Coherent approach to Turbulence

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Turbulence…

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–Feynman's Lectures, 1961

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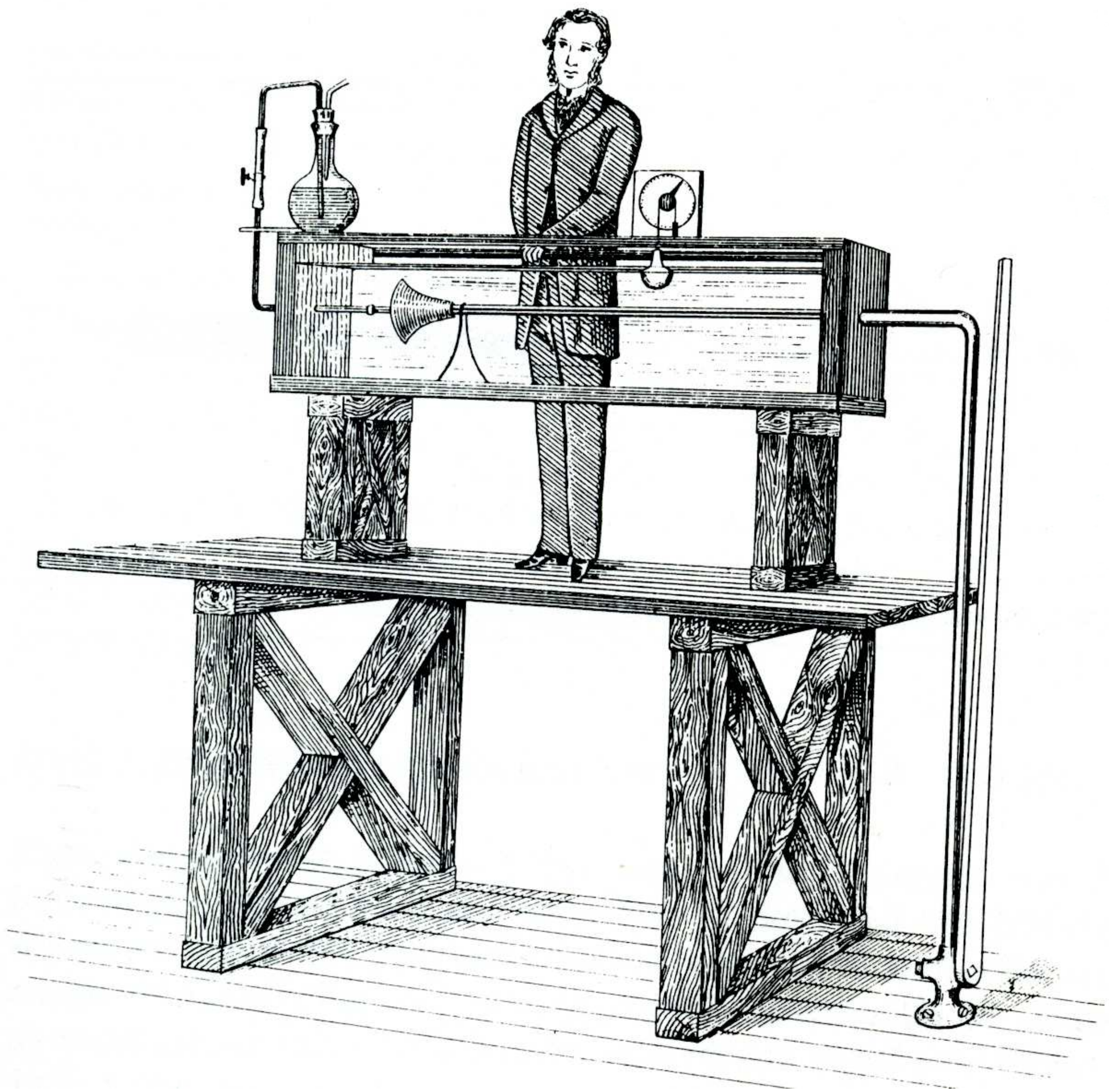
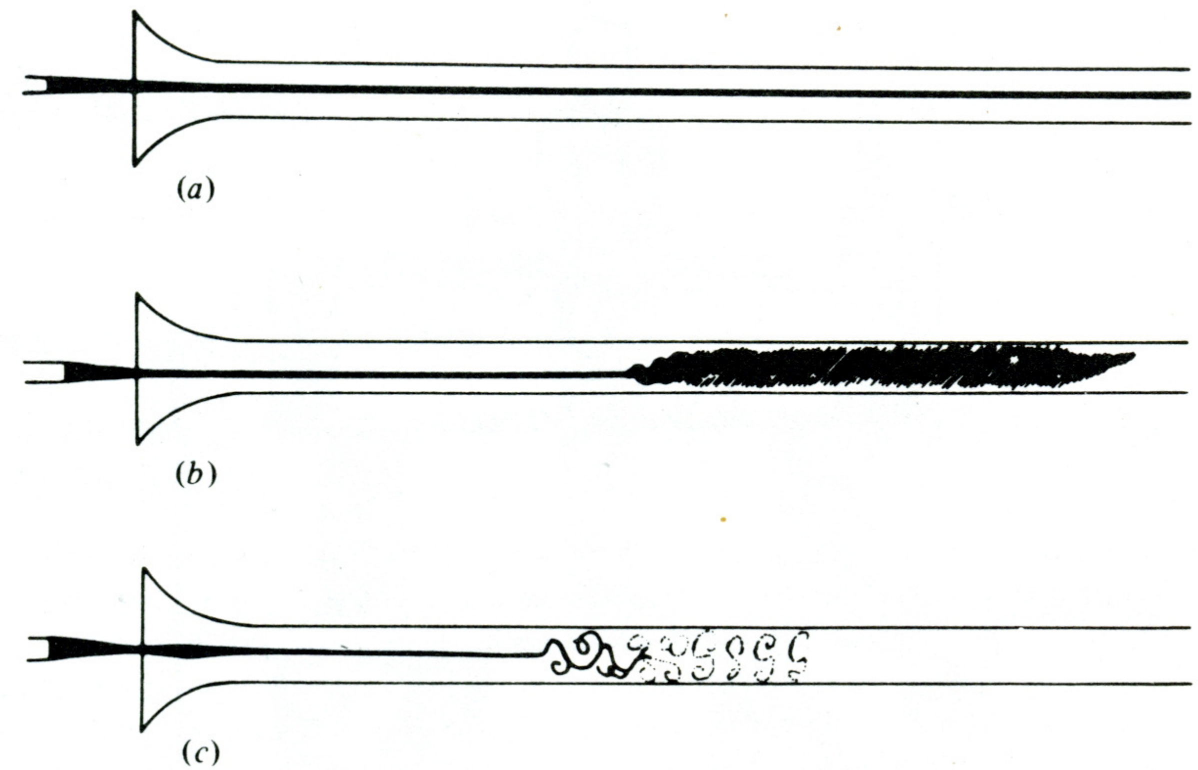
–Feynman's Lectures, 1961

`high speed': > 2 cm/sec for diameter 10 cm!

Reynolds 1883: onset of turbulence in pipe flow





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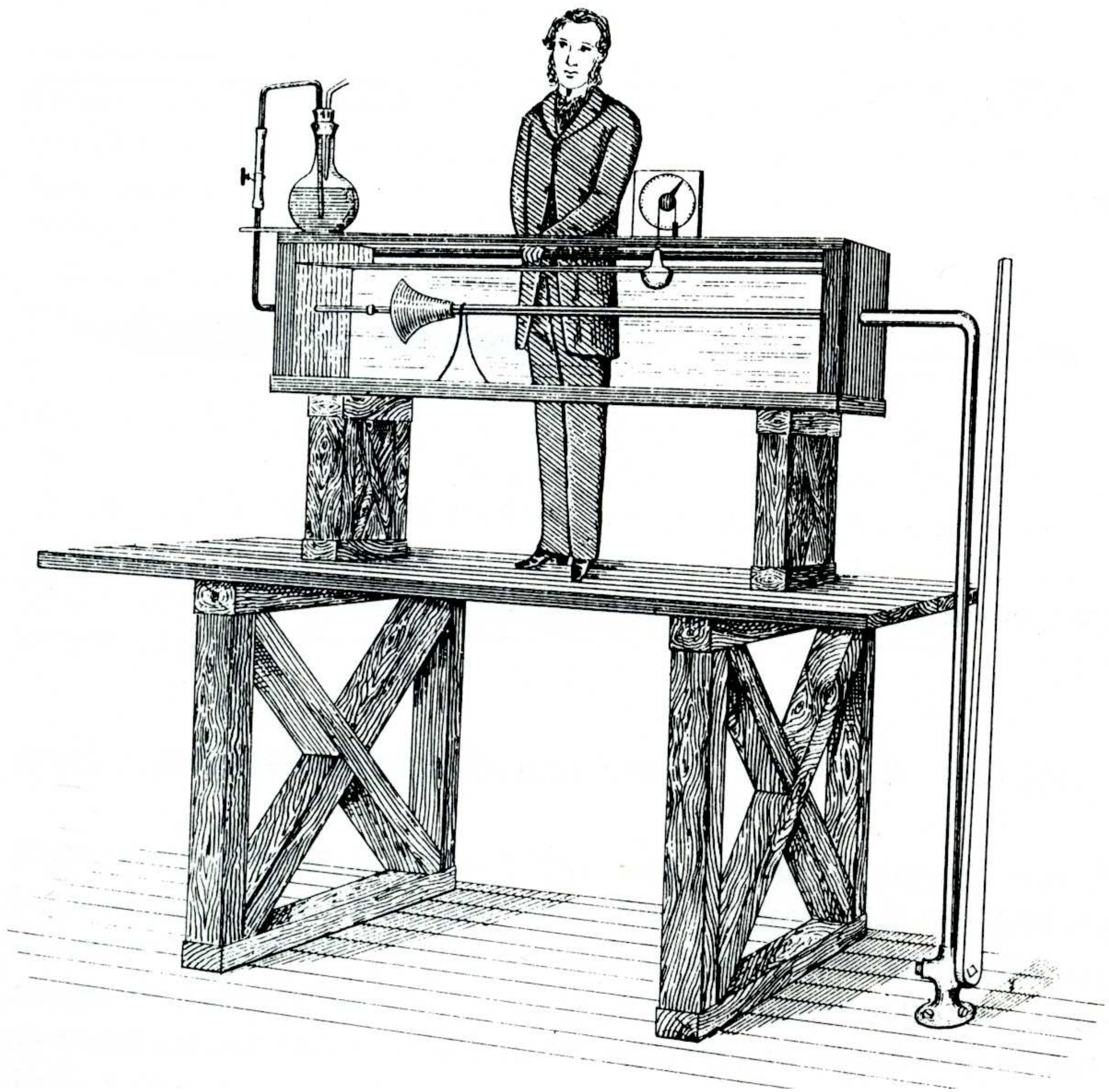
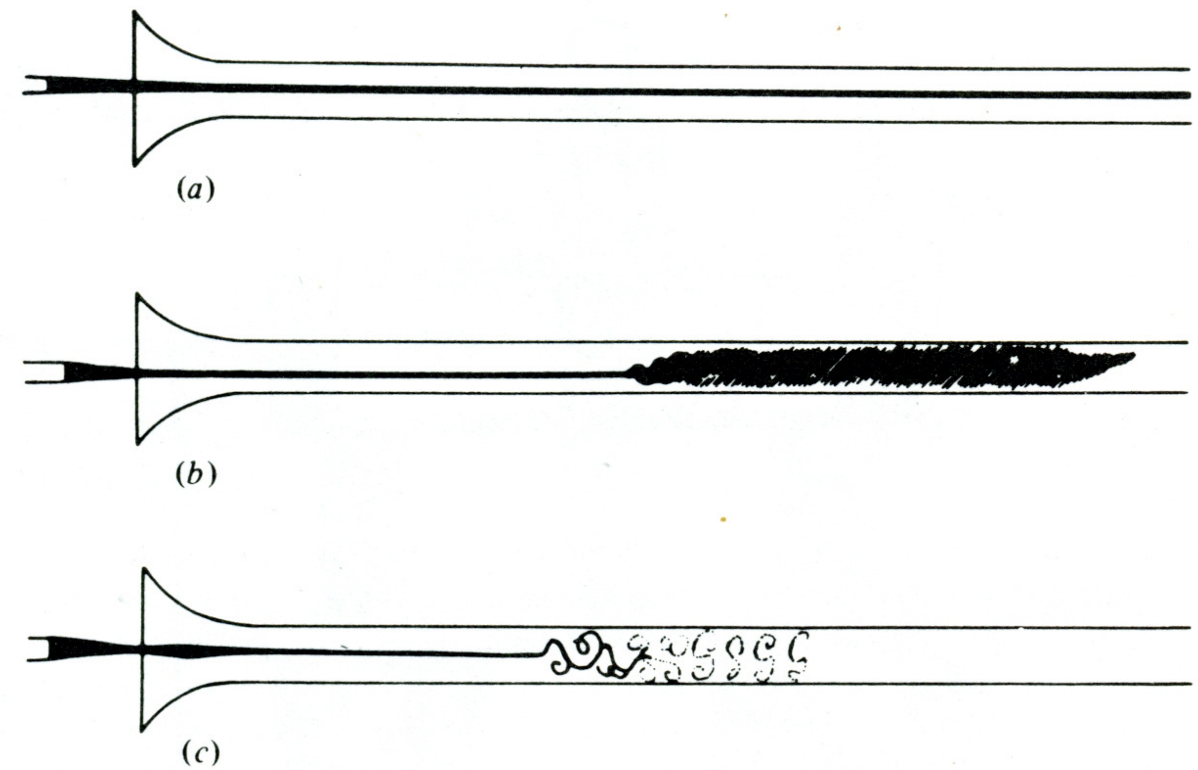




Plumbing?!

Turbulence…

- … `irregularity, or *randomness*, of turbulent flows … makes a deterministic approach impossible; instead one relies on *statistical methods*'

Tennekes & Lumley, A first course in Turbulence, 1972

statistical mechanics of fluids?

Universal small scales?

- homogeneous, isotropic in `inertial range'?

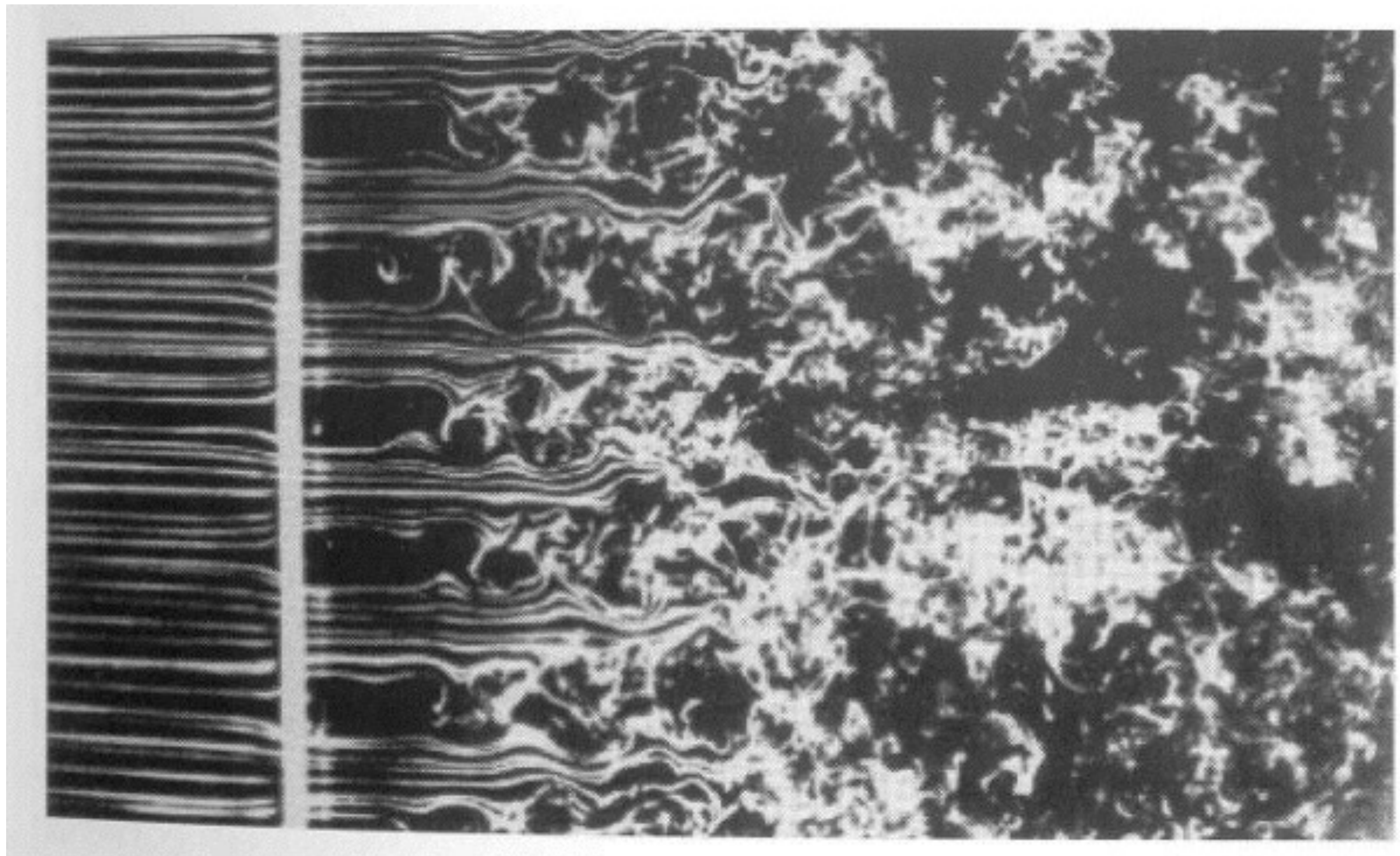
$$L \gg r \gg \eta = (\nu^3/\mathcal{E})^{1/4}$$

- Kolmogorov theory $\langle |\boldsymbol{v}(\boldsymbol{x}+\boldsymbol{r}) - \boldsymbol{v}(\boldsymbol{x})|^2 \rangle \sim \mathcal{E}^{2/3}\, r^{2/3}$

- theoretically appealing, universal, connection with smoothness of Navier-Stokes and Euler equations

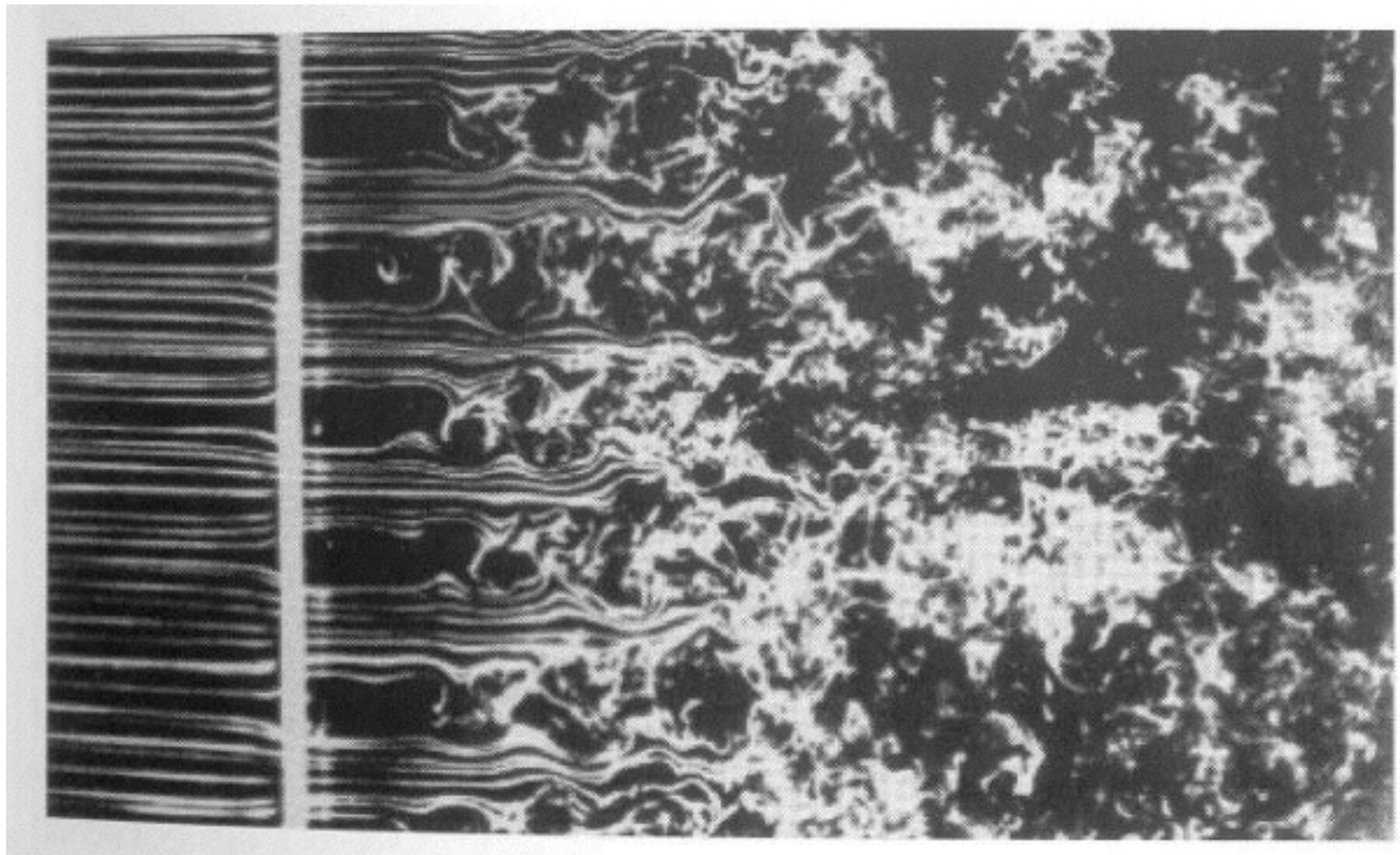
- not `plumbing'!

Grid Turbulence



early `pure' turbulence experiment

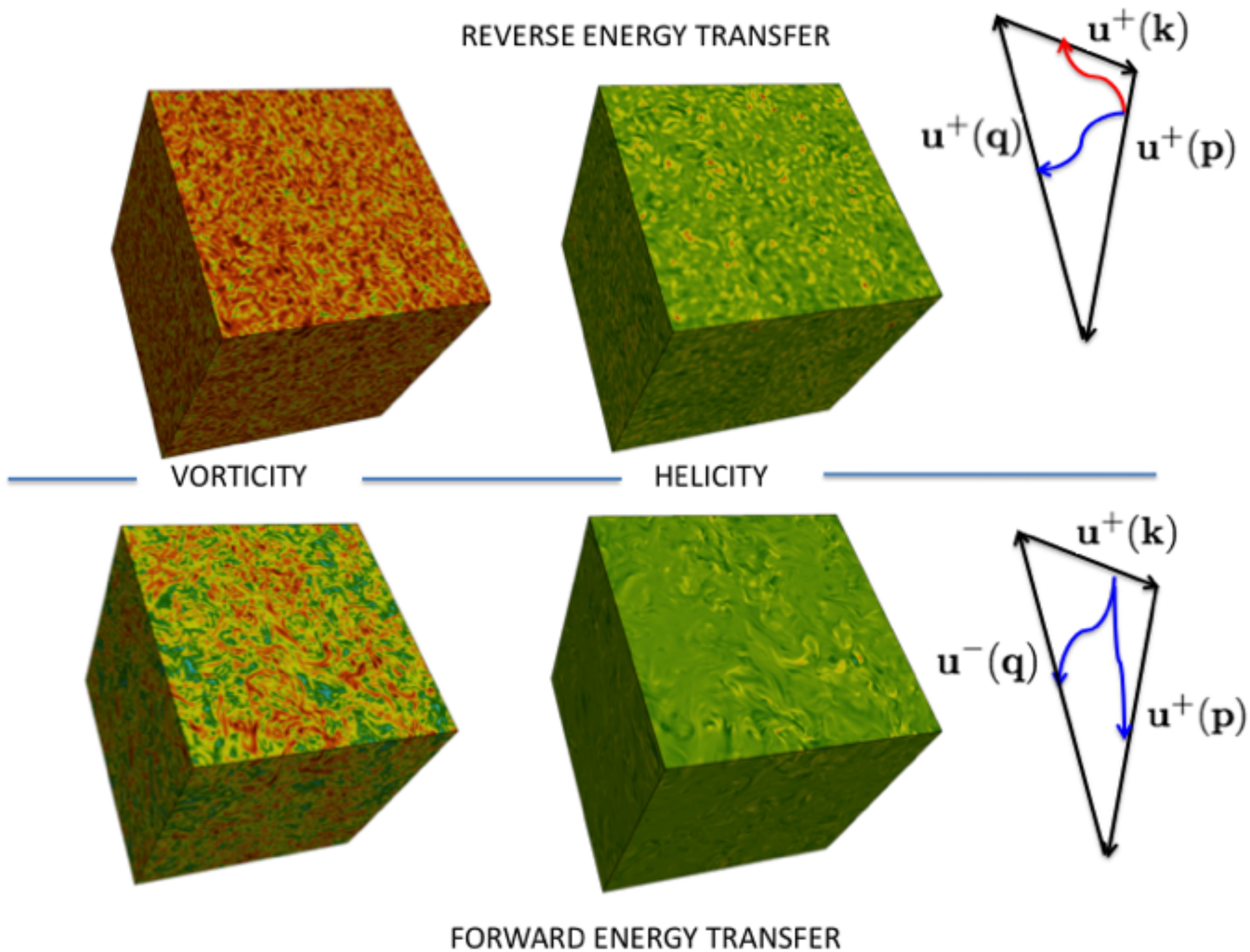
Grid Turbulence



early `pure' turbulence experiment

but decaying…
no `organizing centers'
v=0 global attractor

Turbulence in a periodic box

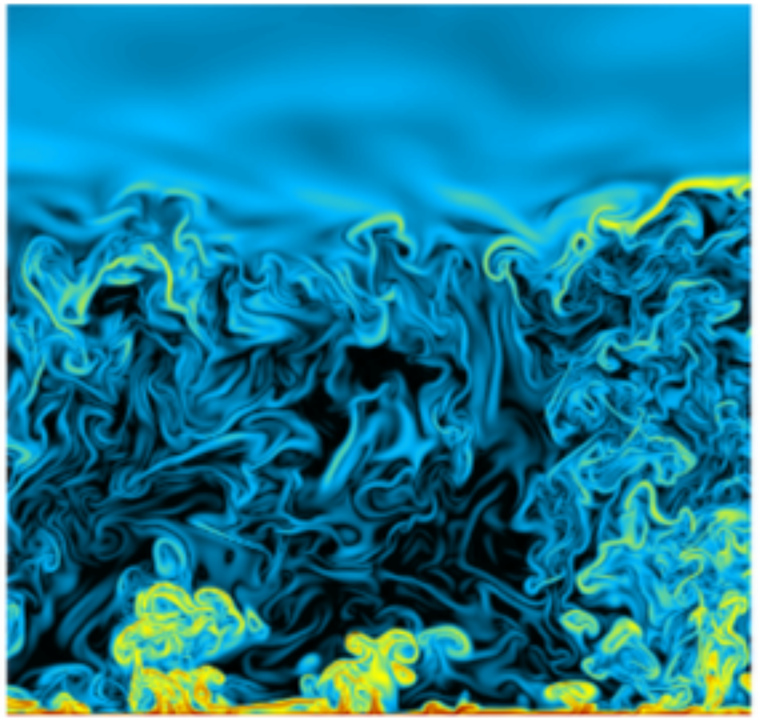


Orszag & Paterson 1972,…,   Biferale, Musacchio, Toschi, JFM 2013

Turbulent Dissipation, Mixing and Predictability

JANUARY 9 - 13, 2017

Overview



Turbulence is a subtle and multi-faceted phenomenon which touches many related areas and its study is widely considered one of the most important fields in classical physics. Recently, rapid progress has been made in the mathematical community towards understanding Onsager's conjecture and anomalous dissipation. Meanwhile, new ideas have emerged from the turbulence physics community regarding spontaneous stochasticity, or breakdown of uniqueness of Lagrangian particle trajectories. Both of these developments are intimately related to applied topics, such as large-eddy simulation of turbulent flows, predictability of turbulent flows, and enhanced mixing by turbulence.

Any significant progress towards true understanding requires close, cross-disciplinary collaboration and communication between the different areas involved. This workshop will bring together various communities working on the topics of turbulence, anomalous dissipation, and spontaneous stochasticity in incompressible fluid mechanics at high and infinite Reynolds number. The goal of this workshop is to increase the dialogue between these communities as the various fields are rapidly developing.

- Kolmogorov scaling/theory:
    - ★ it’s there, if you stir enough! (cf. tidal channel measurements, random forcing in DNS, `French washing machine’,…)
- but not quite: `intermittency’…

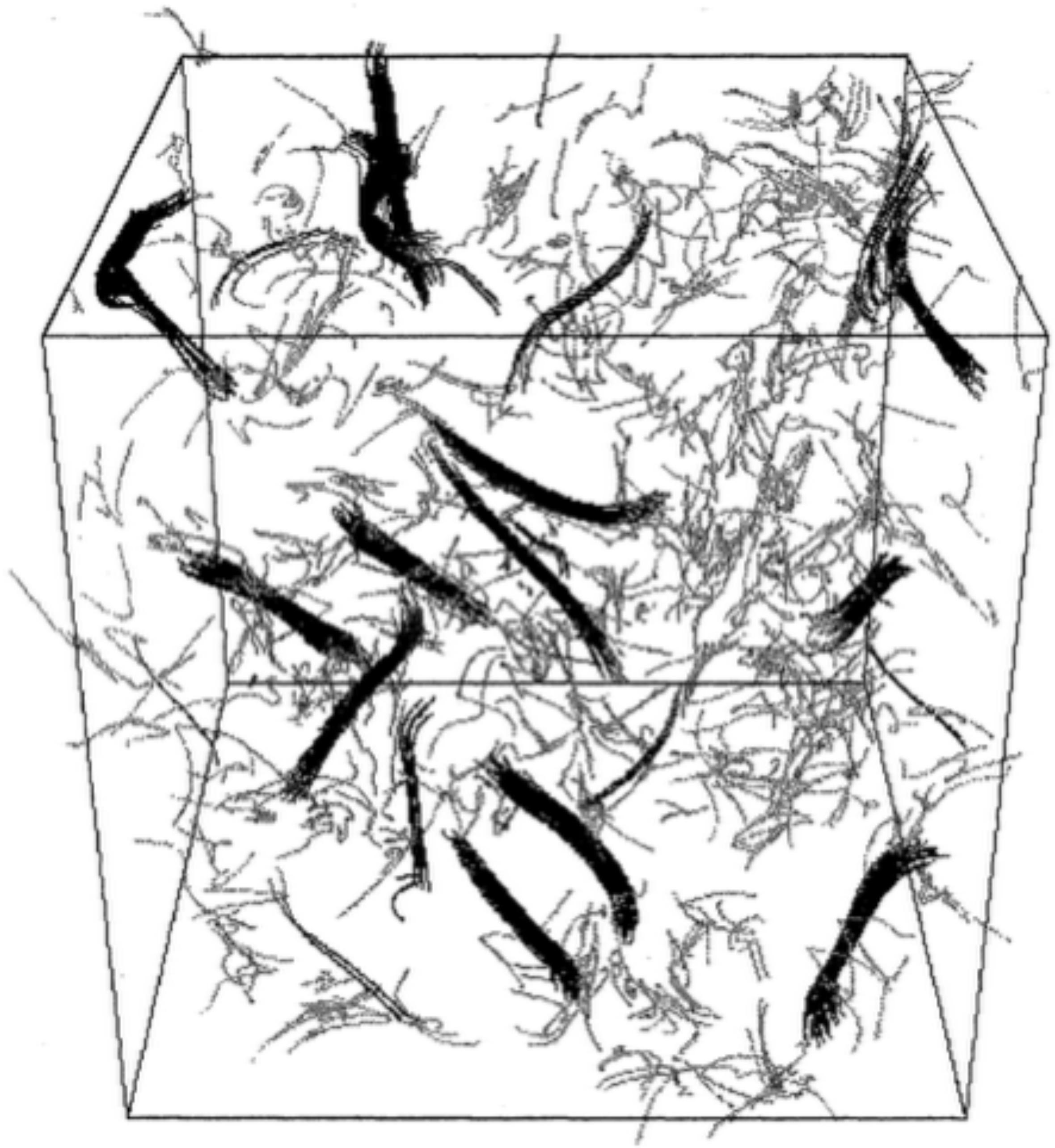


Figure 1. Three-dimensional perspective view of vortex lines in a homogeneous turbulent flow with $R_\lambda \approx 77$ obtained by direct numerical simulation. Local vorticity intensity is keyed to shading, ranging from light grey (low) to black (high).

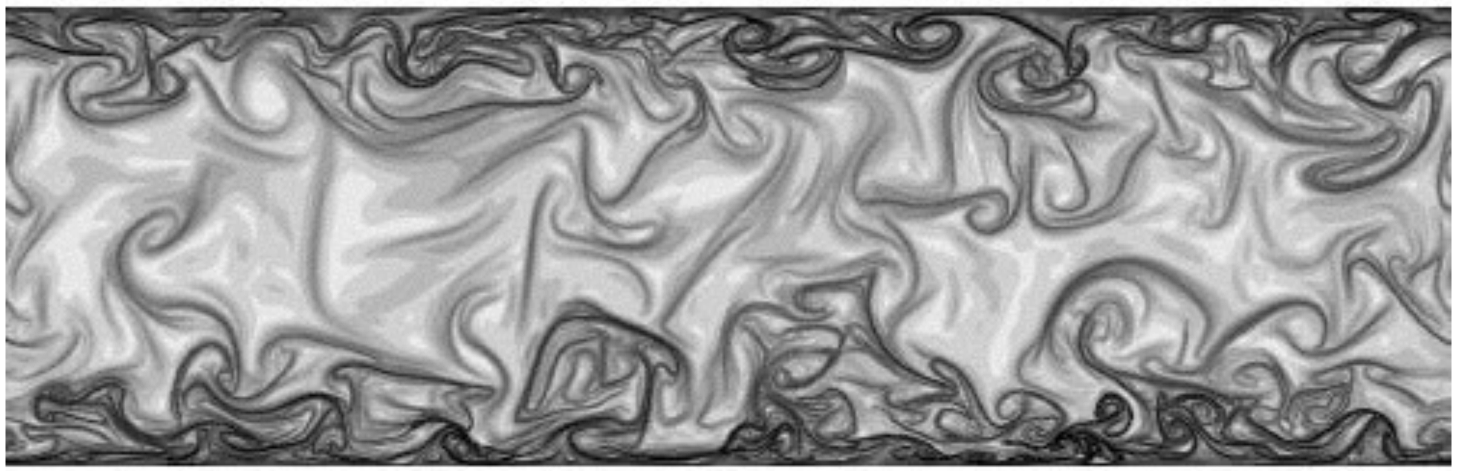
Proc. R. Soc. London A (1991)

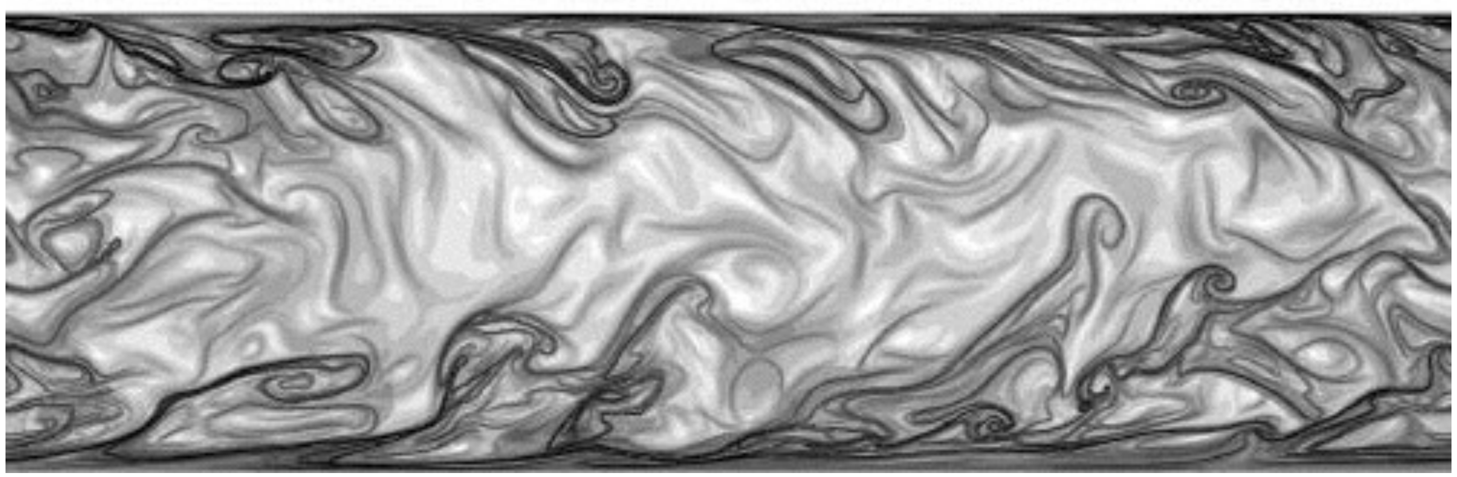
emergence of coherence!

Turbulence in *smooth* channel with *smooth, steady* forcing:

Top

Front



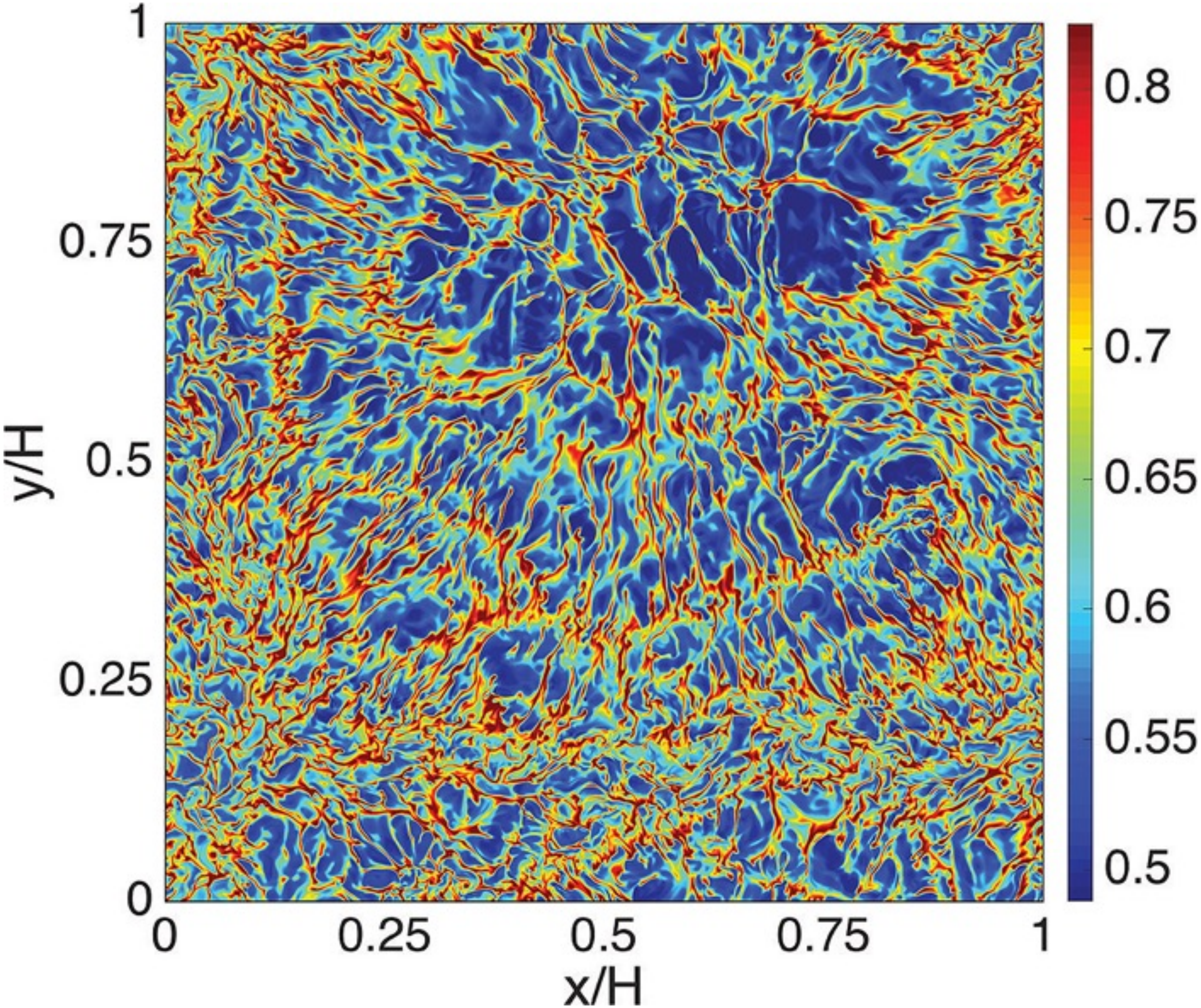


Side

Green, M.A., Rowley, C.W. & Haller, G.
Detection of Lagrangian Coherent structures in three-dimensional turbulence
JFM **572**, 2007, 111-120

Rayleigh-Benard convection

Top view near bottom hot wall



Lohse et al., University of Twente

Coherent approach to Turbulence

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- Turbulence full of poorly defined `coherent structures': vortices, streaks, fronts, plumes,…

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➡ Extract multi-scale *exact* coherent structures *from equations (not the data!):* steady state, traveling wave, time periodic solutions of equations

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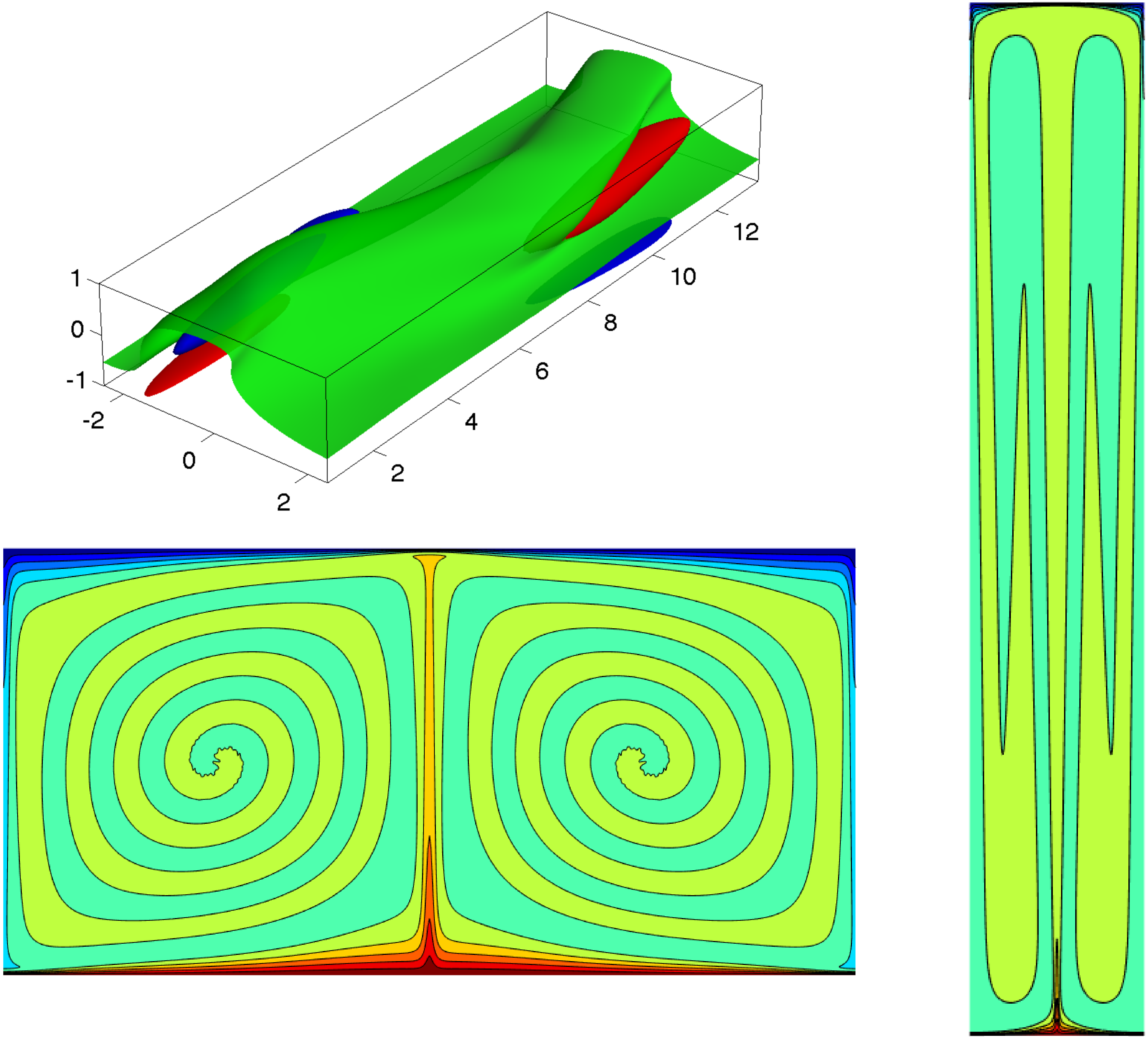
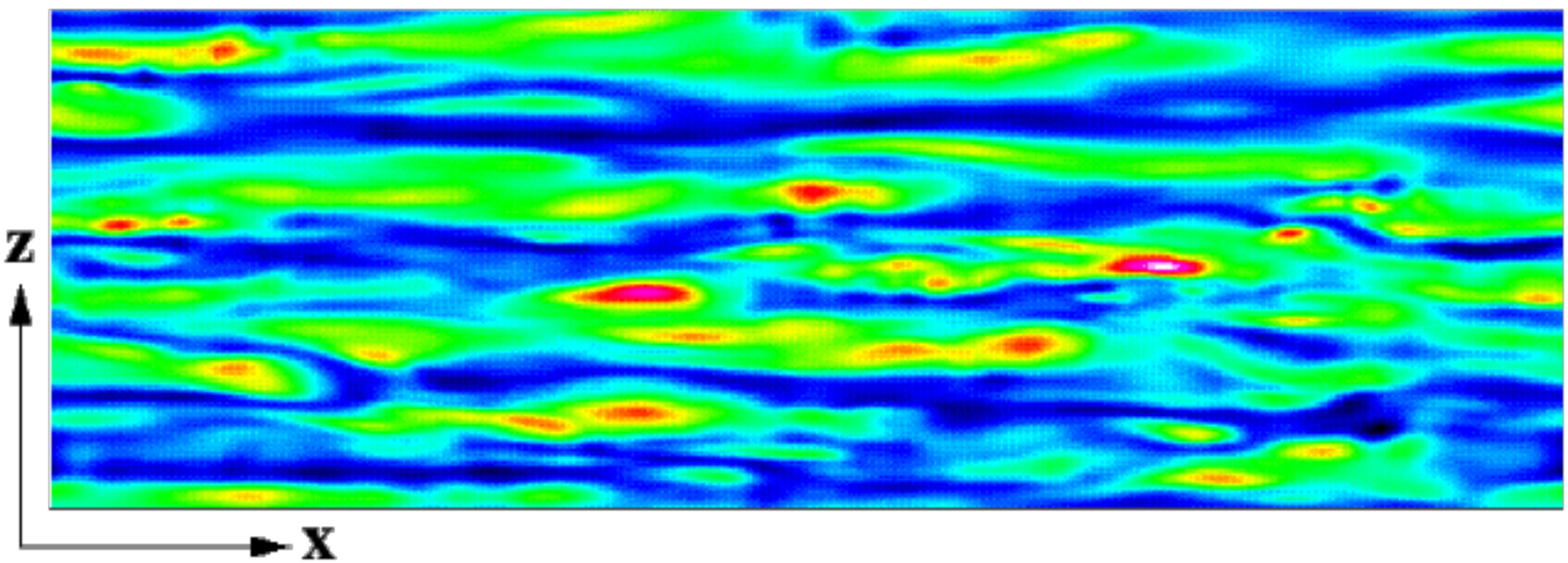
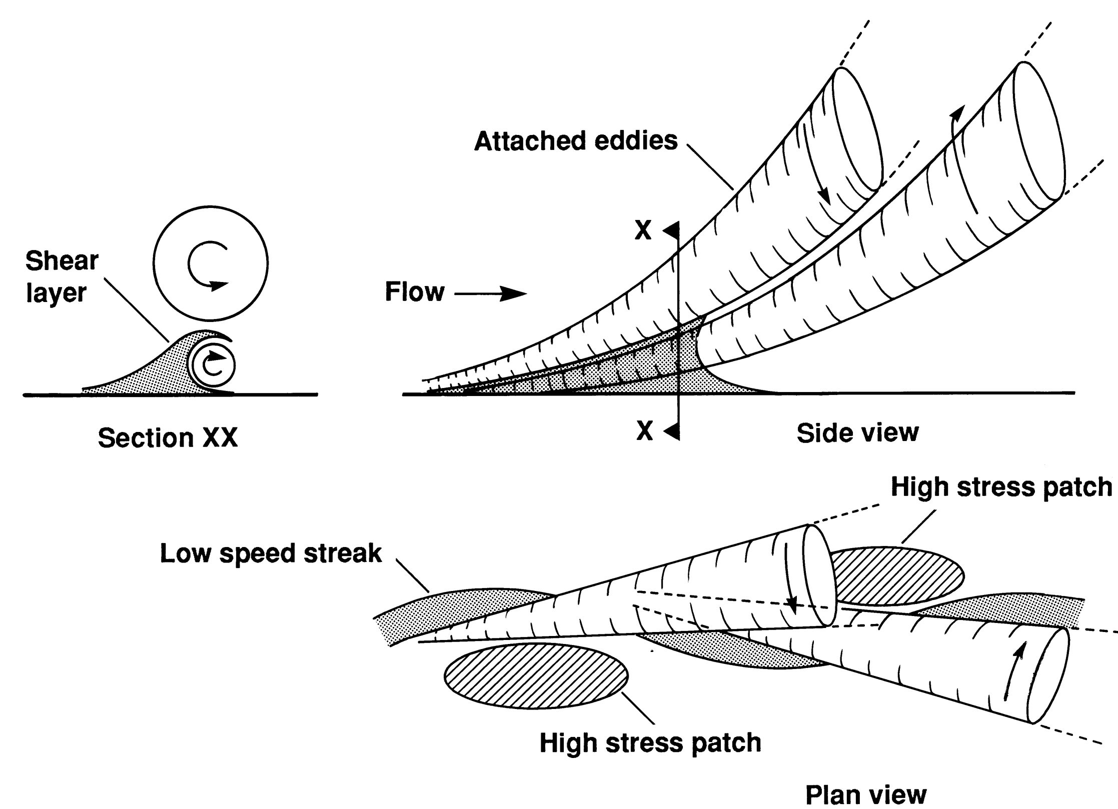
  - Couette + Poiseuille shear flows mostly


1
0
-1
-2
0
2
2
4
6
8
10
12

Skin friction in channel flow

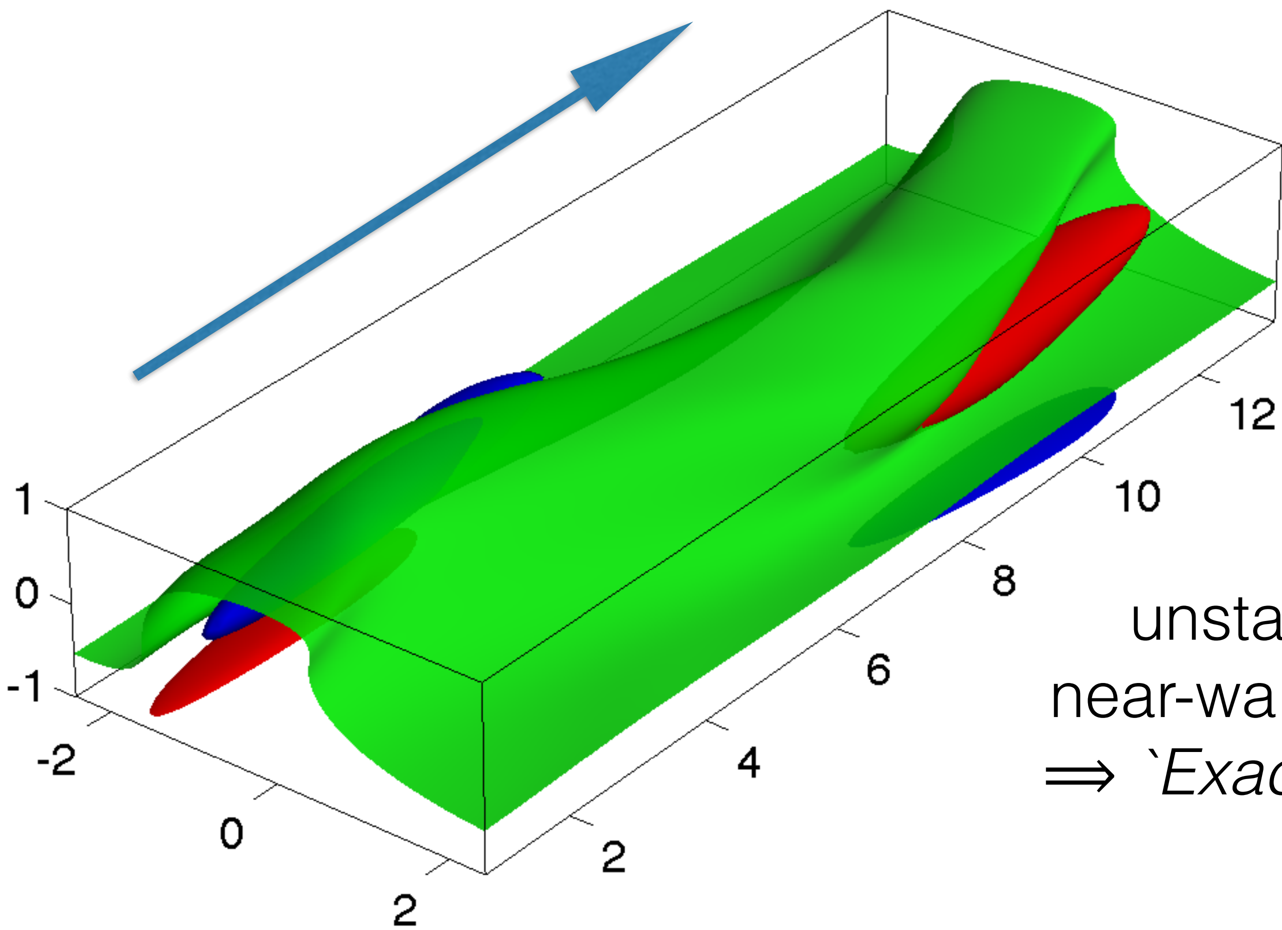


Kim, Moin & Moser
JFM 1987



pattern eduction from KMM hot spots
Derek Stretch 1990

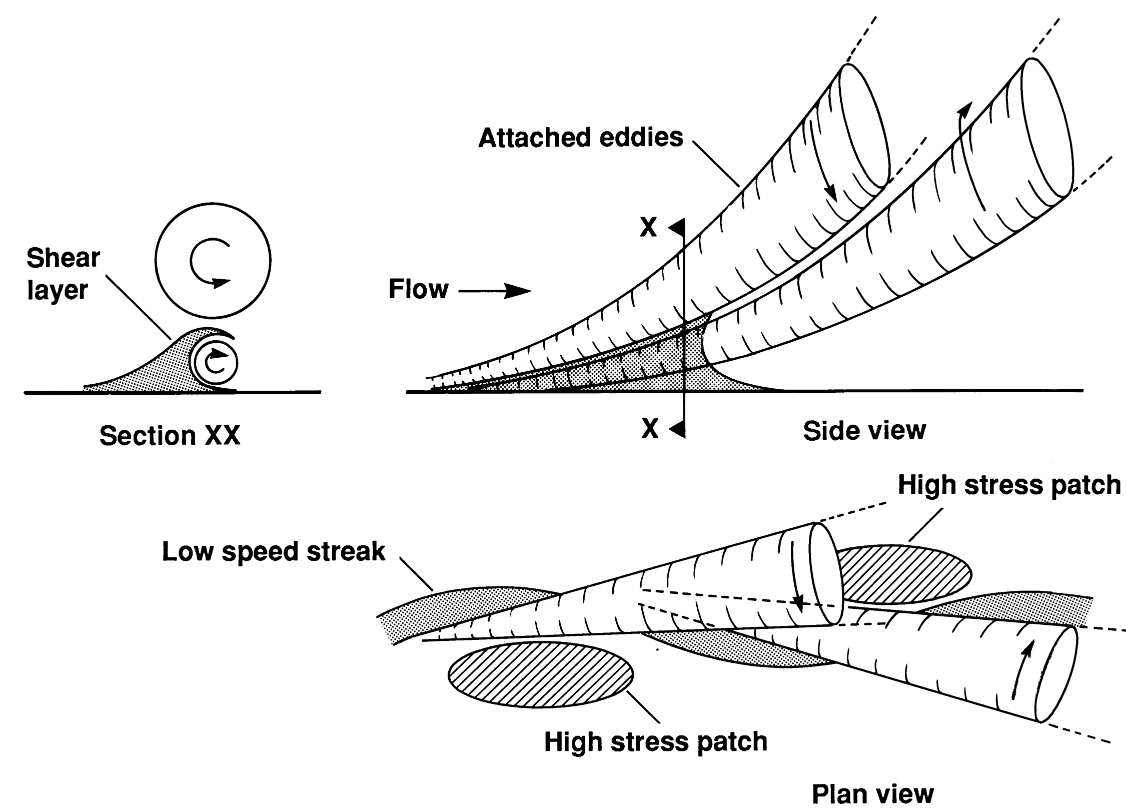
Unstable coherent states in shear flows



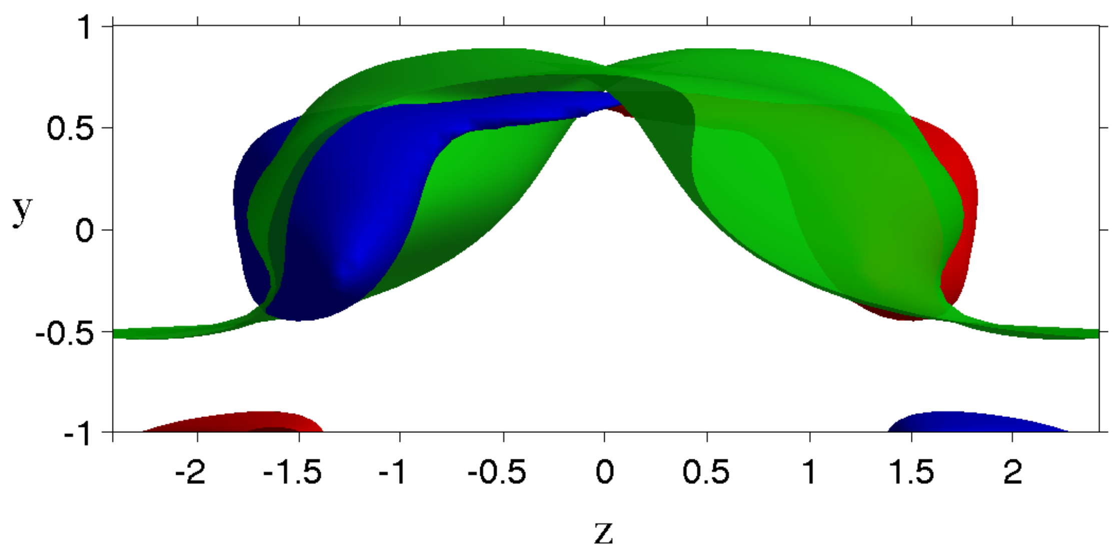
3D Traveling Wave in Plane Poiseuille flow

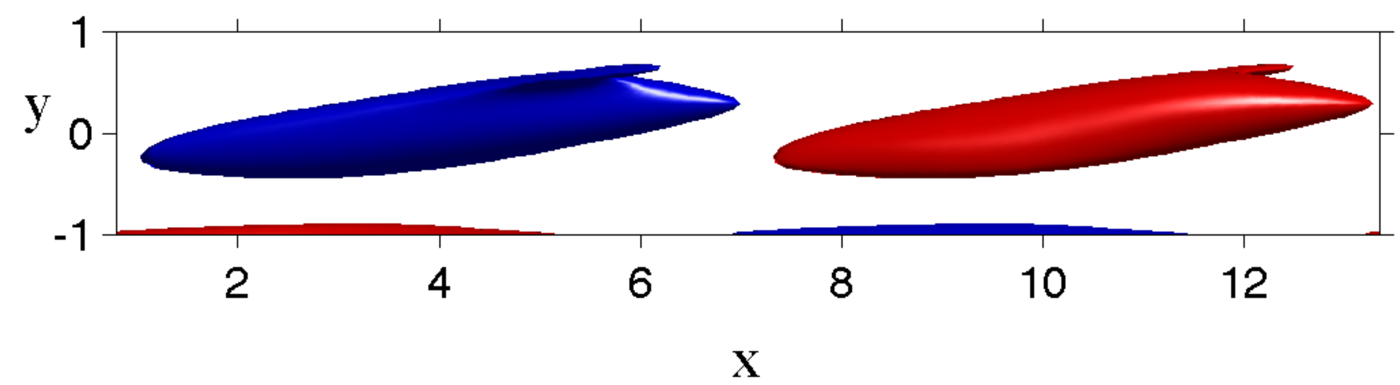
unstable, yet, captures
near-wall structure quite well
$\Longrightarrow$ *`Exact coherent structure'*

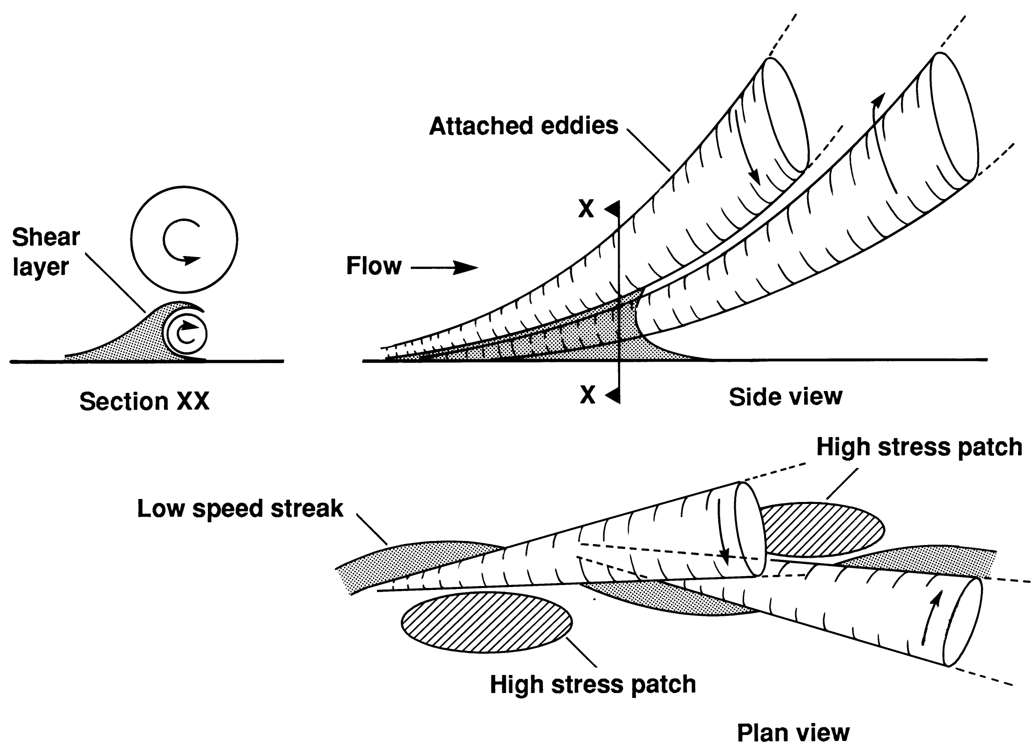
Derek Stretch, CTR 1990
Structure of high drag regions
in turbulent channel flow
(KMM R_τ =180)

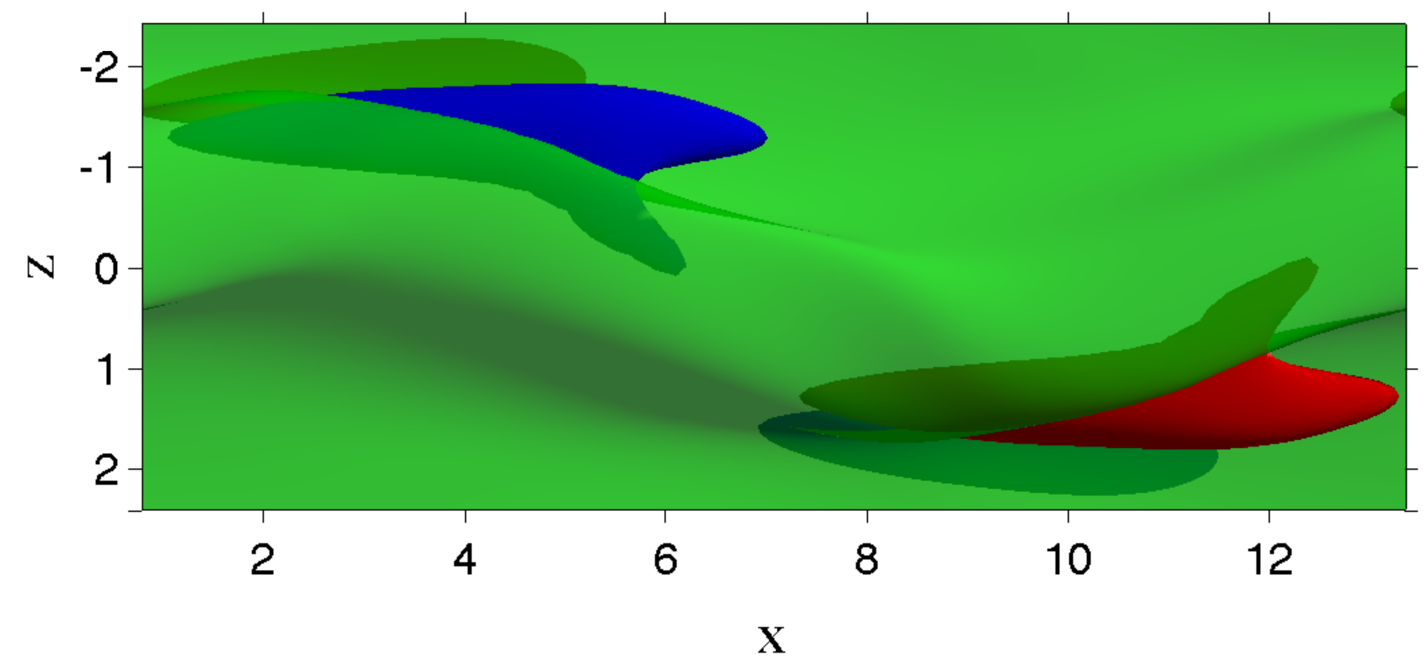


`Optimum' channel flow ECS





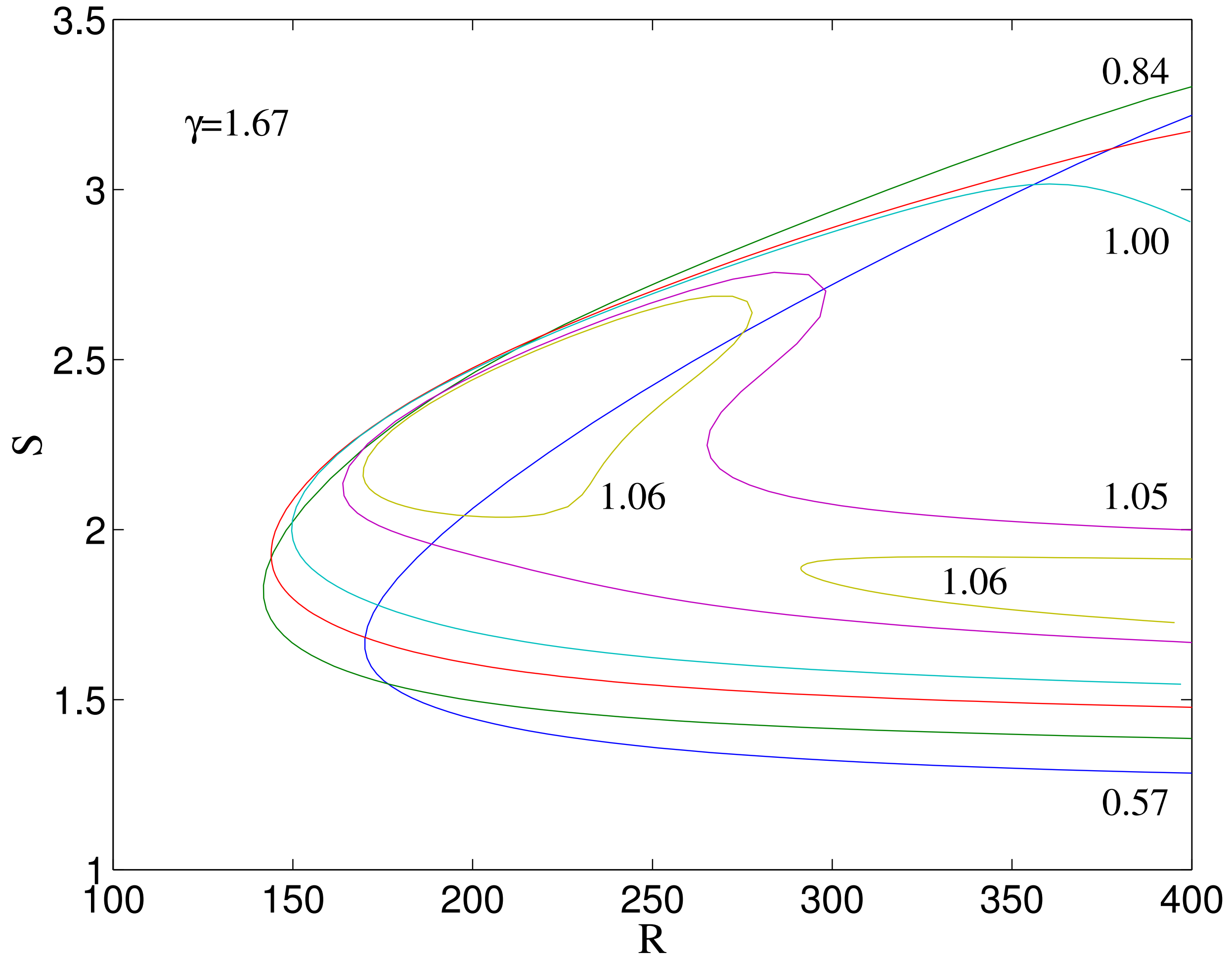


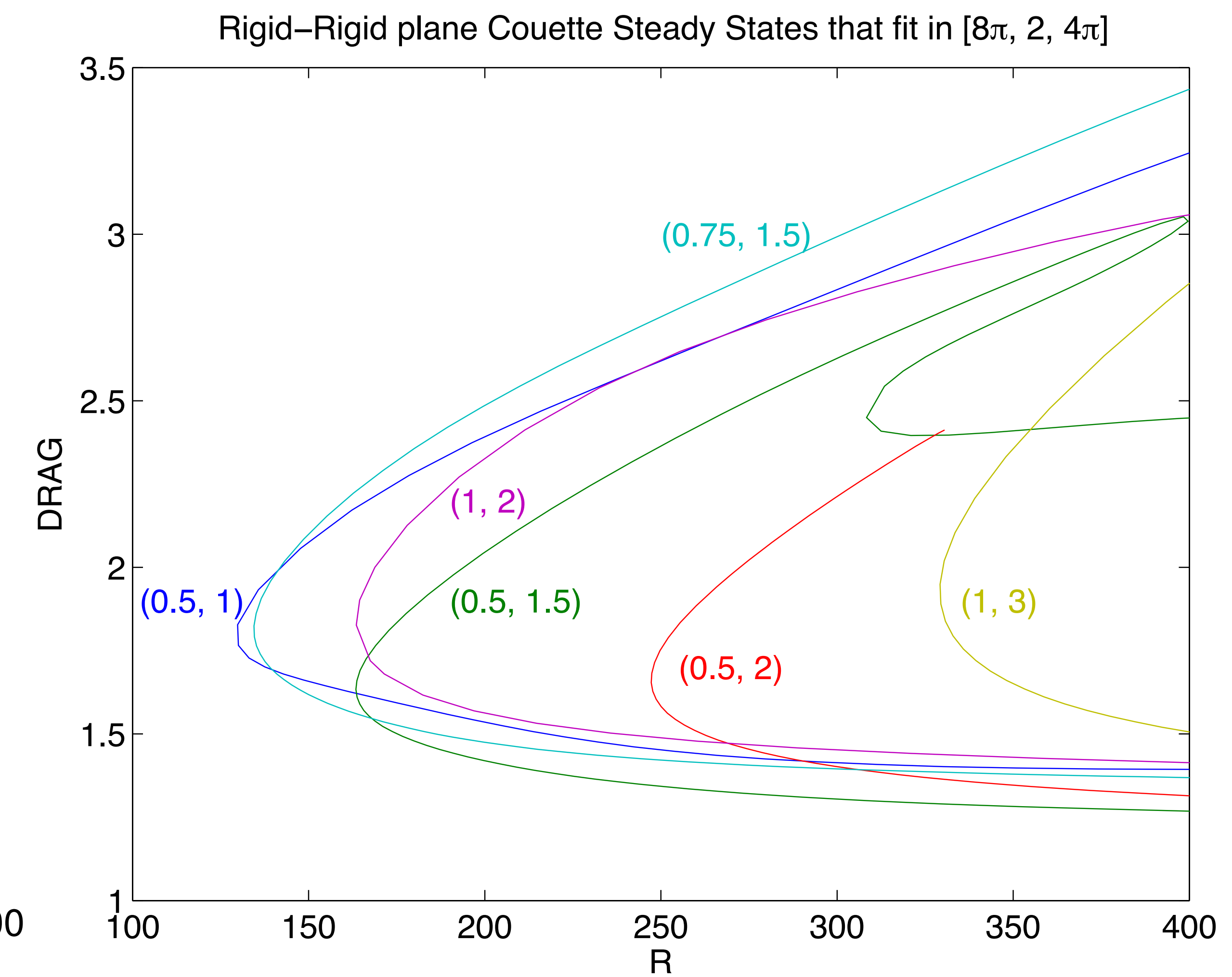


$$\min\ R_\tau = 2h^+ = 44 \quad \text{for } L_x^+ = 274,\ L_z^+ = 105$$

Nice, but more work needed!

Lots of EQs, TWs, POs: which ones matter?





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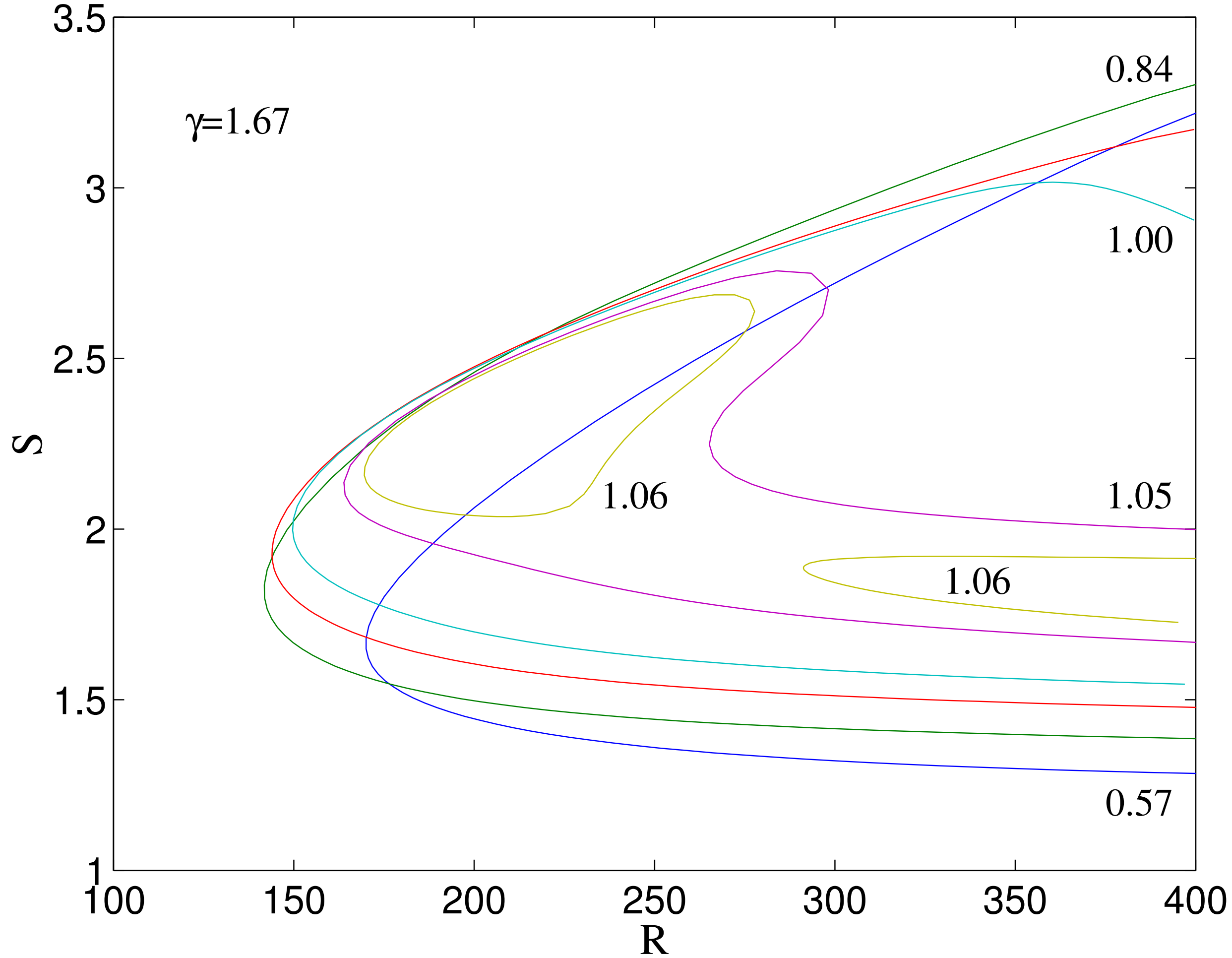
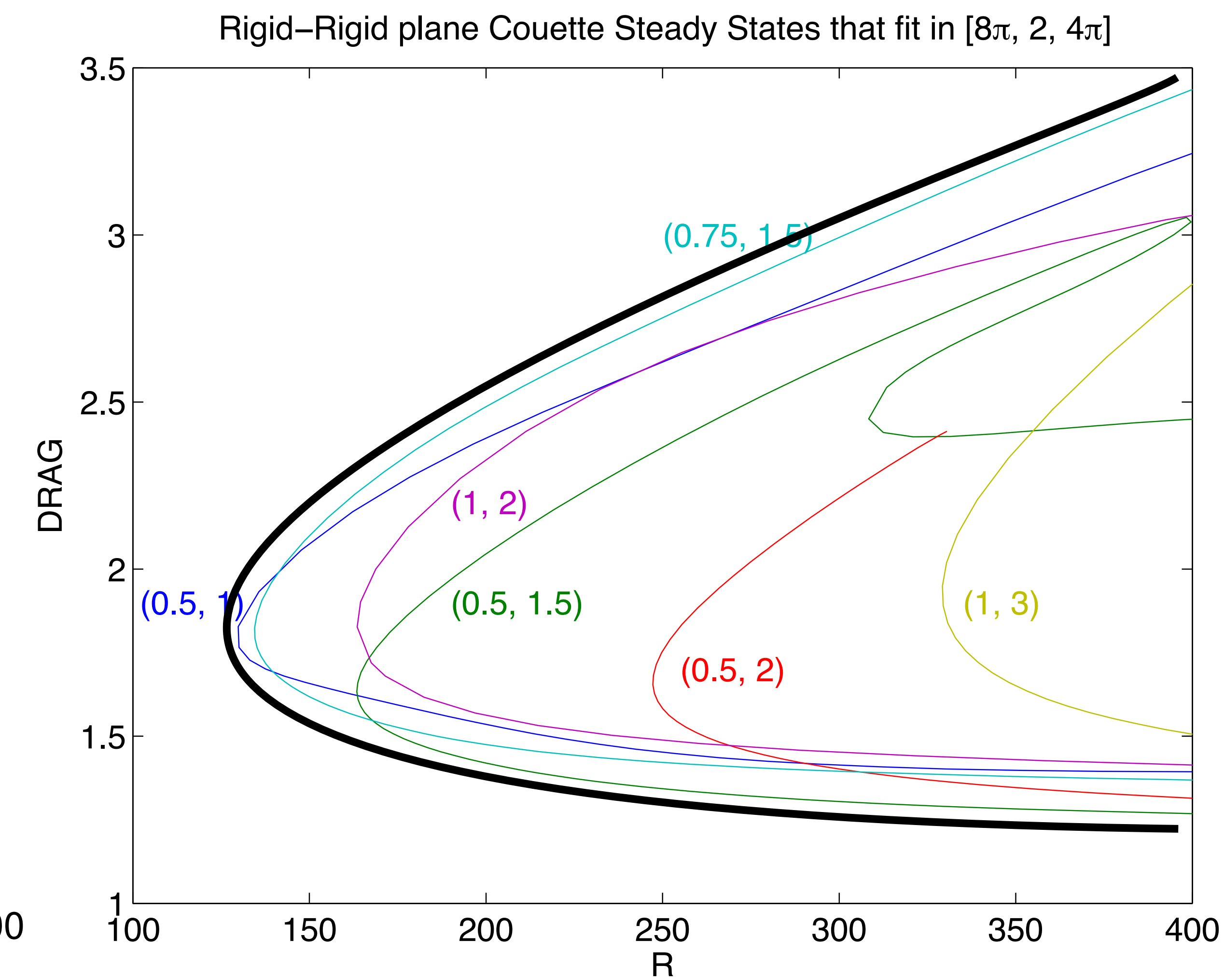




Attempted to compute **envelope**,
largest and smallest shear solutions
optimizing over both horizontal wavenumbers
(Jue Wang & FW, 2003, *abandoned*)

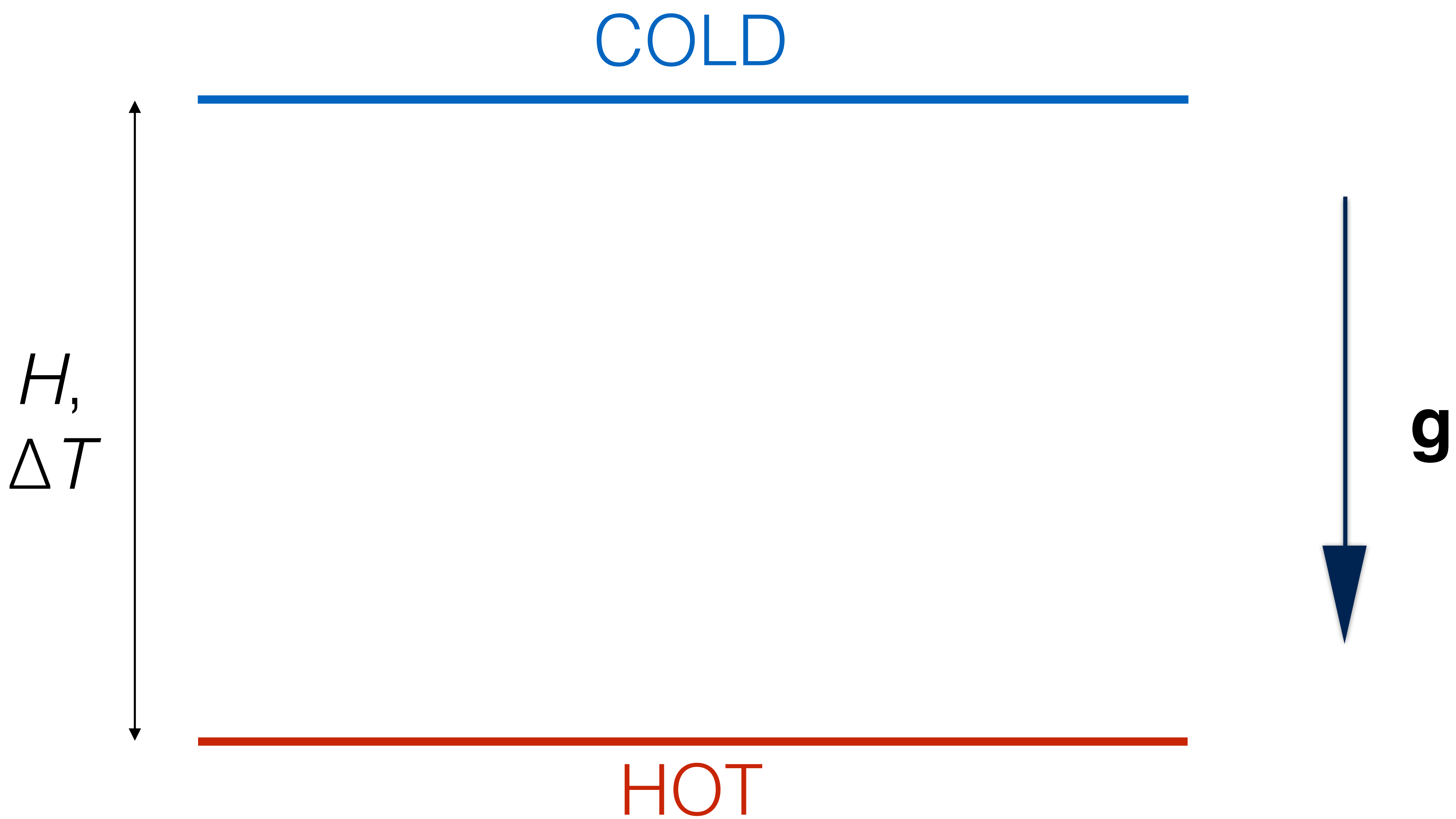
Computing envelope was too hard

(back then in 2003)

Shear flows are difficult:
ECS are 3D traveling waves, periodic orbits,…
Lots of different solutions

Simpler Rayleigh-Bénard problem:

Fluid between 2 horizontal walls: Cold top, Hot bottom



$$\Delta T = T_{bot} - T_{top}$$

Simpler Rayleigh-Bénard problem:
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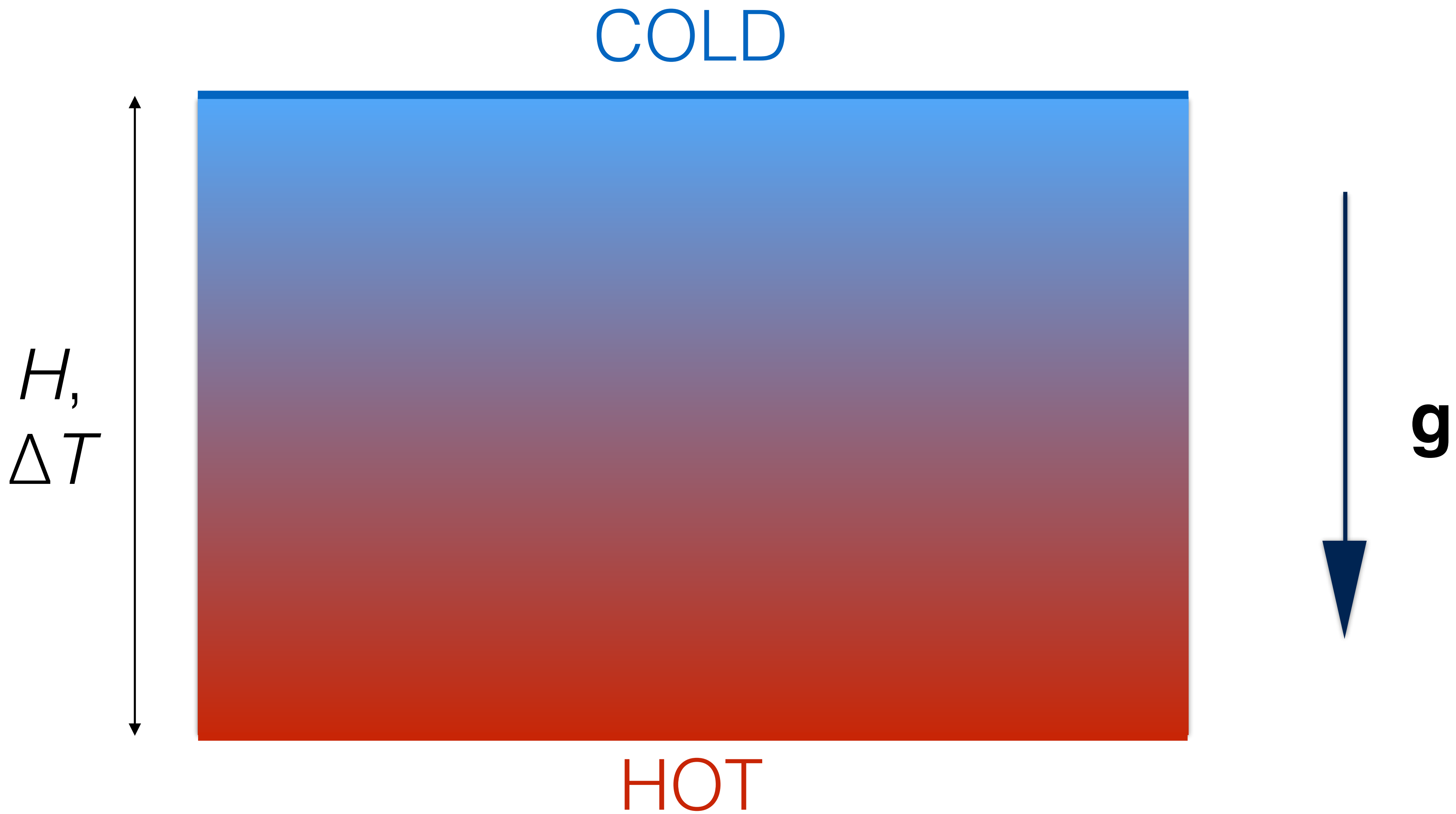


$$\Delta T = T_{bot} - T_{top}$$

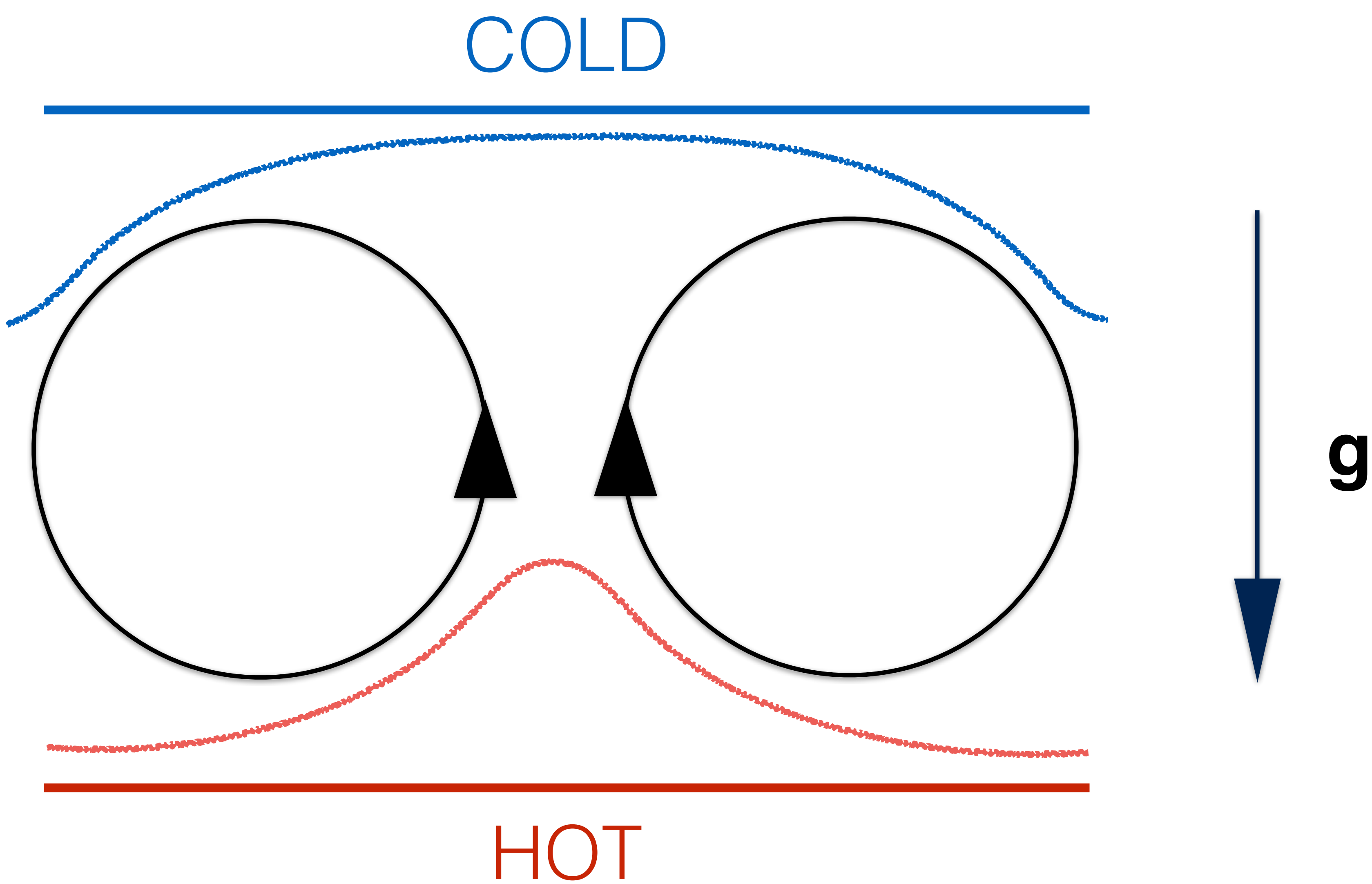
- Boussinesq approximation:

  - density constant except in buoyancy $\rho = \rho_0 \left(1 - \alpha_V T\right)$

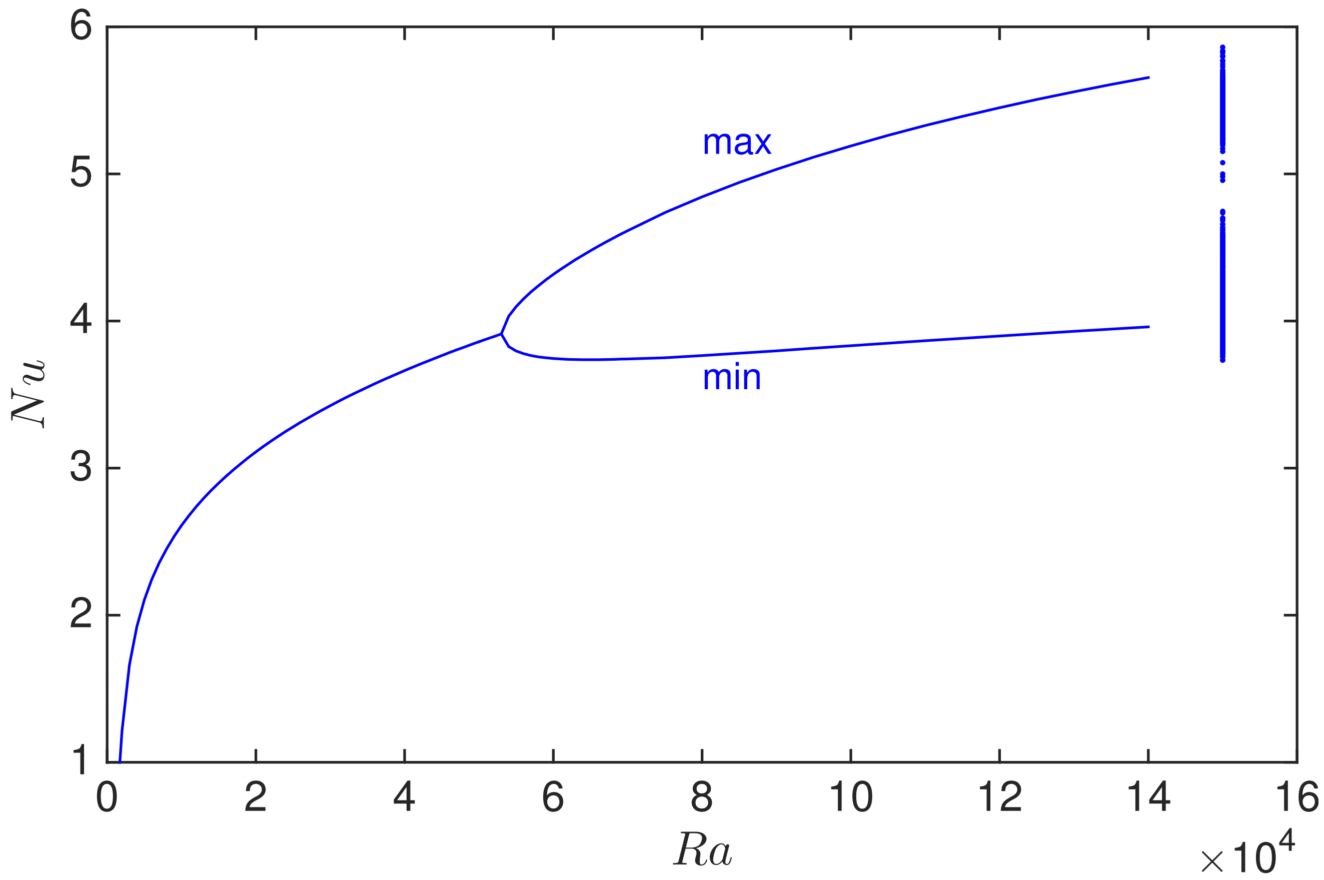
- Governing parameters:

$$Ra = \frac{(g\alpha_V \Delta T)H^3}{\nu\kappa} \qquad Pr = \frac{\nu}{\kappa}$$

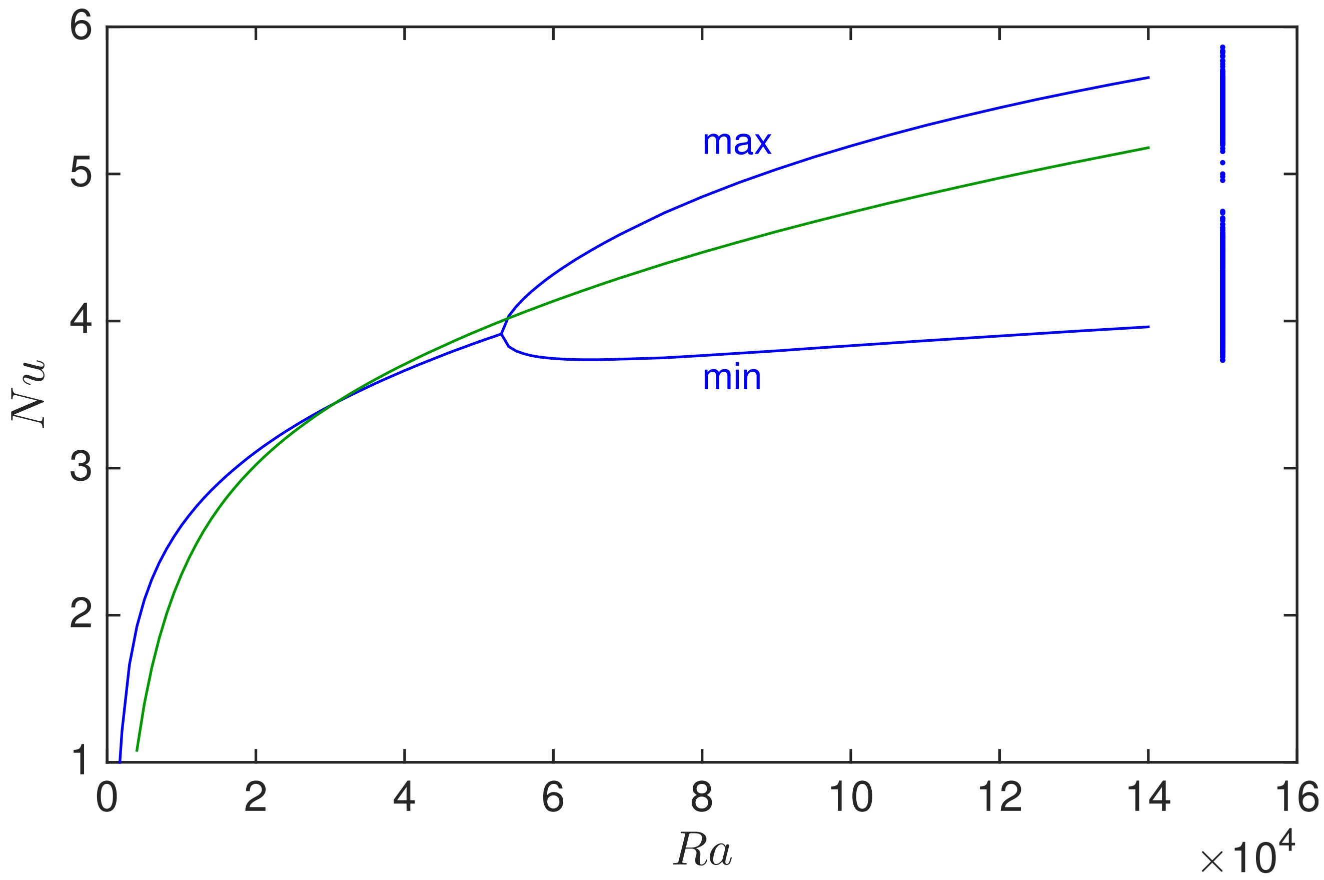
Convection for *Ra* > 1708, L/H ~ 2



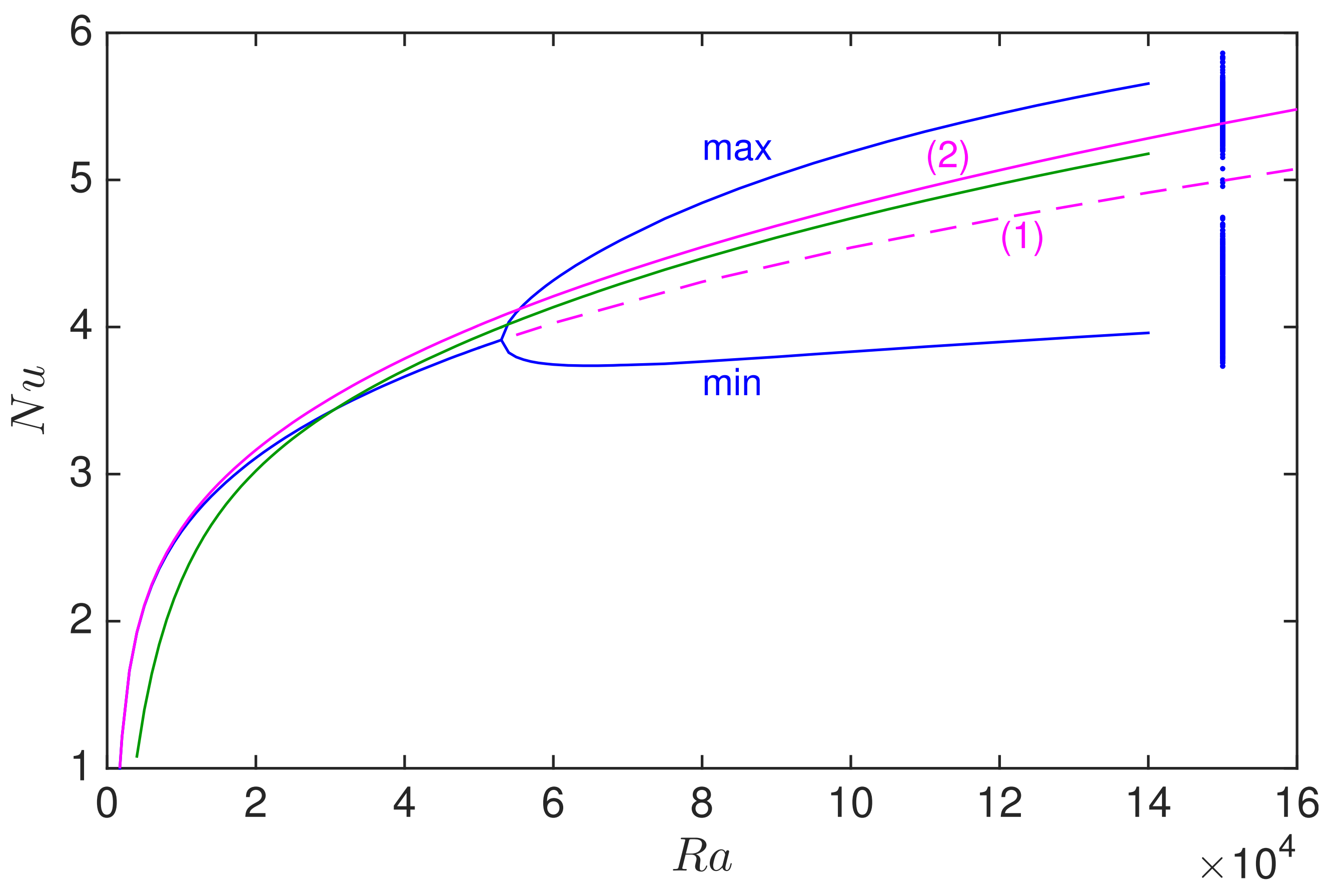
Supercritical bifurcation to steady convection, Hopf bifurcation, Chaos,… L/H=2



L/H=4



unstable L/H=2 & L/H for max Nu

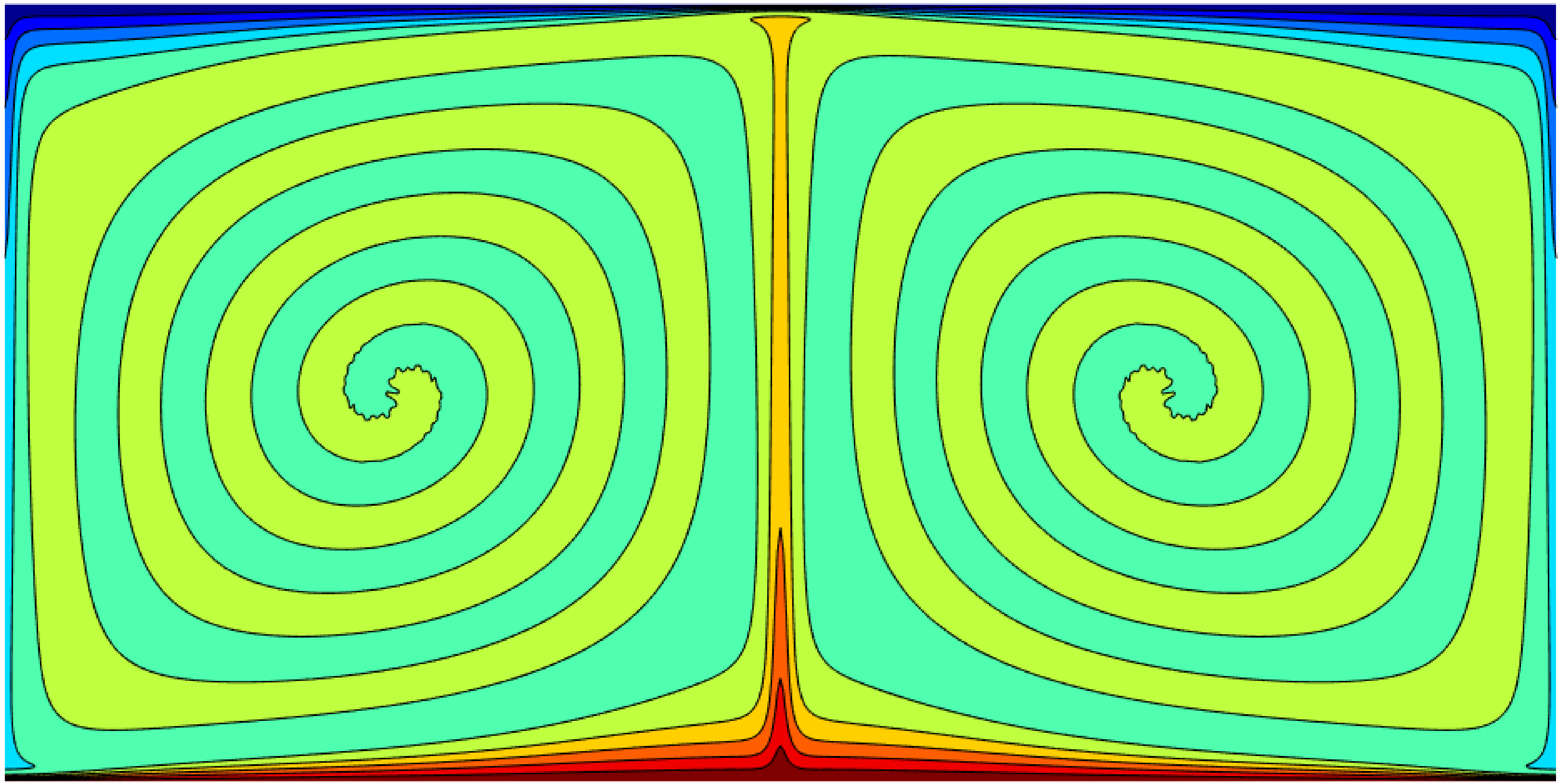


W, Boonkasame & Smith, Phys. Fluids 2015

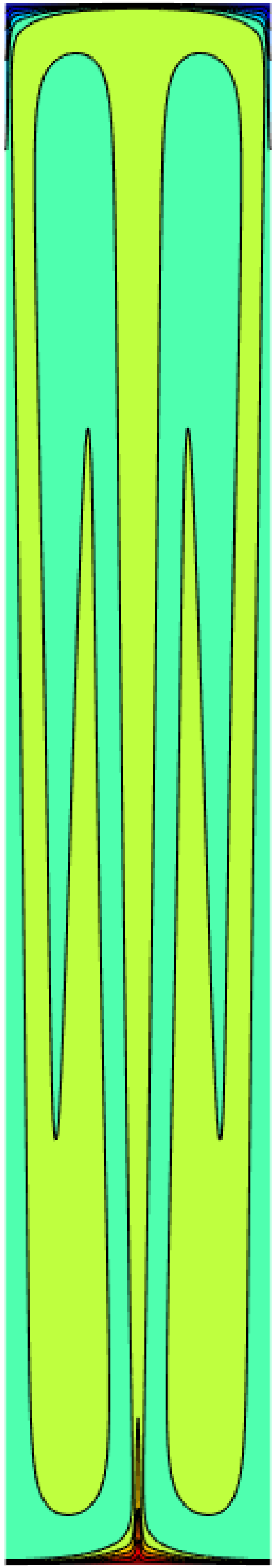
Sondak, Smith & W, JFM 2015

L/H=2 steady state,
Ra=4 x 10^7, Pr=7

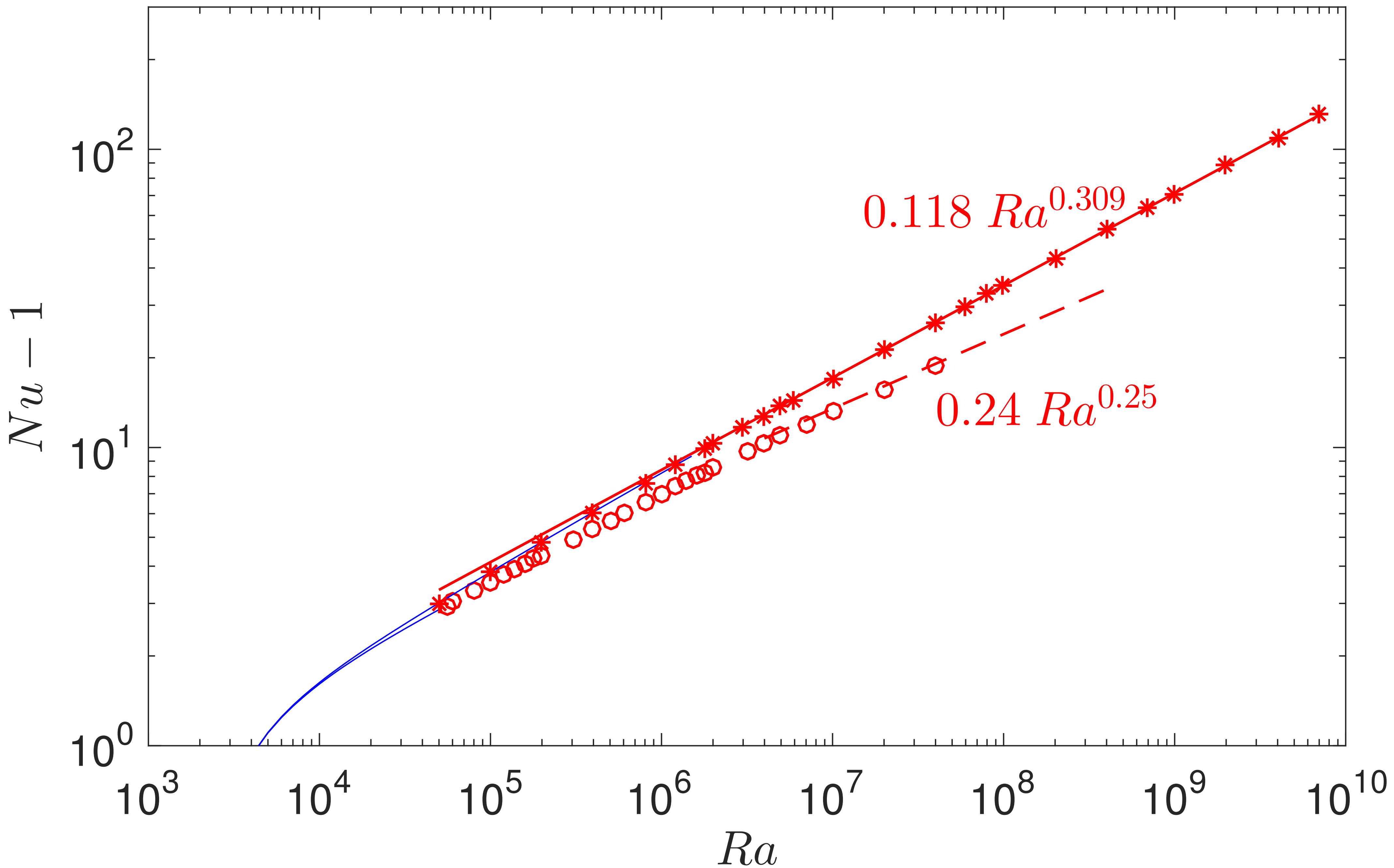
Temperature contours



Optimum steady state,
Ra=7 x 10^9, Pr=7



Nu-Ra scalings



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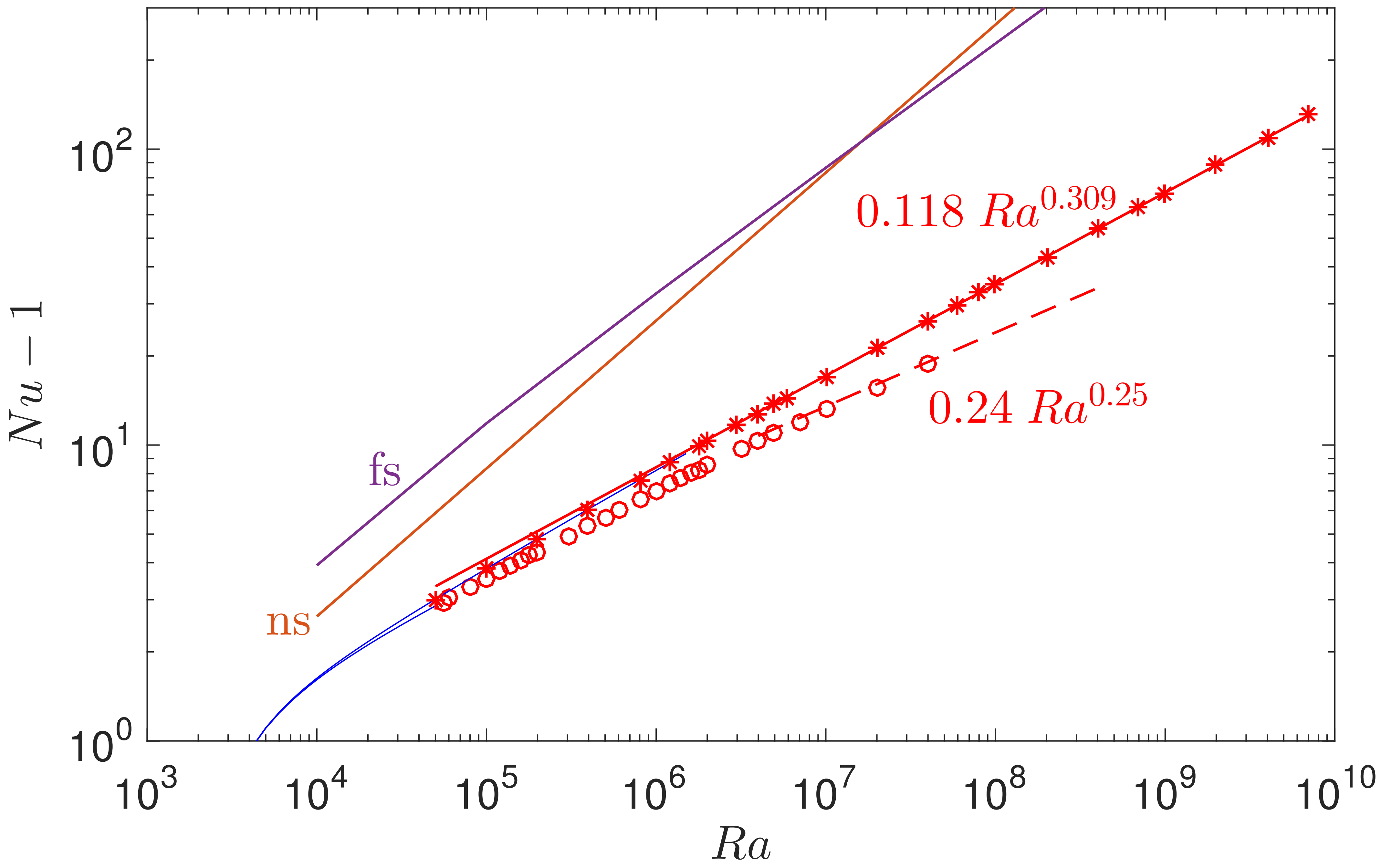
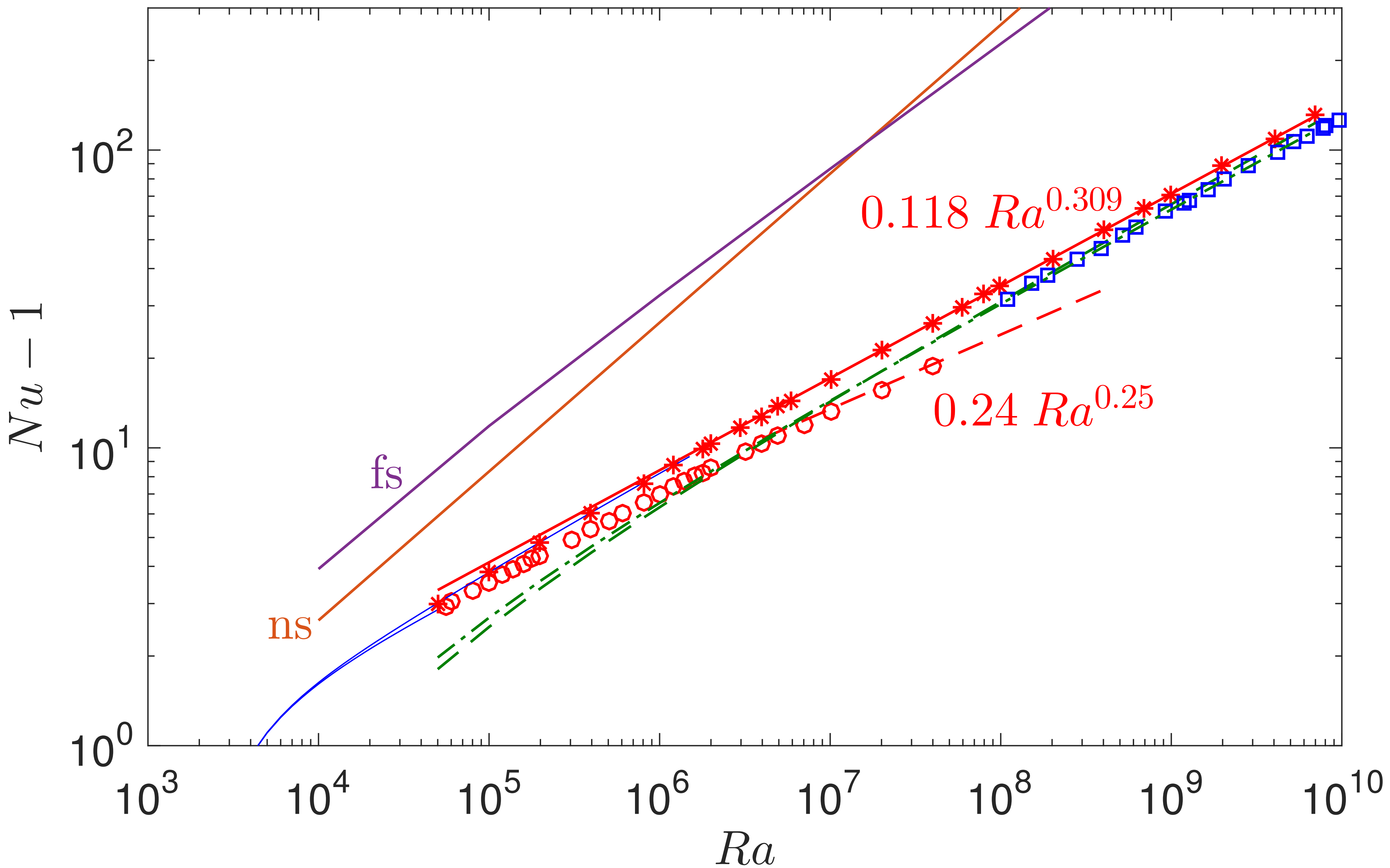


Rigorous free-slip and no-slip bounds

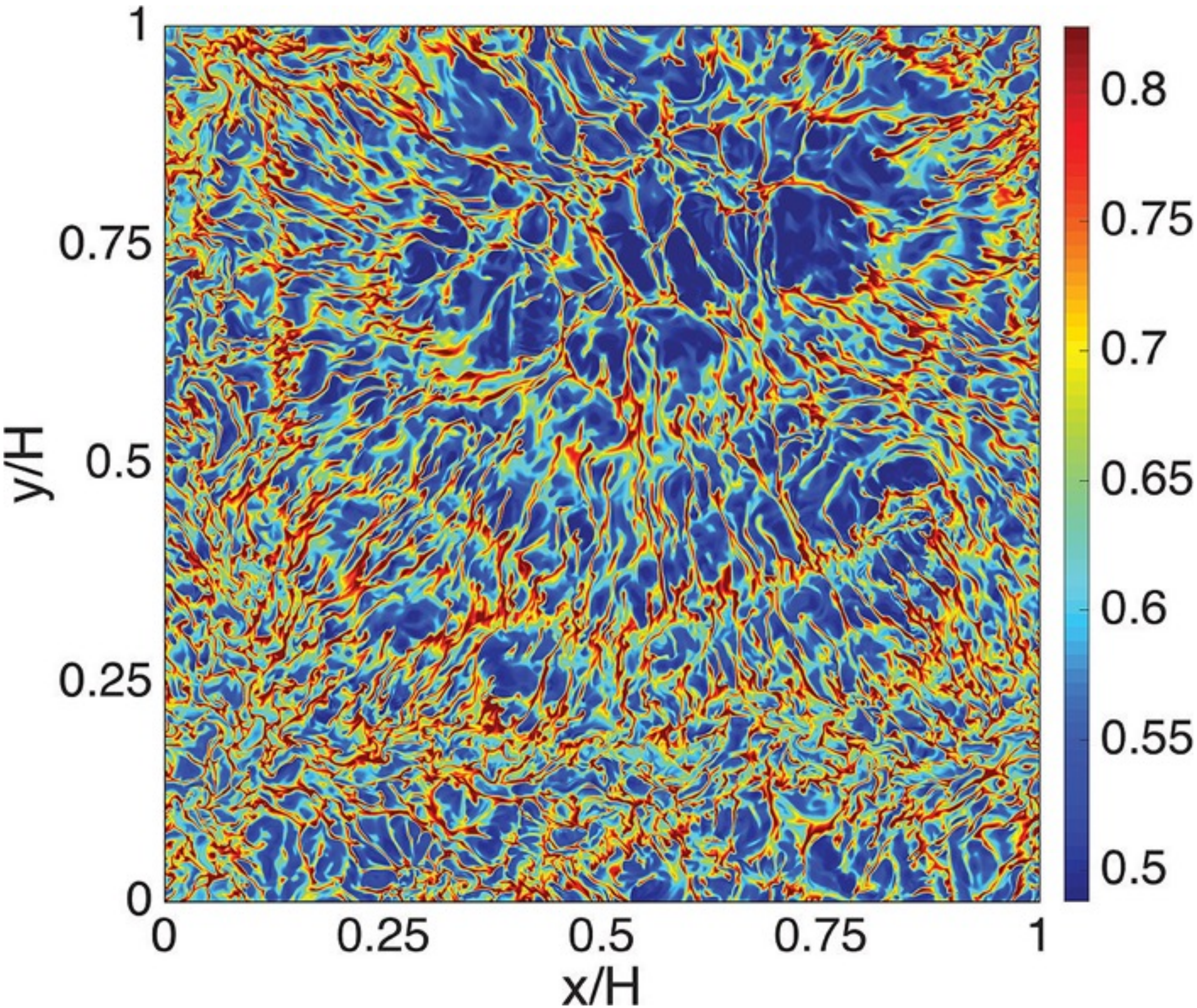
Nu-Ra scalings



3D turbulent data!

Fundamental coherent structure is a 2D sheet

Top view near
bottom hot wall



Lohse et al., University of Twente

Exact Coherent structures

- Convergence of work in wall-bounded shear flows and Taylor-Couette and Rayleigh-Benard and … (e.g. `snaking')
- Not just transition!