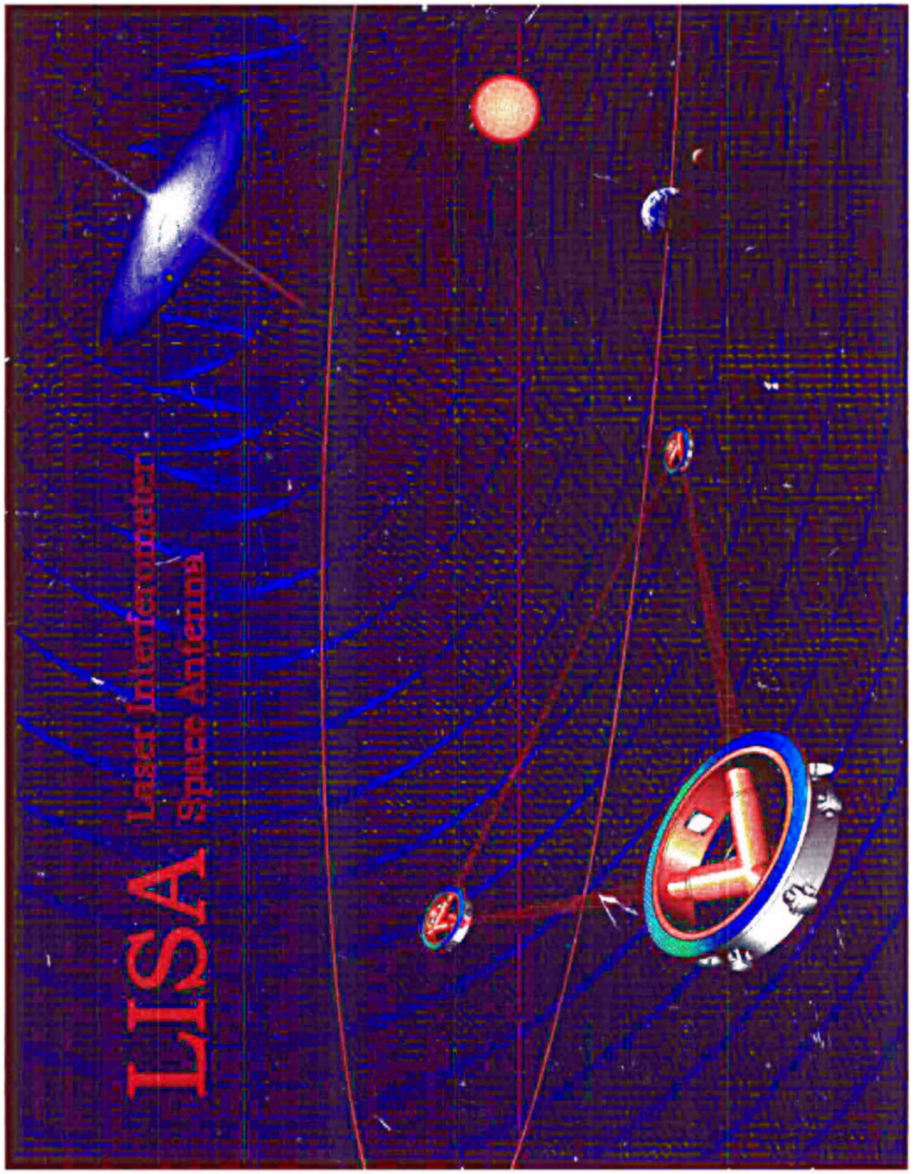
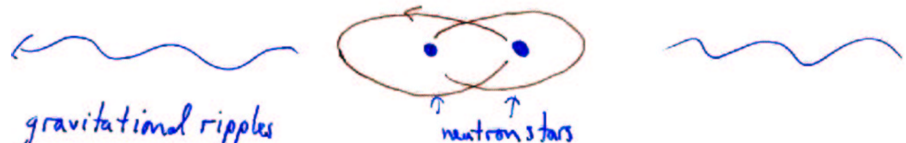


We have not deteted gravitational waves.

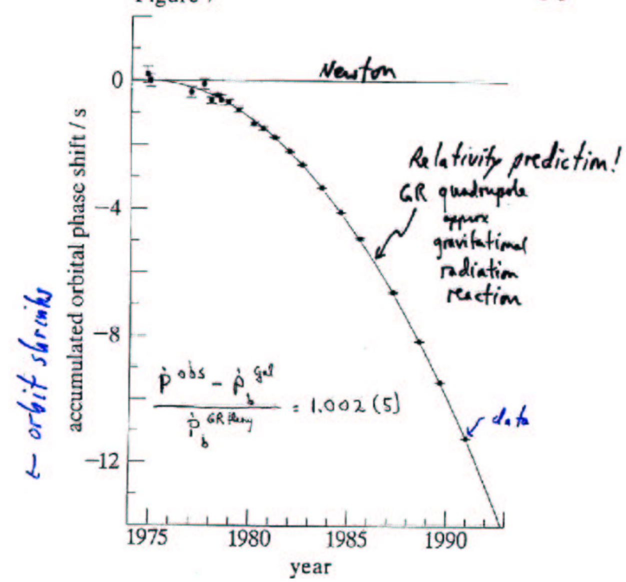
But we know they exist!



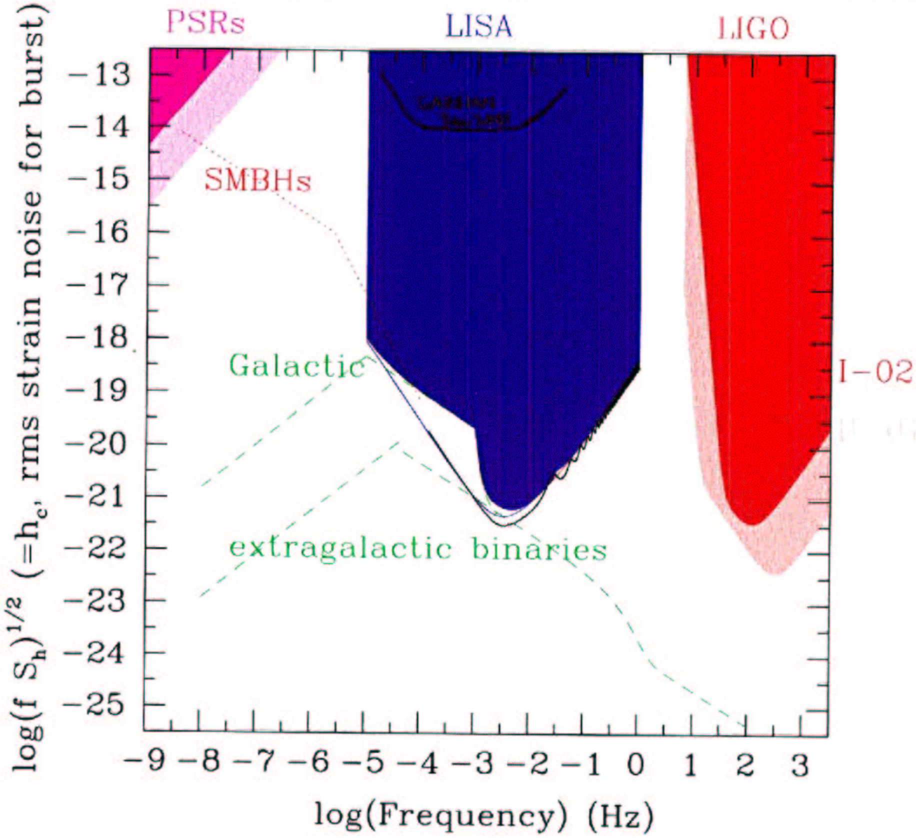
gravitational ripples in spacetime
Carry off orbital energy
→ orbit shrinks

128 PSR 1913+16 *J. H. Taylor* 1992

Figure 7



some agency should seriously support this!
↓ cheap + potential payoff huge - \$500k/y would enable. (10^{-2} LIGO, 10^{-2} LISA)



$$\frac{GM_\odot}{c^3} = 4.925\,\mu s$$

Gravitational radiation from circular binaries (point-like)

$$L_{GW} = \frac{32G}{5}\frac{M^2 m^2 (M+m)}{a^5} = \dot{E}_{orb} = \frac{\partial}{\partial t}\frac{GMm}{2a}$$

$$\frac{\dot{f}_{orb}}{f_{orb}} = \frac{96}{5}\frac{Mm}{(M+m)^{1/3}\,G^{4/3}}(2\pi f_{orb})^{8/3} \qquad a = \left[\frac{G(M+m)}{(2\pi f)^2}\right]^{1/3}$$

$$f = f_{GW} = 2 f_{orb}$$

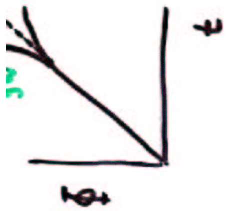
$$f = \frac{1}{\pi \mathcal{M}}\left[\frac{5}{256}\frac{\mathcal{M}}{t_0 - t}\right]^{3/8} \qquad \mathcal{M} = \frac{M^{3/5} m^{3/5}}{(M+m)^{1/5}}$$

chirp mass

$$\Phi = 2\pi \int f\,dt = -2\left[\frac{t_0 - t}{5\mathcal{M}}\right]^{5/8}$$

$$\dot{f} \propto \mathcal{M}^{5/3} f^{11/3}$$

∴ if measure $f, \dot{f} \rightarrow \mathcal{M} \rightarrow t_0 - t$
chirp mass → date of merger



$h_{\times,+}$

$\Phi = ft + \frac{1}{2}\dot{f}t^2 + \frac{1}{3}\ddot{f}t^3 + \ldots$

for π change in phase over mission duration T (1-10y, 3 baseline)

$\Delta\dot{f} \sim \frac{2\pi}{T^2} \sim 7\times10^{-17}\, T_3^{-2}\, s^{-2}$ → $\frac{f}{\Delta\dot{f}} = ?10^6 y$ @ 5mHz

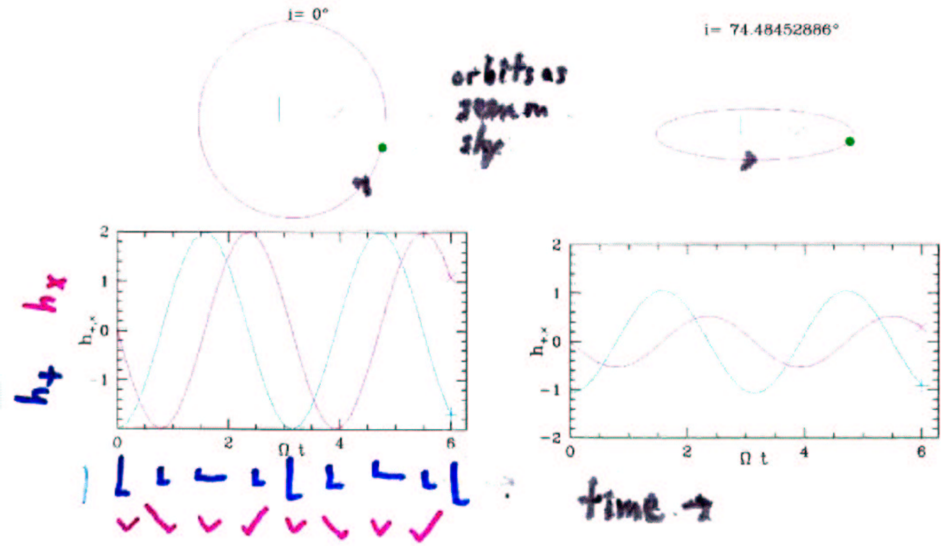
→ <10% $\mathcal{M}$ for all resolved binaries

$\Delta\ddot{f} \sim \frac{3\pi}{T^2} \sim 10^{-23}\, T_3^{-3}\, s^{-3}$

→ detectable for binaries w/ $\tau_{mrg} < 700y$ (a few)

$\frac{f\ddot{f}}{\dot{f}^2} = \frac{5}{3}$ test of point GWR vs tides

Two polarisations of gravitational waves: + and ×



Relative phase and amplitude give inclination and orientation on sky (to 90 deg) of orbit!

Two polarizations relative to arbitrary axes on sky

$+ \; \times$

$$h_+ = \frac{4\mathcal{M}}{r}(\pi \mathcal{M} f)^{2/3}\left\{-\cos 2\phi \,\frac{1+\cos^2 i}{2}\cos\Phi + \sin 2\phi \cos i \sin\Phi\right\}$$

$$h_\times = \;\; '' \;\; \left\{\sin 2\phi \;\; '' \;\; \cos 2\phi \;\; '' \right\}$$

as functions of Φ or t

phasing and relative amplitudes → i, 2ϕ

orbit inclination

position angle on sky (mod $\frac{\pi}{2}$)

$f, \dot{f}, h_+, h_\times$

→ $\boxed{\mathcal{M}, t_0 - t, r, i, 2\phi}$

Satellite orbital modulation of $f = f\left(1 + \frac{v}{c}\cos b \cos\left(\frac{2\pi}{1\,\mathrm{yr}}t + l\right)\right)$ (b: ecliptic latitude; l: ecliptic longitude)

→ l, b → α, δ

↓

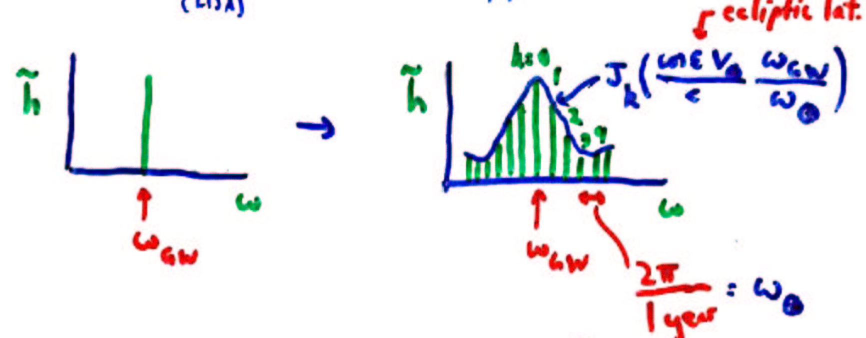
definition of ϕ → precision of i → precision of r.

Determining Source Positions with LISA

- Most sources detected for > 1 year
- Method similar to pulsar timing positions (high freq)

Two effects: – antenna pointing (low freq)

1. FM – modulation of GW frequency by "earth" (LISA) orbital doppler shift



No of strong side bands ~ $\cos\epsilon\left(\frac{f_{GW}}{3\cdot 10^{-4}\,\mathrm{Hz}}\right)$

≈ char freq = $\frac{c}{v_\oplus}\frac{\omega_\oplus}{2\pi} = \frac{c}{v_\oplus}\frac{1}{1\,\mathrm{y}}$

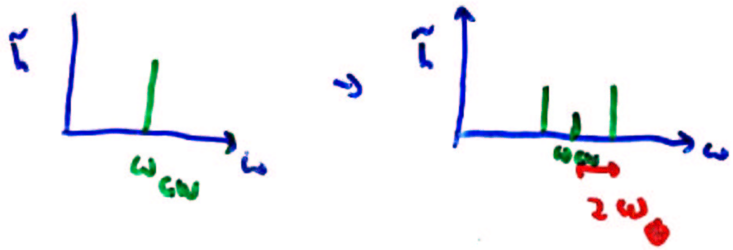
dominates for $f_{GW} > 10^{-3}$ Hz

Angular resolution ~1° for $f = 10^{-2}$ Hz $S/N = 10$

~8° for $f = 10^{-3}$ Hz $S/N = 10$

~1° $f = 10^{-3}$ Hz $S/N = 10^3$

2. AM - modulation of signal amplitude due to annual change in orientation of interferometer arms



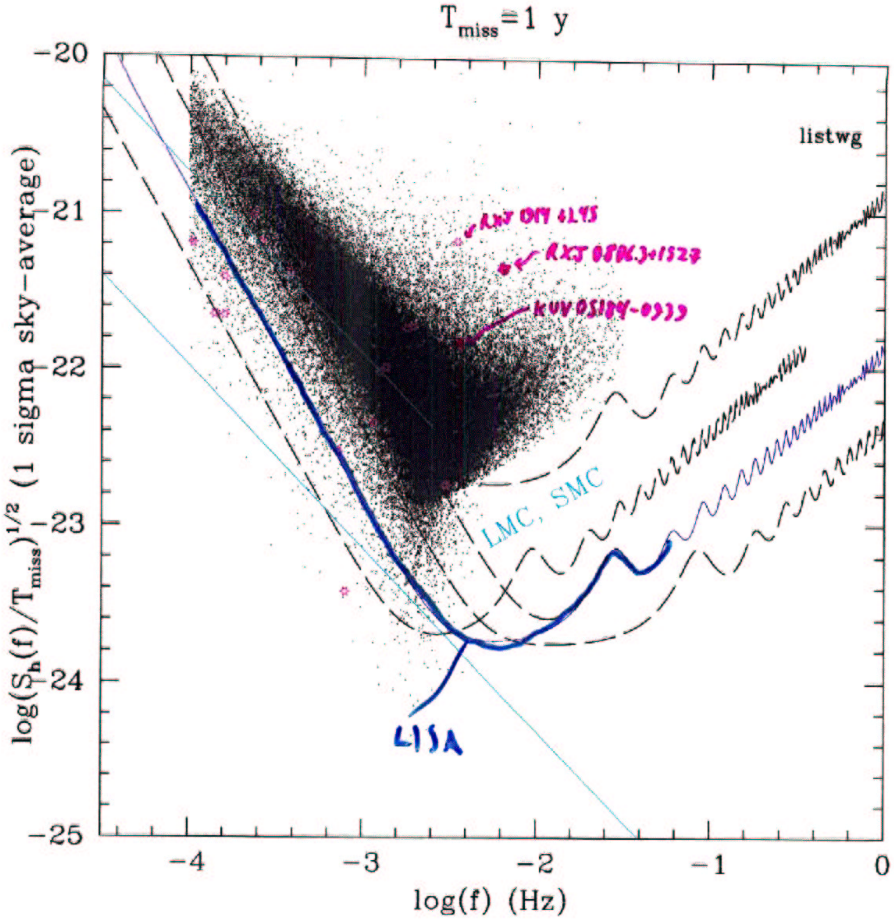
dominates for $f_{GW} < 10^{-3}$ Hz

Angular resolution for $f < 10^{-3}$ Hz:

$\sim 10°$ $S/N = 10$

$\sim 1°$ $S/N = 10^3$

class	source	dist pc	$f = 2/P_b$ mHz	M_1 $M_\odot$	M_2 $M_\odot$	τ_{mrg} 10^8y	h
WD+WD	WD 0957-666	100	0.38	0.37	0.32	2	4×10^{-22}
	WD 1101+364	100	0.16	0.31	0.36	20	2×10^{-22}
	WD 1704+481	100	0.16	0.39	0.56	13	4×10^{-22}
	WD 2331+290	100	0.14	0.39	>0.32	<30	$>2 \times 10^{-22}$
WD+sdB	KPD 0422+4521	100	0.26	0.51	0.53	3	6×10^{-22}
	KPD 1930+2752	100	0.24	0.5	0.97	2	1×10^{-21}
AM CVn	RXJ0806.3+1527	300	6.2	0.4	0.12	–	4×10^{-22}
	RXJ1914+245	100	3.5?	0.6	0.07	–	6×10^{-22}
	KUV05184-0939	1000	3.2	0.7	0.092	–	9×10^{-23}
	AM CVn	100	1.94	0.5	0.033	–	2×10^{-22}
	HP Lib	100	1.79	0.6	0.03		2×10^{-22}
	CR Boo	100	1.36	0.6	0.02		1×10^{-22}
	V803 Cen	100	1.21	0.6	0.02		1×10^{-22}
	CP Eri	200	1.16	0.6	0.02		4×10^{-23}
	GP Com	200	0.72	0.5	0.02		3×10^{-23}
LMXB	4U1820-30	8100	3.0	1.4	<0.1	–	2×10^{-23}
	4U1626-67	3-8000	0.79	1.4	<0.03	–	6×10^{-21}
W UMa	CC Com	90	0.105	0.7	0.7	–	6×10^{-22}



The population of Galactic binary stars in

LISA frequency band ($\gtrsim 10^{-4}$ Hz $\Rightarrow P_{orb} < 5.5$ hours)

Mostly double degenerates - created by spiral-in at $P_{orb} < 10$h $\quad \dot{N} \simeq \frac{1}{20\text{-}100\,y}$

Then evolve by gravitational radiation losses

$$t_{decay} \sim 6\times10^5\,y \left(\frac{f}{3\times10^{-3}\,\text{Hz}}\right)^{-8/3} \qquad f = \frac{2}{P_{orb}} \text{ quadrupole}$$

All in galaxy detectable in h

but LISA frequency resolution $\sim \frac{1}{T}$ ← mission duration 1-10y

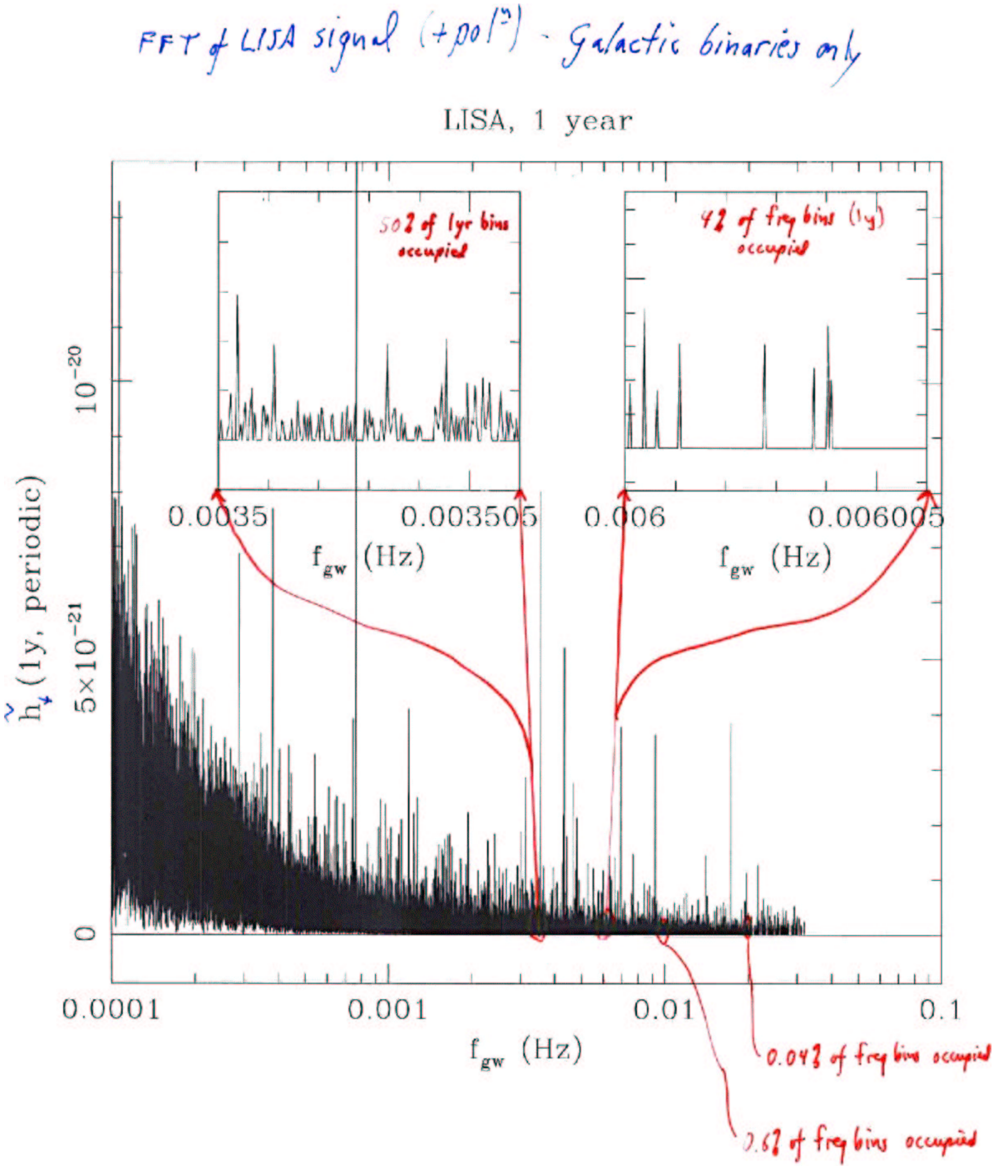
⇒ Confusion (>1 binary per frequency bin) if

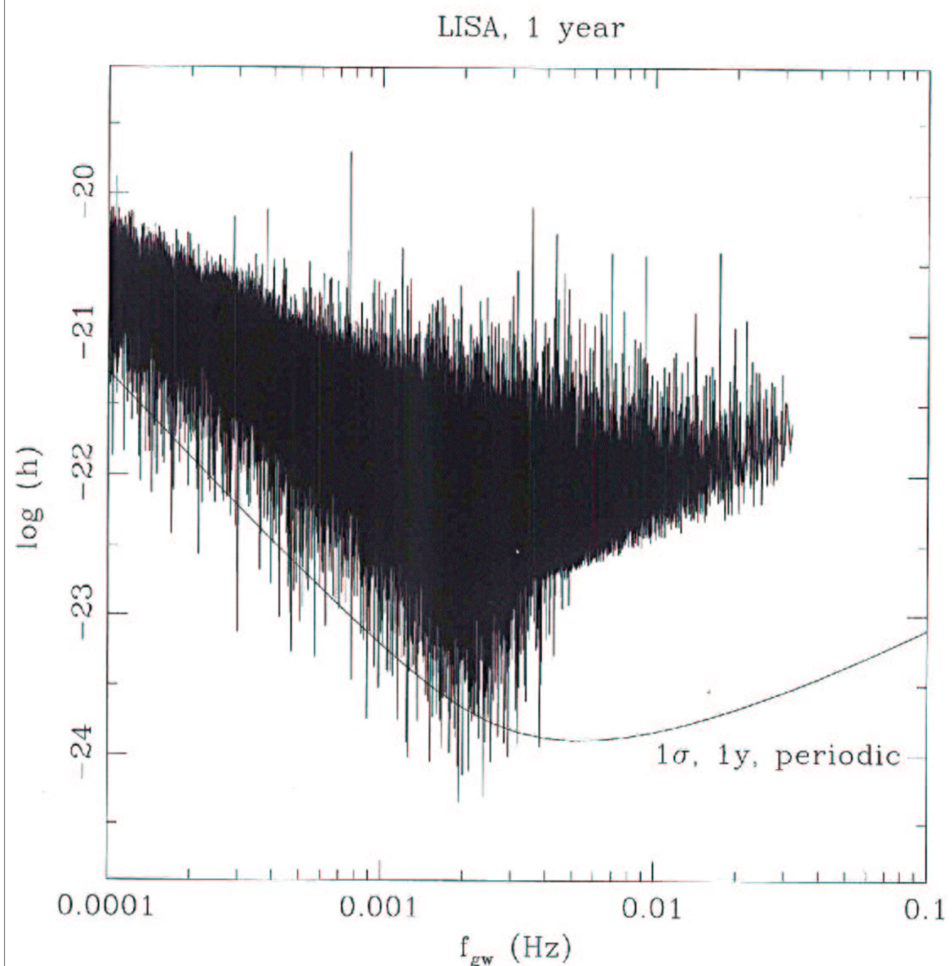
$$\underbrace{N(>f)}_{\#\, f\to 2f} \simeq \dot{N}\, t_{decay} > \underbrace{fT}_{\#\text{ bins } f\to 2f} \quad \Leftrightarrow \quad \frac{dN}{df}\cdot\overbrace{\frac{1}{T}}^{\Delta f} > 1$$

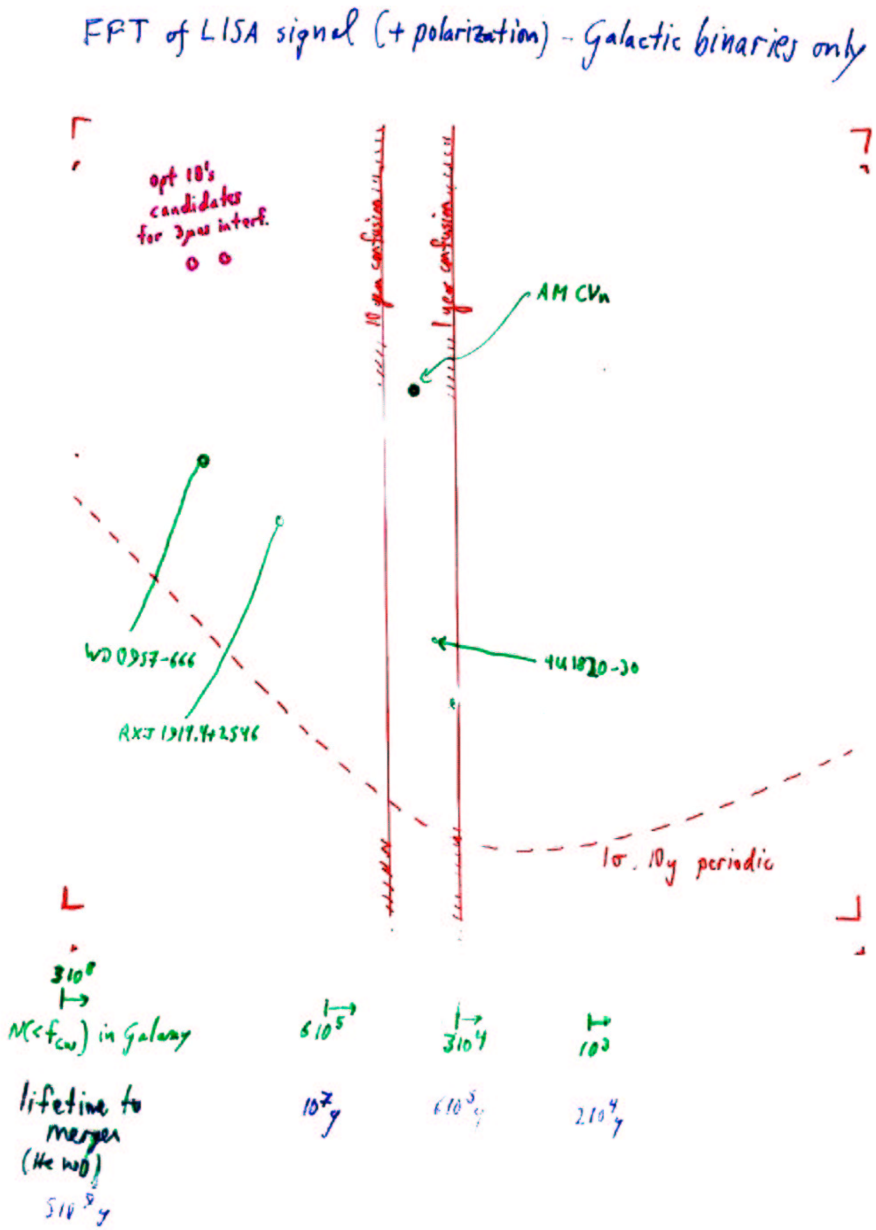
⇒ no confusion for

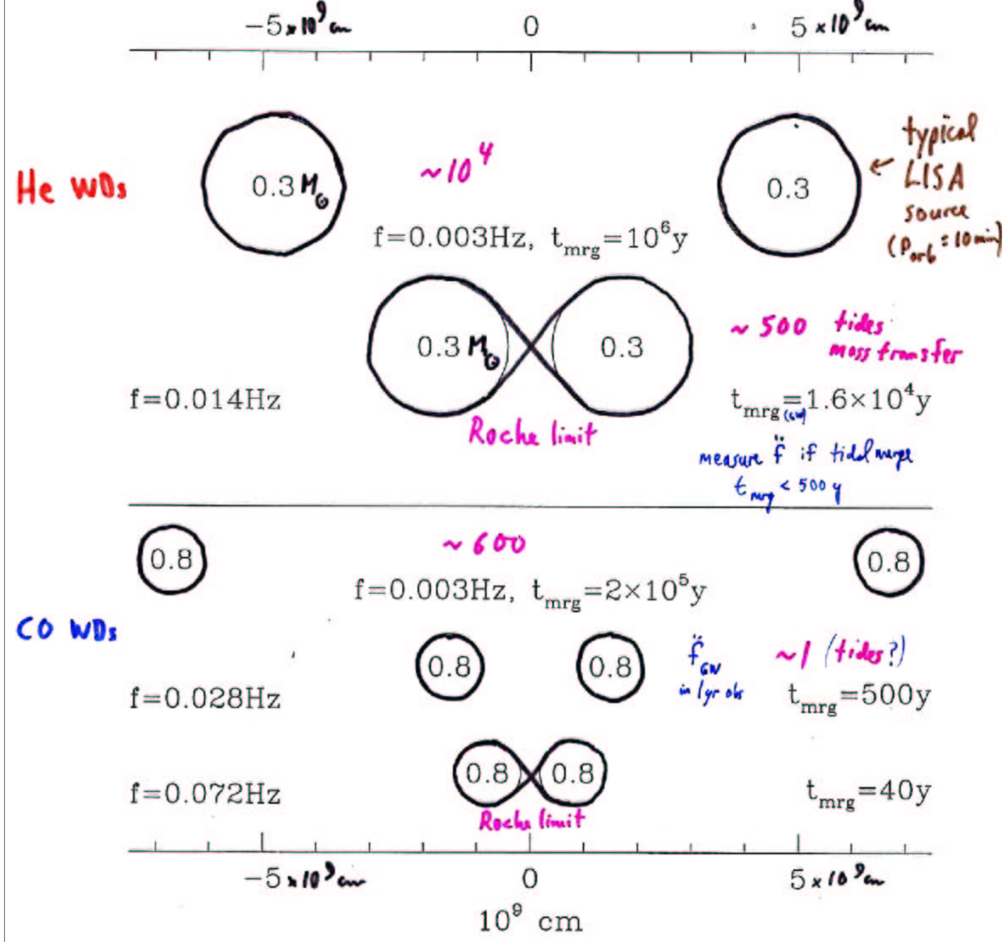
$$f \gtrsim f_{conf} \sim 2\,10^{-3} \left(T/\text{year}\right)^{-\frac{3}{11}} \left(\frac{\dot{N}}{1/30\,y}\right)^{3/11} \text{ Hz}$$

for $f < f_{conf}$ - only nearby strong sources above the din identifiable


FFT of LISA signal (+pol?) - Galactic binaries only
LISA, 1 year
50% of 1yr bins occupied
4% of freq bins (1y) occupied
0.0035
0.003505
0.006
0.006005
f_{gw} (Hz)
$\tilde{h}_+$ (1y, periodic)
10^{-20}
5×10^{-21}
0
0.0001
0.001
0.01
0.1
0.04% of freq bins occupied
0.6% of freq bins occupied


LISA, 1 year
log (h)
−20
−21
−22
−23
−24
1σ, 1y, periodic
0.0001
0.001
0.01
0.1
f_{gw} (Hz)





LISA can measure $\ddot{f}_{GW}$ for $f > 0.027$ Hz - ie for a couple of CO WDs approaching Roche limit - test for tidal heating - if pure GW loss $\ddot{f} f \dot{f}^{-2} = 5/3$

Dominant Galactic LISA sources

IN - **Double degenerates** (WD+WD) dominate power

from Double Common envelope evolution of wide binaries (30-1000 R_*)

2-4 $M_\odot$ primaries dom, but 1-10 contribute

and Roche lobe overflow in Hertzsprung gap ($\lesssim 10 R_*$) followed by common envelope in 2nd mass transfer.

Mix of He-He, He-CO, CO-CO (dominate $L_{galaxy,gw}$) CO-ONe, ONe-ONe less important, rare

5 mHz, $\tau \sim 10^5$y, $h(1kpc) \sim 4\,10^{-22}$

Chirp masses $\mathcal{M}$ up to $\sim 1 M_\odot$

$\mathcal{M} = 0.3 - 0.5\ M_\odot$ typical

0.4 0.6

OUT - **AM CVn** (WD + Roche filling WD He) dominate numbers

0.12 0.6

5 mHz, $h(1kpc) \sim 10^{-22}$, $\tau_{GW} \sim 4\,10^5$y, $\tau \sim ?$

Roche lobe filling

- backing out if conservative (J) transfer ??

direct impact tides in small disks

$h \propto m$

$L_{gw} \propto m^2$

$\tau_{gw} \propto m^{-1}$

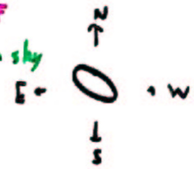
if back out on τ_{GW} $\Rightarrow$ $\begin{cases} N(f)_{out} \simeq \left(\frac{m_{in}}{m_{out}}\right)^{-4} N(f)_{in} \\ \text{power at } f : \left(\frac{M_{out}}{M_{in}}\right)^{-0.25} \times (\text{DD Power}) \end{cases}$

Science payoffs with the 'pesky' Galactic binaries

1. Nearest identifiable WD binary at $D \simeq 10$pc, $P_{orb} \simeq 1.8$h
   $a \simeq 5 \times 10^{10}$cm $\Rightarrow \frac{a}{D} \simeq 3\ \mu$as   15th-18th mag

   challenge for SIM TPF [reflex depends on mag. diff, $\frac{m_1}{m_2}$]

   Determine orbit on sky i, PA

   Compare with LISA solution from GW polarization ratio, orientation.

   Test of LISA calibration, GR, SIM/TPF systematics!

2. The $3\,10^3 - 3\,10^4$ identifiable steady sources should all lie near Galactic plane.

   Unresolved background also (modulation of stochastic signal as interferometer rotates – cf Giampieri + Polnarev 1997)

3. If unusually strong + dipole - halo of binary white dwarfs
   - MACHO problem solved! (Hiscock et al astro-ph 0005134)

   but Pop II, III binary fraction low?

4. ~3000 highest freq binaries - measure $\dot{f}$, h to < 10% (1% in 10y)
   $\Rightarrow \mathcal{M}_{chirp}$, r, i, PA
   ↑ optical masses

   3-D obscuration-free map of galaxy!

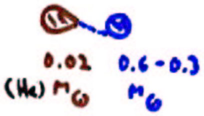
   $f(\mathcal{M}_{chirp})$ for comparison with binary evolution models.

5. ~30 WD binaries of highest f_{GW} and lowest $\mathcal{M}_{chirp}$ will have anomalous non-gw $\dot{f}$, measurable $\ddot{f}$

- due to tidal excitation + dissipation in white dwarfs (cf C.G. Campbell 1984 - needs revisiting!)
- optical heating ⇒ bright even if old (17^{th} mag @ 500pc) (cf Iben, Tutukov + Fedorova 1998)

6. New kinds of source

(cf recent recognition of AM CVn sources $P, \dot{P} + \dot{M}$



post-contact evolution of double-degs.
WD analog of PSR 1957+20 'black widow')

- Planets in near-contact orbit around WDs (cf N. Soker theory of Planetary nebula asymmetries!)

- white dwarfs with strong internal magnetic fields (~10^{11} G - same ratio to external fields ~10^{7-9} G as proposed for pulsars/AXPs)

AE Aqr, WZ Sag - synchronized P_{rot} = 33, 28 s

Detect gravitational radiation from orbit <u>and</u> spin of magnetically distorted white dwarf!

Heyl astro-ph 0001343

- Galactic black hole - black hole binaries (how else?)

⋮