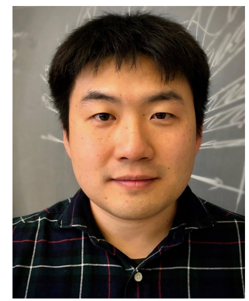
Measurement induced Entanglement transitions in quantum circuits with decoherence

Ehud Altman – UC Berkeley

Zack Weinstein

Yimu Bao

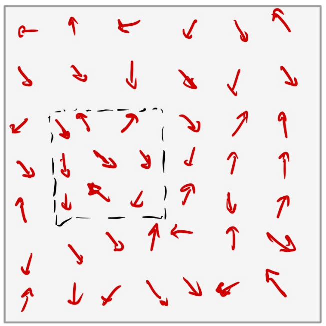


Soonwon Choi

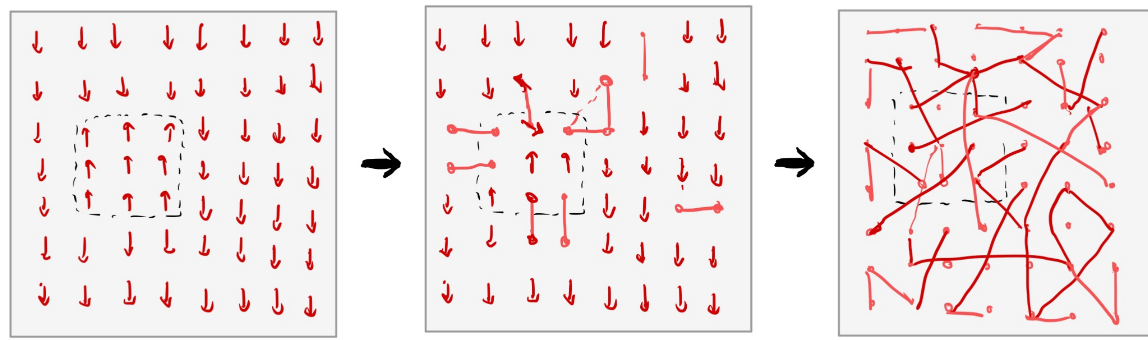
Zala Lenarčič

Classical versus quantum thermalization

Classical



Quantum



$$S_A \to \sigma_{max} V_A$$

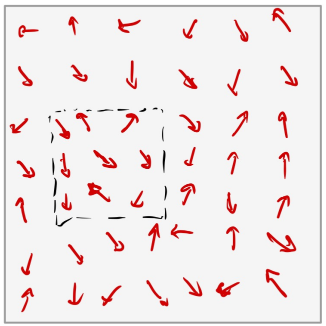
- The state becomes random.
- All information encoded locally

- Almost all information is encoded non-locally

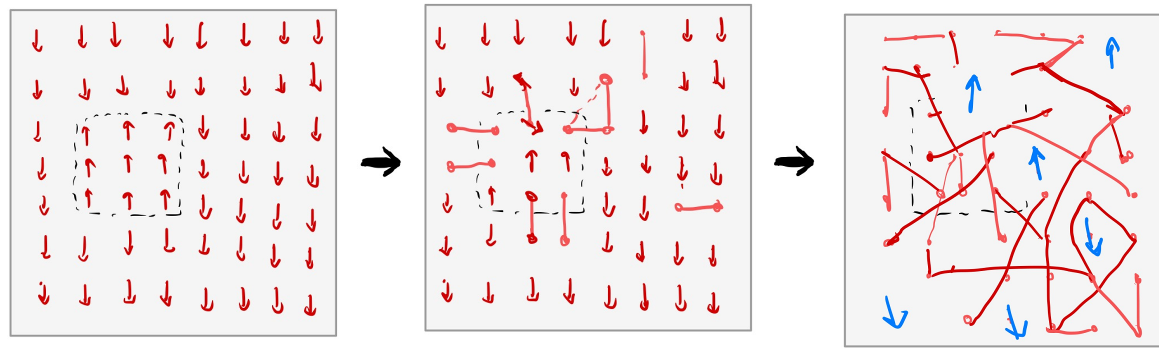
How is this picture affected by the presence of an observer ?

Classical versus quantum thermalization

Classical



Quantum



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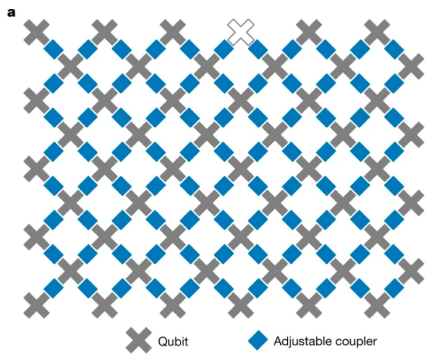
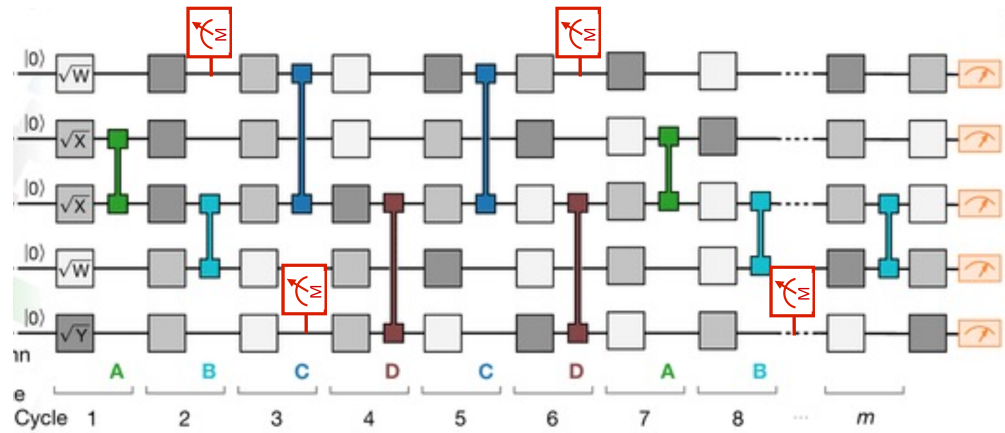
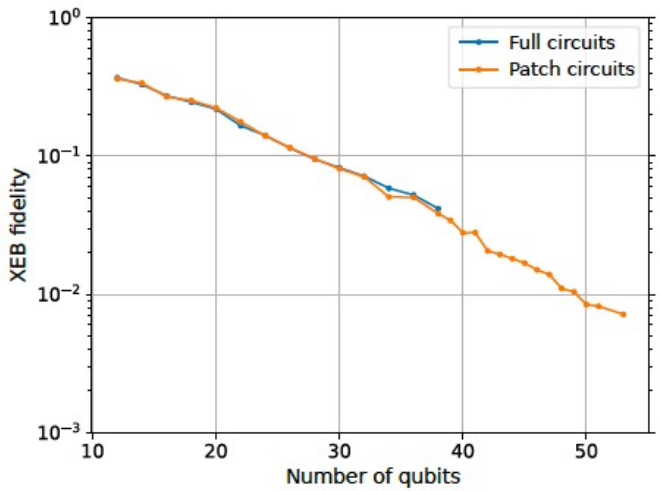
Google's quantum supremacy experiment is in a sense a demonstration of quantum thermalization.





Sampling of local measurements at the output can test for presence of non-local encoding.

How this would the result be affected by intermittent measurements ?

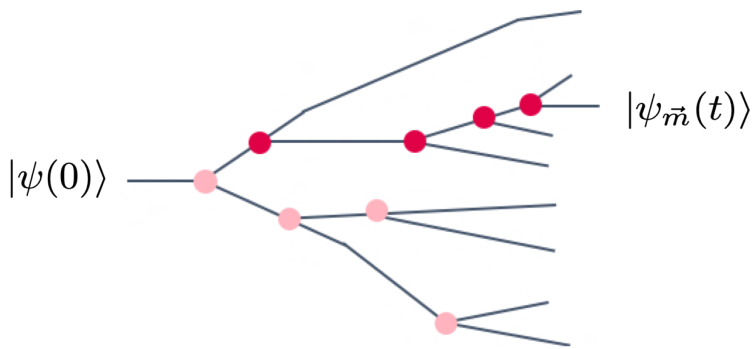


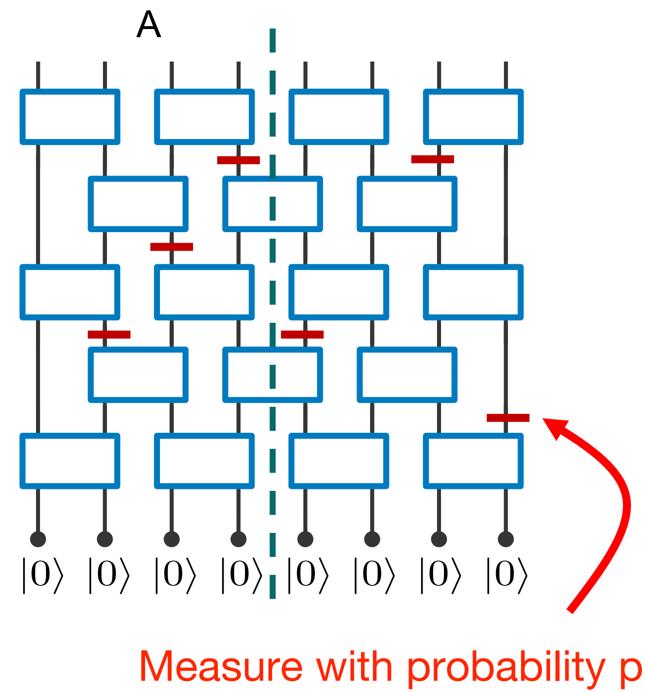
Measurement induced phase transition in hybrid quantum circuits

Skinner, Ruhman, Nahum PRX 2019;
Li, Chen, Fisher PRB 2018,
Chan et. al. PRB 2019, …

The circuit generates an ensemble of quantum trajectories corresponding to sequences of measurement outcomes

$$\vec{m} = \{m_1, m_2, m_3, \ldots\}$$





Ensemble averaged entanglement entropy

$$\langle S_A \rangle_{\mathcal{U}} = \langle \sum_{\vec{m}} p_{\vec{m}} S_{A,\vec{m}} \rangle_{\mathcal{U}}$$

Project on measurement result:

$$|\psi\rangle \mapsto \frac{\hat{P}_\mu |\psi\rangle}{\sqrt{\langle\psi|\hat{P}_\mu|\psi\rangle}} \text{ with prob.} \langle \hat{P}_\mu \rangle$$

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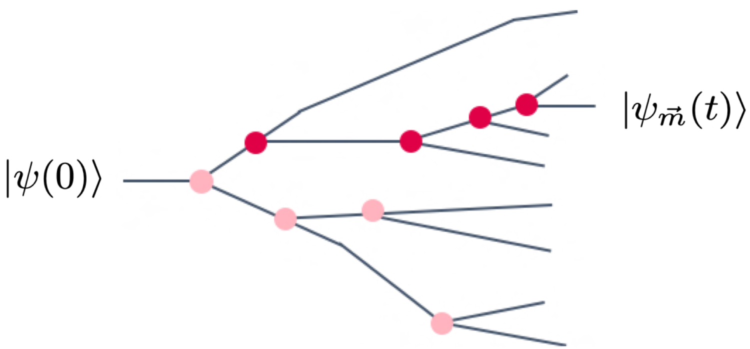

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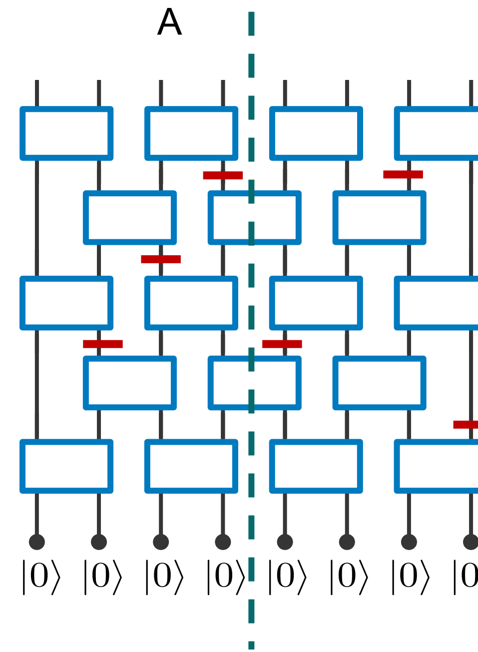

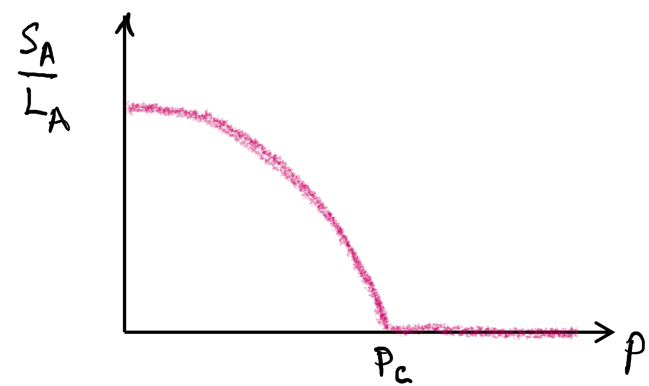

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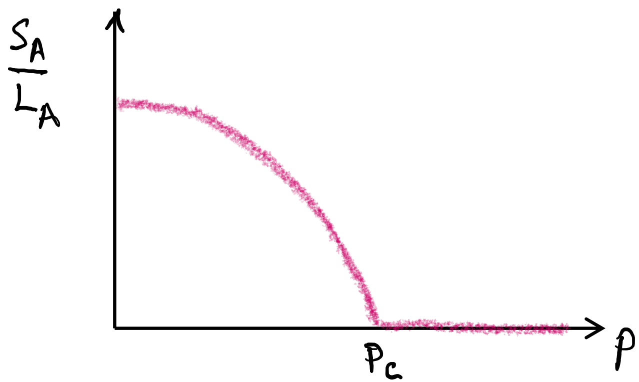
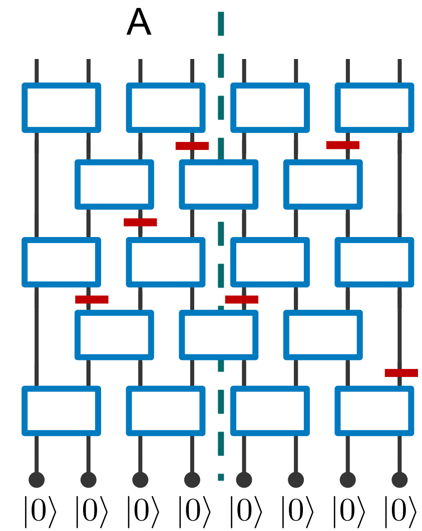




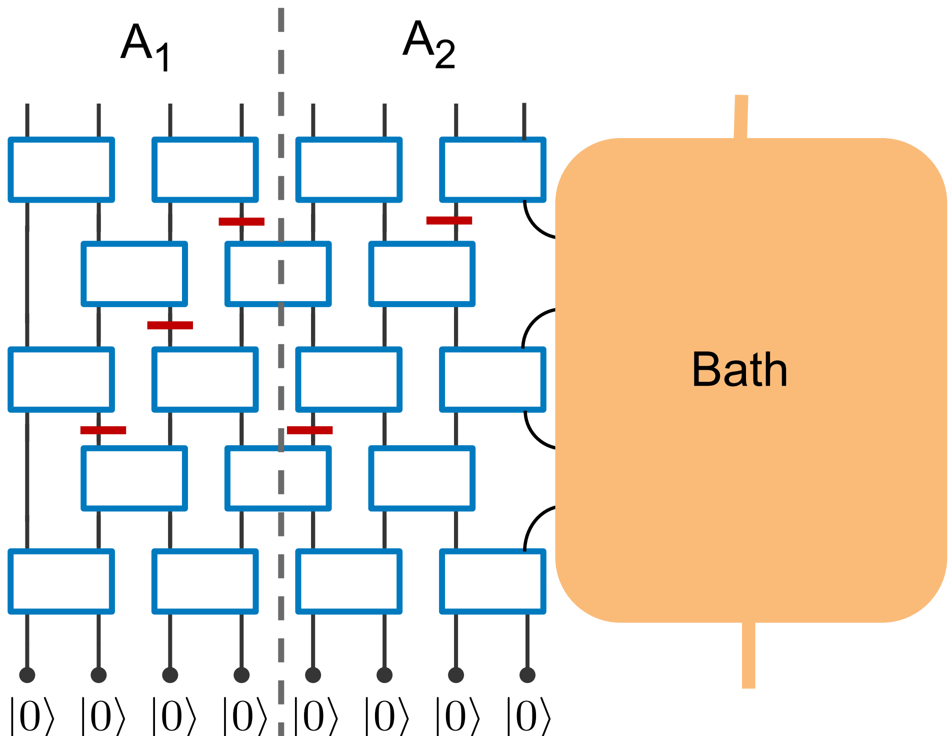
<u>Volume law phase:</u> Scrambling unitary evolution encodes a finite density of logical qubits non locally thereby protecting them from being revealed by the measurements.

➡ Emergent error correcting code finite code density (quantum channel capacity)

Choi, Bao, Qi and EA, PRL 2020

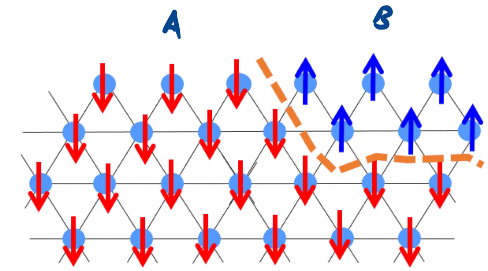
Real circuits also suffer from decoherence !

Can we have large scale entanglement and entanglement transitions in presence of decoherence?

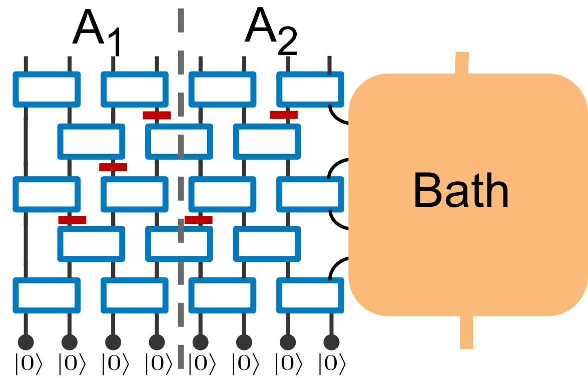


This talk

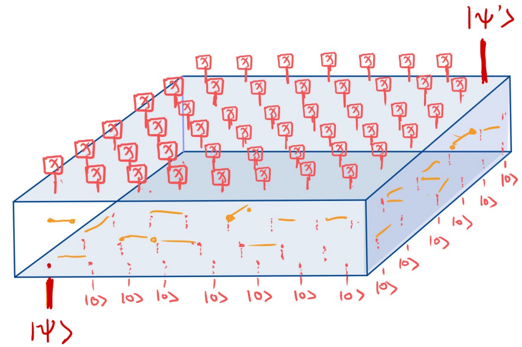
- Preliminaries – mapping to stat-mech



- Entanglement transitions in circuits with decoherence



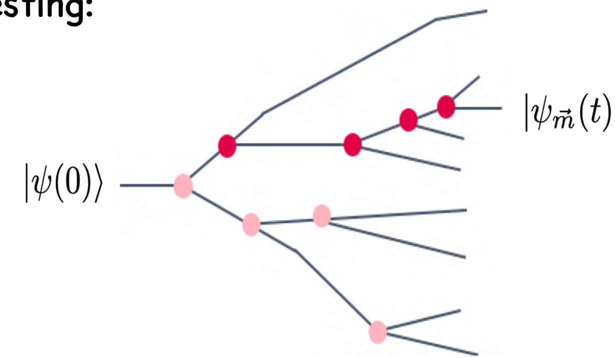
- ~~Finite time teleportation transitions~~



How to characterize the ensemble of trajectories

Expectation values of observables over the ensemble are not interesting:

$$\langle \hat{O} \rangle = \overline{\sum_m \mathrm{tr}\left(|\psi_m\rangle\langle\psi_m|\hat{O} \right)} = \mathrm{tr}\left(\rho_{\mathrm{av}} \hat{O} \right) \qquad \rho_{\mathrm{av}} \xrightarrow[t\to\infty]{} \mathbb{1}$$



We need to consider fluctuations over the trajectories:

$$\mathcal{O}_k = \overline{\sum_m p_m \left(\frac{\langle\psi_m|\hat{O}|\psi_m\rangle}{\langle\psi_m|\psi_m\rangle} \right)^k}$$

Can be captured by the dynamics of n-copies of the density matrix*:

$$\rho^{\otimes n} = \sum_m |\psi_m\rangle\langle\psi_m| \otimes |\psi_m\rangle\langle\psi_m| \otimes \ldots \otimes |\psi_m\rangle\langle\psi_m| \equiv |\rho^{(n)}\rangle\rangle$$

This also captures purities and Renyi entropies

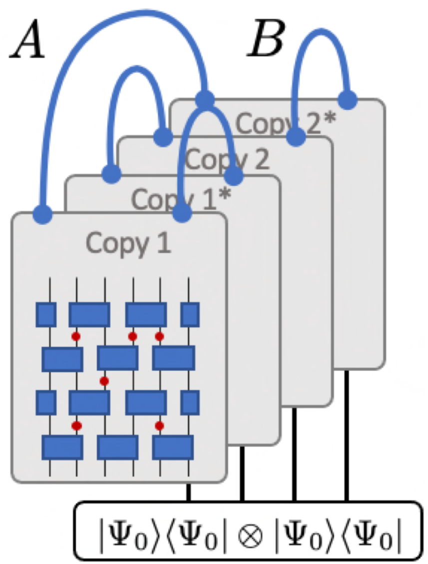
* need auxiliary replicas for correct averaging

Simplest example: calculation of the purity

Dynamics of the doubled density matrix = contracting a tensor network

Computation of purity dictates a top boundary condition.

$$\sum p_{\vec{m}}^2 \mathrm{tr}\{\rho_A^2\} =$$



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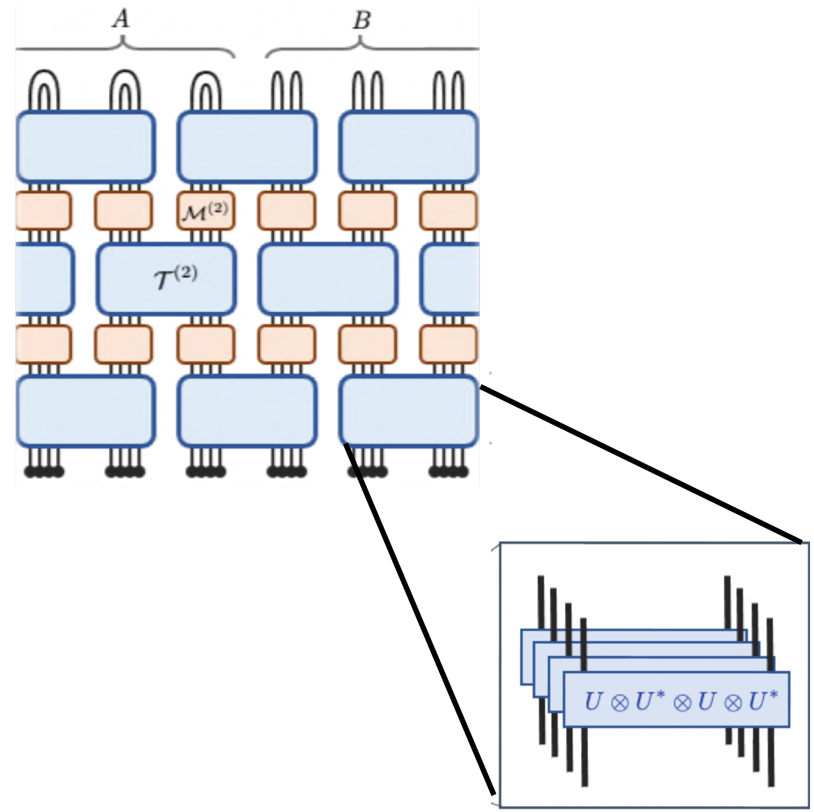


This is in general a very hard problem !

Intrinsic dynamical symmetry

The unitary gates and measurements preserve purity of the state

➡ Global symmetry to permutations between kets and separately between bras

$$\rho^{\otimes n} = \sum_m |\psi_m\rangle\langle\psi_m| \otimes |\psi_m\rangle\langle\psi_m| \otimes \ldots \otimes |\psi_m\rangle\langle\psi_m| \equiv |\rho^{(n)}\rangle\rangle$$

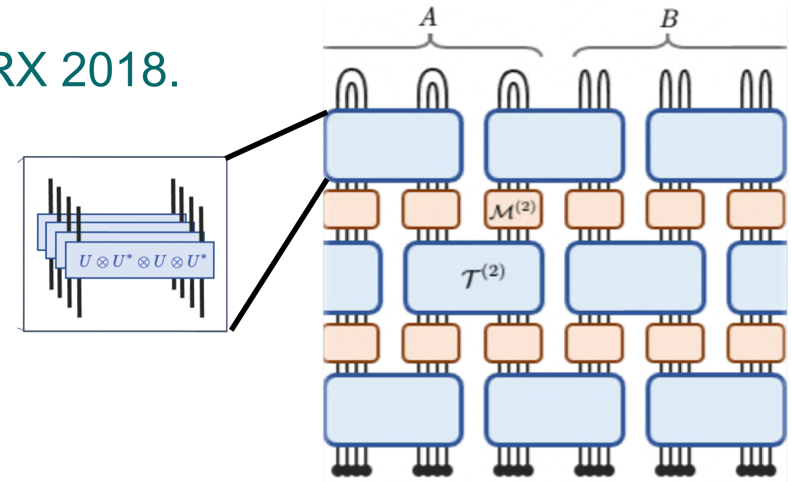
$$(S_n \times S_n) \rtimes \mathbb{Z}_2^H$$

$\mathbb{Z}_2^H$ Due to conservation of hermiticity

Mapping to an effective classical "spin model"

For random unitaries: Nahum, Vijay and Haah, PRX 2018.

Extension to Born measurements:
Bao Choi and EA PRB 2020;
Jian, You, Vasseur and Ludwig PRB 2020

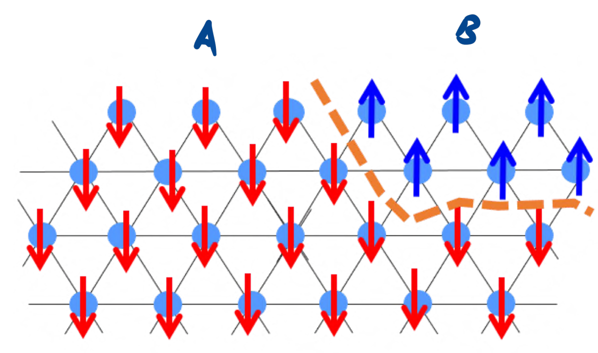


$$\overline{U \otimes U^\star \otimes \ldots \otimes U \otimes U^\star}$$

Averaging over the unitaries enforces local pairing (identifying) forward-back branches of different copies. There are n! ways to pair

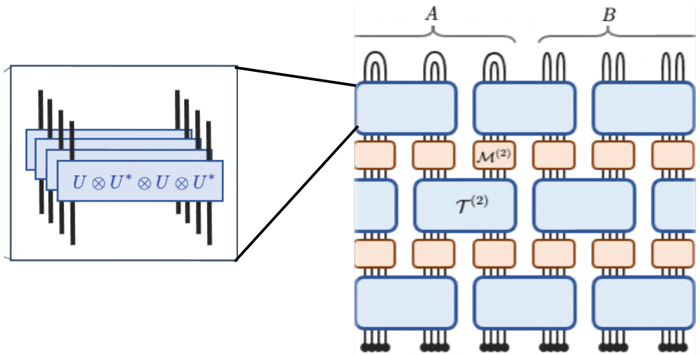
Contracting the tensor network = partition function of a classical "spin" model.

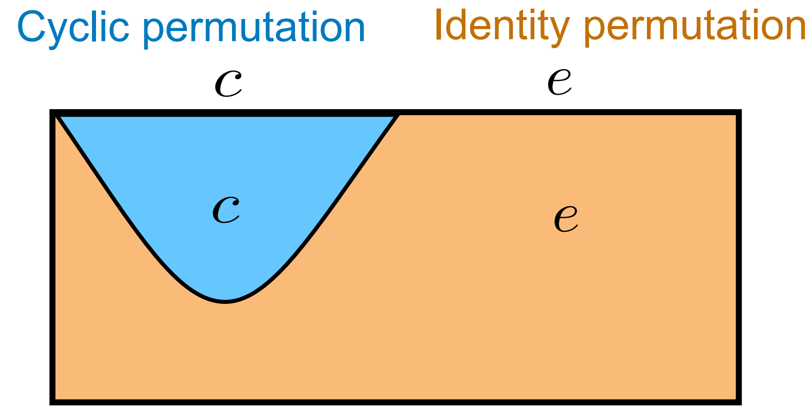
States of the spin = elements of the permutation group



Broken symmetry phase → volume law entanglement

Mapping to an effective classical "spin model"

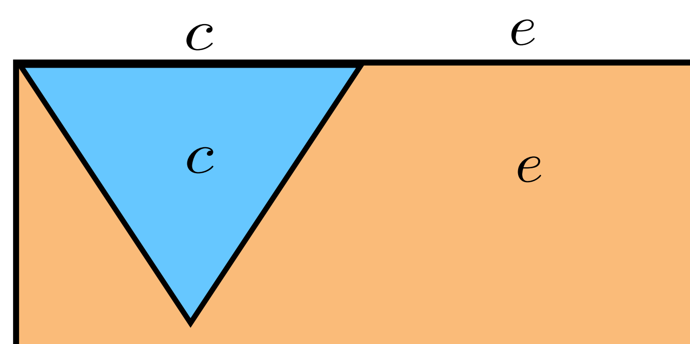




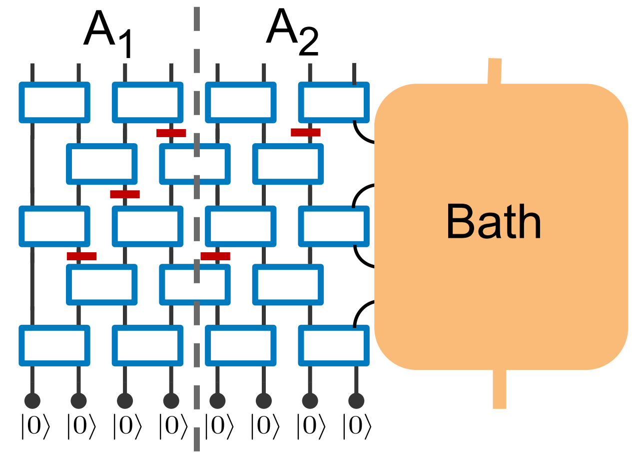
Entanglement entropy = domain wall free energy $S_A^{(n)} = F_{c,e}^{(n)} - F_0^{(n)}$

Important difference from purely unitary circuits:

In unitary circuits the domain walls must obey light cone constraint.
(spin-model is at T=0)



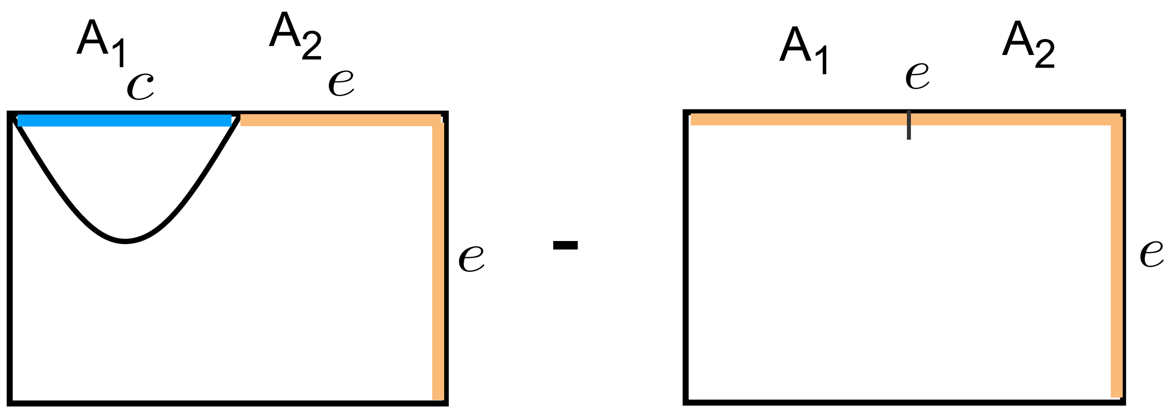
Hybrid circuit coupled to a bath



Coupling to a bath/decoherence breaks S_n symmetry
= magnetic field favoring the identity permutation (e)

Coupling to a bath at the edge imposes
= identity boundary condition on that edge:

$$S^{(n)}_{A_1} =$$



There is still a measurement induced transition. But von the Neumann entropy is not a measure of quantum entanglement in the mixed state.

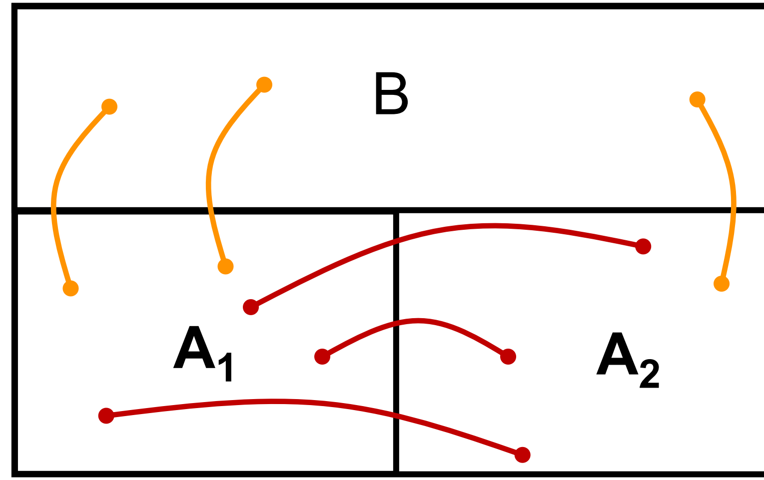
What is the quantum entanglement in the mixed volume law phase?

Logarithmic negativity – a measure of tri-partite entanglement

$$\mathcal{E}_{A_1:A_2} = \log \|\rho_A^{T_2}\|_1$$

Partial transpose in A_2

A measure (upper bound) of the number of distillable Bell-pairs between A_1 and A_2



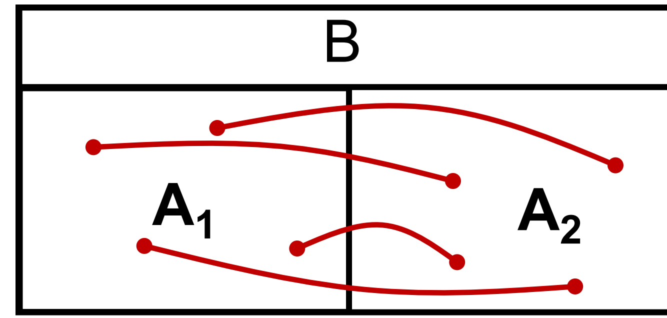
Reyni Negativities (n>2) :

$$\mathcal{E}^{(n)}_{A_1:A_2} = \frac{1}{2-n} \log \left\{ \frac{\mathrm{tr}[(\rho_A^{T_2})^n]}{\mathrm{tr}\rho_A^n} \right\}$$

Page transition in negativity of random states (thermal states)

$$|B| < |A| \quad \Rightarrow \quad \mathcal{E}_{A_1:A_2} \sim \alpha N_{A_1}$$

Volume law entanglement !

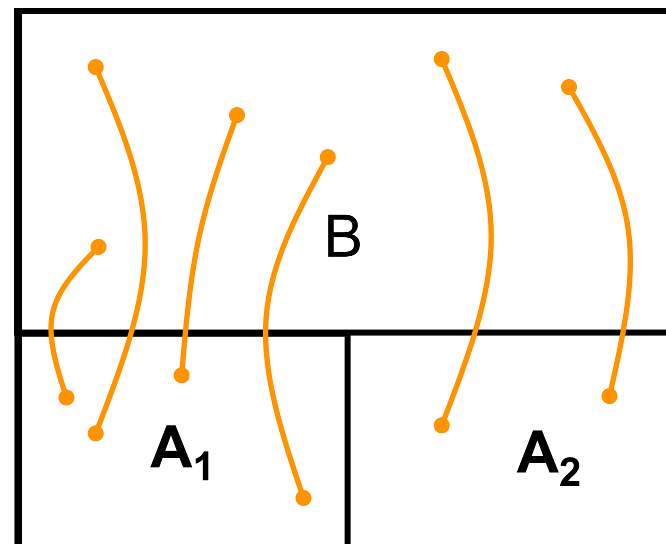


$$|B| > |A| \quad \Rightarrow \quad \mathcal{E}_{A_1:A_2} = 0$$

No quantum entanglement!

A large bath sucks the quantum life out of the system (region A).

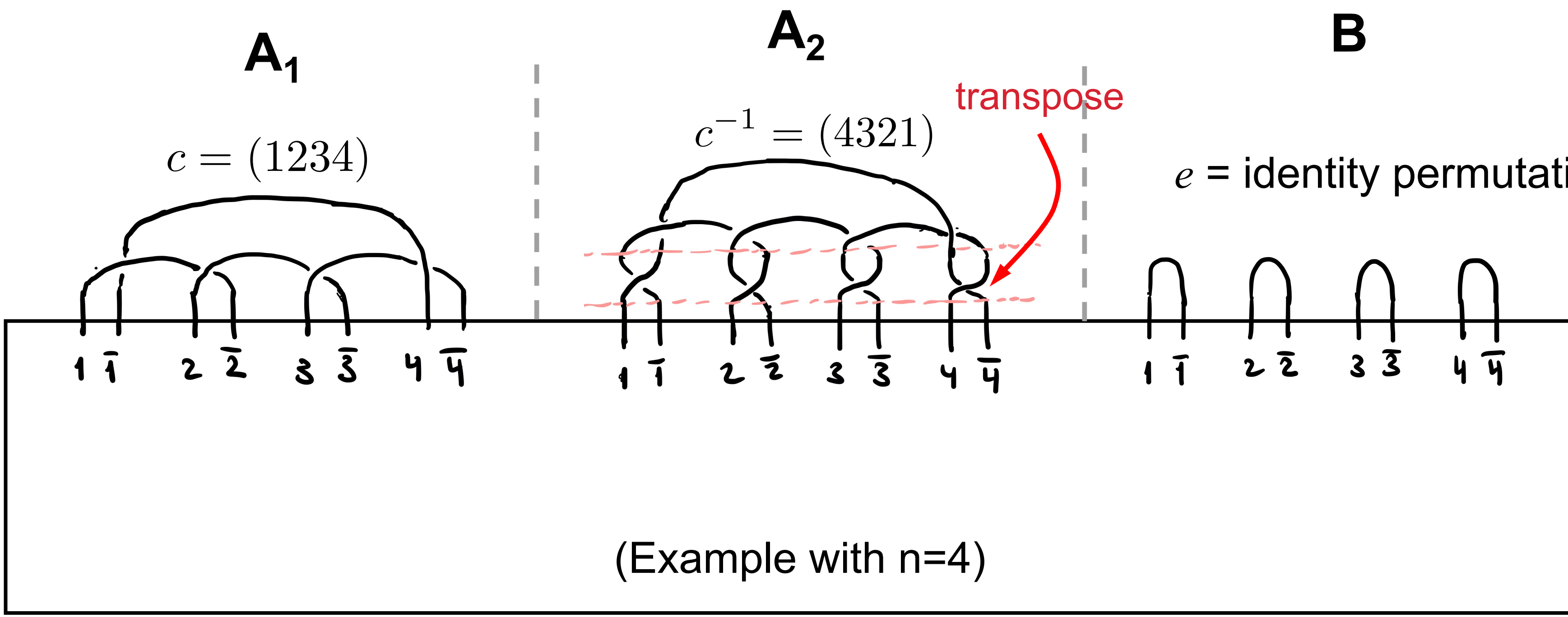
Is this still true in the state produced by random unitaries and measurements?



Computing negativity in the effective stat-mech model

Different top boundary conditions than entropy:
$$\mathcal{E}^{(n)}_{A_1:A_2} = \frac{1}{2-n}\log\left\{\frac{\mathrm{tr}[(\rho_A^{T_2})^n]}{\mathrm{tr}\rho_A^n}\right\}$$

Numerator:



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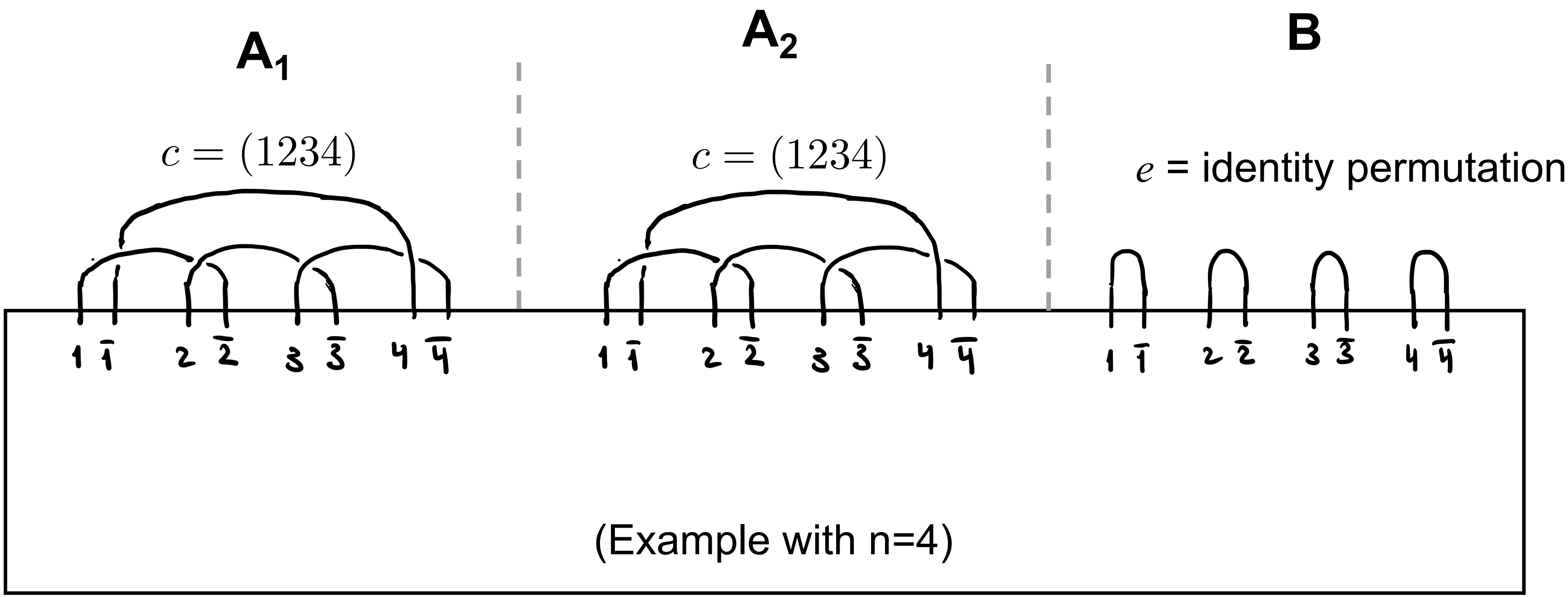
Denominator (reference state):

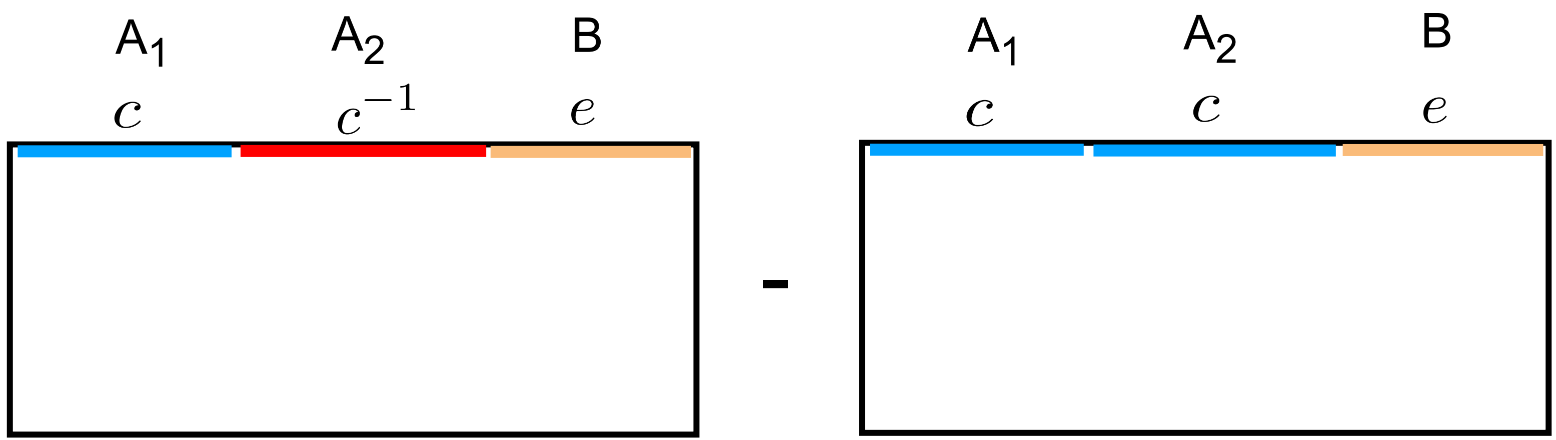


Computing negativity in the effective "spin" model

Different top boundary conditions than entropy: $$\mathcal{E}^{(n)}_{A_1:A_2} = \frac{1}{2-n}\log\left\{\frac{\mathrm{tr}[(\rho_A^{T_2})^n]}{\mathrm{tr}\rho_A^n}\right\}$$

Imposes a (c, c^{-1}) domain wall between A_1 and A_2

$$\mathcal{E}^{(n)}_{A_1:A_2} =$$



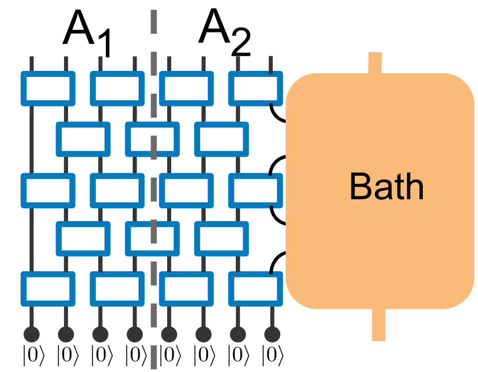
Negativity page transition in random unitary circuits

- Domain walls must follow light cone constraint

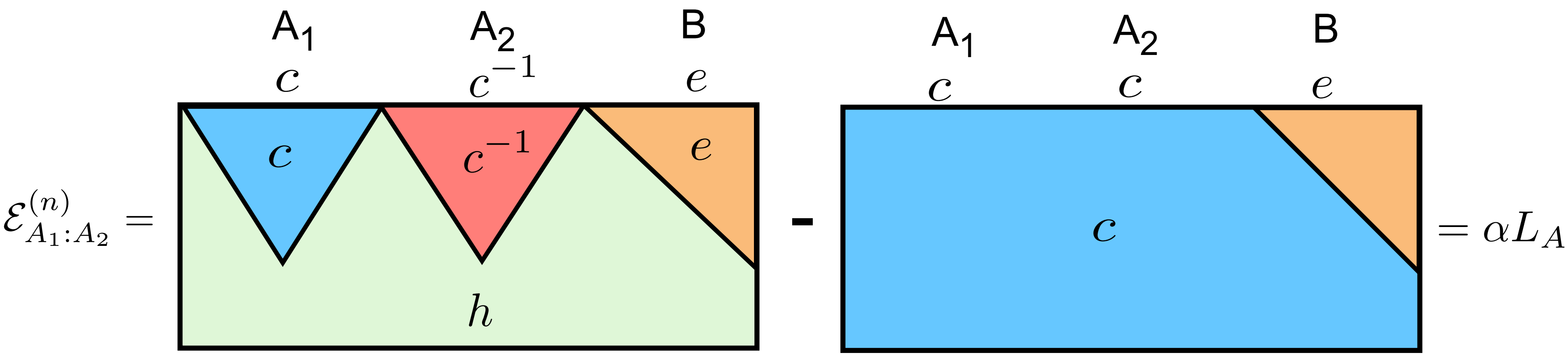
  Nahum, Vijay and Haah, PRX 2018

- Ansatz for domain wall tension between σ and τ domains:

$$f_{\sigma\tau} = f_0 \cdot |\sigma^{-1}\tau|$$ ← number of transpositions



Case |A|>|B| :



➡ Volume law

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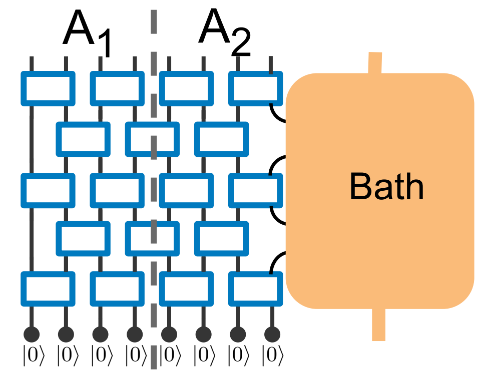

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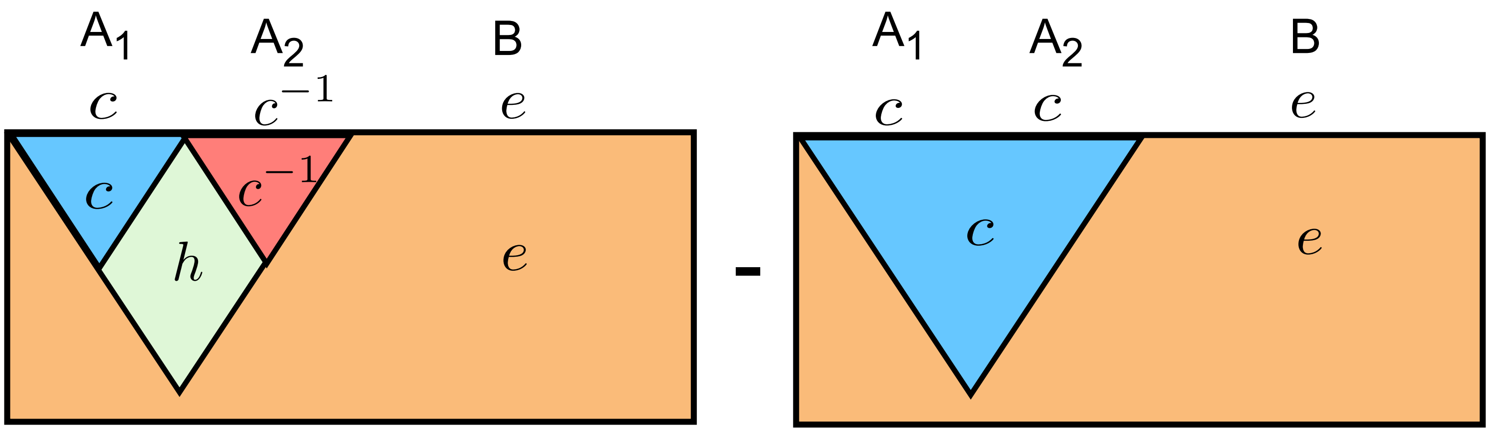
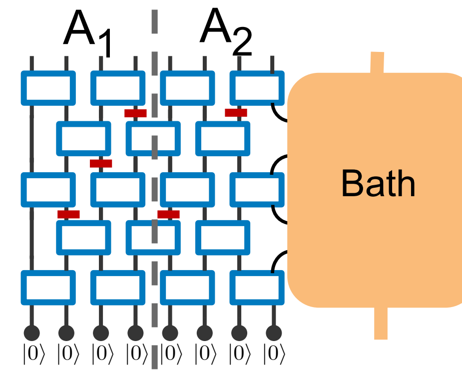
$\mathcal{E}^{(n)}_{A_1:A_2} =$



$= 0$

➡ No extensive negativity

Negativity “Page” transition in **hybrid** quantum circuits (mean field theory)



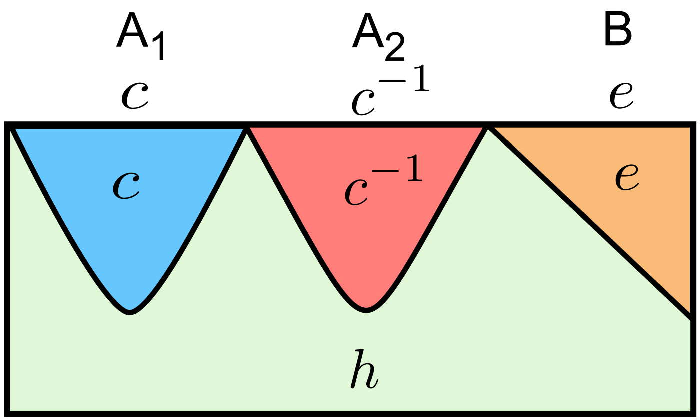
- No strict light cone

- Ansatz for domain wall tension between σ and τ domains:

$$f_{\sigma\tau} = f_0 \cdot |\sigma^{-1}\tau| \cdot \lambda(p)$$

Case |A|>|B| :

$\mathcal{E}^{(n)}_{A_1:A_2} =$



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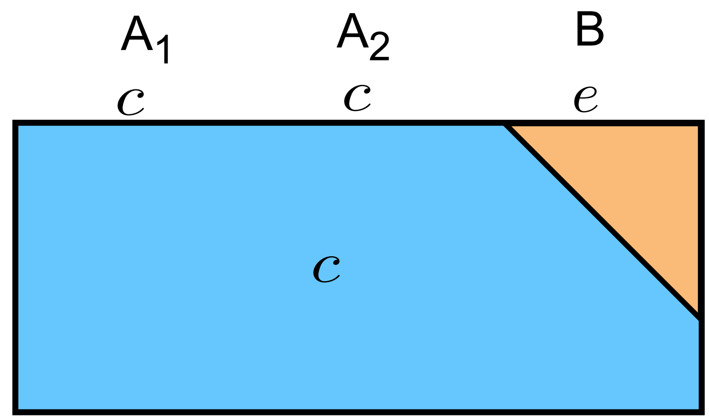
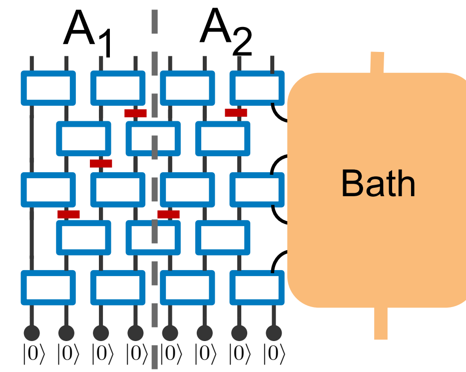


$= \alpha_{>} L_{A_1}$

➡ Volume law

This is the same as the unitary circuit. More interesting is the large bath case:

Negativity “Page” transition in **hybrid** quantum circuits (mean field theory)

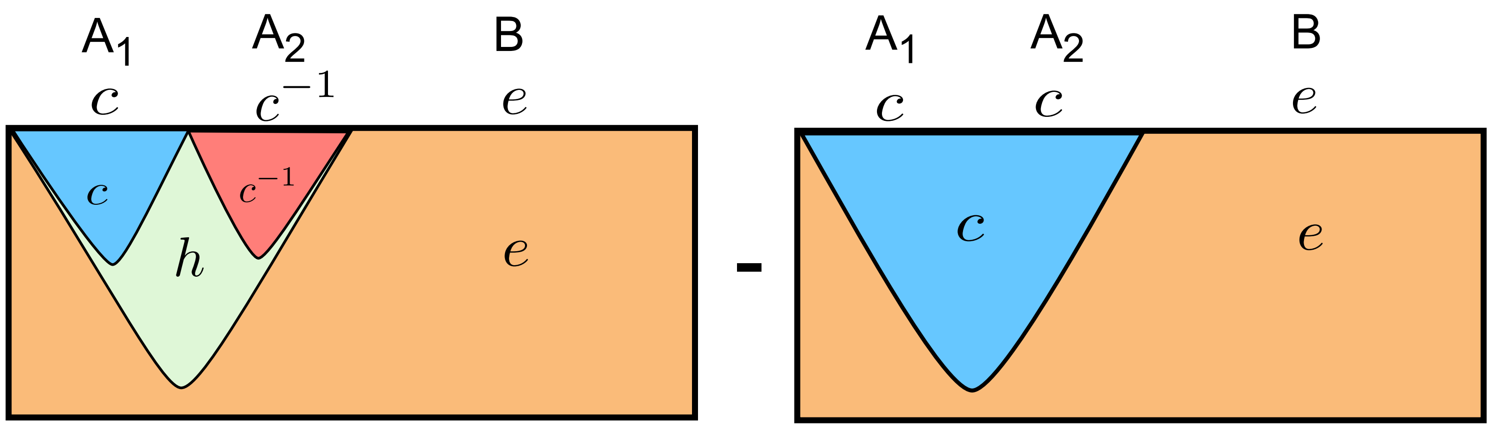


- No strict light cone
- Ansatz for domain wall tension between σ and τ domains:

$$f_{\sigma\tau} = f_0 \cdot |\sigma^{-1}\tau| \cdot \lambda(p)$$

Case |A<|B| :

$\mathcal{E}^{(n)}_{A_1:A_2} =$

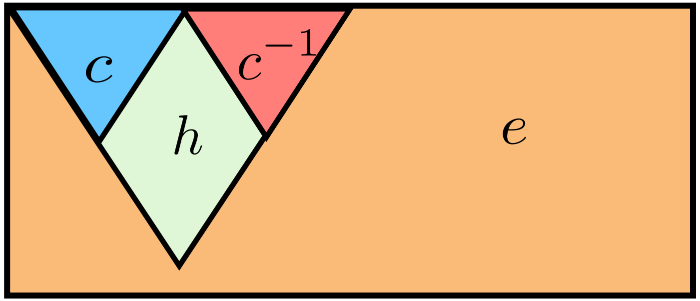


$= \alpha_< L_{A_1}$

Compare with the unitary circuit (|A|<|B|)

Unitary:

$\mathcal{E}^{(n)}_{A_1:A_2} =$



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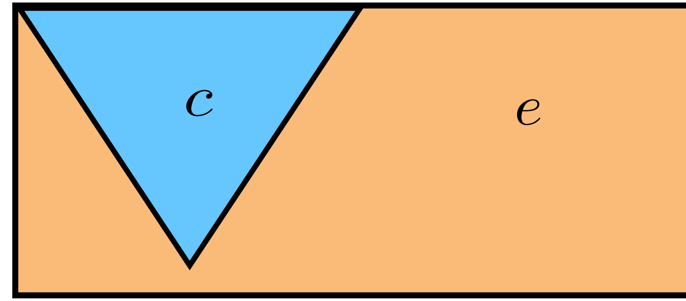
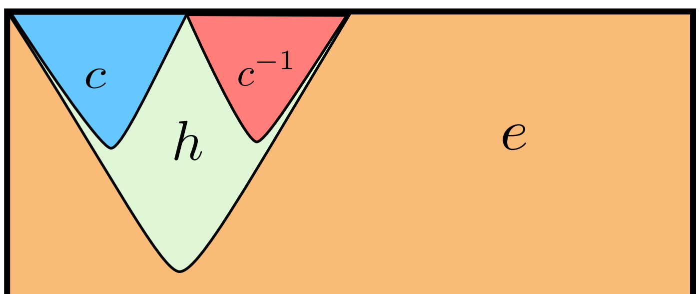


$= 0$

Unitary + measurement:

$\mathcal{E}^{(n)}_{A_1:A_2} =$



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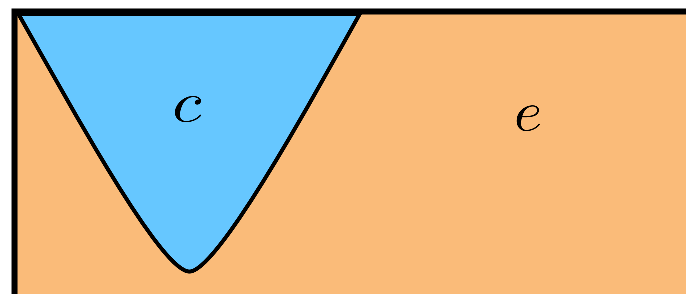
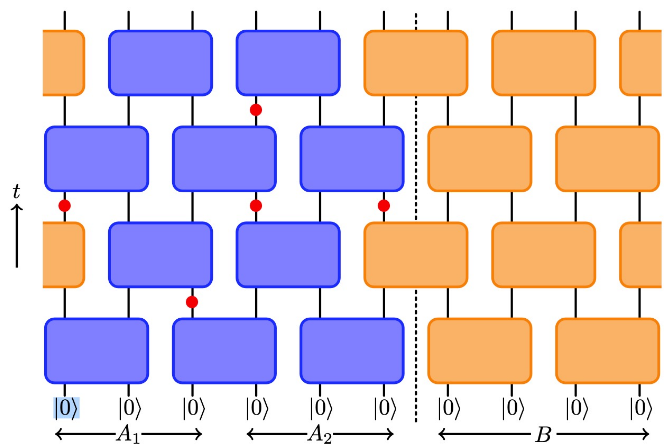


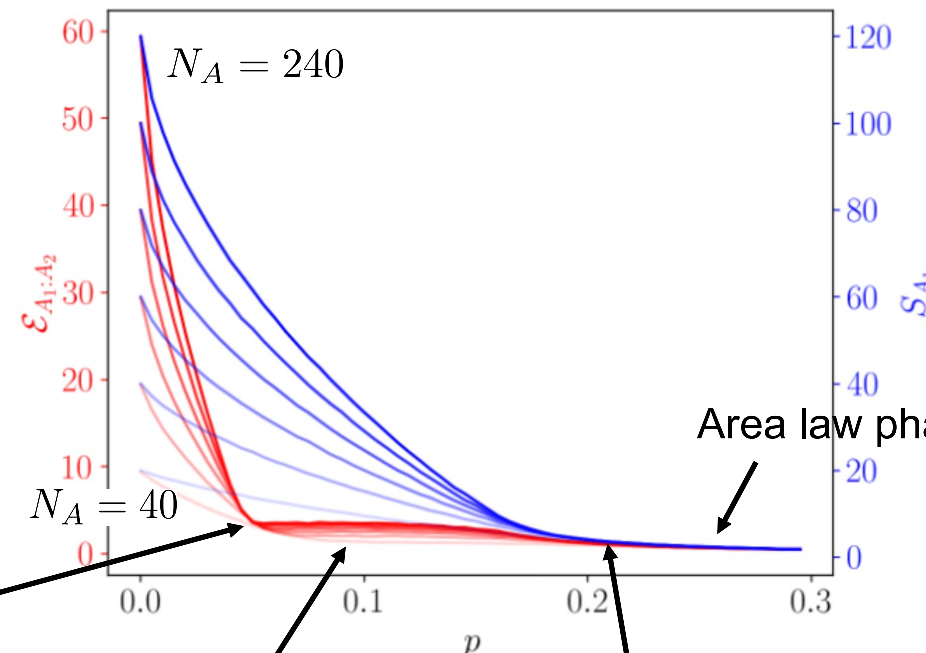
$= \alpha_< L_{A_1}$

Mean field picture suggests a small non-vanishing volume law coefficient even for infinite bath

Numerical simulation of a Clifford circuit

We keep $N_A = 2N_B$. Increasing p (measurement rate on A) effectively decreases the size of A by decreasing the code density = number of logical qubits.





Area law phase

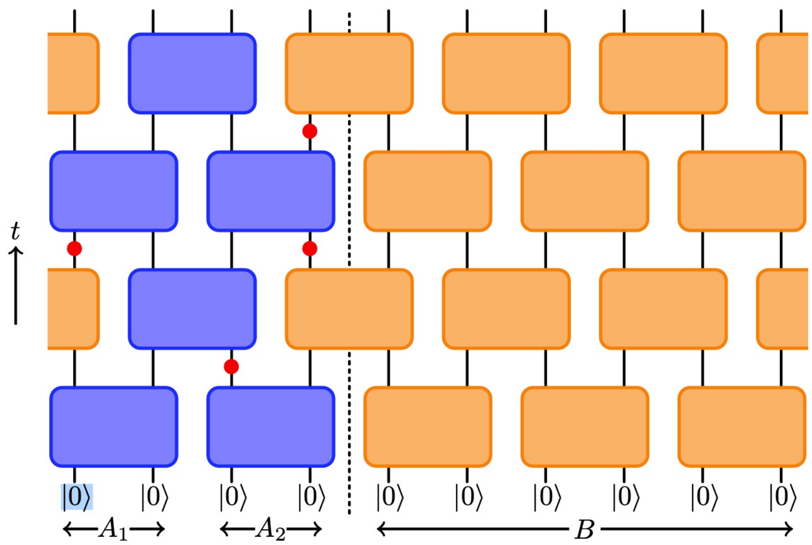
Negativity "Page" transition due to reduction in code density

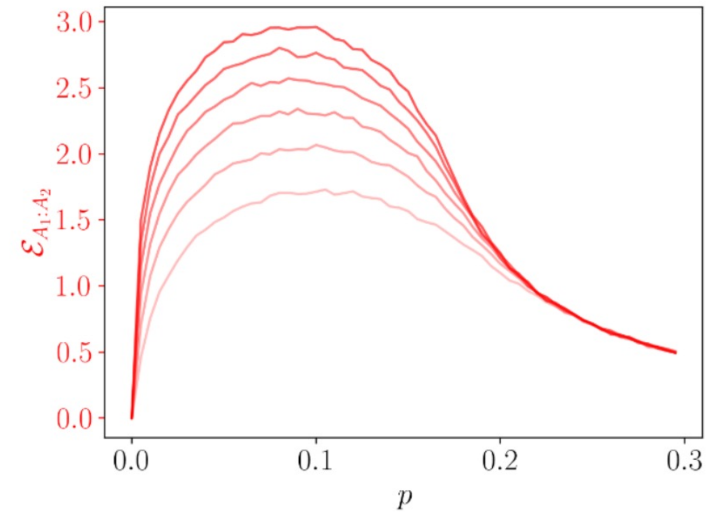
"Infinite" bath volume law phase (?)

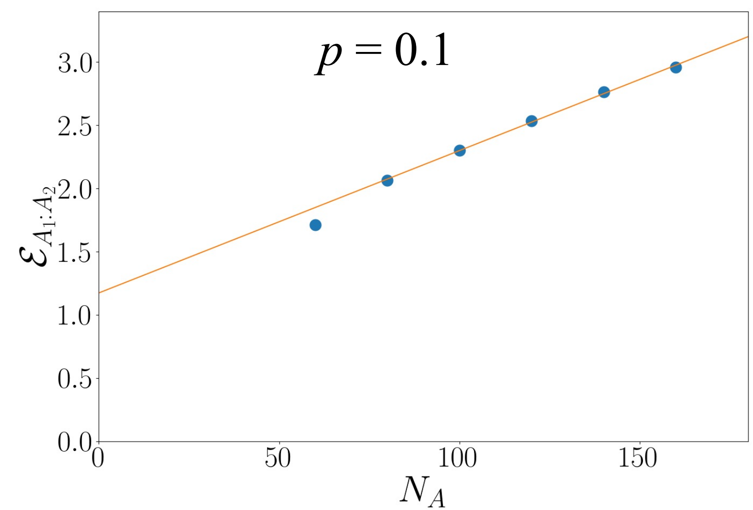
Measurement induced entanglement transition

The infinite bath volume law phase

Take a large bath: $N_B = 2\,N_A$







Suggestive of a volume law

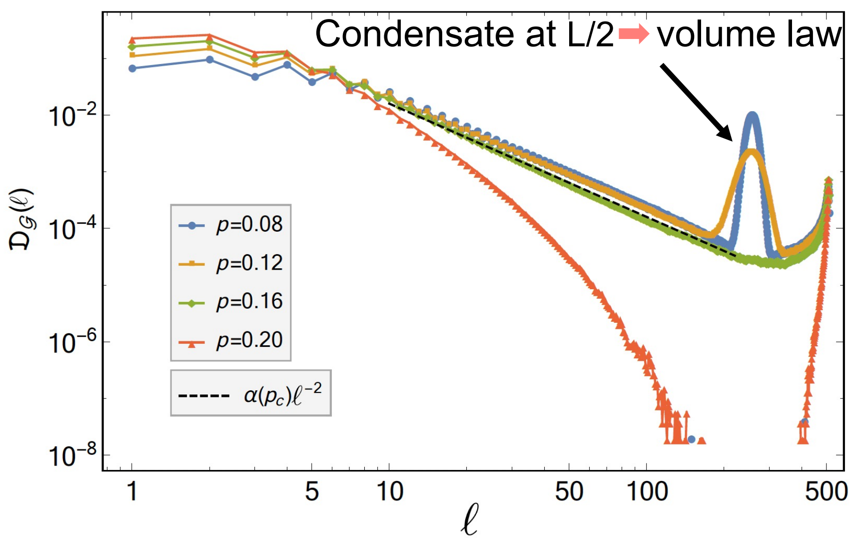
Speculation on stabilizer length distribution with edge decoherence

Large scale entanglement is carried by long stabilizers

Hybrid circuits: Li, Chen and Fisher PRB 2019

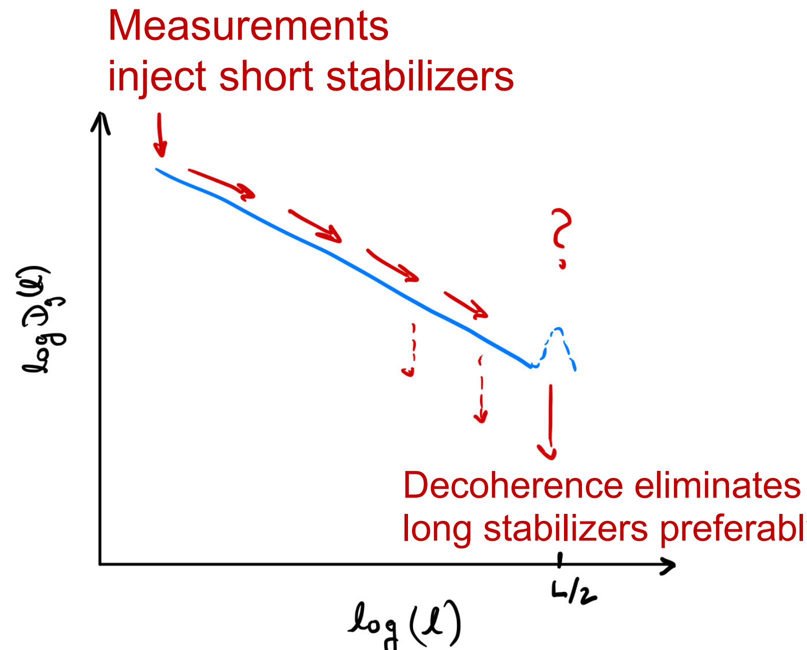
Pure state ➡ N stabilizers = N qubits

Measurements destroy long stabilizers and create short ones



Coupling to decoherence at the edge:

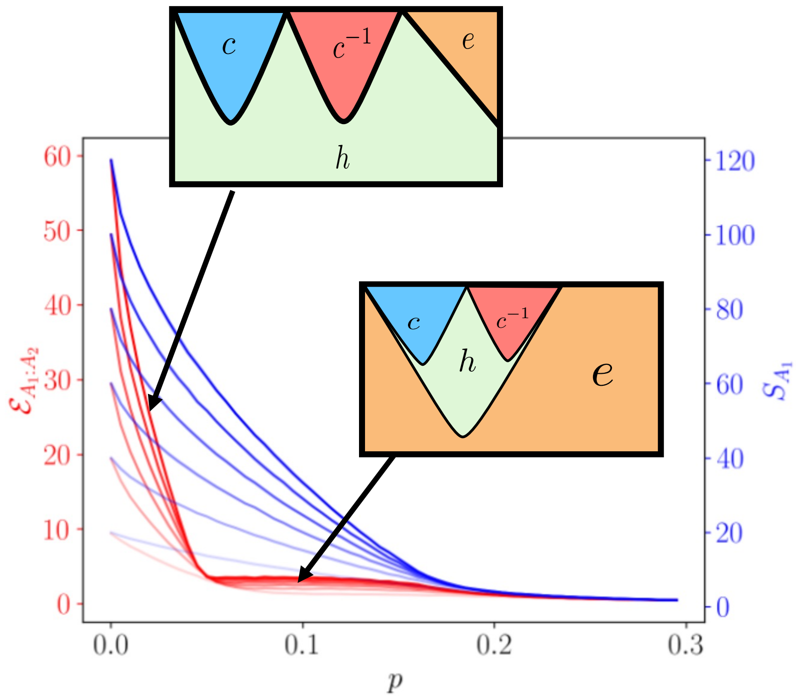
Mixed state ➡ N stabilizers < N qubits

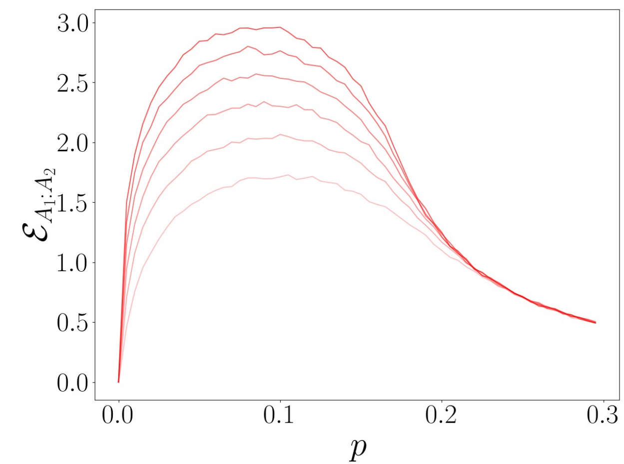


stabilizer cascade

Summary

Measurement induced phases and phase transitions in the entanglement negativity in presence of a bath





Possible volume law negativity phase despite coupling to an **infinite bath (?)**