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Phenomenology of Extreme Type-II Superconductors in the Mixed State

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Background

- liberal arts college, residential, suburban, co-educational
- undergraduate enrollment ~1500, faculty full-time~130
- Department: 3 physicists (1 theorist) + astronomer + non-teaching lab staff
- Physics: 1-4 majors/year, currently total of 30 majors
- History: physics major reinstated 2001

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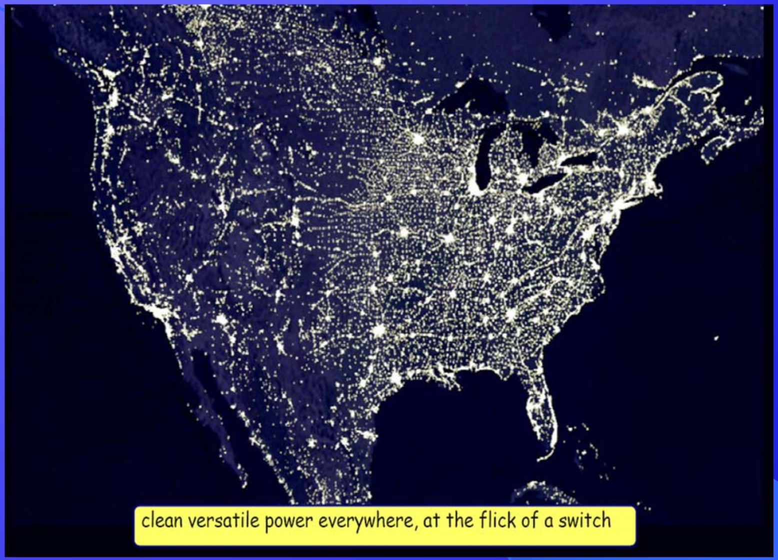
Collaborative Faculty/Student Research

- Goucher Science Student Summer Research Experience: 10-week, support for student + faculty (endowments + NSF, NASA, Research Corp., PRF, NIH etc.)
- Participation: 20% of students in sciences/math
- Theoretical Physics: 16 student collaborators since 1998 (physics+math+CS)
- Outcome: 9 grad school (4 Physics Ph. D.) + 7 (industry, government etc.)

Research with Undergraduate Students

Educational Goals:

- to engage physics, mathematics and/or computer science students together in an active learning environment of theoretical physics research
- to expose students to analytical and computational methods of quantum physics by providing open-ended investigative projects
- to foster learning through peer discussions across disciplines by involving students from two different majors (typically physics and computer science or math);
- to develop an awareness of scientific research in theoretical physics and its impact on emerging technologies



Environmental sustainability: "Extreme type-II superconductors have been identified as one of the technologies that offer powerful new opportunities for restoring the reliability, capacity and efficiency of the power grid". (DOE)

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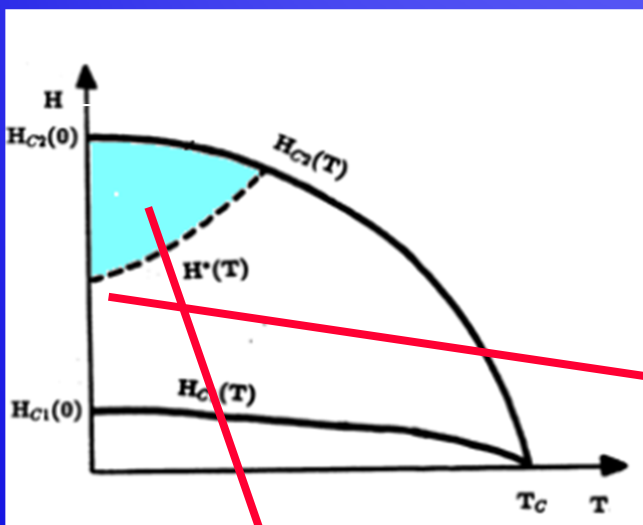
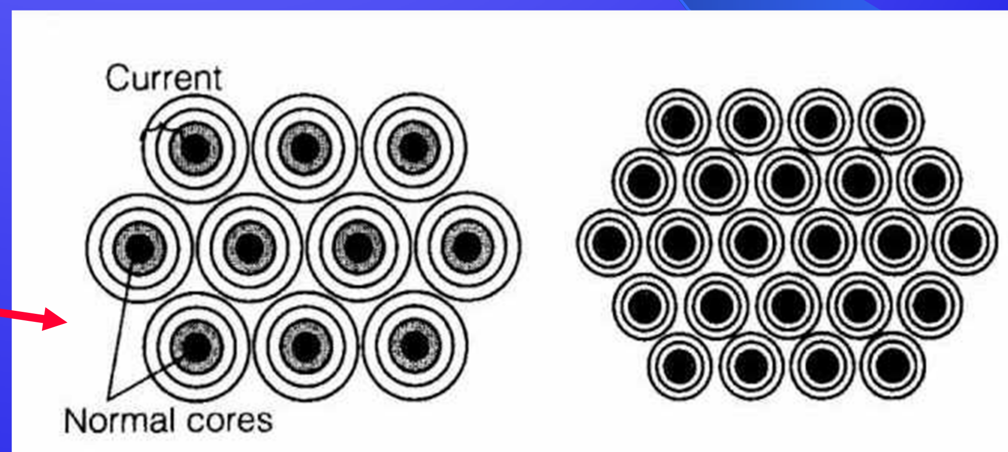
Extreme Type-II Superconductors in High Magnetic Fields

- $H_{c2}(0)$ in Tesla comparable or large than T_c in Kelvins
  (HTS, nonmagnetic nickel borocarbides, MgB_2, A-15, iron arsenides)



Vortex (Mixed) State



high fields+low temperature physics differs from low-field Abrikosov-Gorkov theory

Low Fields ($l \sim l_{mean}$)

High Fields ($l << l_{mean}$)

$l = \sqrt{\hbar / eH}$ Is a magnetic length

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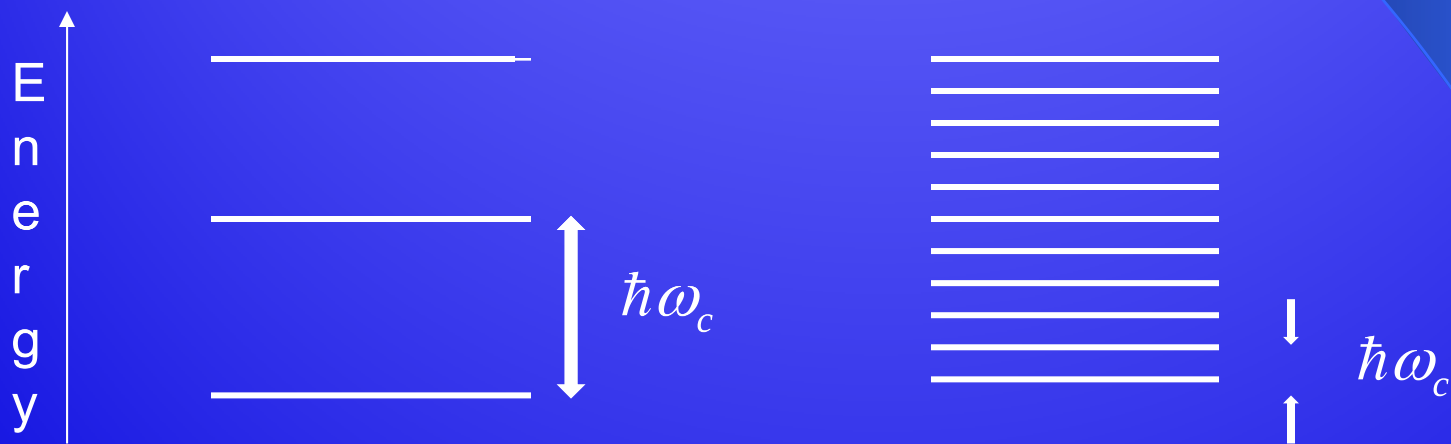
Landau Levels

• Landau level (LL) quantization of electronic energies in a magnetic field *within* the superconducting state is well defined at high fields

High fields+low temparatures   **Low fields+high temperature**



$$\omega_c = \frac{eH}{m^*}$$

$$k_B T, \Delta_{BCS}(H,T), \Gamma < \hbar\omega_c \qquad k_B T, \Delta_{BCS}(H,T), \Gamma > \hbar\omega_c$$

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Description of the model

- MF-Hamiltonian for a 3-dim, weakly coupled (s-wave) superconductor in high magnetic field

$$H = \sum_{\alpha,\beta=1,2} \int \psi_\alpha^\dagger(\vec{r}) \left[\frac{1}{2m^*} \left(-i\hbar\vec{\nabla} + \frac{e}{c}\vec{A} \right)^2 \delta_{\alpha\beta} + U_{\alpha\beta}(\vec{r}) - g\mu_B \vec{\sigma}\cdot\vec{H} - \mu \right] \psi_\beta(\vec{r})\, d^3r + \int \Delta(\vec{r})\psi_\uparrow^\dagger(\vec{r})\psi_\downarrow^\dagger(\vec{r})\, d^3r + H.c.$$

- order parameter $\Delta(\vec{r}) = V\left\langle \psi_\uparrow(\vec{r})\psi_\downarrow(\vec{r}) \right\rangle$ is constructed from LLs for charge *2e*

Assumptions:

- H(**r**) is uniform since closely packed vortex lattice, **A** in Landau gauge
- no Zeeman splitting, g≈0
- $U_{\alpha\beta}(\vec{r}) = \sum_j U_{\alpha\beta}\left(\vec{r} - \vec{R}_j\right)\delta_{\alpha\beta}$ is a non-magnetic random impurity contribution
- Order parameter Δ(**r**) forms a vortex lattice if dirty but homogenous superconductor ($\xi >> \xi_{imp}$)

Clean System in a High Magnetic Field

- Extension of BCS theory to finite temperatures and non-uniform order parameter
- Magnetic sub-lattice representation (MSR) characterized by a quasi-momentum **q** ⊥ **H***
- Eigenfunctions constructed to preserve one electronic flux per unit cell:

$$\varphi_{k_z\vec{q}n}(\vec{r}) = \sqrt{\frac{b_y}{2^n n!\sqrt{\pi} l L^3}} \exp(ik_z z) \sum_k \exp\left(i\frac{\pi b_x}{2a}k^2 - ikq_y b_y\right) \exp\left[i\left(q_x + \frac{\pi}{a}\right)x - \frac{1}{2}\left(\frac{y}{l} + q_x l + \frac{\pi k}{a}l\right)^2\right] H_n\left(\frac{y}{l} + \left(q_x + \frac{\pi}{a}\right)l\right)$$

- BdG transformation:

$$\psi_\uparrow(\vec{r}) = \sum_{k_z\vec{q}n}[u_{k_z\vec{q}n} c_{\uparrow k_z\vec{q}n} - v^\dagger_{-k_z-\vec{q}n} c^\dagger_{\downarrow -k_z-\vec{q}n}]\varphi_{k_z\vec{q}n}(\vec{r})$$

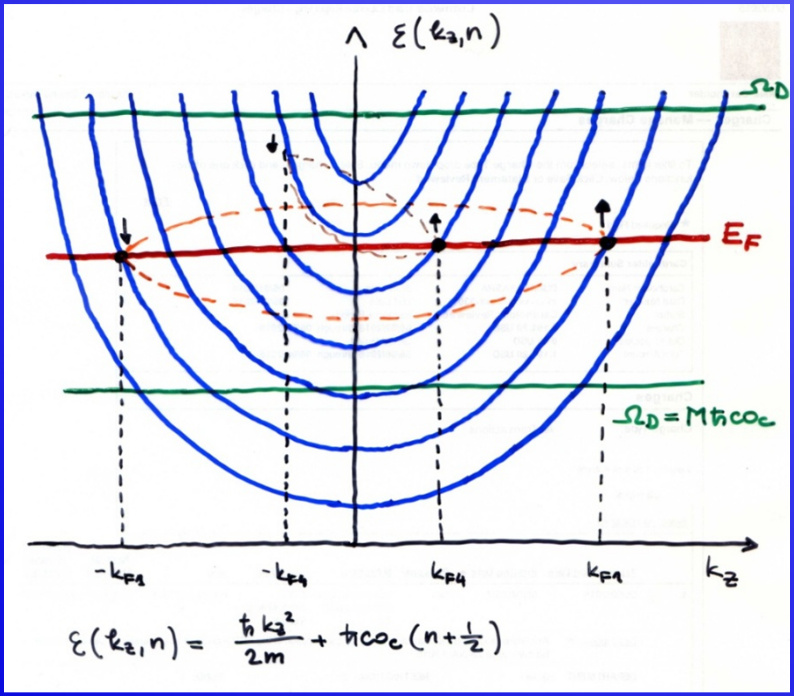
$$\psi_\downarrow(\vec{r}) = \sum_{k_z\vec{q}n}[u_{k_z\vec{q}n} c_{\downarrow k_z\vec{q}n} - v^\dagger_{-k_z-\vec{q}n} c^\dagger_{\uparrow -k_z-\vec{q}n}]\varphi_{k_z\vec{q}n}(\vec{r})$$

*SD and Z. Tesanovic: "Quantized Landau Levels in Superconductors", invited review article (chapter) in the book: "The Superconducting State in Magnetic Fields: Special Topics and New Trends", Edited by Carlos A. R. Sa de Melo, Series on Directions in Condensed Matter Physics –Vol 13, 197, World Scientific, Singapore (1998).

Clean system in high magnetic field

BdG equations:

- formally a two-component Schrödinger equation for quasiparticle amplitudes
- order parameter acts as an off-diagonal potential
- self-consistency condition $\Delta(\vec{r}) = V \left\langle \psi_{\uparrow}(\vec{r}) \psi_{\downarrow}(\vec{r}) \right\rangle$



diagonal: $\left|-k_z - \vec{q}\; n\right\rangle_{\uparrow} \leftrightarrow \left|k_z \vec{q}\; n\right\rangle_{\downarrow}$

off-diagonal: $\left|-k_z - \vec{q}\; n\right\rangle_{\uparrow} \leftrightarrow \left|k_z \vec{q}\; n \pm m\right\rangle_{\downarrow}$

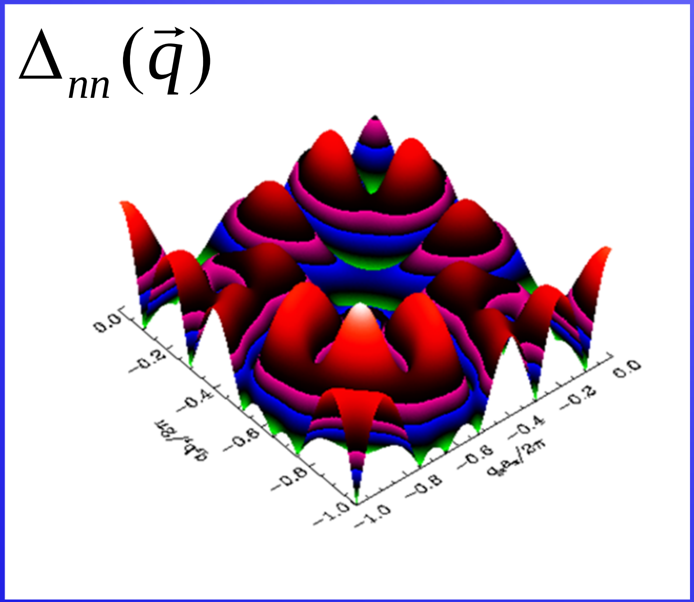
$m = 1, 2, \ldots\ldots M = \mathrm{int}\left(\Omega_D / \hbar\omega_c\right)$

diagonalizing $2(n_c+M)$ x $2(n_c+M)$ matrix

Realistic systems: $n_c = \mathrm{int}\left(E_F / \hbar\omega_c\right)$ can be few tens ($H \leq H_{c2}$) to few thousands ($H \sim 0.5 H_{c2}$)

Quasiparticle Excitation Spectrum

High field: formation of gapless or near gapless excitations at the Fermi level



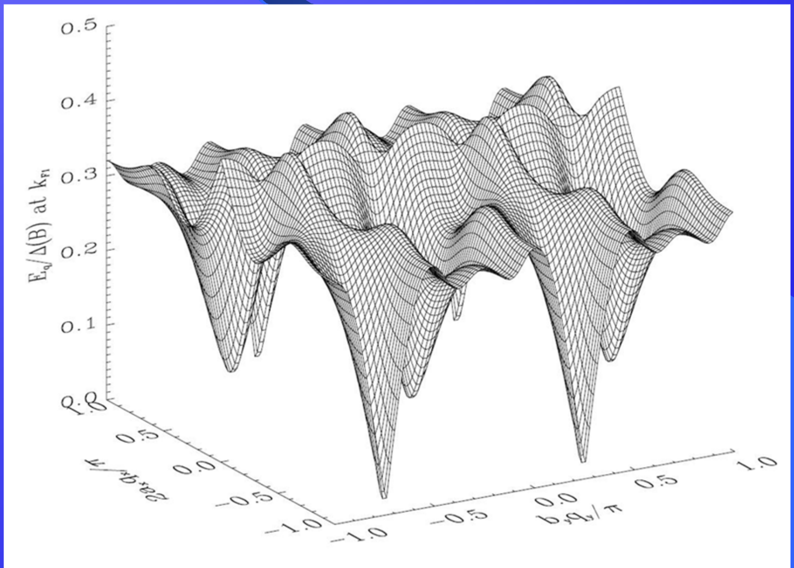


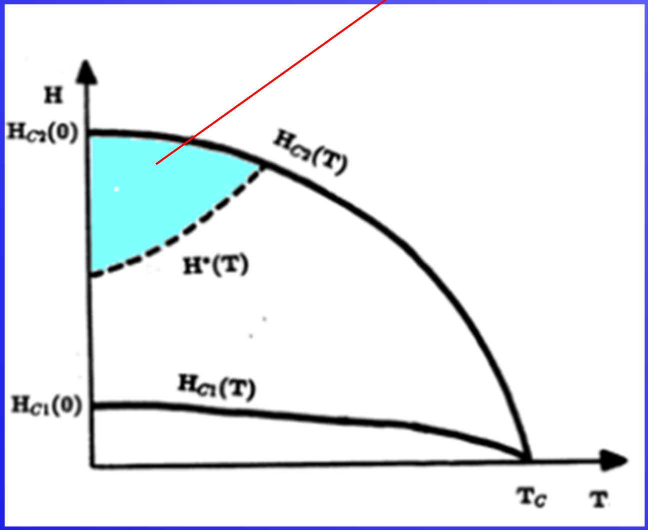
Figure 2: Quasiparticle energies in 48th Landau level (diagonal approximation*) (graphics by Michael Garmin, Goucher '10)

Figure 3: Quasiparticle energies obtained when off-diagonal pairing included** (graphics by T. Villazon Goucher '14)

SD, T. P. Powell and Z. Tesanovic , PRB **66** (2002); L. Carr*, J. J. Trafton*, SD and Z. Tesanovic, PRB, **68**, (2003).
**SD, J. Irwin* and T. Villazon* , in preparation.

Quasiparticle Excitation Spectrum

novel gapless superconductivity: coherent gapless excitations



- center-of-mass motion of Cooper pairs in magnetic field (s-wave and d-wave*)
- in 3-dim *gaplessness* persists to H*~(0.2-0.5)H_{c2}
- H* can be estimated from dHvA experiments in the mixed state
- Below H* gaps open-up, localized states in the vortex core (s-wave) or extended states (d-wave)

- gapless quasiparticle excitations lead to qualitatively different thermodynamics, transport, acoustic attenuation, tunneling etc.

* K. Yasui and T. Kita, Phys. Rev. B **66**, 184516 (2002)

Disorder Effects in High Magnetic Fields

- Perturbative approach*: Green's functions 2 x 2 Nambu matrix $\hat{G}(\vec{r};i\omega)$ for a clean superconductor is dressed via scattering

$$i\omega \rightarrow i\tilde{\omega} = i\omega - \Sigma_{nn}^{N}(i\omega)$$

$$\Delta_{nn}(\vec{q}) \rightarrow \tilde{\Delta}_{nn}(\vec{q}) = \Delta_{nn}(\vec{q}) + \Sigma_{nn}^{A}(\vec{q},i\omega)$$

- dirty but homogenous superconductor
- non-magnetic short-range impurity potential
- diagonal approximation since no qualitative difference in excitation spectrum
- Scattering does not mix LLs ($U_0 << \hbar\omega_c$)

- *T*-matrix approach:

$$\hat{T}\left(\vec{r},\vec{r}';\omega\right) = U\left(\vec{r}\right)\delta\left(\vec{r}-\vec{r}'\right)\sigma_z + \int d\vec{r}_1 U\left(\vec{r}\right)\sigma_z \hat{G}\left(\vec{r},\vec{r}_1;\omega\right)\hat{T}\left(\vec{r}_1,\vec{r}';\omega\right)$$

- Self-energies are diagonal (with respect to MSR) *T*-matrix elements averaged over impurity random positions

SD, J. Tenebaum, J .Porembski* and K. Tata*, PRB **82** (2010)

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Disorder Effects on DOS in High Magnetic Fields

- non-linear complex integral equation for self-energies

$$u = \frac{\omega}{\Delta} + g \frac{\sum_n \frac{m^*}{4\pi^3 k_{Fn} N(0)} \int d\vec{q} \frac{(1-\sqrt{2}\left|f_{nn}(\vec{q})\right|^2)u}{\sqrt{u^2 + \left|f_{nn}(\vec{q})\right|^2}}}{c^2 - \left[\sum_n \frac{m^*}{4\pi^3 k_{Fn} N(0)} \int d\vec{q} \frac{u}{\sqrt{u^2 + \left|f_{nn}(\vec{q})\right|^2}}\right]^2}$$

where $u = \frac{\tilde{\omega}}{\tilde{\Delta}}$ *and* $f_{nn}(\vec{q}) = \frac{\Delta_{nn}(\vec{q})}{\tilde{\Delta}}$

g measures inverse scattering rate

c measures scattering strength

- Solution *u* determines density of states (DOS) of a superconductor in presence of disorder

$$N(\omega)/N(0) = \frac{1}{N(0)} Im \sum_{n=0}^{n_c} \frac{m}{4\pi^3 k_{Fn}} \int d\vec{q} \frac{u}{\sqrt{|f_{nn}(\vec{q})|^2 - u^2}}$$

Phenomenology of Superconductors at High Fields: Tunneling (STM) Current

Figure 1: Differential conductance σ(V) for a disordered $LuNi_2B_2C$ superconductor at zero temperature rescaled by a normal state value[6].

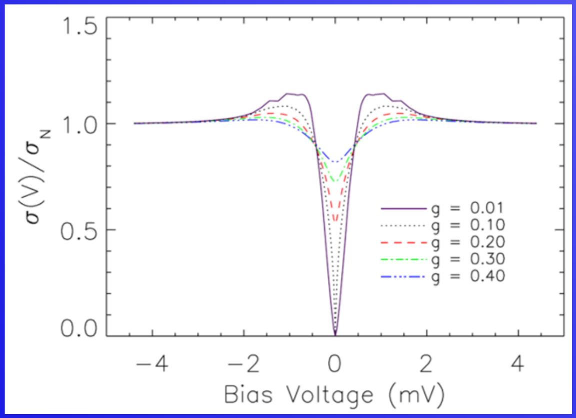

Fig 1a: In a field H = 5.5 Tesla in the weak-scattering limit (c=1) vs. disorder parameter g .

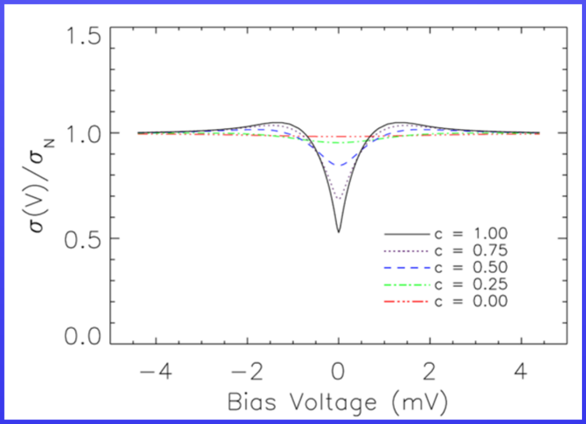

Fig 1b: In a field H = 5.5 Tesla as a function of disorder parameter c with g=0.2.

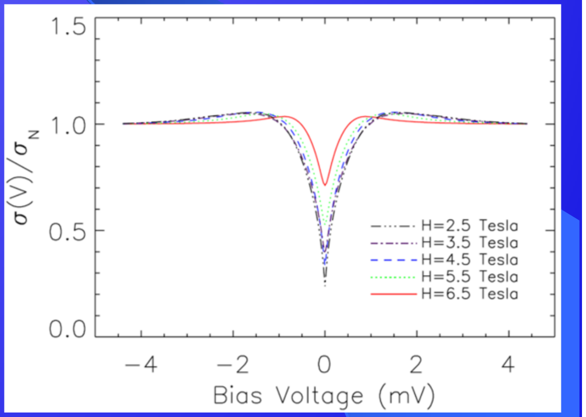

Fig 1c: At different fields in the weak-scattering limit (c=1) and with disorder parameter g=0.2.

6. SD, J. Tenebaum*, J .Porembski* and K. Tata*, PRB **82** (2010)

Acknowledgement

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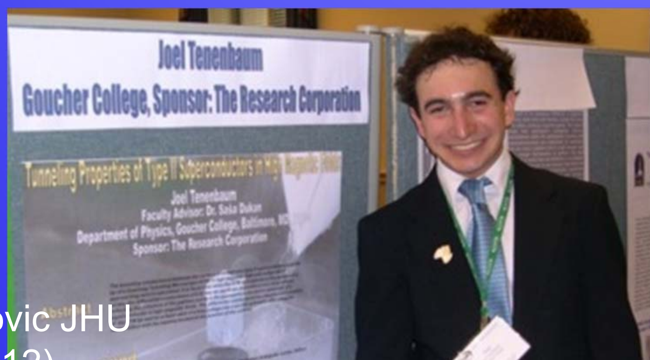

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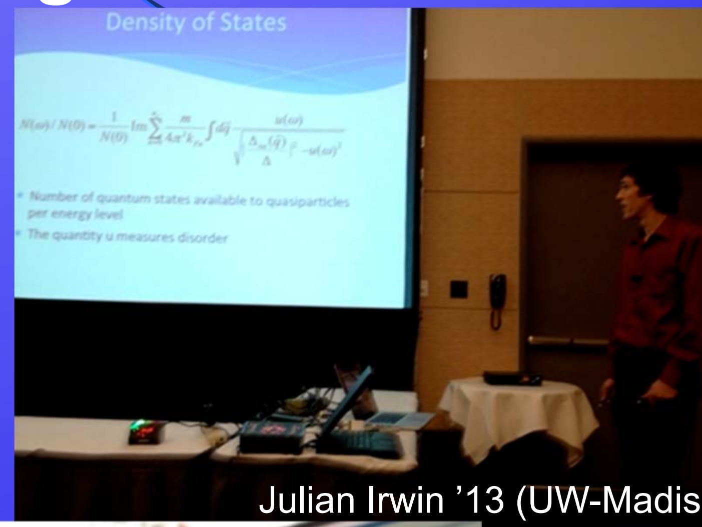

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