SEMI-TELEPORTATION:
"Doing More by trying Less"

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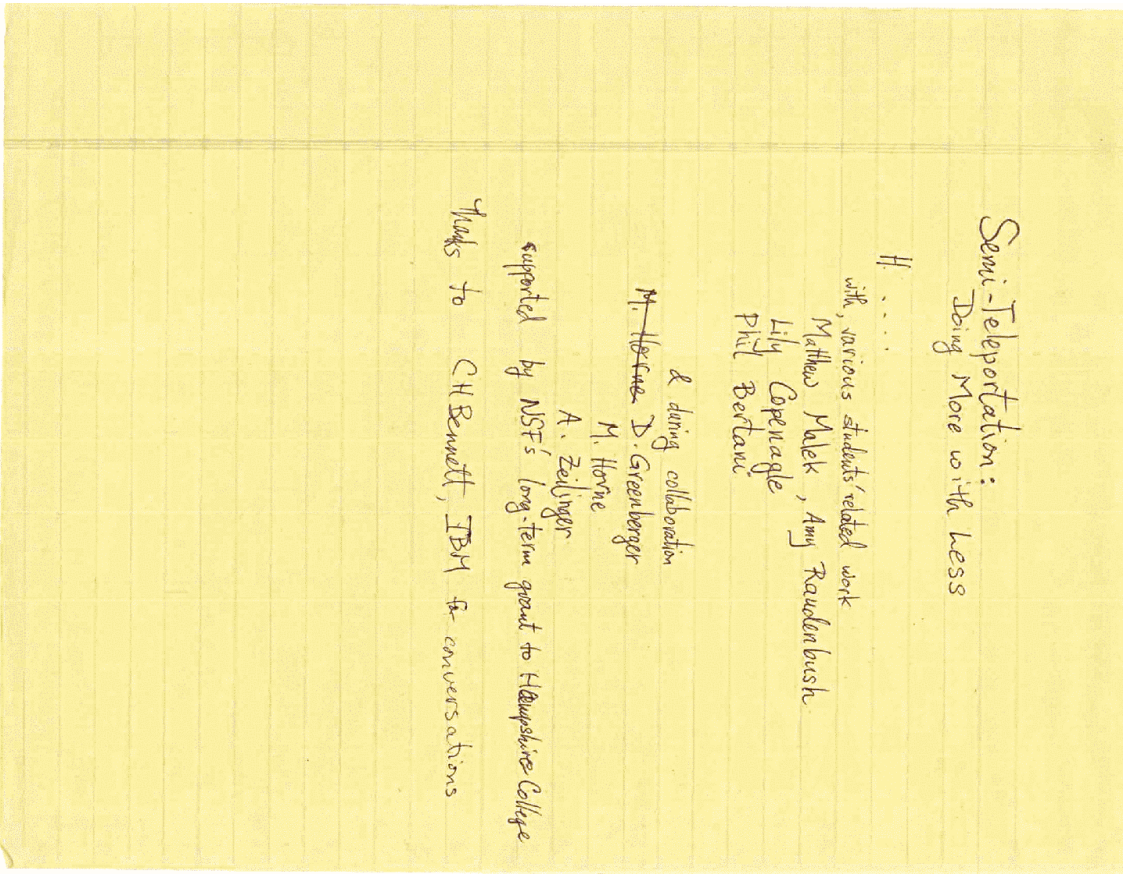
KITP Workshop on Theoretical Physics at Primarily Undergraduate Institutions

July 17, 2007 2:15 PM

with various students' related work
Matthew Malek & Amy Raudenbush
Lily Copenagle
Phil Bertani

during collaboration w/ D. Greenberger, M. Horne and A. Zeilinger
as supported by NSF long-term grant to Hampshire College

with Thanks to CH Bennett, IBM, for many private conversations...



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MAIN POINTS

1. Quantum Teleportation in ordinary information terms – why it is already "Super" Compression

2. The four characters of REMOTE STATE EXCHANGE: Alice & Bob serve Charles & Di

3. REMOTE STATE PREPARATION

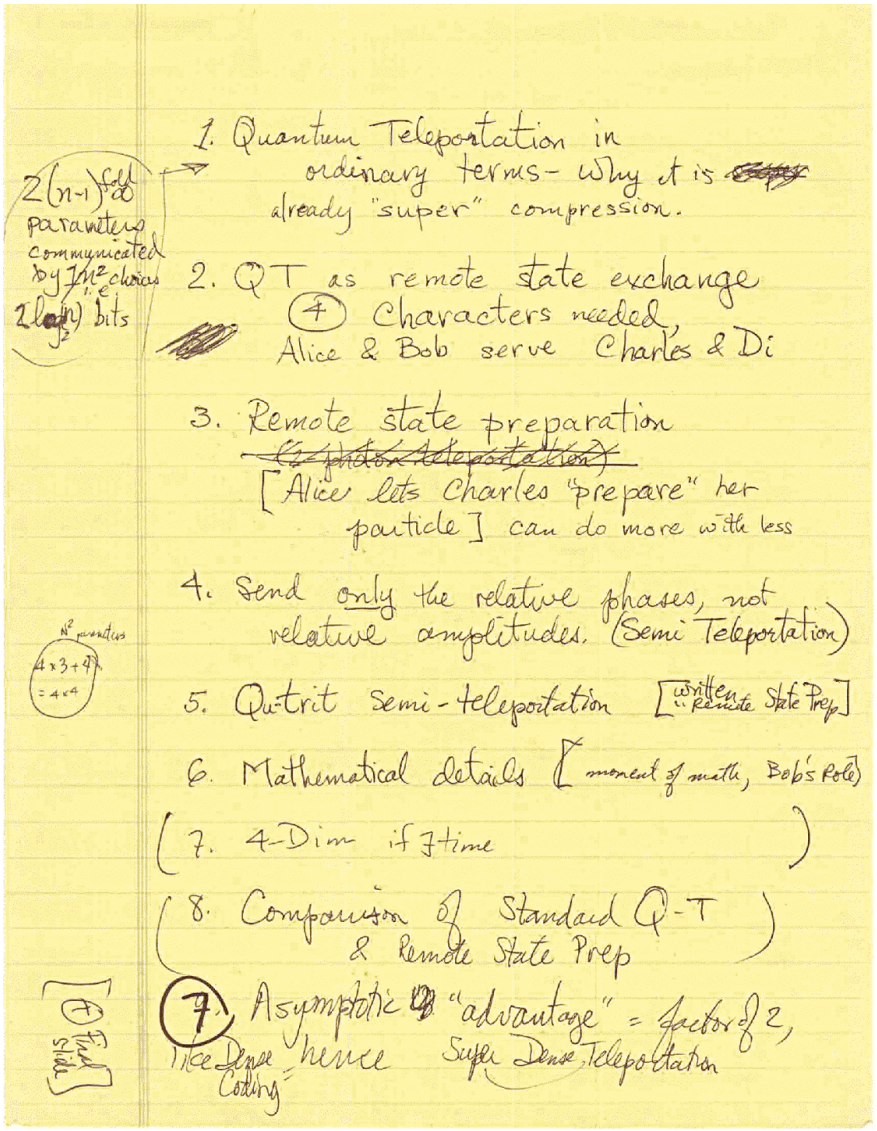
If Alice lets Charles "prepare" her particle she can do more with less (fewer bits)

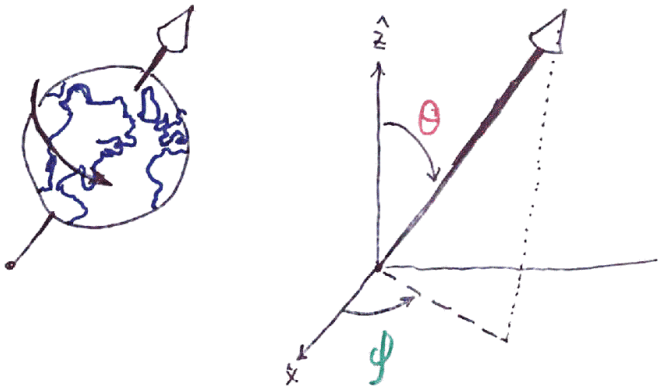
4. SEND only the relative phases of a state: SEMI-TELEPORTATION

5. Qutrit SEMI-TELEPORTATION 2 parameters sent using < 2 bits!

6. Details, mathematical & physical – what Bob must do

7. Asymptotic "Advantage" = factor of 2 is like DenseCoding, Hence SuperDense Teleportation (Bennett)

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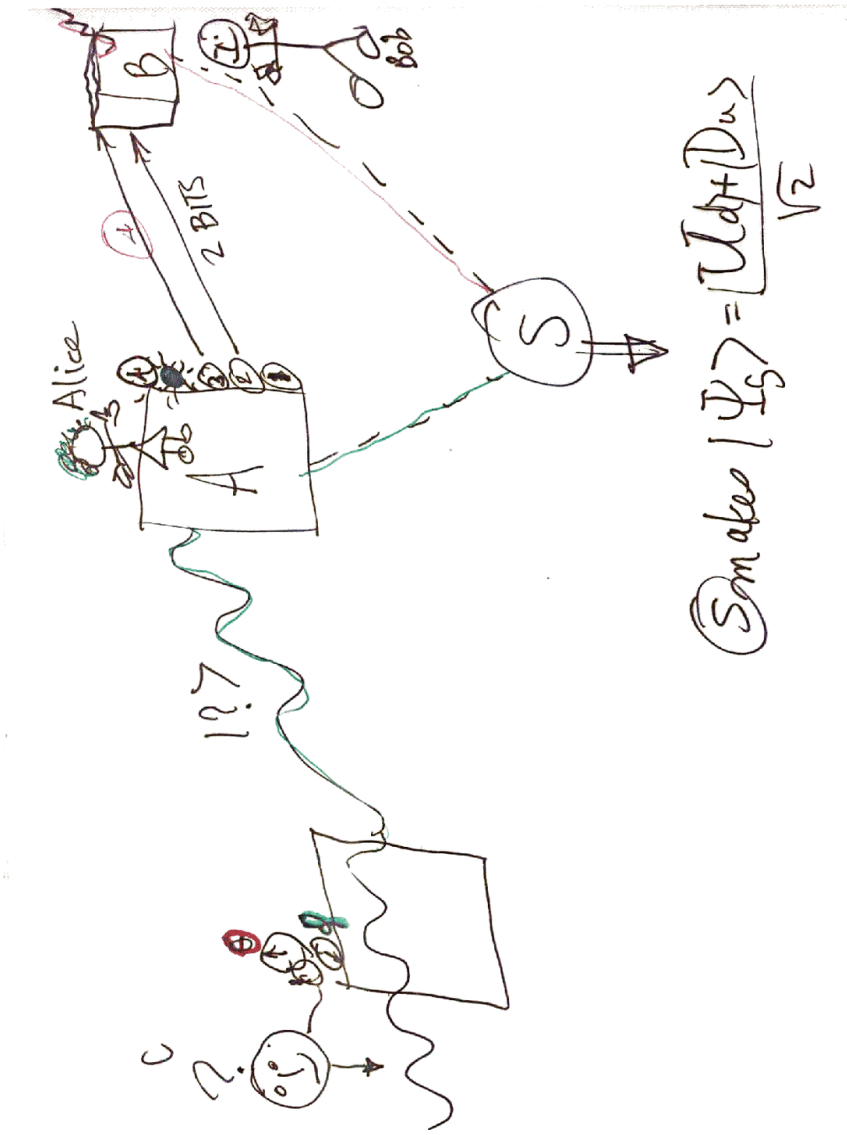
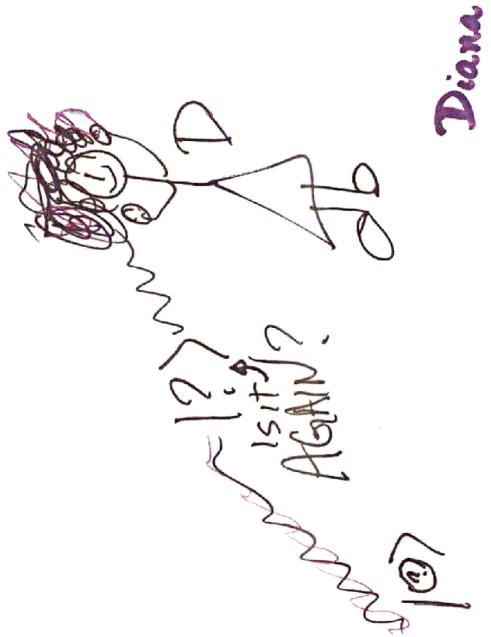

B
2 BITS
Alice
A
S
$|\Psi_S\rangle = \frac{|U_A\rangle + |D_A\rangle}{\sqrt{2}}$
|??⟩
?



Superdense QuTrit Teleportation

$$|\psi\rangle = \frac{1}{\sqrt{3}}\left[|1\rangle + e^{i\alpha}|2\rangle + e^{i\beta}|3\rangle\right]$$

Note $|\psi\rangle = U_{Charles}|e\rangle$

where $|e\rangle = \frac{1}{\sqrt{3}}[|1\rangle + |2\rangle + |3\rangle]$

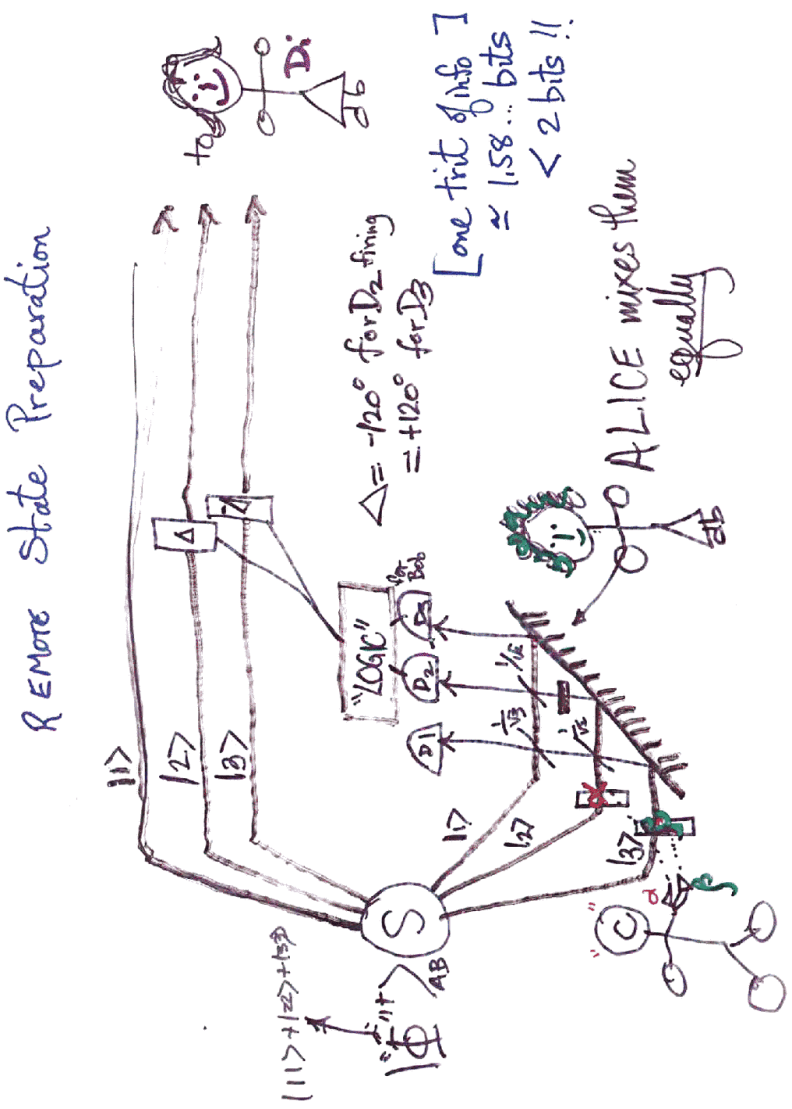
standard equimodular state

and

$$U_C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

How teleport?

$$|S\rangle_{A\&B} = |\text{"}\Phi\text{"}^+\rangle = \frac{|11\rangle + e^{i\alpha}|22\rangle + e^{i\beta}|33\rangle}{\sqrt{3}}$$



A moment of "Math" for QuTrit SDT

$$U_C|\Phi^+\rangle = \frac{1}{\sqrt{3}}\left[|11\rangle + e^{i\alpha}|22\rangle + e^{i\beta}|33\rangle\right]$$

Alice then mixes her $|1\rangle$, $|2\rangle$, & $|3\rangle$ COMPLETELY e.g. FT by U_A

$$U_A = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \tau & \tau^* \\ 1 & \tau^* & \tau \end{pmatrix} \qquad \tau = \sqrt[3]{1} = \frac{-1+\sqrt{3}\,i}{2}$$

Then SHE measures in the standard $\{|1\rangle, |2\rangle, |3\rangle\}$ basis

IN EFFECT, alice is measuring in a new basis, mutually unbiased w.r.t. the original $\{|1\rangle, |2\rangle, |3\rangle\}$ set.

If she gets $|1\rangle$ (D_1) it is easy to see Bob has $|\Psi\rangle_{Charles} = |1\rangle + e^{i\alpha}|2\rangle + e^{i\beta}|3\rangle$

less easy: if Alice gets $|2\rangle$ (D_2)

$|\Psi'\rangle_{Bob} = |1\rangle + e^{i\alpha}\tau|2\rangle + e^{i\beta}\tau^*|3\rangle$

if $|\Psi''\rangle$ = switch τ & τ^* (D_3)

Bob's Role

EASY: If Alice signals "1" for D1
he does NOTHING
If Alice signals "2" for D2
he applies

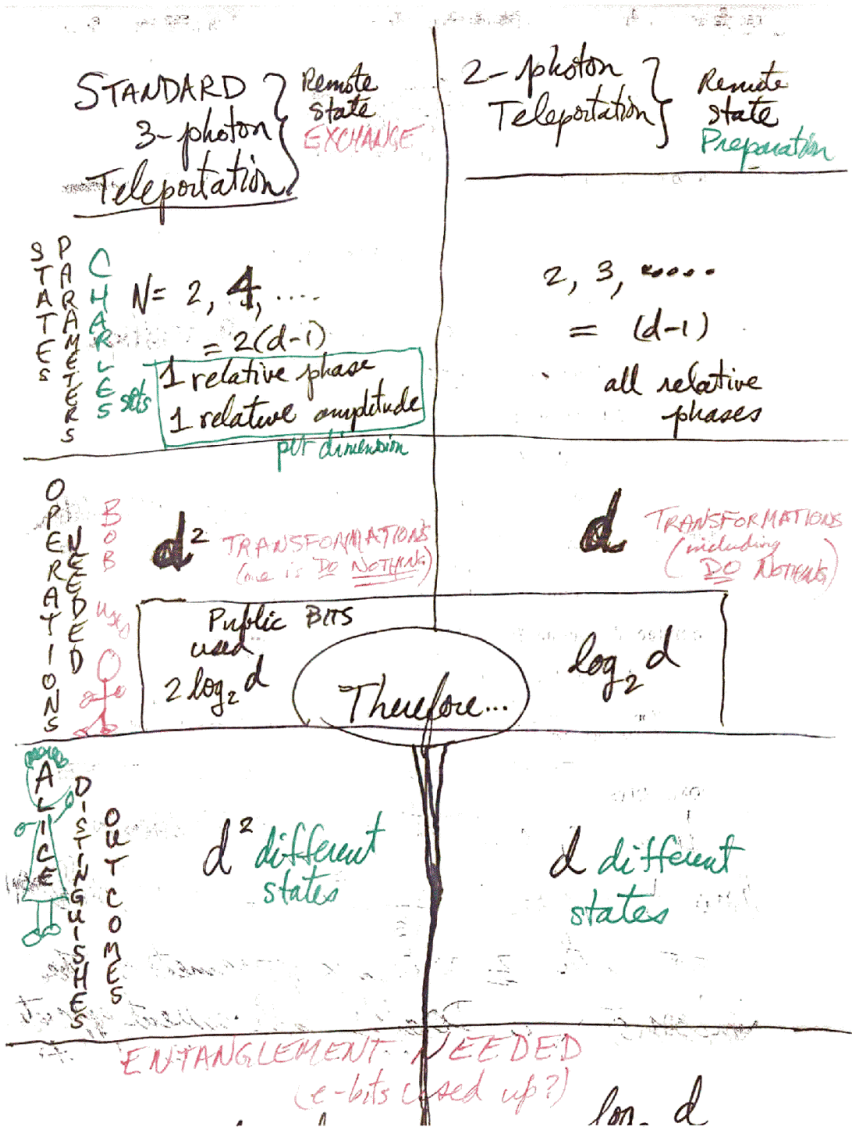
$$U_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \tau^* & 0 \\ 0 & 0 & \tau \end{pmatrix}$$

&

If Alice signals "3" for D3
he applies

$$U_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \tau & 0 \\ 0 & 0 & \tau^* \end{pmatrix} \qquad = U_2^2$$

3 distinct signals, 1 trit of info
< 2 bits yet 2 parameters are sent.



The Assymptotic Benefit is 2

Let $N = (2n-2)$ where n = dimension

QT uses $2 \log_2[(N+2)/2]$ bits to send N parameters

SDT uses $2n-1$ bits to send same #

$$r = \frac{2 \log_2[(N+2)/2]}{\log_2(N+1)}$$

$$\lim_{N \to \infty} r = 2$$