

# The Quantum Cost of a Nonlocal Measurement (work in progress)

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## The General Problem: How “Nonlocal” Is a Measurement?



Each participant has a quantum object.

They want to make some measurement  $M$  on the pair.

How much quantum communication is needed?

$$Q(M) = ?$$

## How Does One Specify a Measurement?

- Hermitian operator? No, the *eigenvalues* are unnecessary.

- Complete orthogonal measurement:

$$M = (|\phi_1\rangle\langle\phi_1|, \dots, |\phi_d\rangle\langle\phi_d|)$$

$$p_i = |\langle\psi|\phi_i\rangle|^2$$

- Most general measurement:

$$M = (E_1, \dots, E_n), \text{ with } \sum E_i = I.$$

$$p_i = \langle\psi|E_i|\psi\rangle$$

The dependence of the probabilities on  $|\psi\rangle$  is all we care about.

## How Does One Quantify Quantum Communication?



$Q(M)$  = Number of maximally entangled qubit pairs (ebits) needed per run of the measurement  $M$  (asymptotically).

## Our Main Example So Far

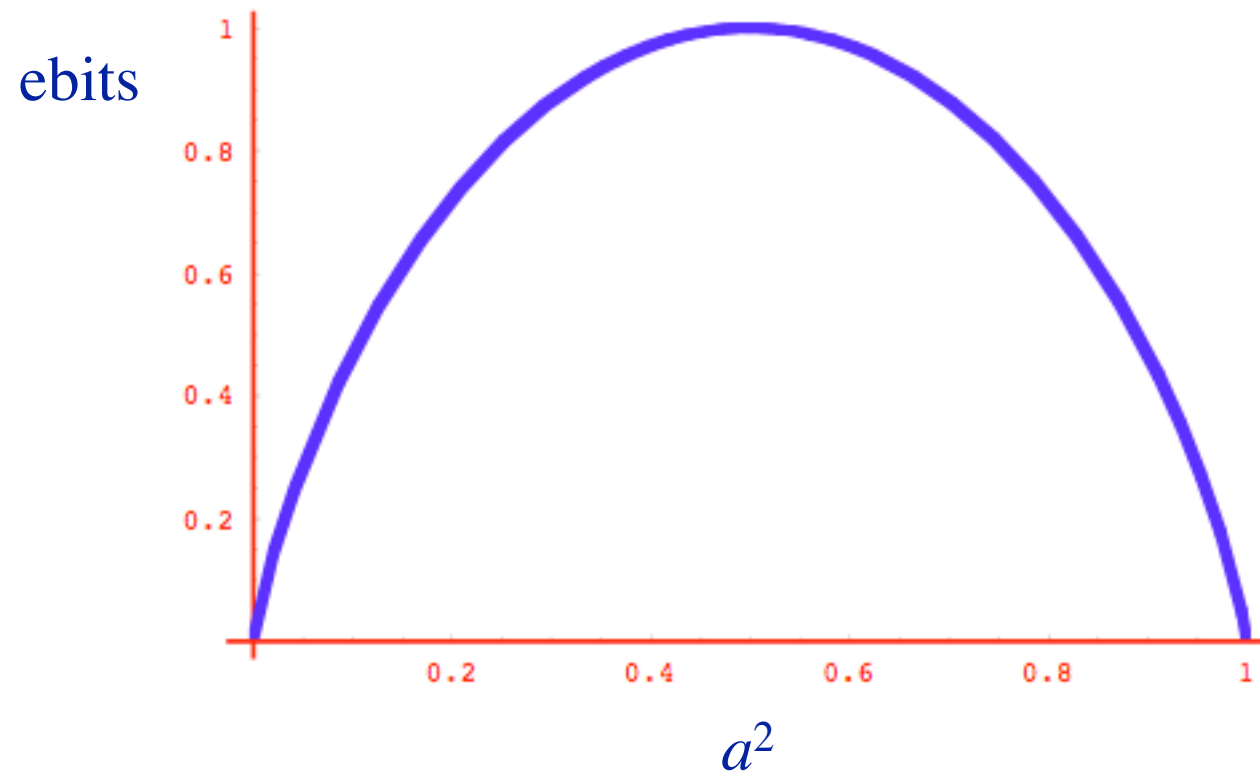


Alice and Bob want to distinguish these four states:

$$\begin{aligned} & a |\uparrow\uparrow\rangle + b |\downarrow\downarrow\rangle \\ & b |\uparrow\uparrow\rangle - a |\downarrow\downarrow\rangle \\ & a |\uparrow\downarrow\rangle + b |\downarrow\uparrow\rangle \\ & b |\uparrow\downarrow\rangle - a |\downarrow\uparrow\rangle \end{aligned}$$

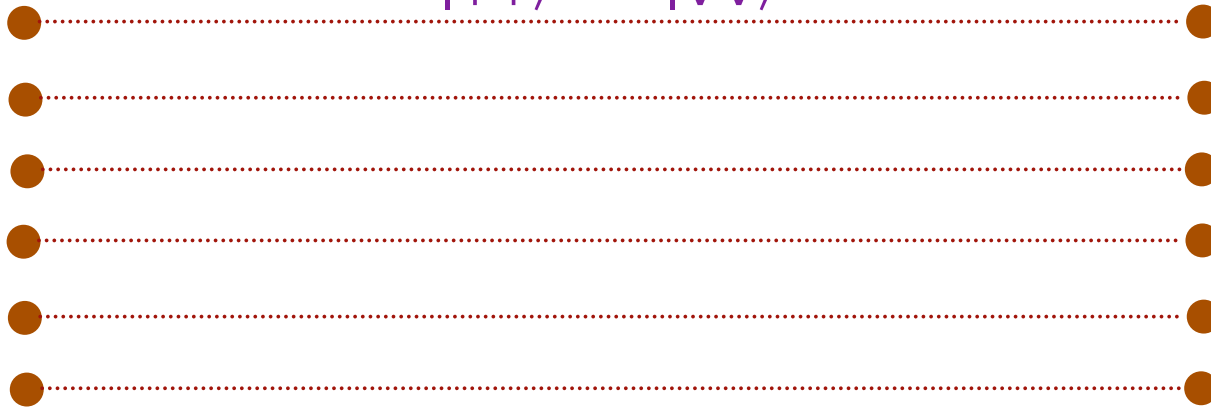
These states all have the same degree of entanglement.

Amount of entanglement in the state  $a|\uparrow\uparrow\rangle + b|\downarrow\downarrow\rangle$



What do we mean by, say, “half an ebit of entanglement”?

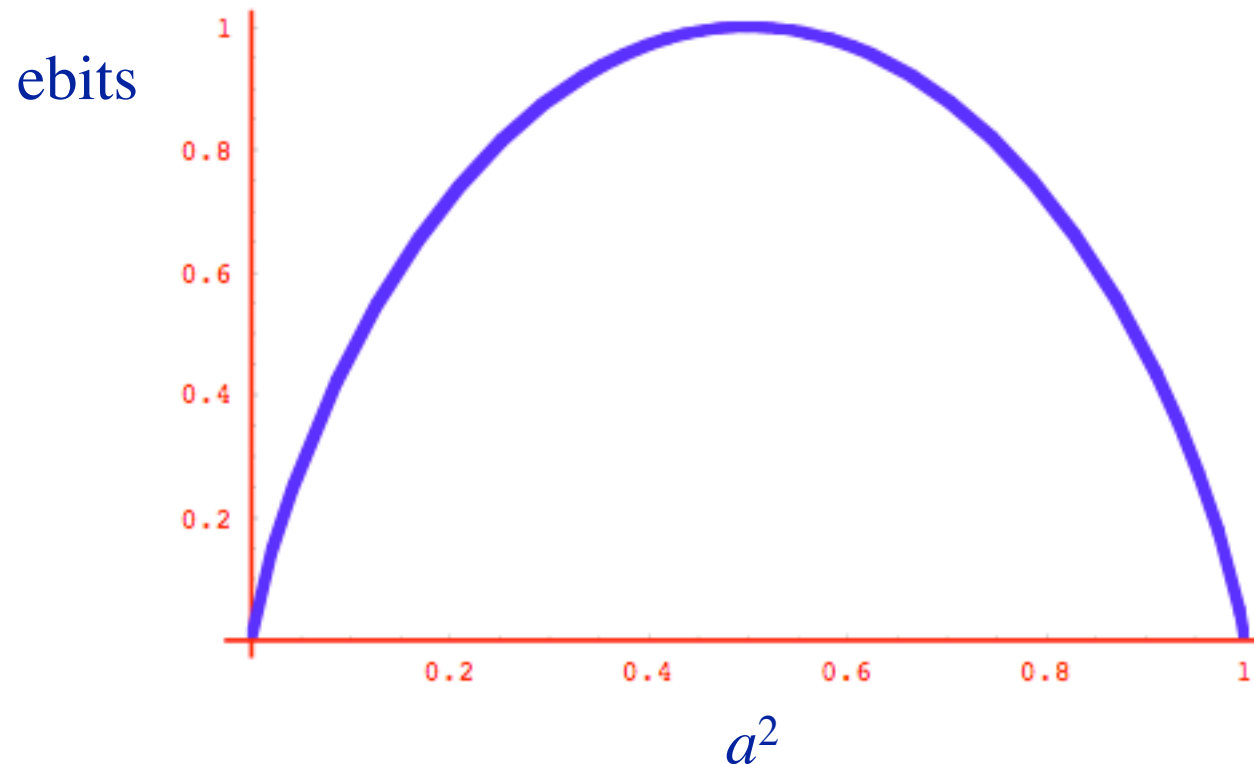
$$a|\uparrow\uparrow\rangle + b|\downarrow\downarrow\rangle$$



$$(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)/\sqrt{2}$$



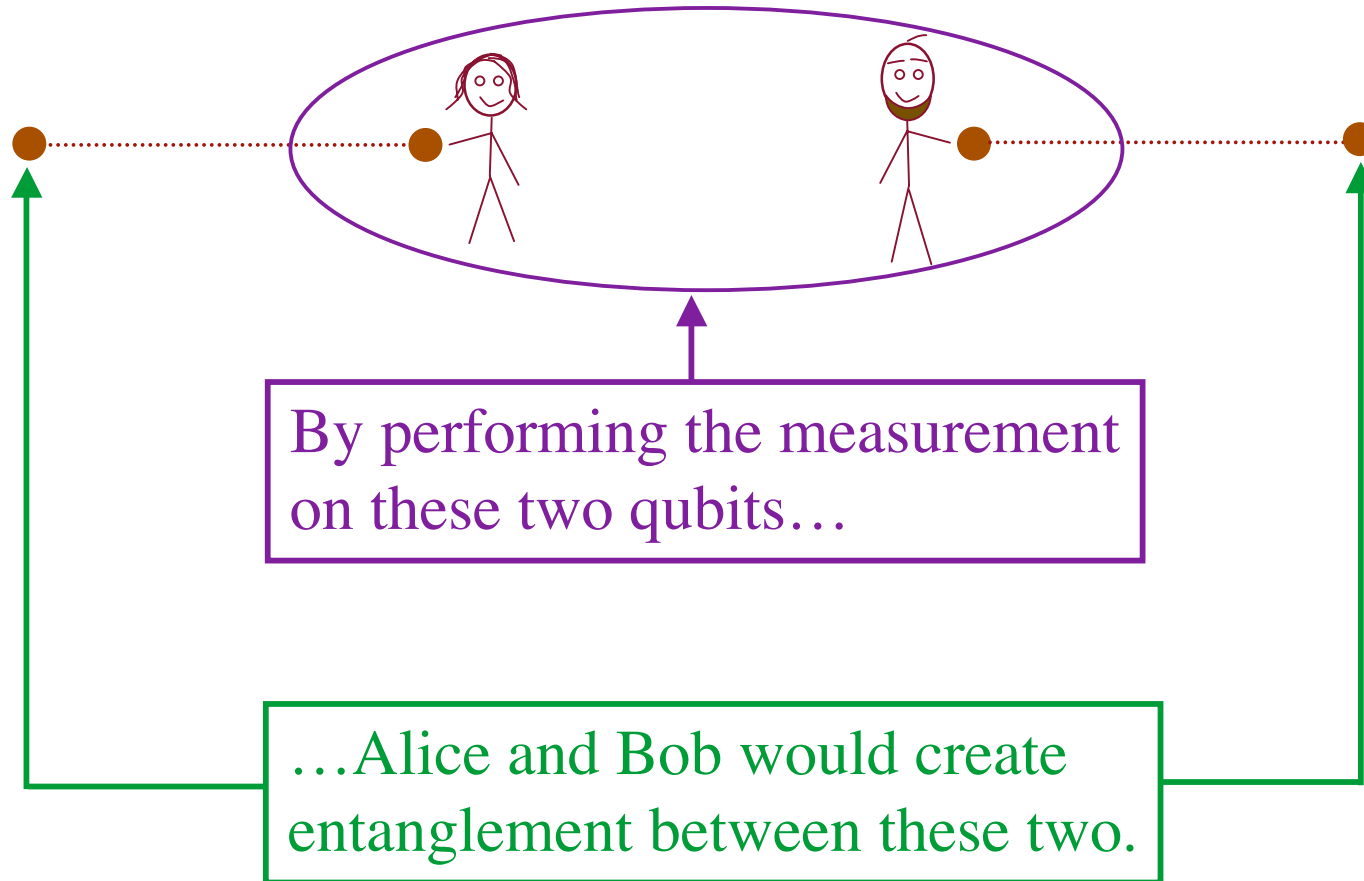
Again, the amount of entanglement in the state  $a|\uparrow\uparrow\rangle + b|\downarrow\downarrow\rangle$



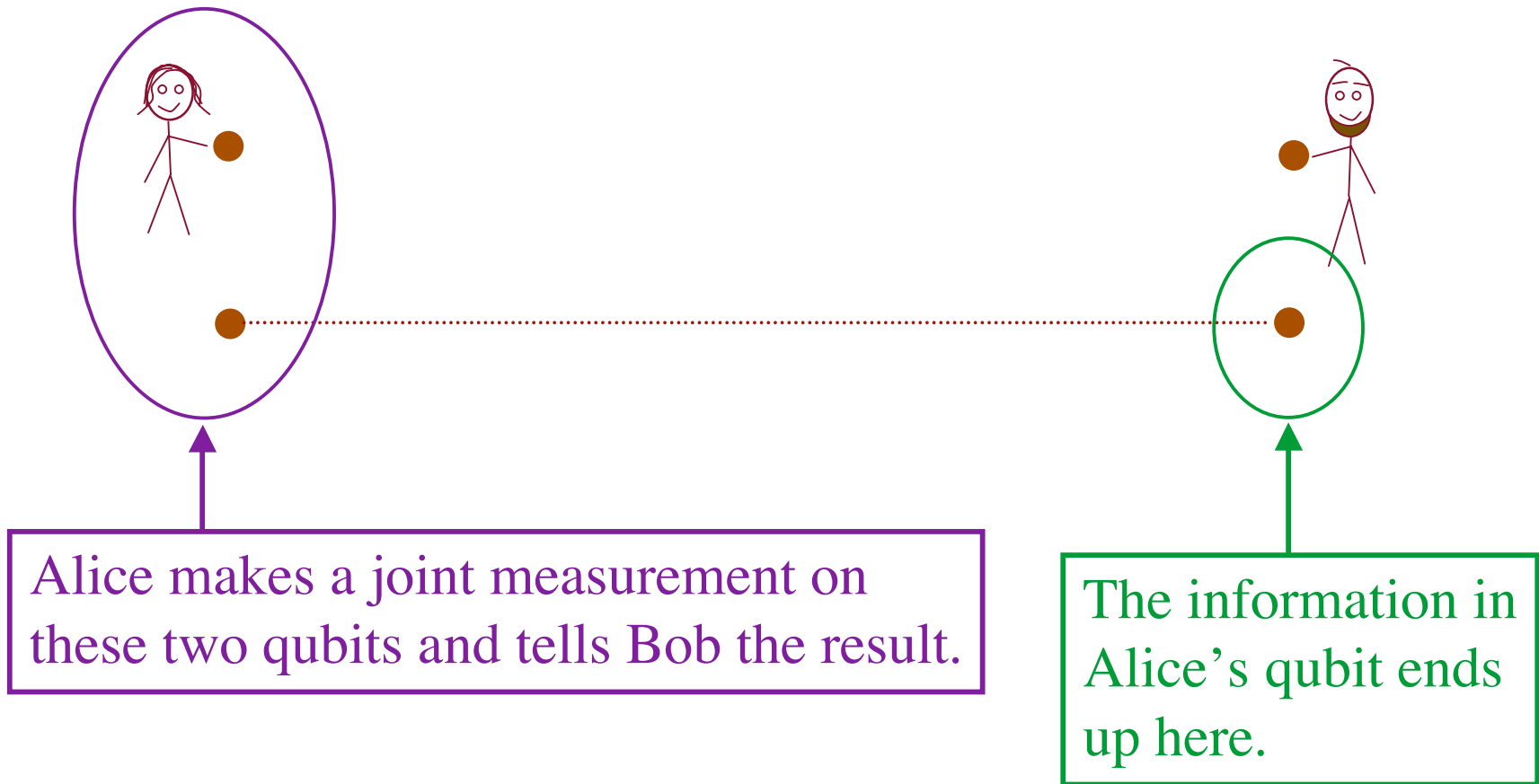
This curve is a *lower bound* on  $Q(M)$  for this measurement, because the measurement could *create* this much entanglement.



## How could the measurement create entanglement?



## One Way To Do the Measurement: Teleportation



Now Bob can just do the measurement. Cost = 1 ebit

It's much easier if  $a = 0$  or  $a = 1$

Then the states to be distinguished are  $|\uparrow\uparrow\rangle$ ,  $|\uparrow\downarrow\rangle$ ,  $|\downarrow\uparrow\rangle$ ,  $|\downarrow\downarrow\rangle$ .



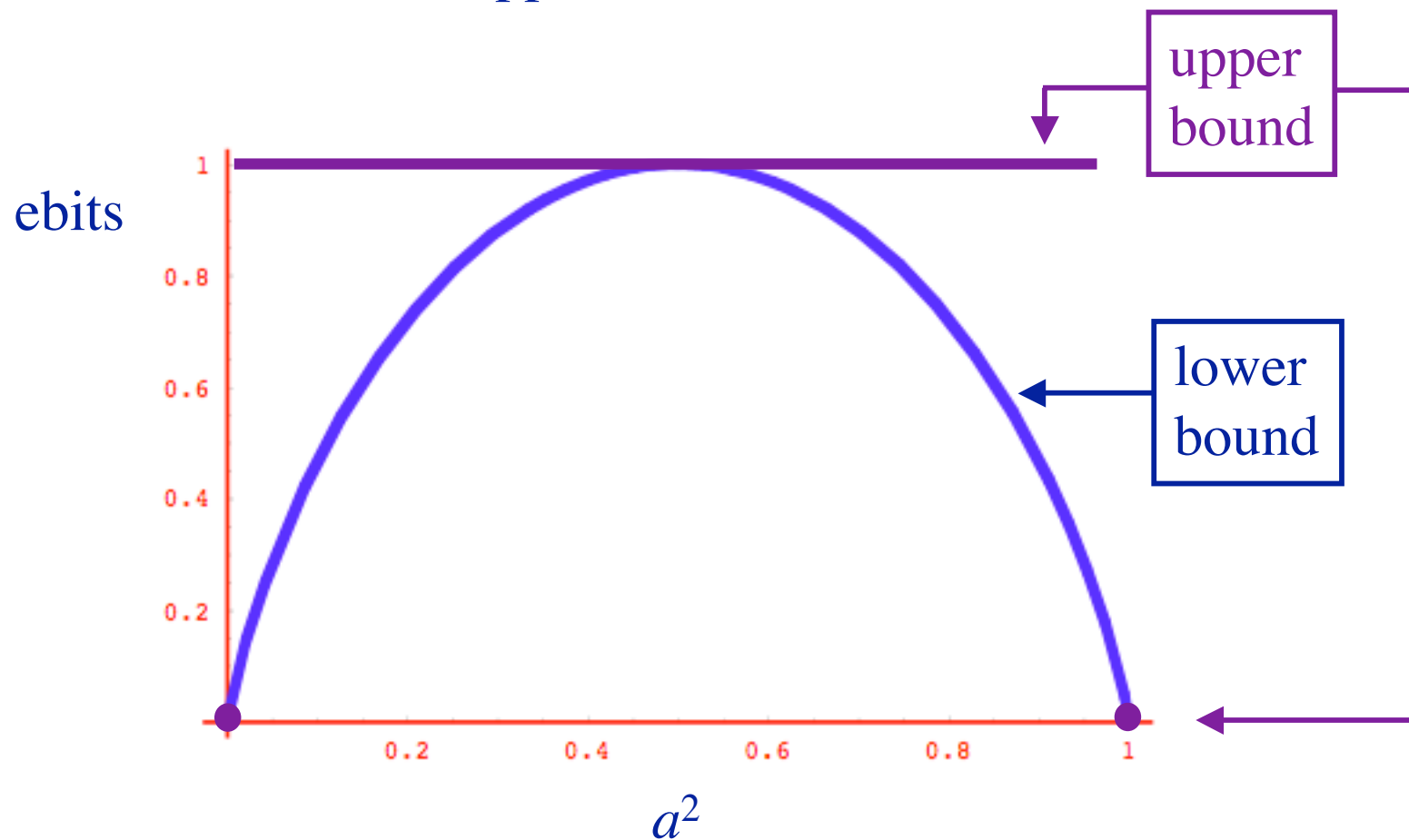
Alice measures  $|\uparrow\rangle$  vs  $|\downarrow\rangle$ .



Bob measures  $|\uparrow\rangle$  vs  $|\downarrow\rangle$ .

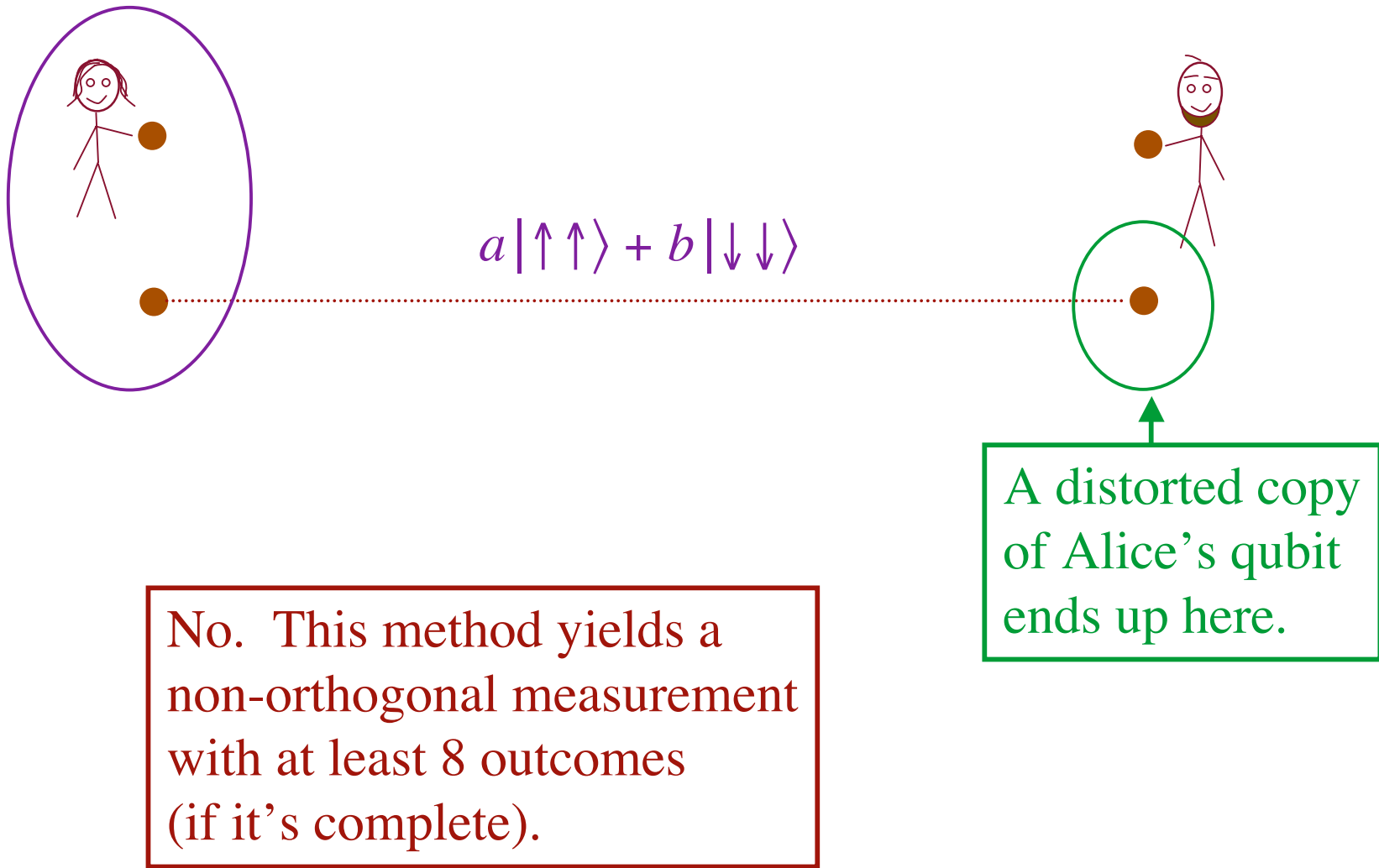
Cost = 0 ebits

## So We Have an Upper Bound on $Q(M)$

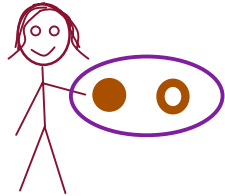


But do we really need a whole ebit to distinguish among states that are barely entangled?

## Can we do it in the following way? Unfaithful teleportation



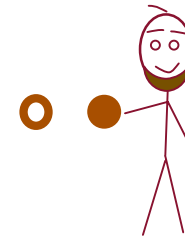
## A Method That Works



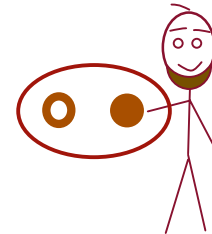
Alice moves *some* of the information from her qubit into another qubit (reversibly).



Alice teleports a qubit to Bob (cheaply).

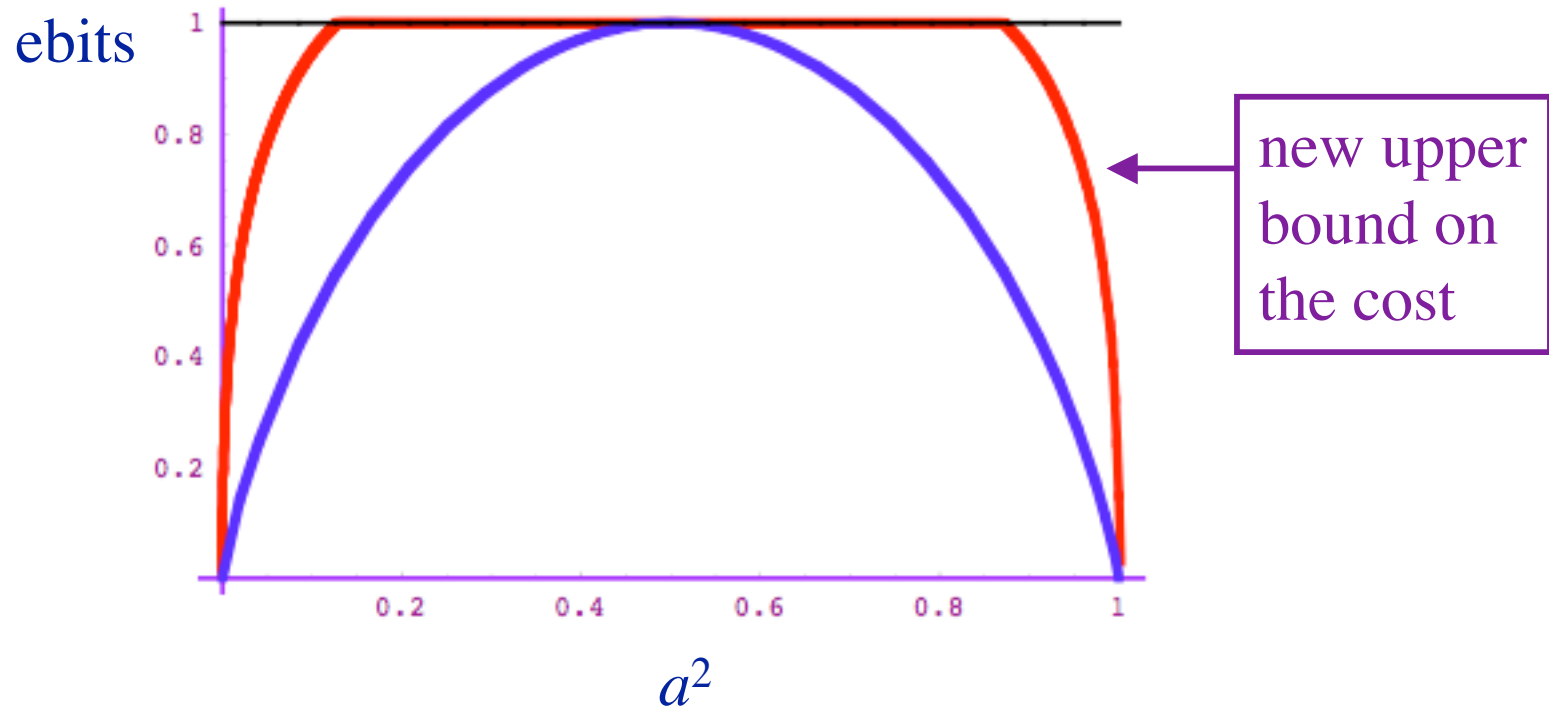


Bob attempts to finish the measurement using what he has, plus classical communication from Alice.



If he fails, Alice teleports the other qubit and Bob finishes the measurement.

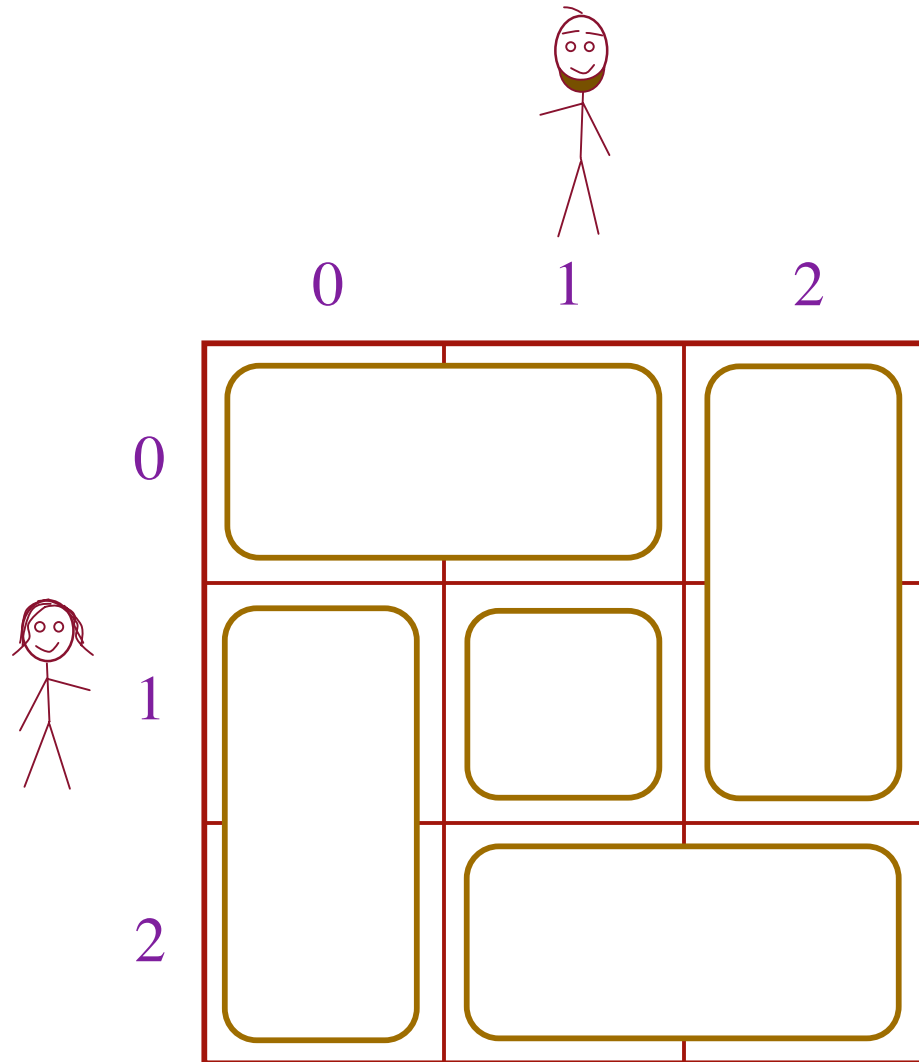
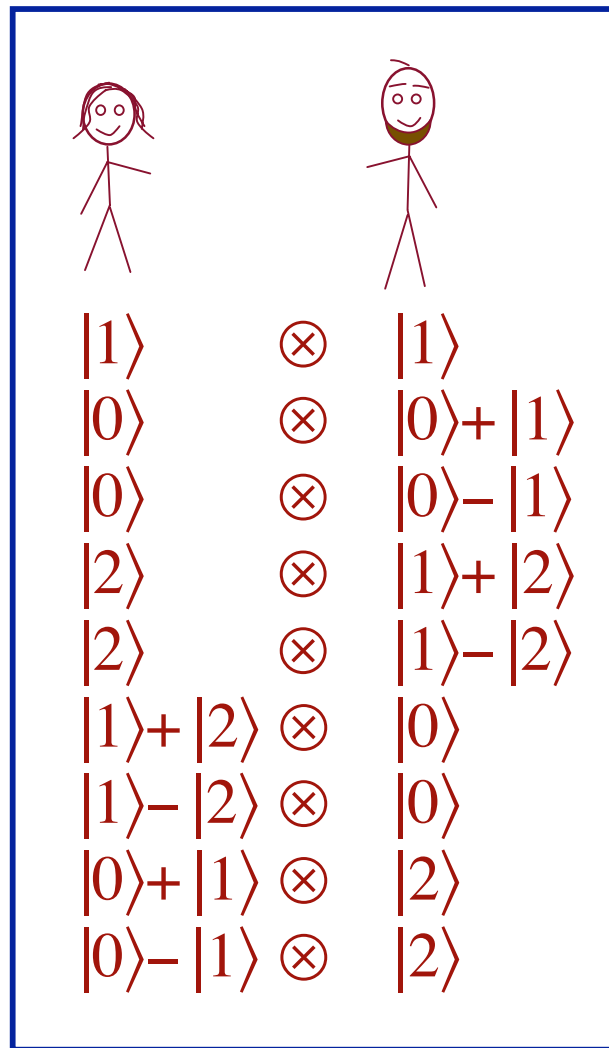
How Well Does This Work? (Shelby Kimmel, 4 days ago)



I think we can do better!

Should we expect to achieve the blue curve? Not necessarily.

An example in which the cost exceeds the entanglement of the outcome states (Bennett et al, 1999)





## Conclusions

- A nonlocal measurement with *partially* entangled outcome states can sometimes be done with less than maximal shared entanglement.
- But the entanglement cost of the measurement can exceed the entanglement of the outcome states.
- The function  $Q(M)$  seems to be hard to find, even for a pair of qubits. It depends not just on the entanglements of the outcome states, but also on the *relations* among these states.