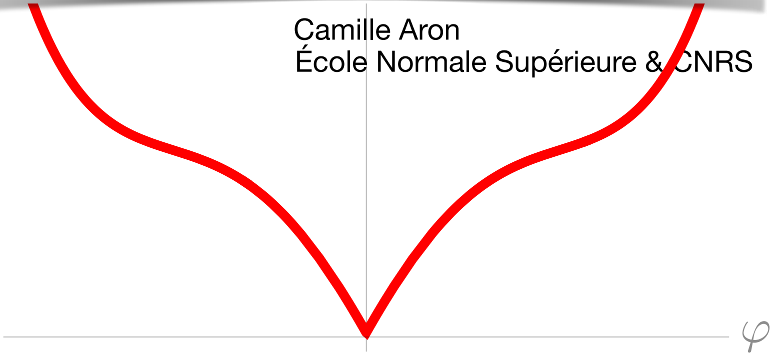
Non-analytic non-equilibrium field theories?



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Ginzburg-Landau's Effective Field Theory

Classification of equilibrium phase transitions

- identify order parameter
- dimension + symmetry group = universality class

Ex: vector order parameter

$\mathrm{O}(n)$

$n = -2$: loop erased random walk
$n = 0$: polymers
$n = 1$: uniaxial magnets, liquid-gas, binary mixtures, …
$n = 2$: planar magnets, Kosterlitz Thouless, …
$n = 3$: Heisenberg magnets, …
$n \geq 4$: transitions with increase of unit cell, …

$\mathrm{U}(1)$ superconductivity

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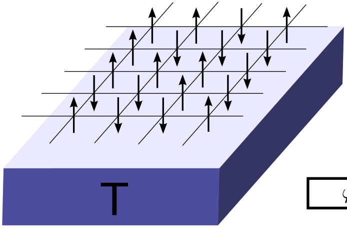
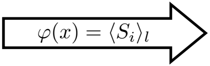
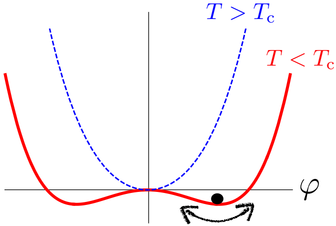
ex: Ising Model $\mathbb{Z}_2$ symmetry $\lambda\,\varphi^4$ theory





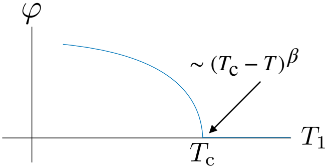


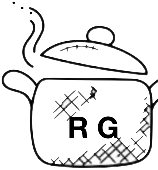
$$P\{S_i\} \sim e^{-H\{S_i\}/T}$$

$$P[\varphi] \sim e^{-\mathscr{F}[\varphi]}$$

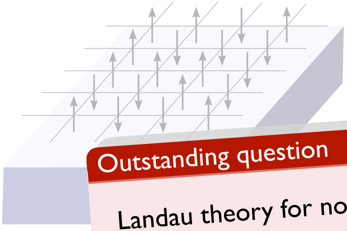
$$H = -\sum_{\langle ij \rangle} S_i\, S_j$$

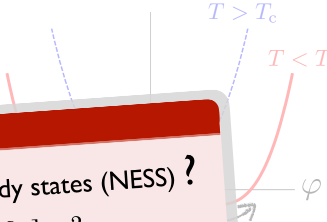
$$\mathscr{F}[\varphi] = \int \mathrm{d}^d x \; \frac{1}{2}(\nabla\varphi)^2 + \mu\varphi^2 + \lambda\varphi^4 + \dots$$





ex: Ising Model | $\mathbb{Z}_2$ symmetry | $\lambda\,\varphi^4$ theory





Outstanding question

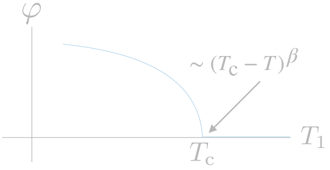
Landau theory for non-equilibrium steady states (NESS) ?

$P_{\mathrm{NESS}}[\varphi] \sim \mathrm{e}^{-\mathscr{F}_{\mathrm{NESS}}[\varphi]}$ $\qquad$ $\mathscr{F}_{\mathrm{NESS}}[\varphi] = ?$

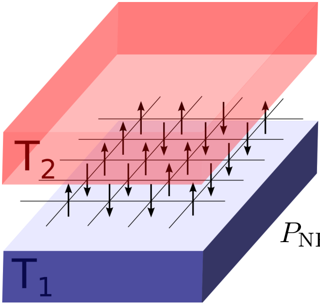
$P\{S_i\} \sim$

$H = -\sum_{\langle ij \rangle} S_i S_j$

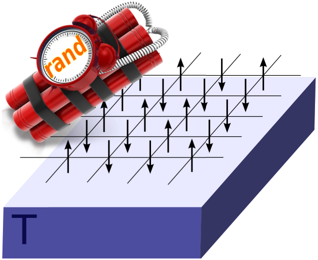
$$\mathscr{F}[\varphi] = \int \mathrm{d}^d x\ \frac{1}{2}(\nabla\varphi)^2 + \mu\varphi^2 + \lambda\varphi^4 + \dots$$



2-bath Ising Model	Stochastic Reheating

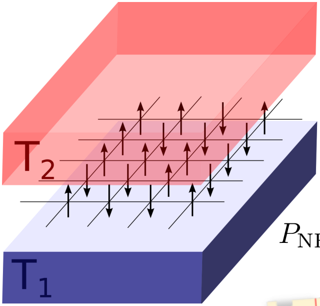


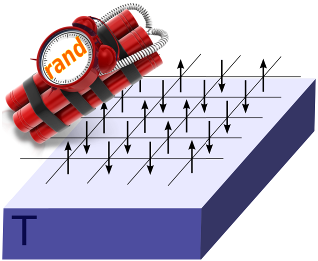
$$P_{\mathrm{NESS}}[\varphi] \sim \mathrm{e}^{-\mathcal{F}_{\mathrm{NESS}}[\varphi]}$$



Non-Equilibrium Steady States

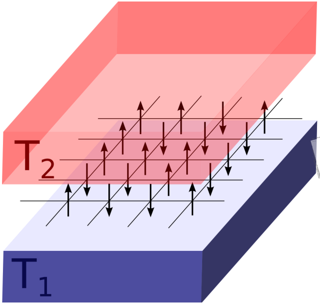
2-bath Ising Model | **Stochastic Reheating**





$$P_{\text{NESS}}[\varphi] \sim \mathrm{e}^{-\mathcal{F}_{\text{NESS}}[\varphi]}$$

2-bath Ising Model



$$P_{\mathrm{NESS}}[\varphi] \sim \mathrm{e}^{-\mathscr{F}_{\mathrm{NESS}}[\varphi]}$$

Self-consistent equation:

hybridizations with baths

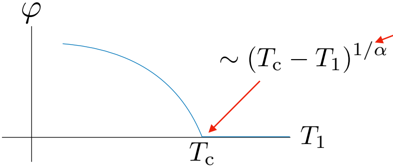
MEAN FIELD

$$\varphi = \frac{\nu_1(2|\varphi|) + \nu_2(2|\varphi|)}{\nu_1(2|\varphi|)\ \coth(\varphi/T_1) + \nu_2(2|\varphi|)\ \coth(\varphi/T_2)}$$

Non-equilibrium free energy

$$\mathscr{F}_{\mathrm{NESS}}[\varphi] = \int \mathrm{d}^d x\ \ \mu\varphi^2 + c_\alpha |\varphi|^\alpha + \lambda\varphi^4 + \ldots$$

$\alpha \notin \mathbb{N}$
low energy
features of baths



Questions

Do the non-analytic operators of the non-equilibrium Landau potential

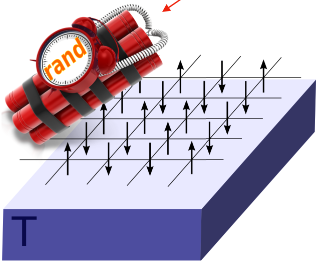
1. survive away from the mean-field approximation?

2. survive in the infrared (IR)?

3. survive the fluctuations in low dimensions?

Stochastic Reheating of the Ising Model

reset to infinite temperature with rate r



Renewal formula

$$P_{\mathrm{NESS}}[\varphi] = r \int_0^\infty \mathrm{d}t \; \mathrm{e}^{-rt} \; P_0([\varphi]; t)$$

quench from infinite temperature

Equilibrium

FIELD THEORY

Model A relaxation dynamics

$$\eta\, \partial_t \varphi(x,t) = -\frac{\delta \mathscr{F}_{\mathrm{EQ}}[\varphi]}{\delta\varphi(x,t)} + \xi(x,t)$$

$$\varphi(x,0) = 0$$

$$\mathscr{F}_{\mathrm{EQ}}[\varphi] = \int \mathrm{d}^d x \left(\frac{1}{2}\mu\,\varphi^2 + \frac{1}{4}\lambda\,\varphi^4 + \ldots + \frac{1}{2}(\nabla\varphi)^2 + \ldots \right)$$

MEAN FIELD

Langevin relaxation dynamics

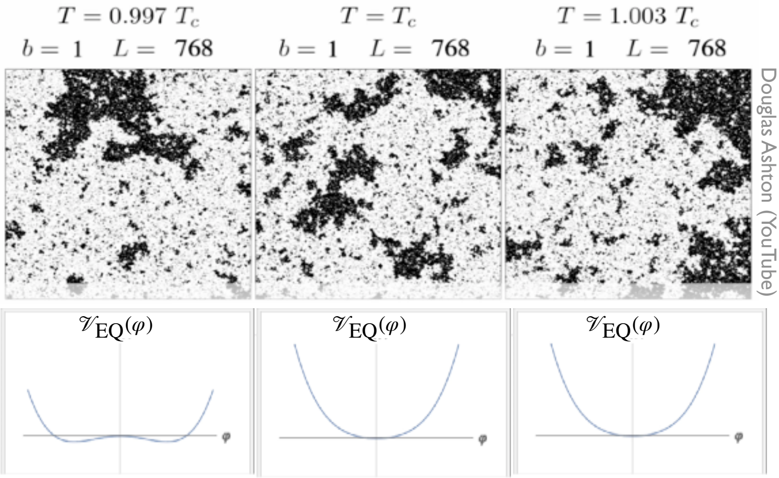
$$\eta\, \partial_t \varphi(t) = -\,\partial_\varphi \mathscr{V}_{\mathrm{EQ}}(\varphi) + \xi(t)$$

$$\varphi(0) = 0$$

$$\mathscr{V}_{\mathrm{EQ}}[\varphi] \sim \frac{1}{2}\mu\,\varphi^2 + \frac{1}{4}\lambda\,\varphi^4 + \ldots$$

Real-space Renormalization Group

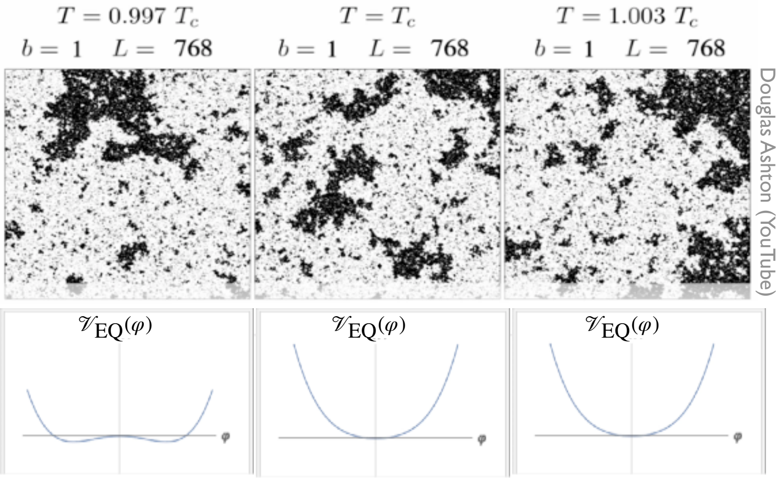
Equilibrium RG flow



Infrared fixed point in $d > d_{\mathrm{uc}}$

$\mathscr{V}^*_{\mathrm{EQ}}(\Phi) = -\frac{1}{2}\,\Phi^2$ $\qquad$ $\mathscr{V}^*_{\mathrm{EQ}}(\Phi) = 0$ $\qquad$ $\mathscr{V}^*_{\mathrm{EQ}}(\Phi) = +\frac{1}{2}\Phi^2$

Equilibrium RG flow



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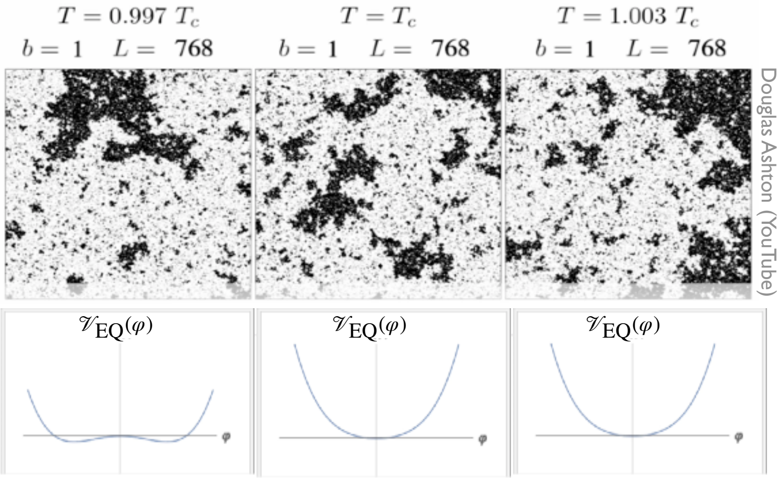

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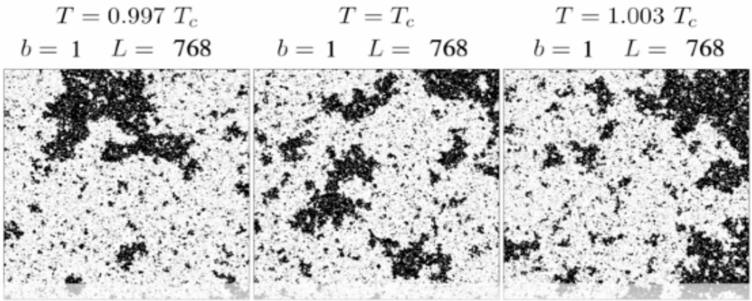
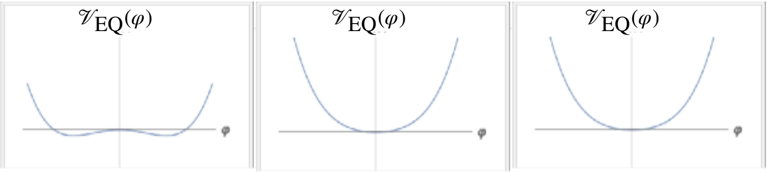


Douglas Ashton (YouTube)



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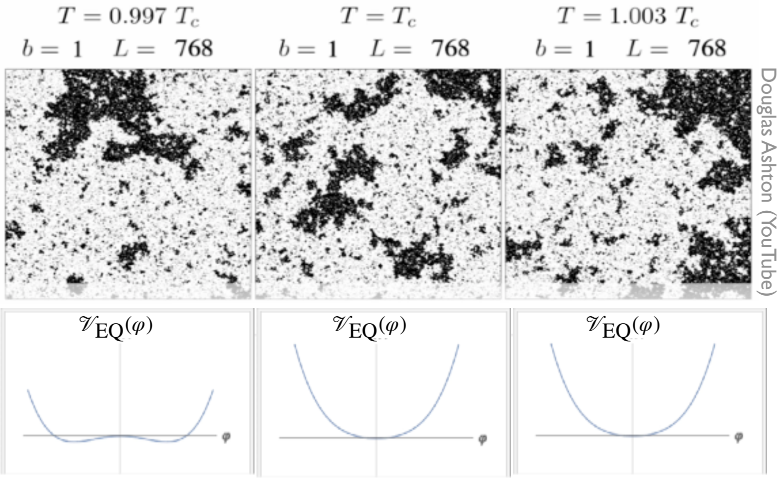
Equilibrium RG flow



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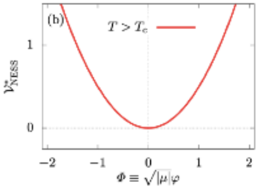
Non-equilibrium RG flow: $d > d_{\mathrm{uc}}$

$T < T_c$

$T = T_c$

$T > T_c$

$$\mathscr{V}^*_{\mathrm{NESS}}(\Phi) = +\frac{1}{2}\Phi^2$$

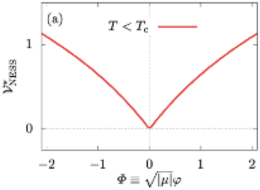


equilibrium universality

Non-equilibrium RG flow: $d > d_{uc}$

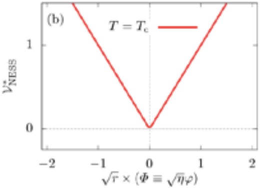
$T < T_c$

$$\mathscr{V}^*_{\mathrm{NESS}}(\Phi) = \sqrt{\frac{2}{\pi}}\,|\Phi| - \left(\frac{1}{2} - \frac{1}{\pi}\right)\Phi^2 + \frac{4/\pi - 1}{3\sqrt{2\pi}}\,|\Phi|^3 + \dots$$



$T = T_c$

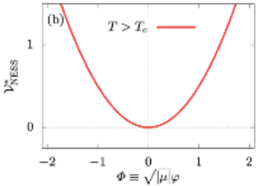
$$\mathscr{V}^*_{\mathrm{NESS}}(\Phi) = \sqrt{r}\,|\Phi|$$



Non-analytic operators are IR relevant!

$T > T_c$

$$\mathscr{V}^*_{\mathrm{NESS}}(\Phi) = +\frac{1}{2}\Phi^2$$

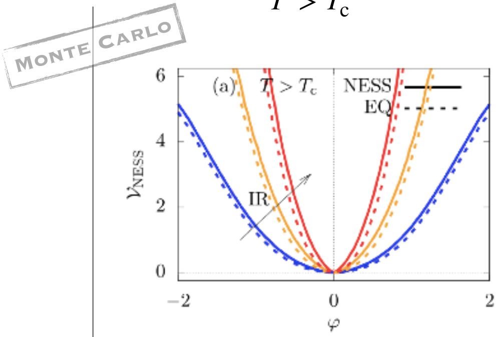


equilibrium universality

Non-equilibrium RG flow: $d = 2 < d_{uc}$

$T < T_c$

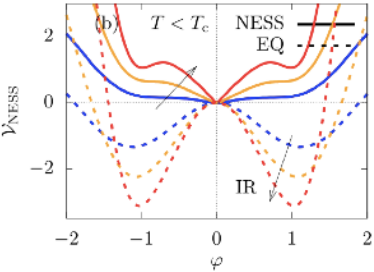
$T > T_c$



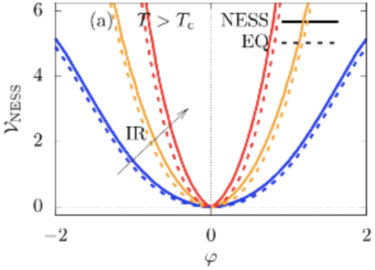
equilibrium universality

Non-equilibrium RG flow: $d = 2 < d_{\mathrm{uc}}$

$T < T_c$



$T > T_c$



Non-analytic operators are IR relevant!

equilibrium universality

Non-analytic Non-Equilibrium Field Theories?

Summary

- 2 concrete examples with non-analytic operators in $\mathscr{F}_{NESS}[\varphi]$
- involve low-energy features of the environment $\longleftarrow$ universality?
- RG analysis in the NESS
    - exact above d_{uc}
    - numerics at $d = 2$

➔ non-analytic operators

- exist away from mean-field approximation
- do survive fluctuations even below d_{uc}
- can be IR relevant