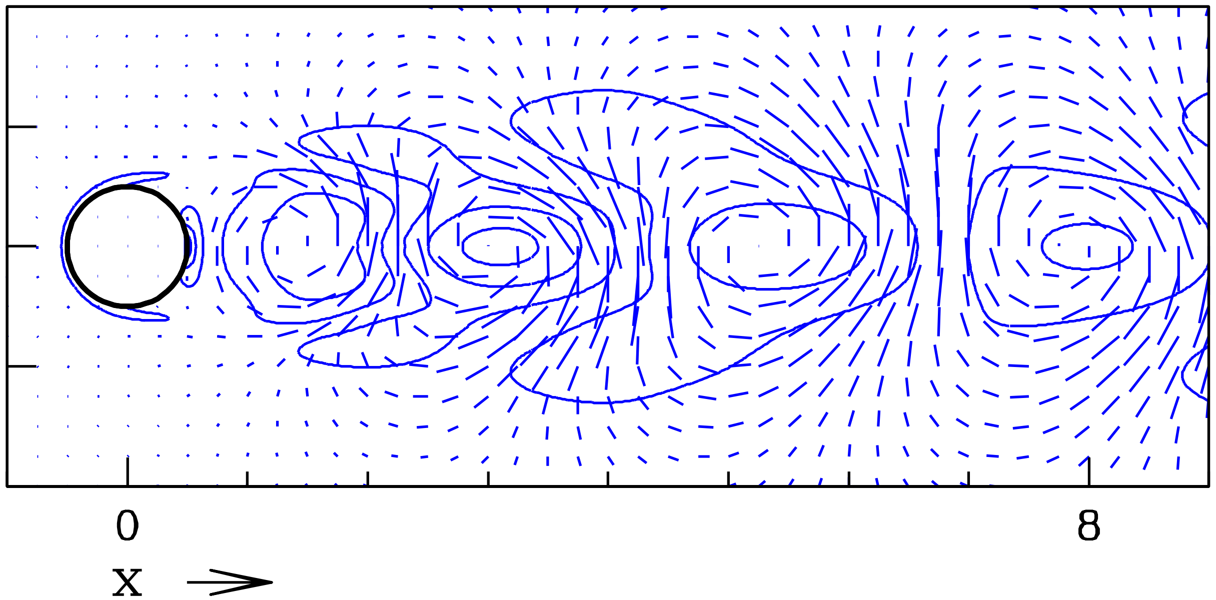
$$\psi = \psi_1 \ \sin kx \ \sin \pi z \ \sin \omega t$$

$$\omega = -\nabla^2 \psi = (k^2 + \pi^2)\psi$$

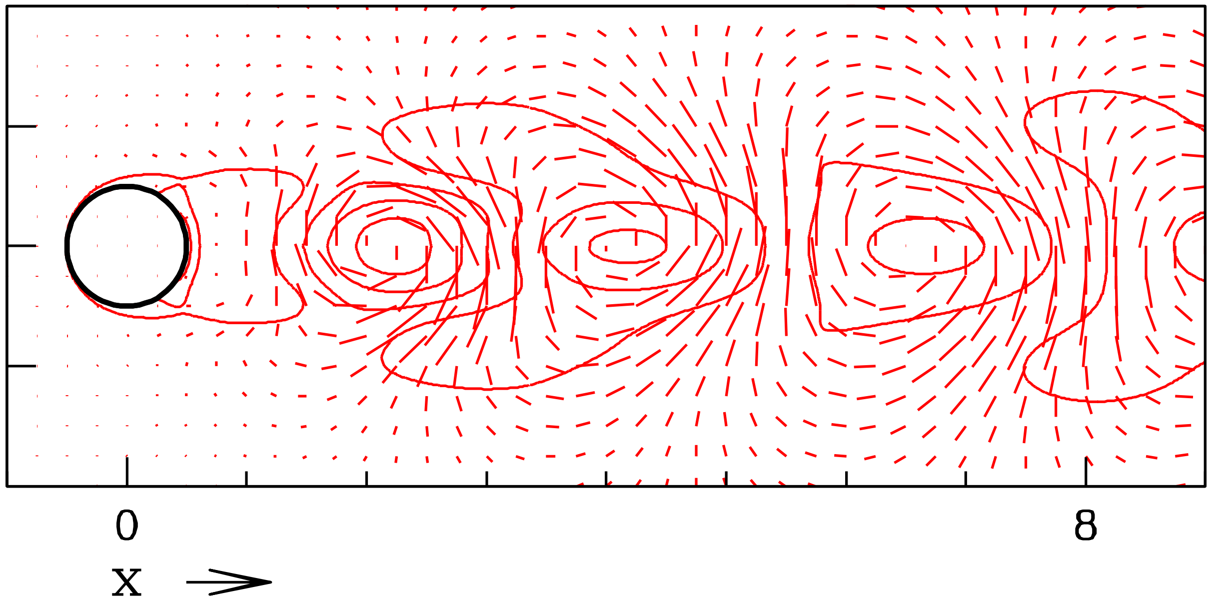
$$\nabla \psi \times \nabla \omega = 0$$

no generation of mean flow

Cylinder wake



$\nabla\psi_R \times \nabla\omega_R$ small



$\nabla\psi_I \times \nabla\omega_I$ small

J. Fluid Mech. (2007), vol. 593, pp. 333–358.
doi:10.1017/S0022112007008907 Printed in the United Kingdom

Global stability of base and mean flows: a general approach and its applications to cylinder and open cavity flows

DENIS SIPP AND ANTON LEBEDEV
ONERA, 8 rue des Vertugadins, 92190 Meudon, France

Multiple scale expansion near Hopf threshold

$$Re^{-1} = Re_c^{-1} - \epsilon \qquad \boldsymbol{U}(t) = \boldsymbol{U}_0 + \sqrt{\epsilon}\boldsymbol{U}_1(t, t_1) + \epsilon \boldsymbol{U}_2(t, t_1) + \epsilon\sqrt{\epsilon}\boldsymbol{U}_3(t, t_1) + \cdots$$

Asymptotic/numerical calculation of mean flow, limit cycle, eigenvectors, …

Counter-example of open-cavity flow:
eigenvalues of mean flow do NOT predict the frequency.

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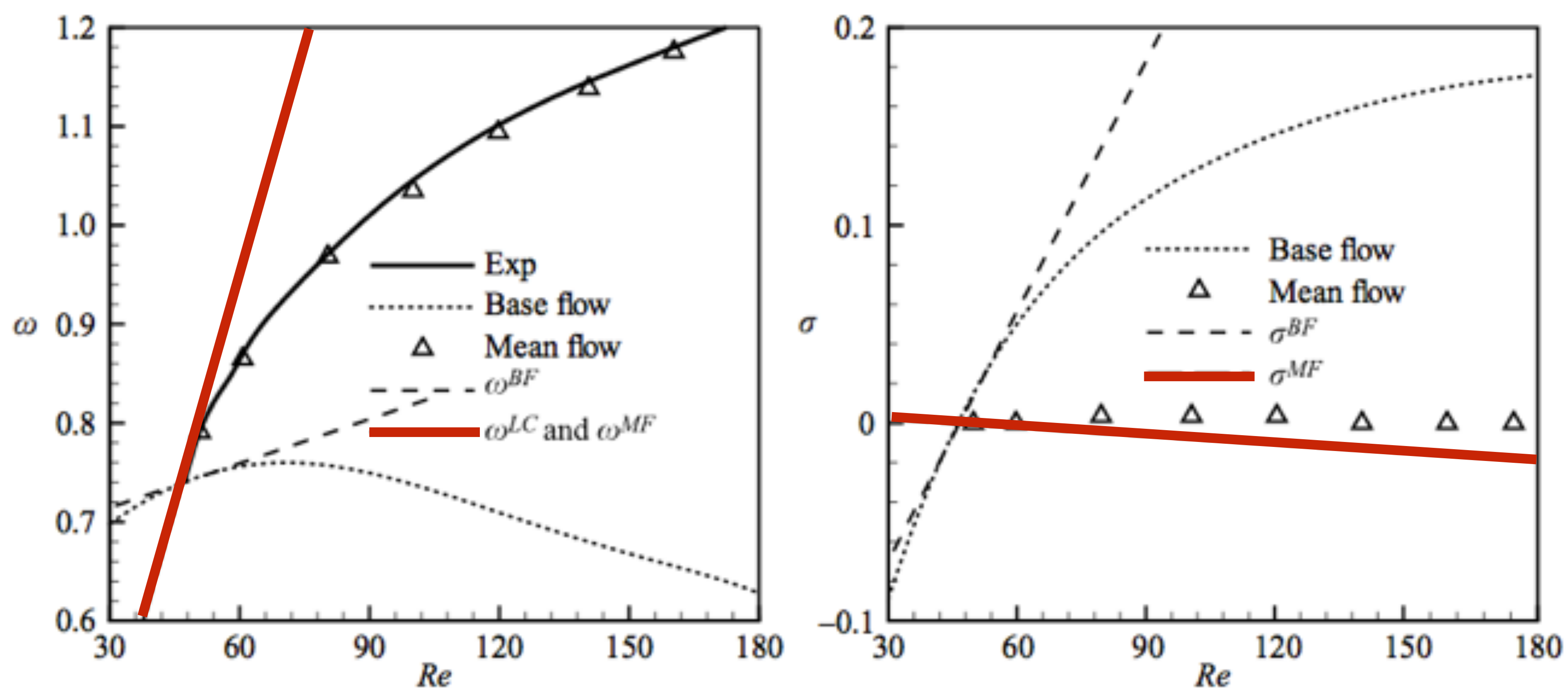
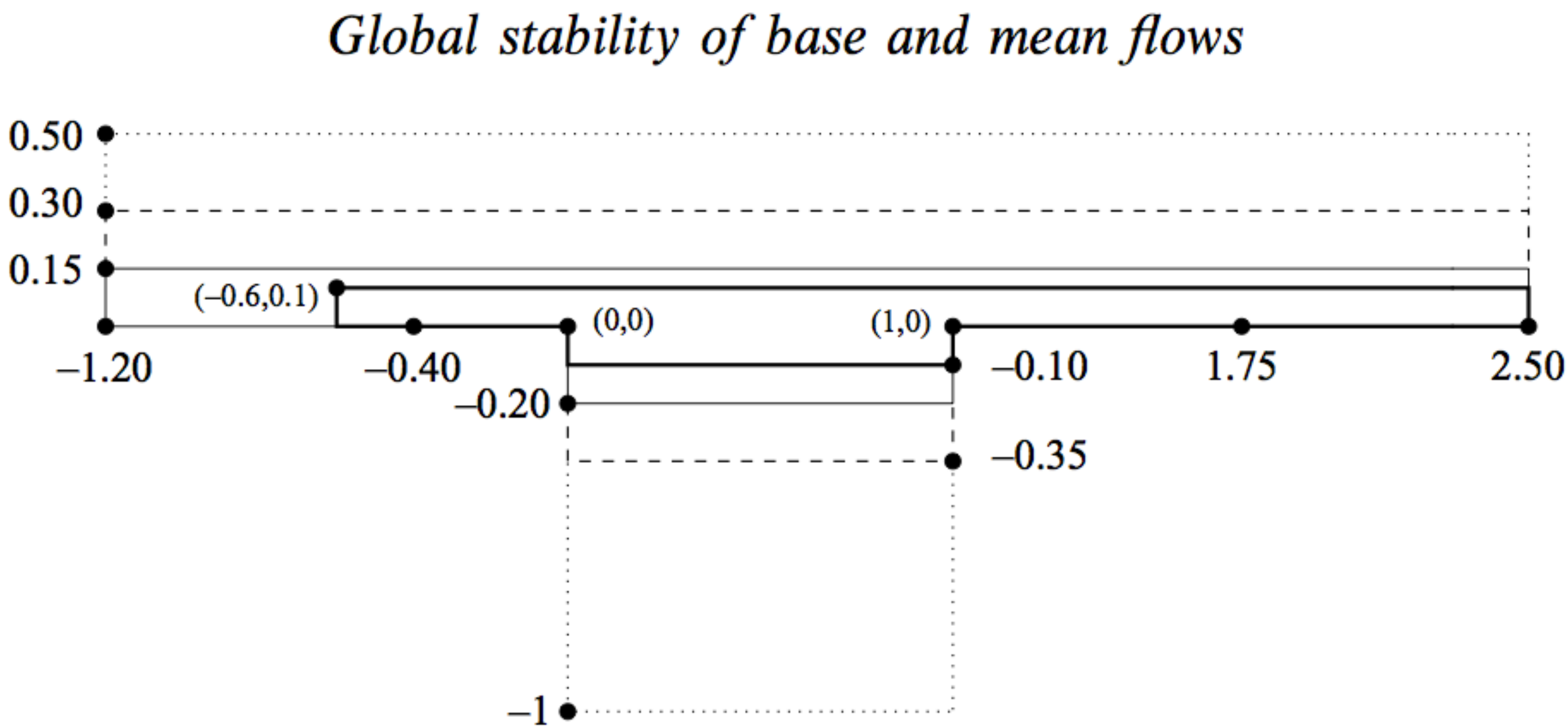


Counter-example of open-cavity flow:
eigenvalues of mean flow do NOT predict the frequency.



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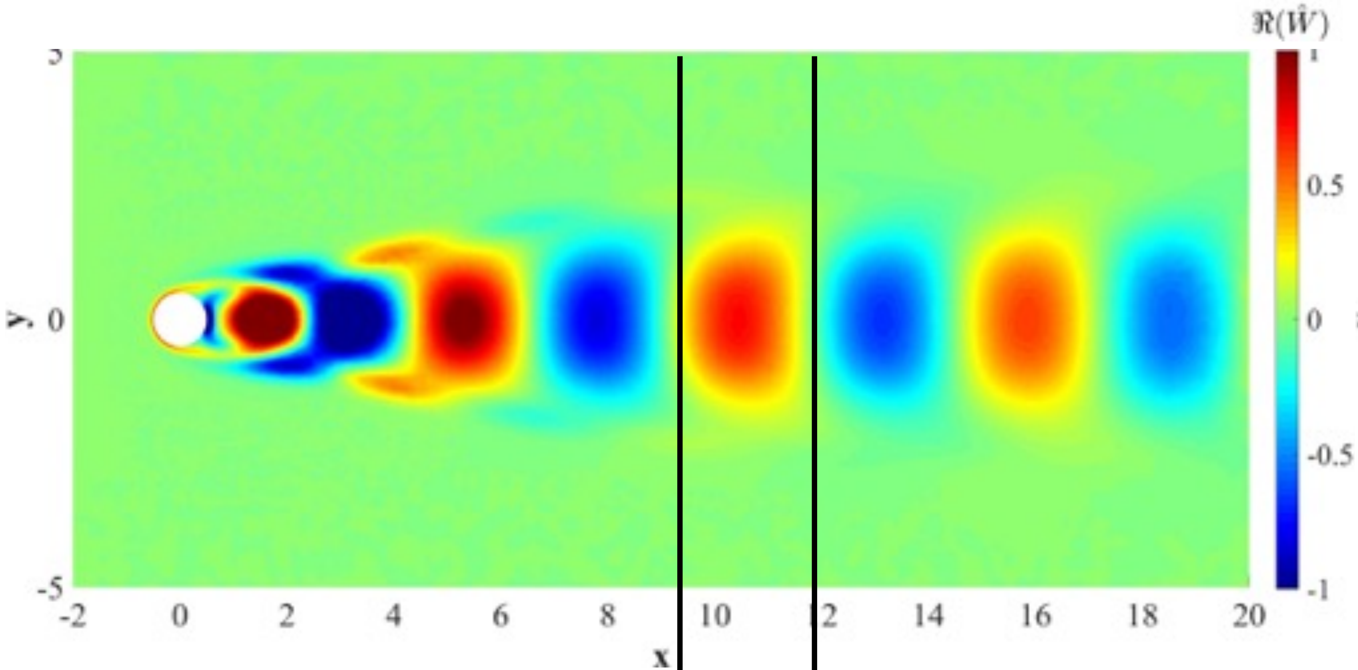
Thermosolutal		Open flows	
Traveling waves	**OK**	Cylinder wake	**OK**
Standing waves	**NO**	Open cavity	**NO**

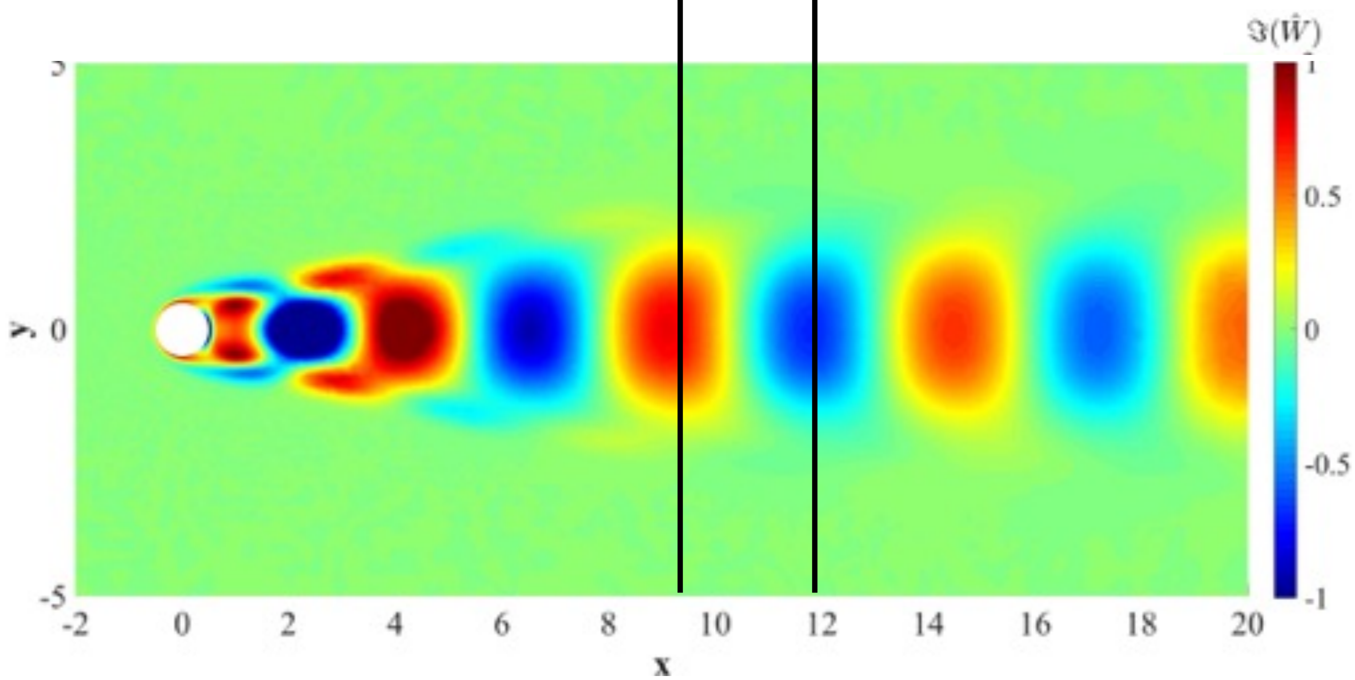
Is there an analogy between the

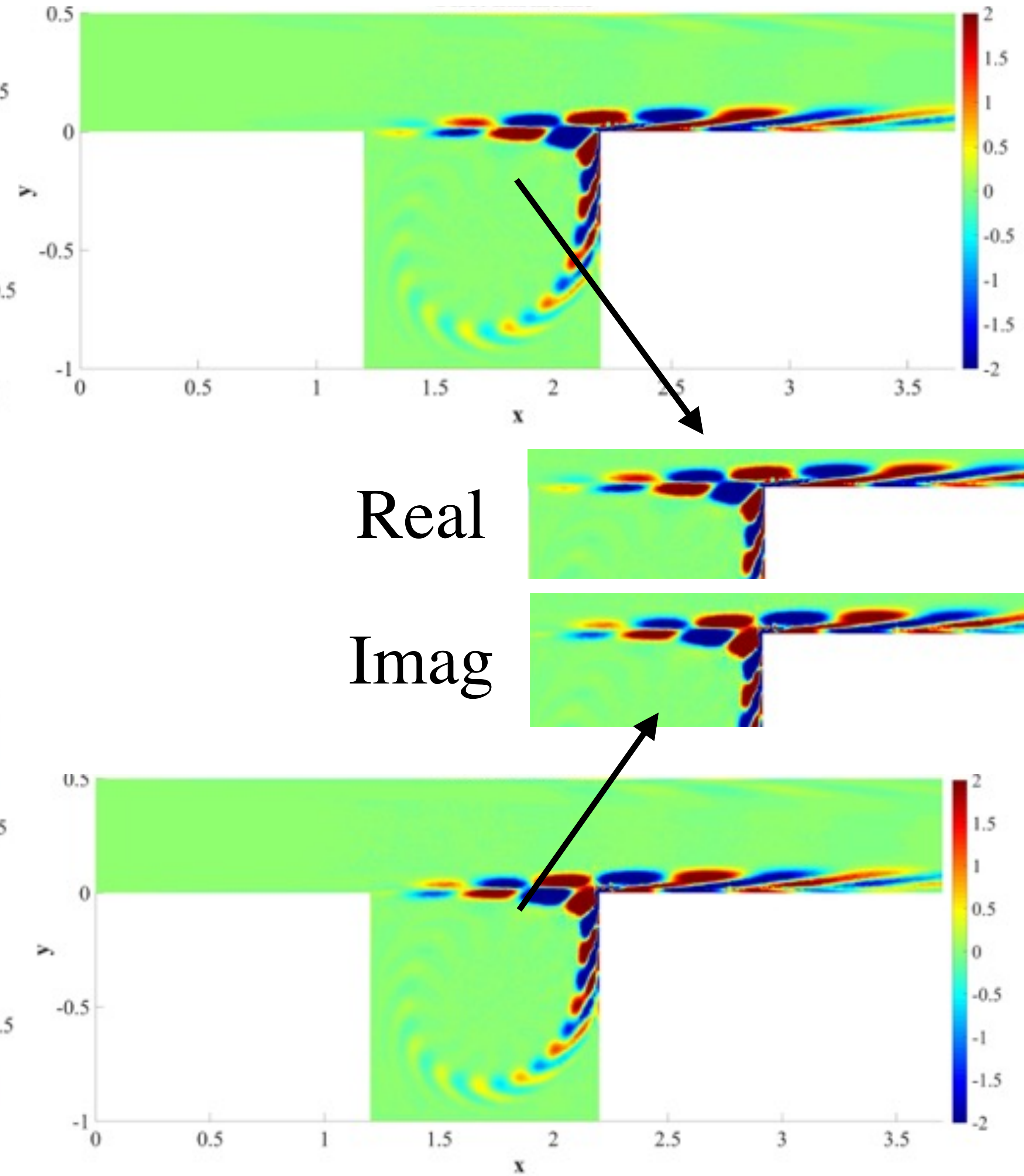
Cylinder wake with TW and open cavity with SW ????

Real part of u_1



Imaginary part of u_1



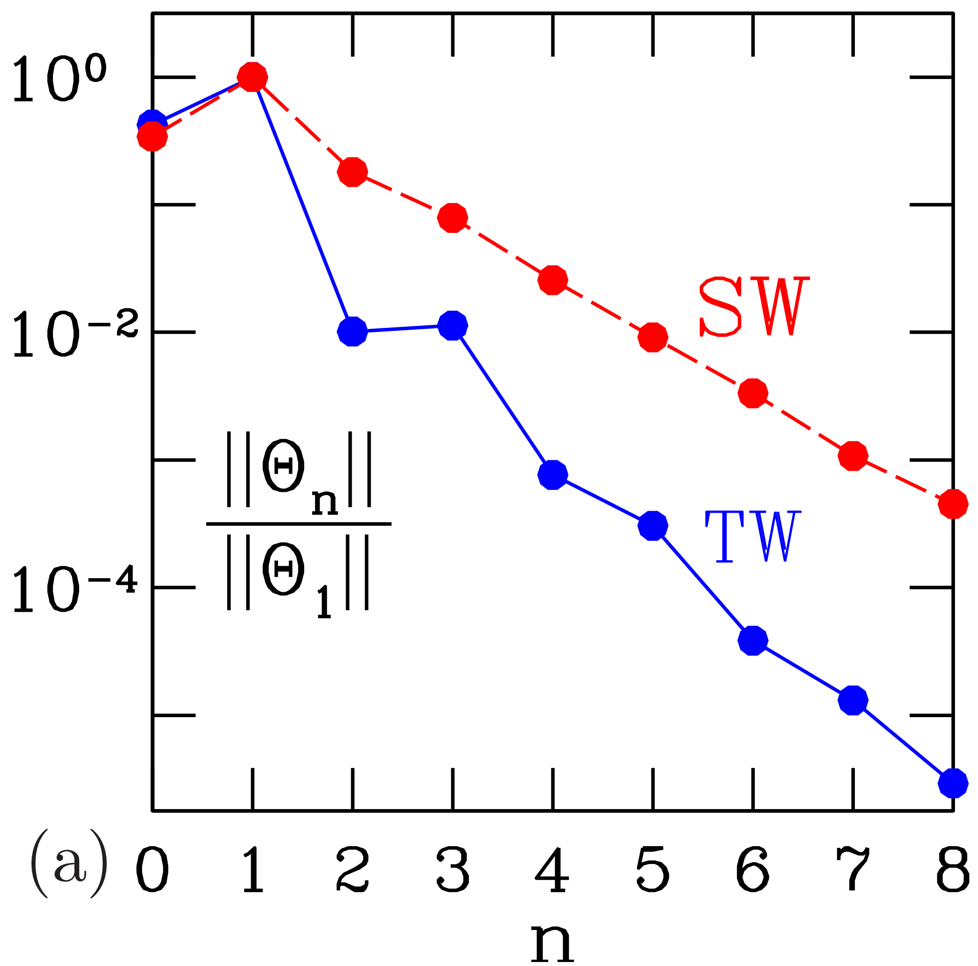


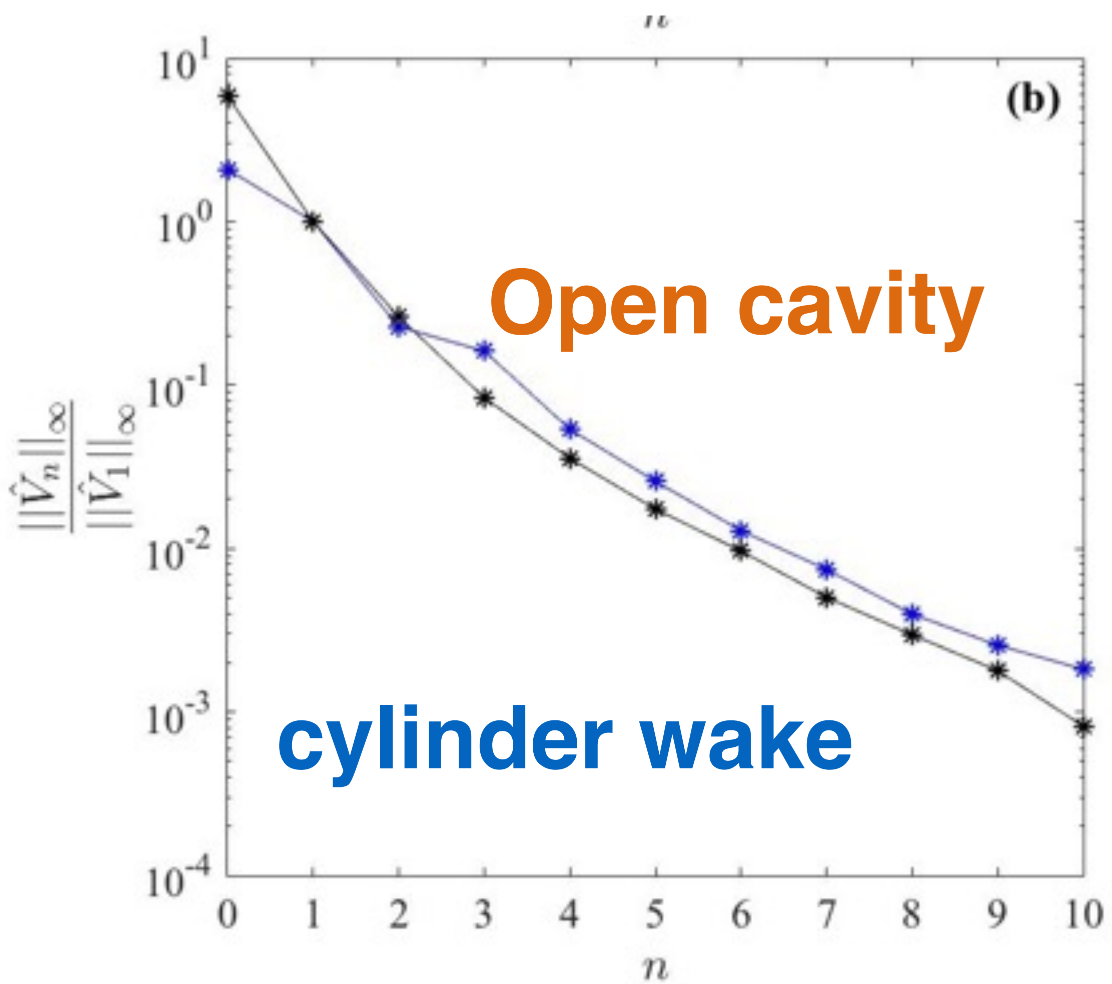
Traveling wave behavior :
real and imaginary parts are $L_x/4$ out of phase

Thermosolutal

Open flows

Spectrum normalized by the first harmonic





Similar spectrum !!!

PRL 113, 084501 (2014) PHYSICAL REVIEW LETTERS week ending 22 AUGUST 2014

Self-Consistent Mean Flow Description of the Nonlinear Saturation of the Vortex Shedding in the Cylinder Wake

Vladislav Mantič-Lugo,[*] Cristóbal Arratia,[†] and François Gallaire
Laboratory of Fluid Mechanics and Instabilities, École Polytechnique Fédérale de Lausanne, EPFL-STI Lausanne CH-1015, Switzerland
(Received 19 December 2013; published 20 August 2014)

J. Fluid Mech. (2016), *vol.* 793, *pp.* 777–797. 777
doi:10.1017/jfm.2016.109

Self-consistent model for the saturation mechanism of the response to harmonic forcing in the backward-facing step flow

V. Mantič-Lugo[1,†] and F. Gallaire[1]

J. Fluid Mech. (2016), *vol.* 800, *pp.* 327–357. 327
doi:10.1017/jfm.2016.390

A self-consistent formulation for the sensitivity analysis of finite-amplitude vortex shedding in the cylinder wake

P. Meliga[1,†], E. Boujo[2,‡] and F. Gallaire[2]

PHYSICS OF FLUIDS 27, 074103 (2015)

A self-consistent model for the saturation dynamics of the vortex shedding around the mean flow in the unstable cylinder wake

Vladislav Mantič-Lugo,[1,a)] Cristóbal Arratia,[1,2,b)] and François Gallaire[1,c)]

Evolution equation:

$$\partial_t \mathbf{U} = \mathcal{L}\mathbf{U} + \mathcal{N}(\mathbf{U}, \mathbf{U})$$

Temporal Fourier decomposition:

$$\mathbf{U} = \overline{\mathbf{U}} + \sum_{n \neq 0} \mathbf{u}_n e^{in\omega t}$$

Substitute into evolution equation

Component 0:

$$0 = \mathcal{L}\overline{\mathbf{U}} + \mathcal{N}(\overline{\mathbf{U}}, \overline{\mathbf{U}}) + \sum_{m \neq 0} \mathcal{N}(\mathbf{u}_m, \mathbf{u}_{-m})$$

Component 1:

$$i\omega \mathbf{u}_1 = \underbrace{\mathcal{L}\mathbf{u}_1 + \mathcal{N}(\overline{\mathbf{U}}, \mathbf{u}_1) + \mathcal{N}(\mathbf{u}_1, \overline{\mathbf{U}})}_{\mathcal{L}_{\overline{\mathbf{U}}}\mathbf{u}_1} + \mathcal{N}_1$$

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Self-Consistent Mean Flow Description of the Nonlinear Saturation of the Vortex Shedding in the Cylinder Wake

Vladislav Mantič-Lugo,* Cristóbal Arratia,† and François Gallaire
Laboratory of Fluid Mechanics and Instabilities, École Polytechnique Fédérale de Lausanne, EPFL-STI-IGM-LFMI, Lausanne CH-1015, Switzerland
(Received 19 December 2013; published 20 August 2014)

Mean flows without time integration:

$$0 = \mathcal{L}\overline{\mathbf{U}} + \mathcal{N}(\overline{\mathbf{U}}, \overline{\mathbf{U}}) + \sum_{m \neq 0} \mathcal{N}(\mathbf{u}_m, \mathbf{u}_{-m}) \rightarrow \mathcal{N}(\mathbf{u}_1, \mathbf{u}_{-1})$$

Solve for $\bar{\mathrm{U}}$ with Newton's method

$$(\sigma + i\omega)\mathbf{u}_1 = \mathcal{L}_{\bar{U}}\mathbf{u}_1 + \mathcal{N}_1 \qquad \|u_1\| = A$$

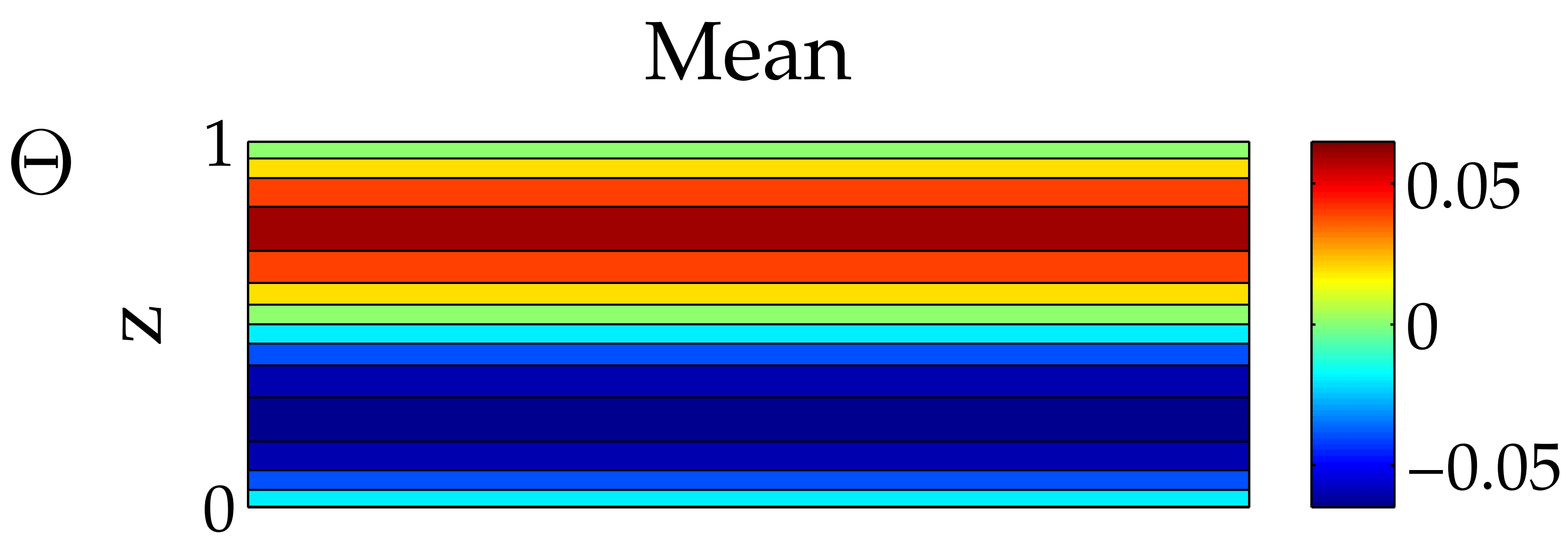
Solve for ($\mathbf{u}_1, \sigma, \omega$) by diagonalisation

Find A such that $\sigma = 0$

Yields almost exact $\bar{\mathrm{U}}, \mathbf{u}_1, \omega$!

Thermosolutal in **x-homogeneous** domain :

Mean flow of TW is independent of x



$$0 = \mathcal{L}\bar{U} + \cancel{\mathcal{N}(\bar{U},\bar{U})} + \color{red}{\mathcal{N}(u_1, u_{-1})}$$

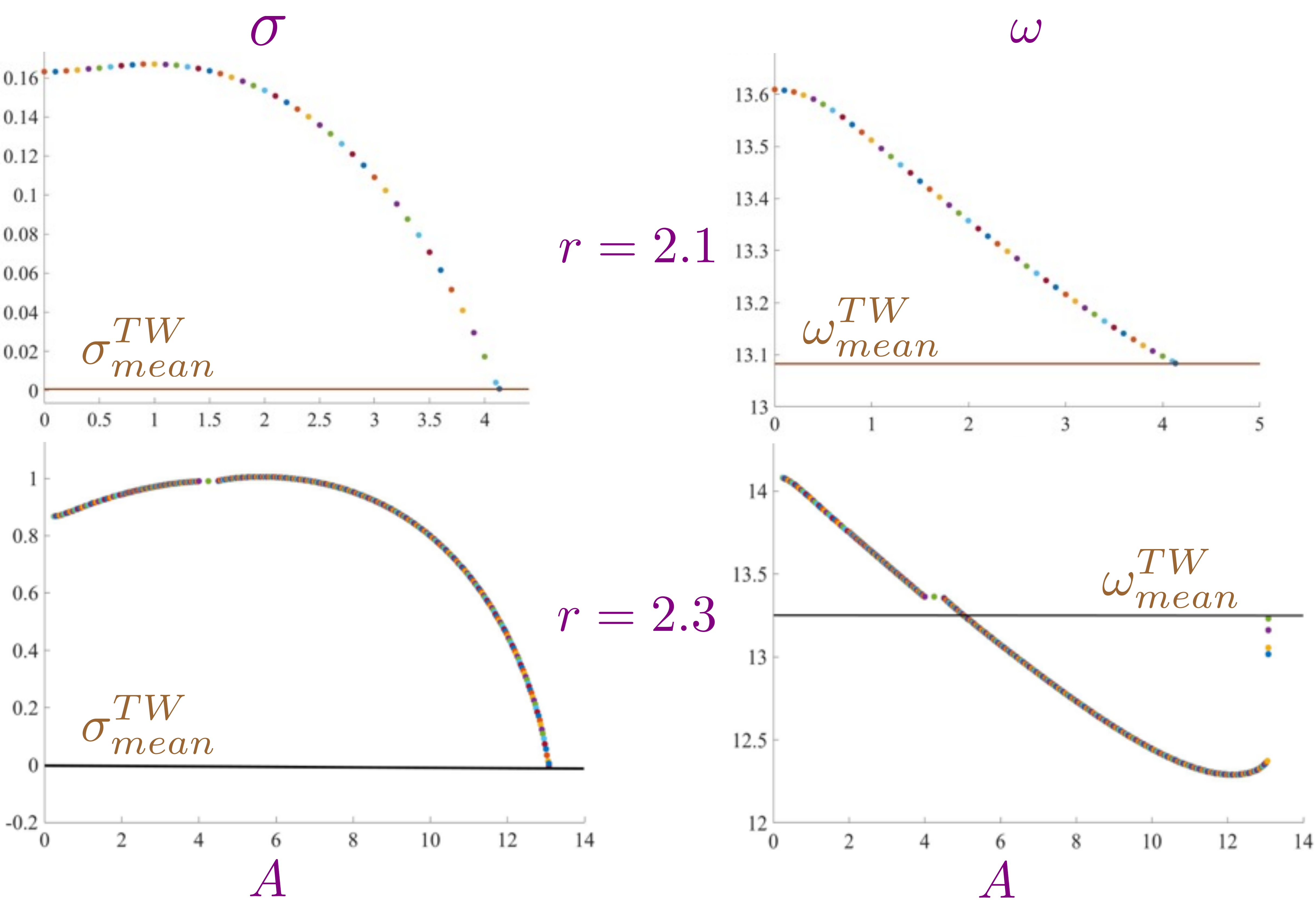
$$\bar{U}(z) = -\mathcal{L}^{-1}\mathcal{N}(u_1, u_{-1})$$

~~Newton~~ → **just invert Laplacian**

$$= -\partial_{zz}^{-1}\mathcal{N}(u_1, u_{-1})$$

→ **just invert** ∂_{zz}

Convergence of SCM equations



Resolvent Analysis

McKeon & Sharma, JFM 2010

$$i\omega \mathbf{u}_1 = \mathcal{L}_{\bar{U}} \mathbf{u}_1 + \mathcal{N}_1$$

$$\mathbf{u}_1 = (i\omega - \mathcal{L}_{\overline{U}})^{-1} \mathcal{N}_1 \equiv \mathcal{R}(\omega)\mathcal{N}_1$$

More generally: $\mathbf{u}(\omega) = \qquad \mathcal{R}(\omega)\mathcal{N}(\omega)$

Singular value decomposition: $\mathcal{R}(\omega)\phi_j(\mathbf{x},\omega) = \mu_j(\omega)\psi_j(\mathbf{x},\omega)$

If resolvent has a highly dominant singular value μ_{dom},

then $\mathcal{R}$ extracts and amplifies the component of mode ϕ_{dom} in $\mathcal{N}$

Independent of the details of $\mathcal{N}$

$$\mathbf{u} = \mathcal{R}(\omega)\sum_j \langle \mathcal{N}, \phi_j \rangle \phi_j = \sum_j \langle \mathcal{N}, \phi_j \rangle \mu_j \psi_j \approx \langle \mathcal{N}, \phi_{\mathrm{dom}} \rangle \mu_{\mathrm{dom}} \psi_{\mathrm{dom}}$$

$$\mathbf{u}(\mathbf{x},\omega) \approx \underbrace{\langle \mathcal{N}, \phi_{\mathrm{dom}} \rangle(\omega)\, \mu_{\mathrm{dom}}(\omega)}_{\text{scalar amplitude } \Lambda(\omega)} \; \underbrace{\psi_{\mathrm{dom}}(\mathbf{x},\omega)}_{\text{spatial dependence}}$$

J. Fluid Mech. (2016), *vol.* 798, *pp.* 485–504. 485
doi:10.1017/jfm.2016.331

Conditions for validity of mean flow stability analysis

Samir Beneddine[1,†], Denis Sipp[1], Anthony Arnault[2], Julien Dandois[2] and Lutz Lesshafft[3]

Fully turbulent flow with broad spectrum,
rather than periodic flow with only $\omega, 2\omega, \ldots$

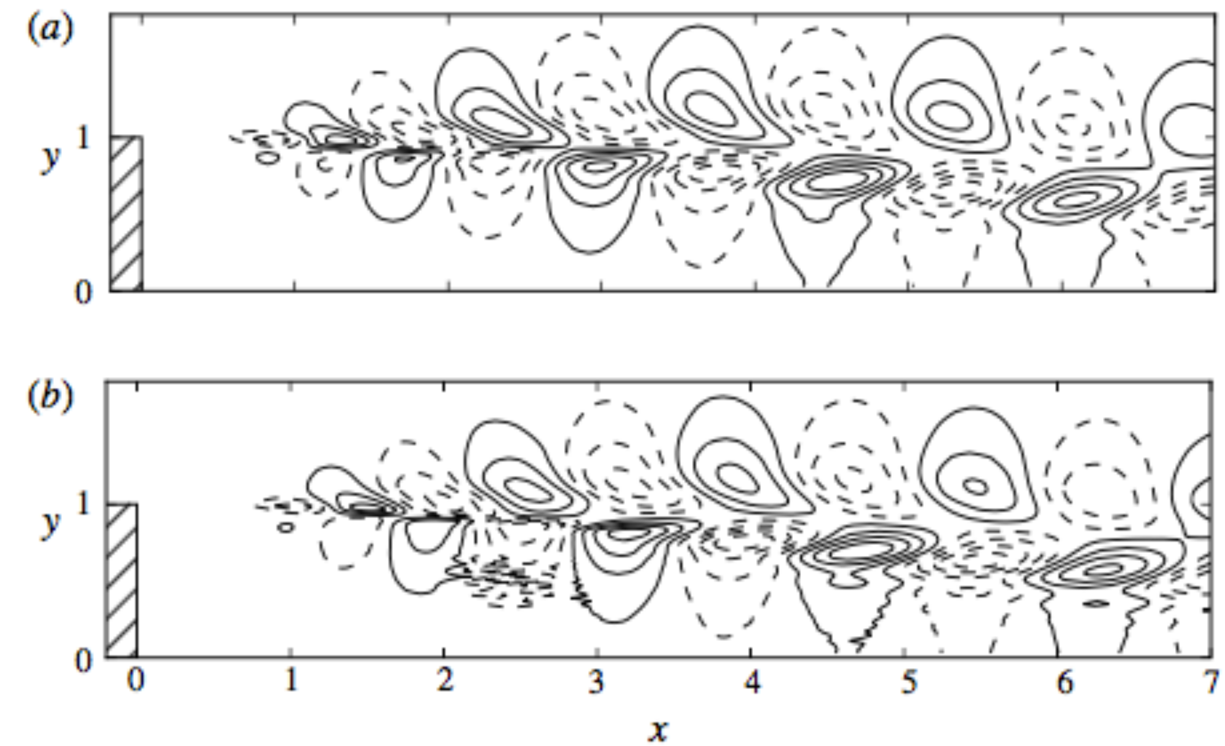
$$\mathcal{N}(\mathbf{x},\omega) \equiv -(\tilde{\mathbf{u}}\cdot\nabla)\tilde{\mathbf{u}} + \langle(\tilde{\mathbf{u}}\cdot\nabla)\tilde{\mathbf{u}}\rangle$$

$$\mathbf{u}(\mathbf{x},\omega) \approx \underbrace{\langle\mathcal{N},\phi_{\rm dom}\rangle(\omega)\;\mu_{\rm dom}(\omega)}_{\text{scalar amplitude }\Lambda(\omega)}\;\underbrace{\psi_{\rm dom}(\mathbf{x},\omega)}_{\text{spatial dependence}}$$

Spatial dependence

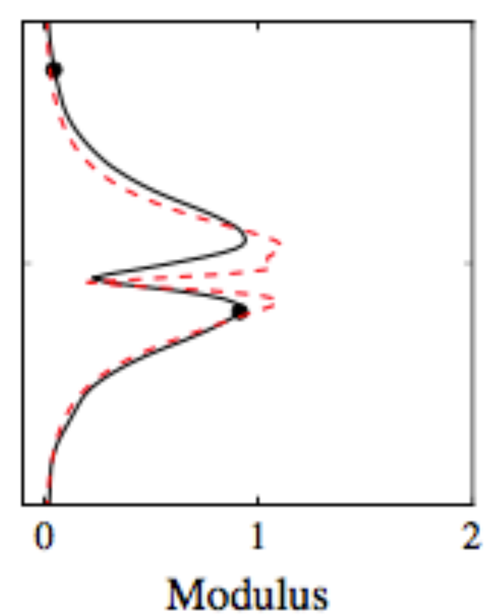
dominant optimal response of resolvent $\psi_{\mathrm{dom}}(\mathbf{x}, \omega)$

simulation $\mathbf{u}(\mathbf{x}, \omega)$

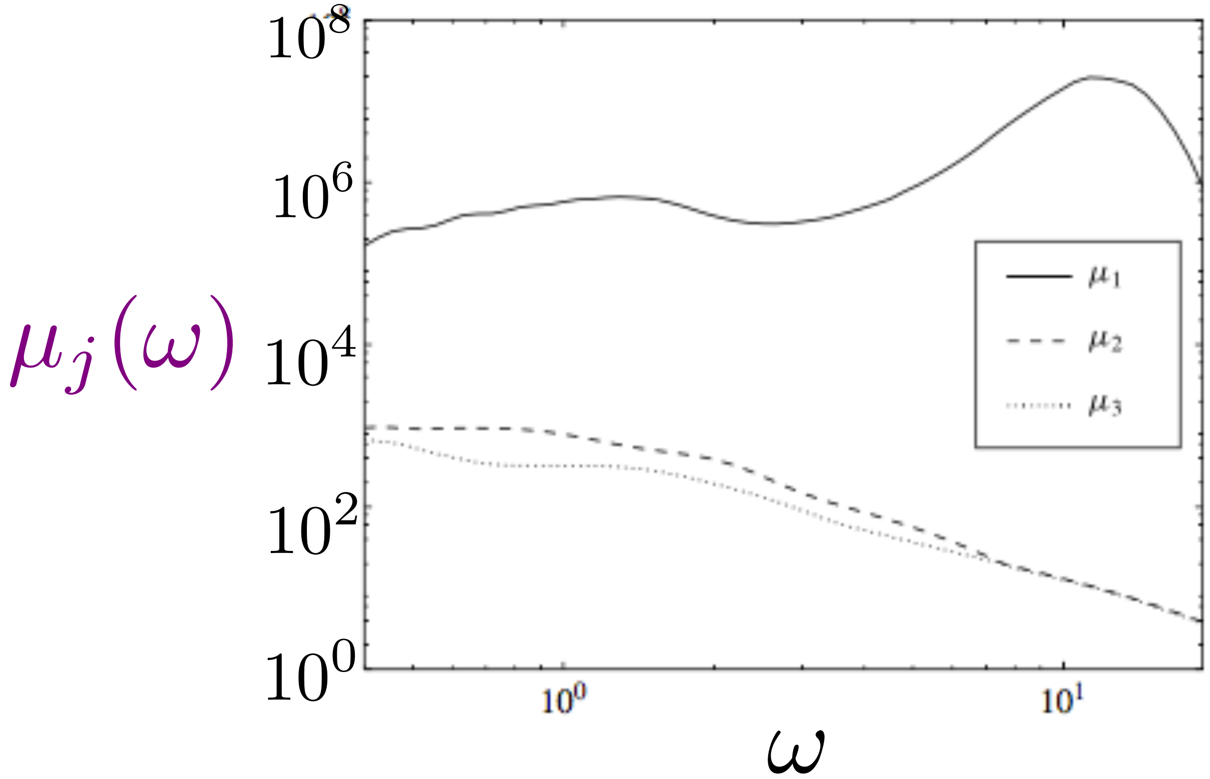


Scalar amplitude

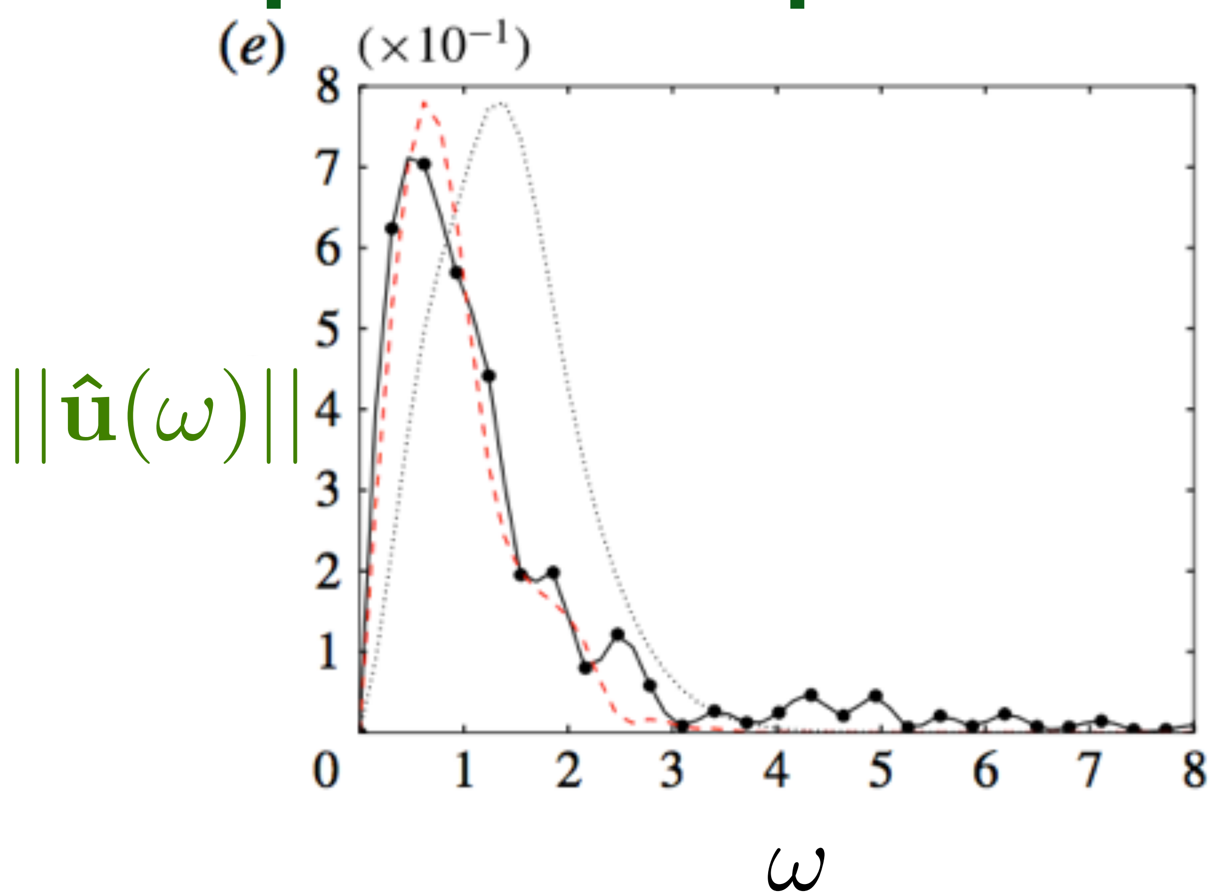
choose amplitude $\Lambda(\omega)$ so that dominant optimal response and simulation agree at two points



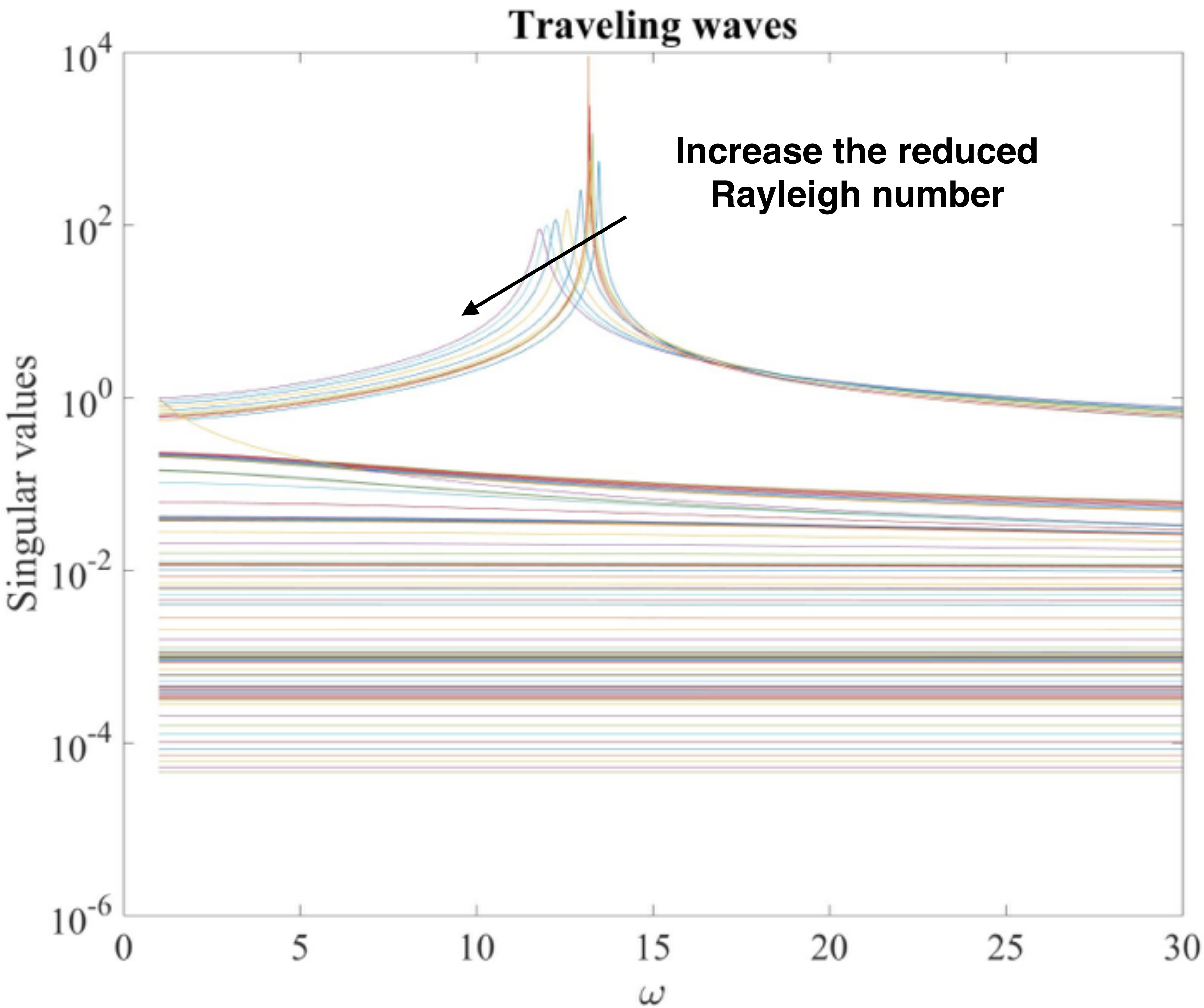
Dominant singular value



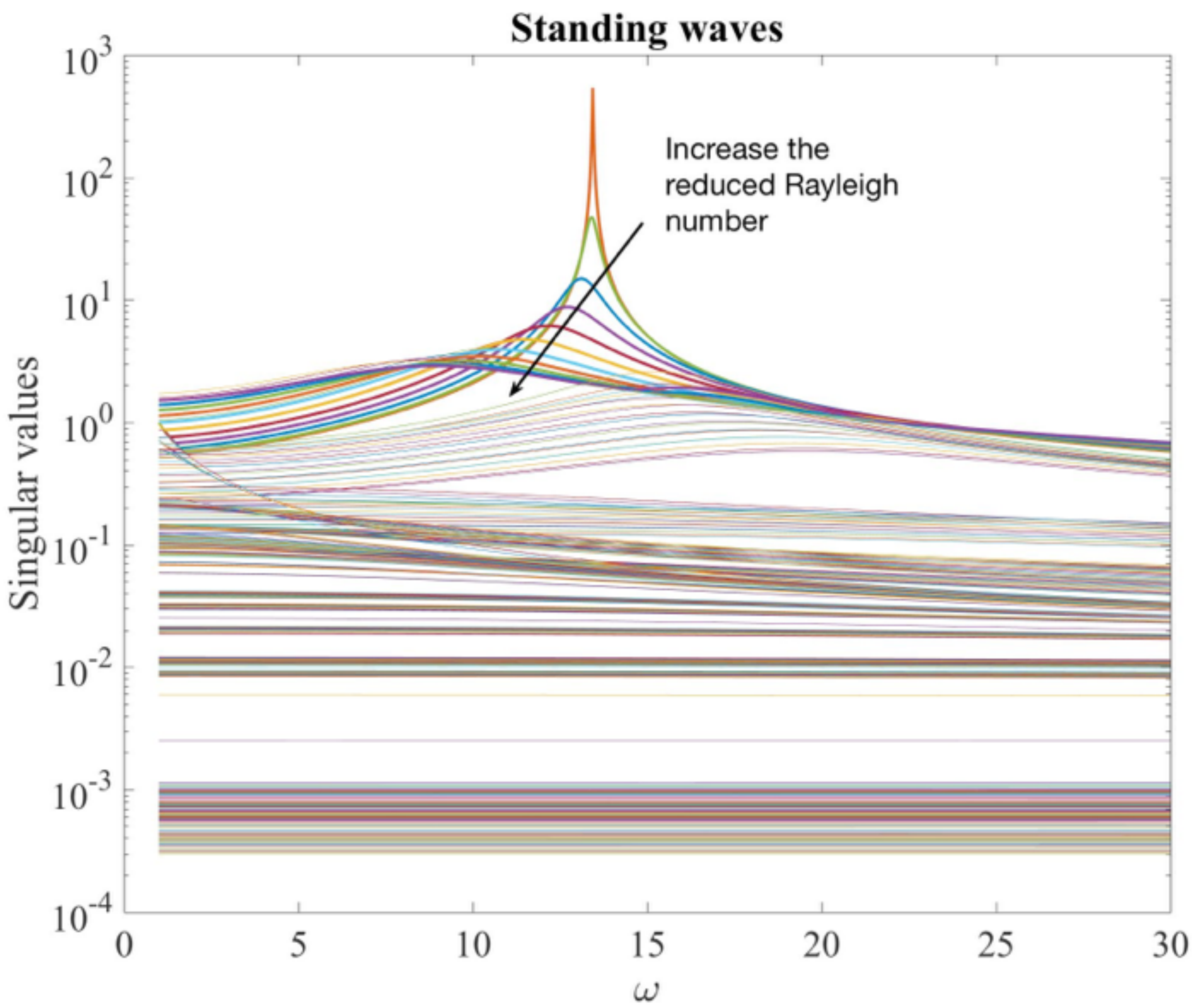
Spectrum reproduced!



Thermosolutal TW: SVD highly peaked



Thermosolutal SW: SVD not highly peaked



Stay tuned …

Thank you!