Dualities, Dimensions, and Uncertainties:
A New Perspective on Quantum Black Holes

LMU|LA Loyola Marymount University Los Angeles

Jonas Mureika
Department of Physics
Loyola Marymount University

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Based on:

B. Carr, J. Mureika, P. Nicolini, arXiv:1504.07637 [gr-qc] (to appear in JHEP)

The Basics

- **Gravitation**
  - General relativistic formulation
  - Black holes, horizons, and singularity problems
- **Quantum Mechanics**
  - Uncertainty and limits of classical / quantum boundary
- **Dualities**
  - Common behavior between seemingly disparate systems
- **Physics of (n+1)-D spacetime**
  - How is the world different with more/less dimensions?

Characteristic Scales of Nature

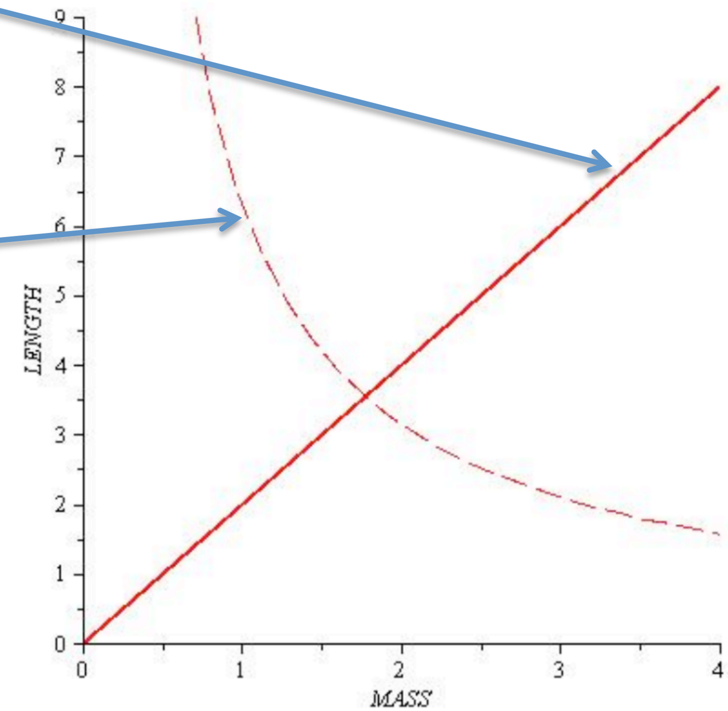
The large- and short-scale characteristics of Nature are defined by different and (apparently) disconnected theories and length scales

Large: General Relativity

$$r_g = \frac{2GM}{c^2} \longrightarrow M$$

Short: Quantum Mechanics

$$\lambda_C = \frac{\hbar}{Mc} \rightarrow \frac{1}{M}$$

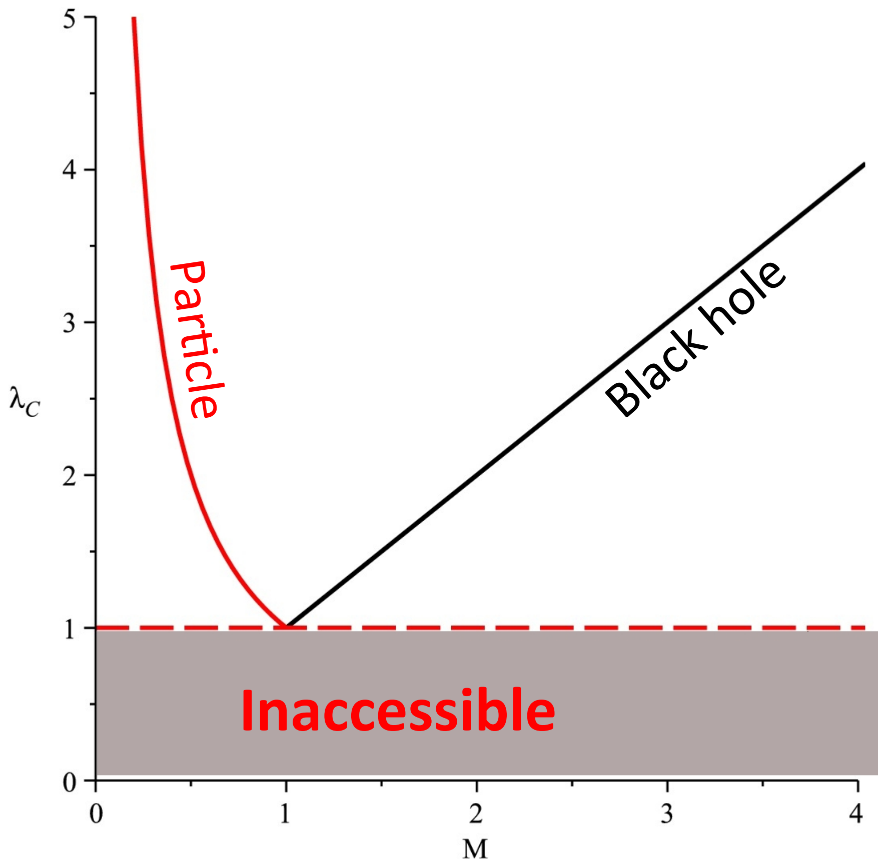


Critical Points in Gravitation

The point where $r_g \approx \lambda_C$ is a critical point

$$\lambda_C = r_g \implies M_{\min} = \frac{1}{\sqrt{G}} = M_{\rm Pl}$$

$$\lambda_{\min} = \ell_{\rm Pl}$$



$$\ell_P = \sqrt{\frac{\hbar G}{c^3}}$$

Common scale for relativity and quantum

Defines the smallest black hole, or alternatively the largest particle

The Uncertainty Principle and Gravity

Quantum mechanics defines its own characteristic length via the Heisenberg Uncertainty Principle (HUP)

$$\Delta x_Q \geq \frac{\hbar}{2\Delta p}$$

Gravitation *also* defines a characteristic length

$$r_G = \frac{2GM}{c^2}$$

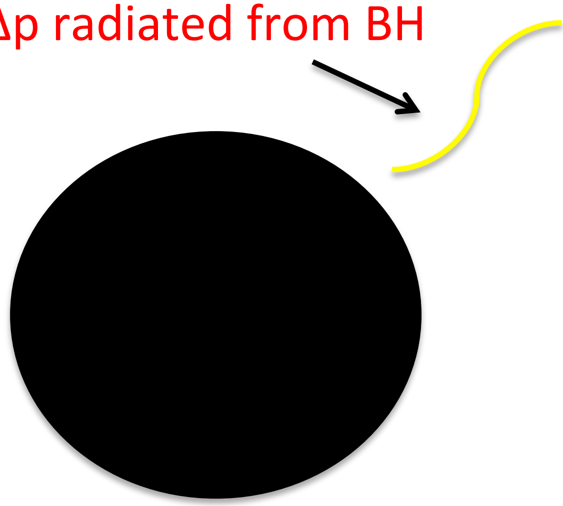
Again, the dependence on mass (momentum) is different!

But curiously, the HUP discloses a direct link to gravitation

Hawking Temperature from HUP

The temperature of a black hole can be derived from the HUP!

Photon of momentum
Δp radiated from BH



$$\Delta p \sim \frac{\hbar}{\Delta x} \quad , \quad \Delta x = \frac{2GM}{c^2}$$

Position uncertainty
must be horizon size

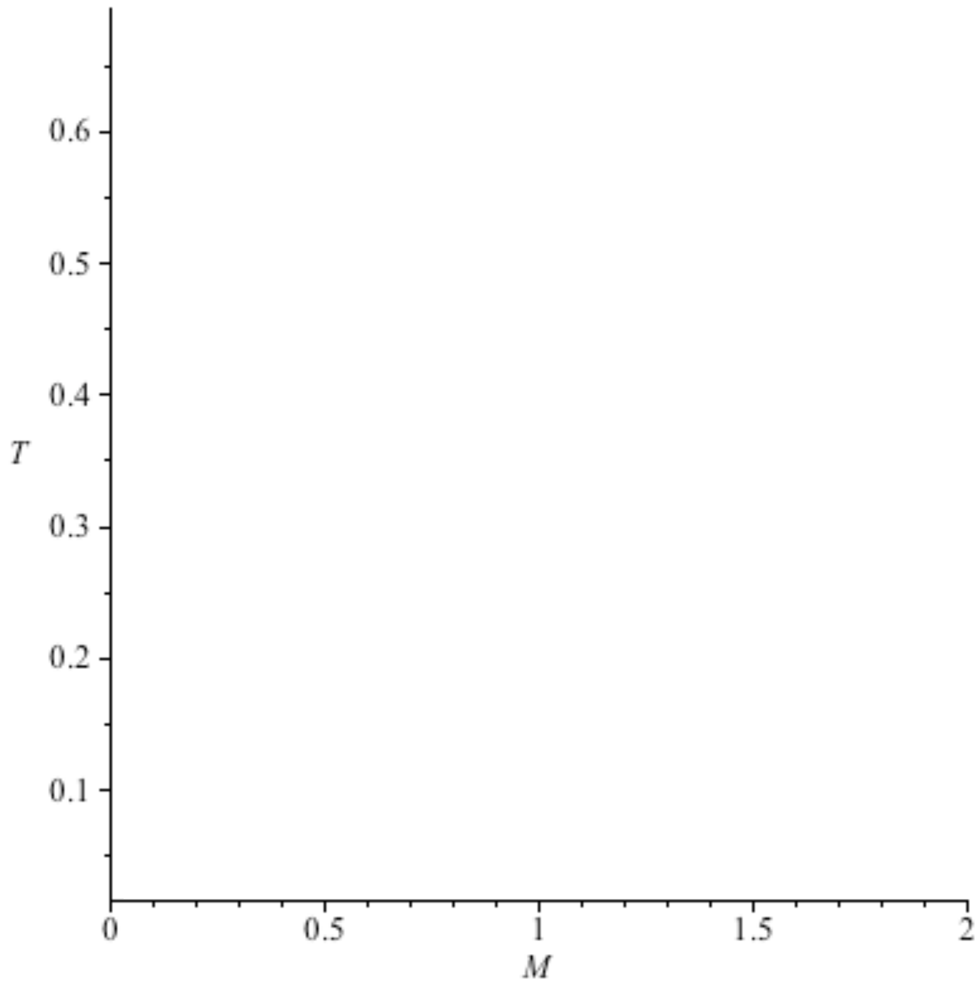
$$\Longrightarrow \quad \Delta p \sim \frac{\hbar c^2}{2GM}$$

Photon energy defines
black hole temperature

$$\longrightarrow \quad T = \Delta p c \quad \Longrightarrow \quad \boxed{T = \frac{\hbar c^3}{2GM}}$$

Black Holes and Quantum Mechanics

Classical black holes (Schwarzschild) don't behave well in the quantum regime (predicted from both QM and GR)



$$T \sim \frac{1}{M}$$

hen $M \to 0$

The *Generalized* Uncertainty Principle

Heisenberg Uncertainty Principle defines ***quantum*** uncertainty:

$$\Delta x_Q \geq \frac{\hbar}{2\Delta p}$$

Schwarzschild radius defines ***gravitational*** uncertainty:

$$\Delta x_G = \frac{2GM}{c^2}$$

***Total* uncertainty is determined by QM, but also gravitation:**

$$\Delta x \sim \Delta x_Q + \Delta x_G \quad \Longrightarrow \quad \Delta x \sim \frac{\hbar}{\Delta p} + G\Delta p$$

Duality! Large and small equally treated!

Dualities in Physics

A duality defines **common physical description** or behavior between two otherwise disparate systems

- **Length scales (T-duality)**
  - Behavior of a system on a scale R is equivalent to the behavior of a system on a scale 1/R
- **Coupling Strength (S-duality)**
  - The physics of a strongly-coupled system in one theory is equivalent to the physics of a weakly coupled system in another theory

What About Mass Duality?

[Carr, JRM, Nicolini, arXiv:1504.07637]

Can black holes exist *below* the Planck scale? $M_{\mathrm{BH}} < M_{\mathrm{Pl}}$

Use the GUP to emphasize the ***duality*** in the black hole mass

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left(1 + \beta \Delta p^2\right)$$

$$\Delta x \sim \frac{1}{\Delta p} + \Delta p$$

$$\Delta x_G \sim \frac{1}{M_{\mathrm{bh}}} + M_{\mathrm{bh}}$$

Can we encode this in the metric?

GUP and Sub-Planckian Black Holes

[Carr, JRM, Nicolini, arXiv:1504.07637]

Assume a duality in the *mass:*

$$M \longrightarrow M\left(1 + \frac{\beta}{2}\frac{M_{\rm Pl}^2}{M^2}\right)$$

Metric is:

$$ds^2 = F(r)dt^2 - F(r)^{-1}dr^2 - r^2 d\Omega^2$$

$$F(r) = 1 - \frac{2}{M_{\rm Pl}^2}\frac{M}{r}\left(1 + \frac{\beta}{2}\frac{M_{\rm Pl}^2}{M^2}\right)$$

Planck mass is now critical point for which…

$$M \gg M_{\rm Pl} \quad \Longrightarrow \quad F(r) \sim 1 - \frac{M}{r}$$

$$M \ll M_{\rm Pl} \quad \Longrightarrow \quad F(r) \sim 1 - \frac{1}{Mr}$$

Black Hole Characteristics: Horizon

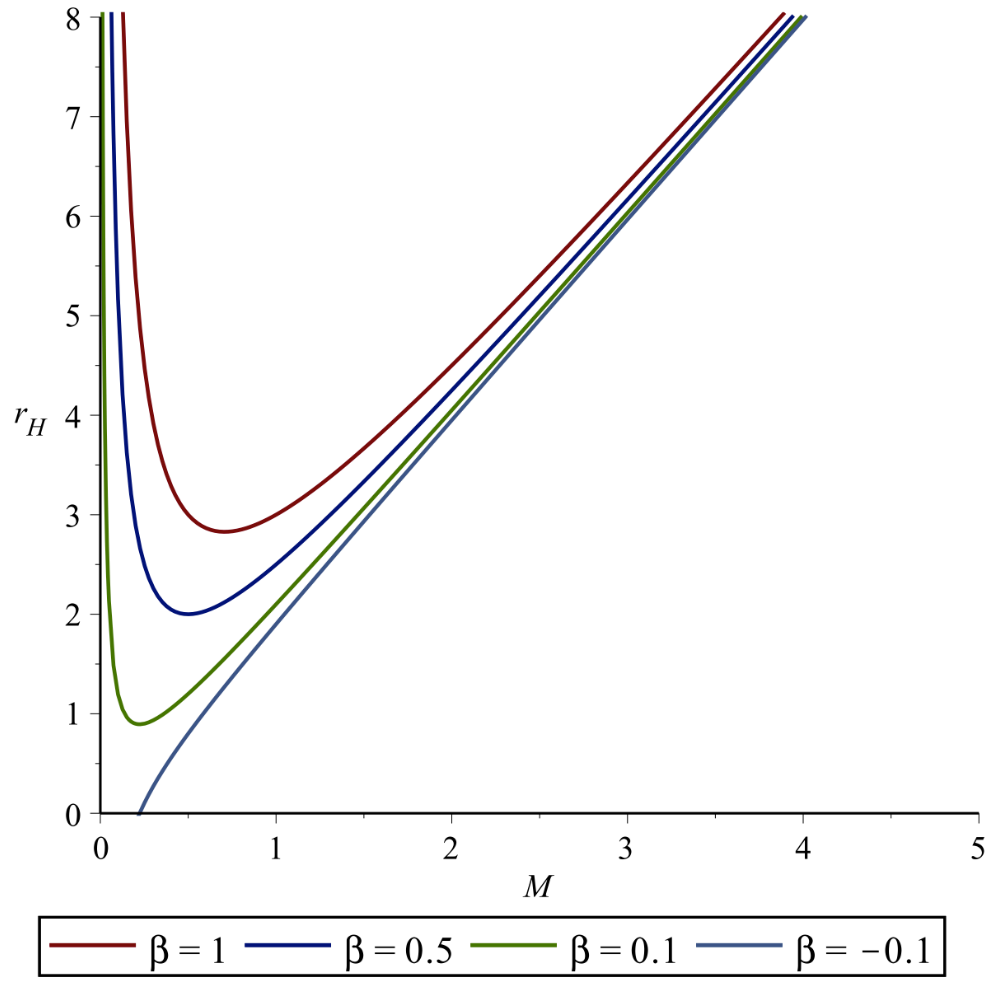
$$F(r_H) = 0$$

$$r_H = \frac{2}{M_{\mathrm{Pl}}^2}\left(\frac{M^2 + \frac{\beta}{2}M_{\mathrm{Pl}}^2}{M}\right)$$

$$M \gg M_{\mathrm{Pl}} \quad \Longrightarrow \quad r_H \approx \frac{2M}{M_{\mathrm{Pl}}^2}$$

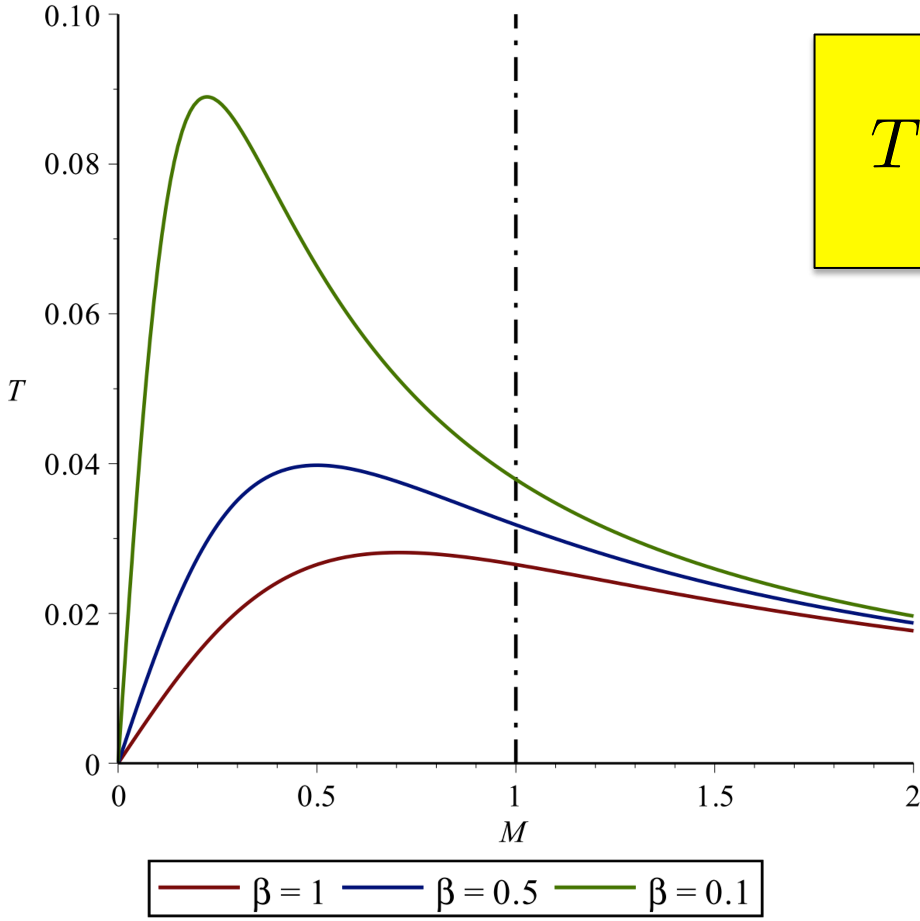
$$M \sim M_{\mathrm{Pl}} \quad \Longrightarrow \quad r_H \sim \frac{2+\beta}{M_{\mathrm{Pl}}}$$

$$M \ll M_{\mathrm{Pl}} \quad \Longrightarrow \quad r_H \approx \frac{\beta}{M}$$



Black Hole Characteristics: Temperature

From surface gravity: $T = \dfrac{\kappa}{2\pi} \quad , \quad \kappa = \dfrac{1}{2}\dfrac{dF}{dr}(r = r_H)$



$$T = \frac{M_{\rm Pl}^2}{8\pi M(1 + \beta M_{\rm Pl}^2/2M^2)}$$

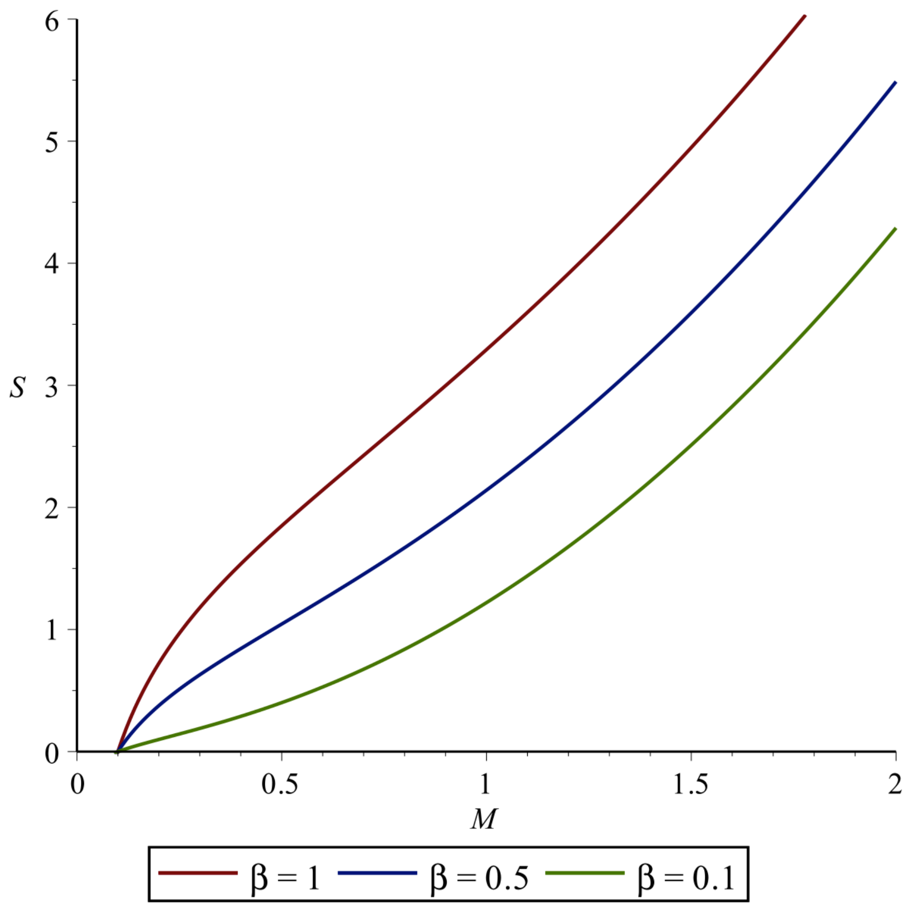
$$M \gg M_{\rm Pl} \quad \Longrightarrow \quad T \approx \frac{M_{\rm Pl}^2}{8\pi M}$$

$$M \sim M_{\rm Pl} \quad \Longrightarrow \quad T \sim \frac{M_{\rm Pl}}{8\pi(1 + \beta/2)}$$

$$M \ll M_{\rm Pl} \quad \Longrightarrow \quad T \approx \frac{M}{4\pi\beta}$$

Black Hole Characteristics: Entropy

$$S = \int_{M_0}^{M} \frac{dM'}{T(M')} \sim 4\pi \left(\frac{M^2}{M_{\rm Pl}^2} - \frac{M_0^2}{M_{\rm Pl}^2} + \beta \log \frac{M}{M_0} \right)$$



$$M \gg M_{\rm Pl} \implies S \sim M^2$$

$$M \ll M_{\rm Pl} \implies S \sim \log\left(M/M_0\right)$$

General Relativity in (n+1)-Dimensions

(n+1)-D

$$ds^2 = f(r)\,dt^2 - \frac{dr^2}{f(r)} + r^2 d\Omega_{n-2}$$

$$f(r) = 1 - \frac{2G_n M}{r^{n-2}}$$

n-dimensional Newton's constant

(3+1)-D

$$f(r) = 1 - \frac{2G_3 M}{r}$$

(1+1)-D

$$f(r) = 1 - 2G_1 M|x|$$

Why Do We Care About Lower Dimensions?

- Extra dimensions are the rage!

- A surprising feature of many different quantum gravity theories is *dimensional reduction*

- The effective dimension of spacetime seems to reduce as one goes to higher energy (lower length scales)

- Could this be related to some kind of duality?...

Thermodynamics of (3+1)-D vs (1+1)-D Black Holes

(3+1)-D

$$g_{tt} = 1 - \frac{2G_N M}{r}$$

$$g_{rr} = -g_{tt}^{-1}$$

$$r_H \sim M$$

$$T \sim \frac{1}{M}$$

$$S \sim M^2$$

(1+1)-D

$$g_{tt} = 1 - G_1 M|x|$$

$$g_{xx} = -g_{tt}^{-1}$$

$$r_H \sim \frac{1}{M}$$

$$T \sim M$$

$$S \sim \log(M)$$

Sub-Planckian regime →

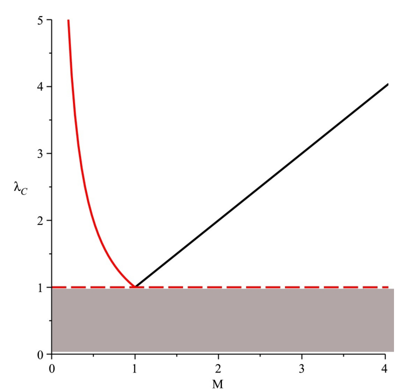
"Dimensional reduction"

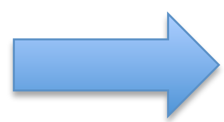
The gravitational physics of the sub-Planckian regime is governed by an *effective* (1+1)-D

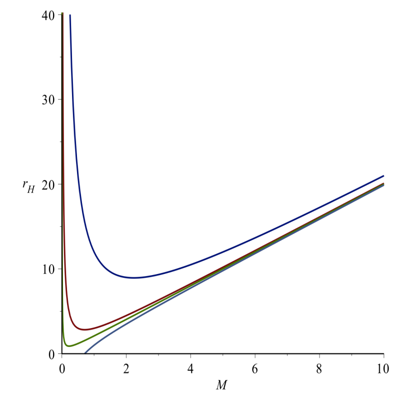
Why Is All This Interesting?

- "Encoding" the GUP duality in the mass gives a metric that exhibits dimensional reduction in the sub-Planckian regime (feature of many quantum gravity theories!)

- Smooths out "GUP" diagram; no critical point:







- Instead of a two regimes governed by different theories (GR and QM), we have a consistent theory (gravity) in two different spacetime dimensions

Thank you!

jmureika@lmu.edu