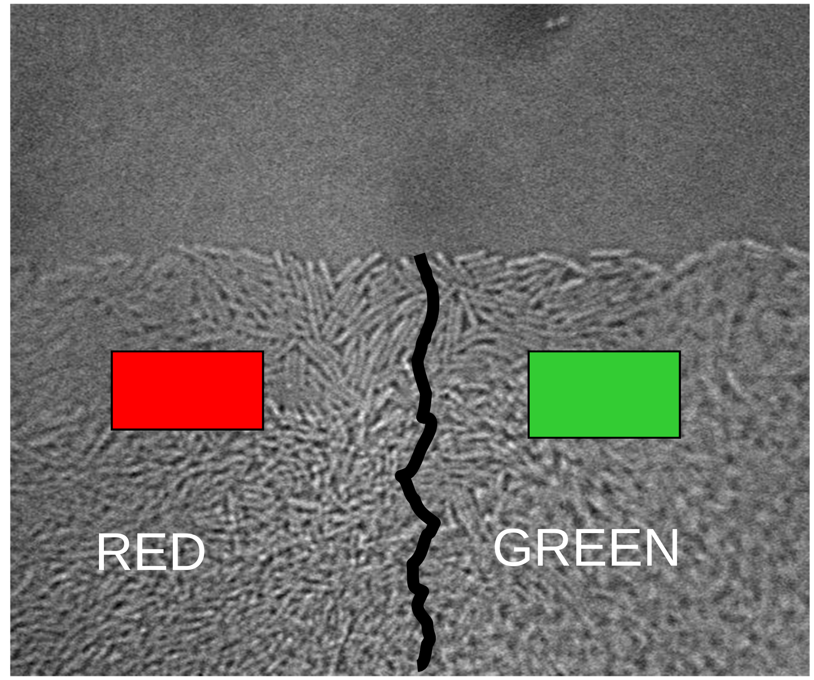
Range Expansions with Competition or Cooperation



In 500 generations….

Large mammals expand over ~10^4 km

Bacteria (in a Petri dish) expand ~ 1 cm

Red and Green Strains….

1. *Could be neutral….*
2. *Could have different doubling times*
3. *One or both could secrete toxins that impede the other…*
4. *One or both could secrete amino acids useful to the other (mutualism)*

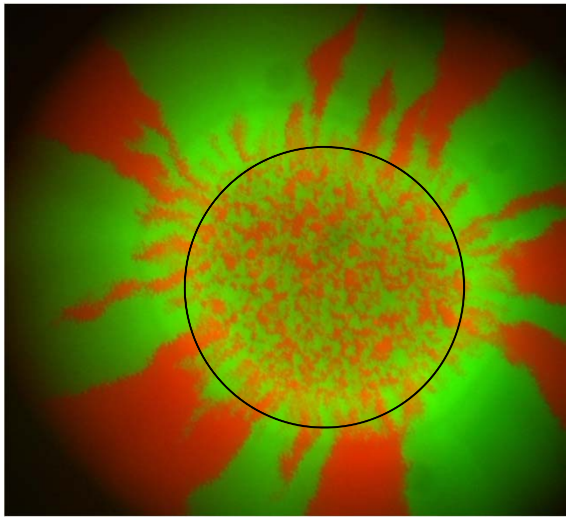
Competition and Cooperation at Frontiers

Why Frontiers?

K. Korolev & drn

--Range expansions are very common in biology… Number fluctuations very large at the edge of a population wave

-- Can we test theories of frontier evolution and cooperation with colored bacterial strains with variable "mutualism"?



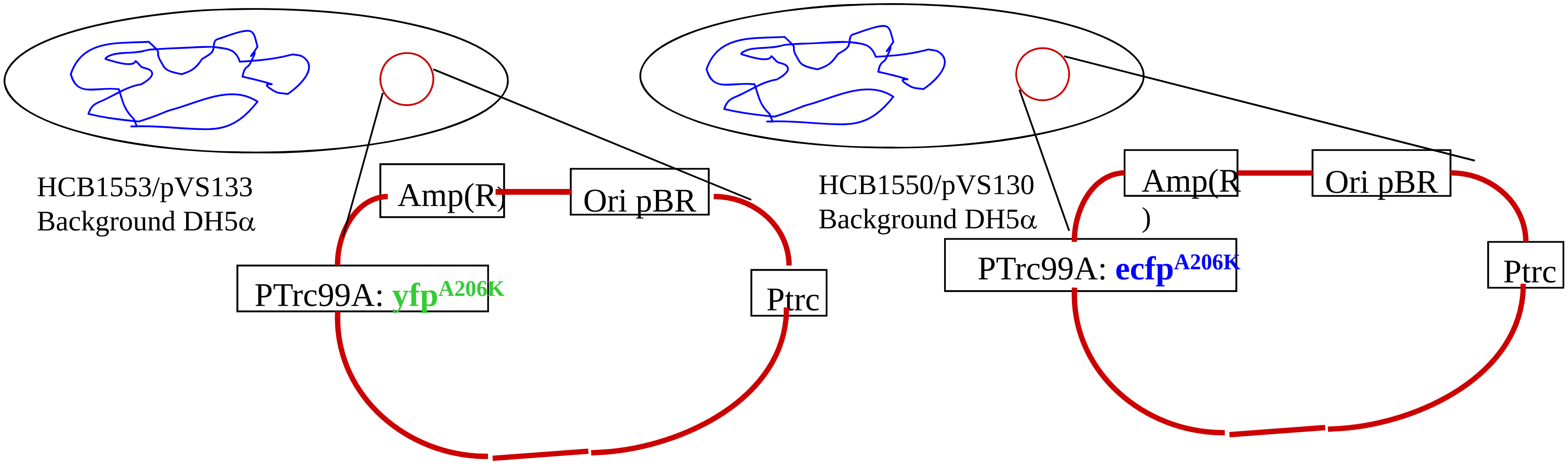
Stepping Stone Models of Competition and Cooperation

-- Frequency-dependent selection (prisoner's dilemma, snow drift, coordination games)

-- Phase transitions in 1+1 dimensions as the degree of cooperation is varied….

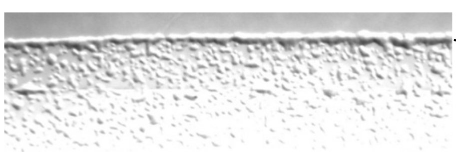
O. Hallatschek
J. Xavier
K. Foster
N. Karohan
A. Murray
M. Mueller
M. Lavrentovich

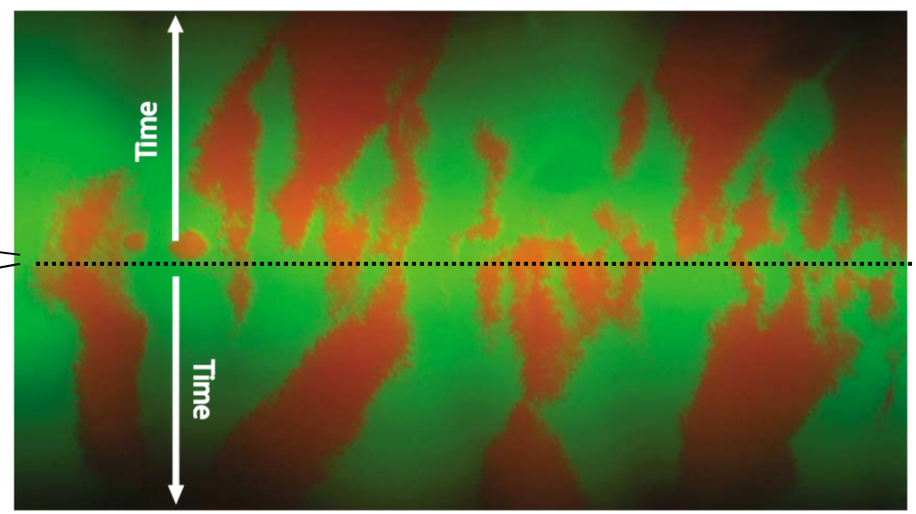
Genetic Demixing of Escherichia coli



Linear inoculants (razor blade inculation) 50%-50% mixtures

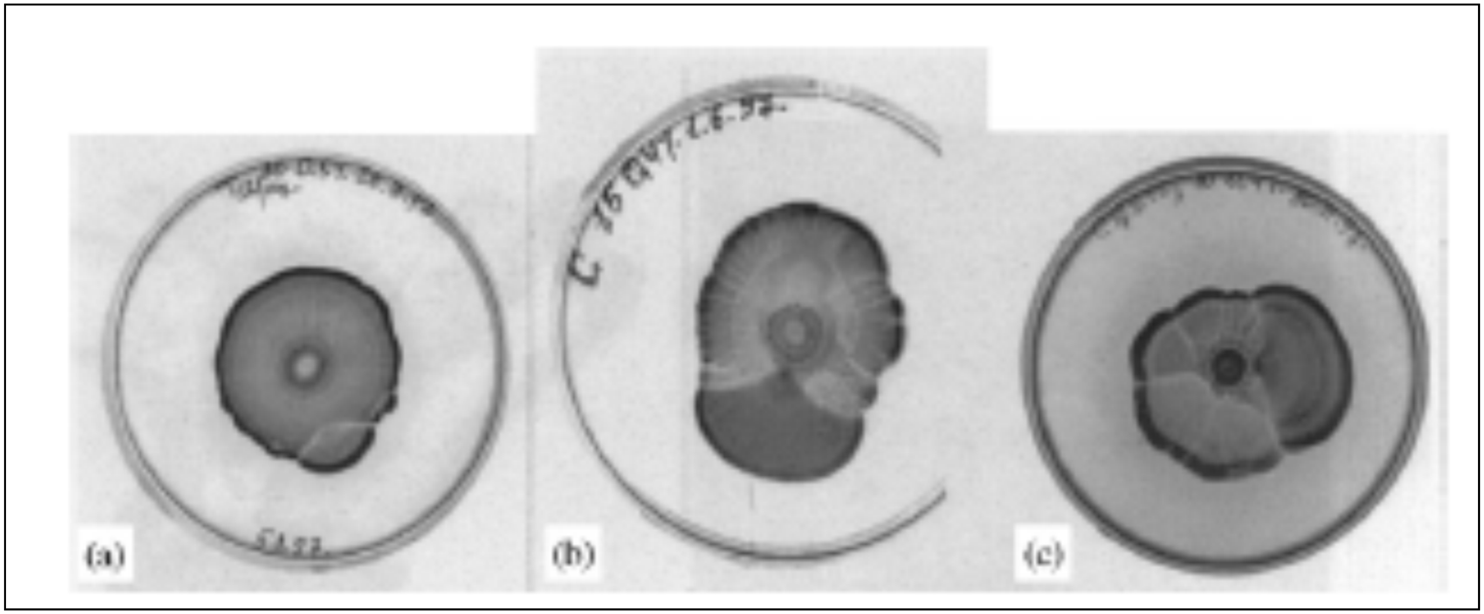
100X, 6h after innoculation





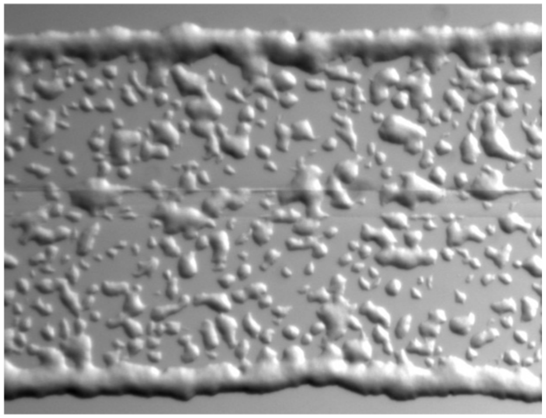
Selective advantages in Paenibacillus dendritiformis:

I. G. Ron et al. Physica A320, 485 (2003)

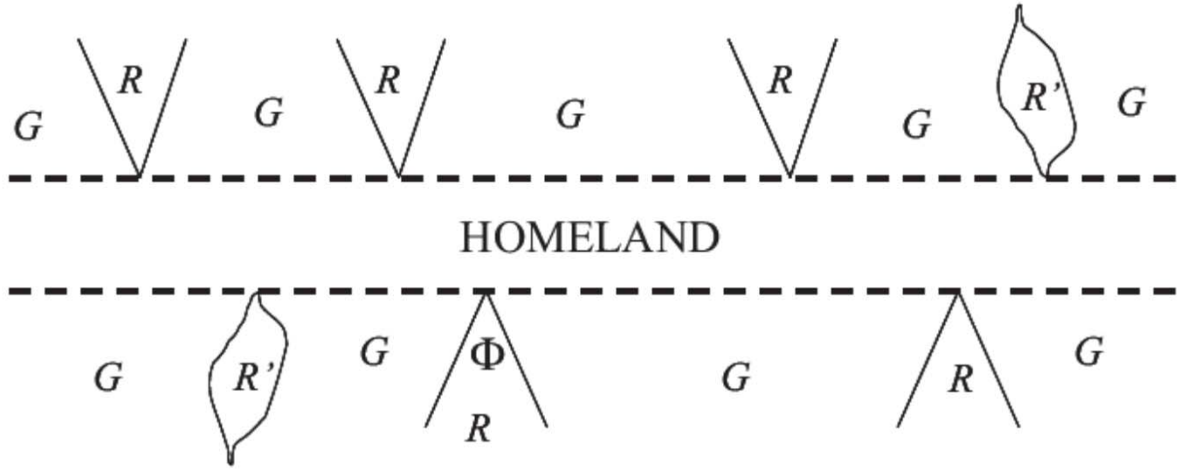


Emerging sectors in compact colonies
of P. dendritiformis.

Selective advantages from opening angles



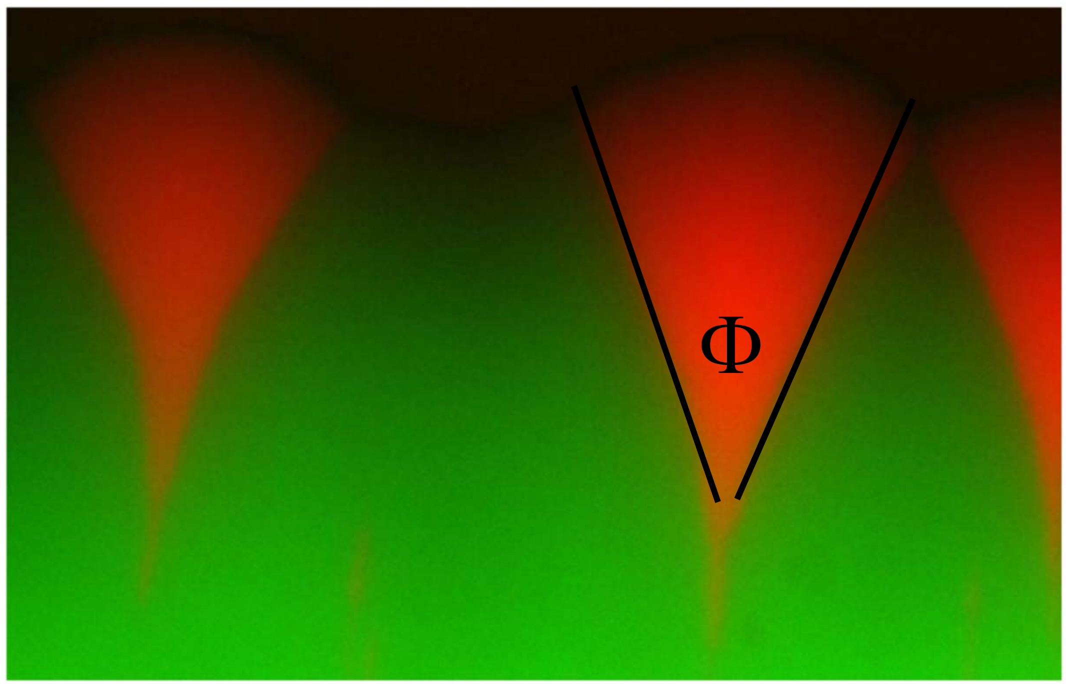
Razor blade innoculation



G is wildtype "indicator strain"
growth velocity v

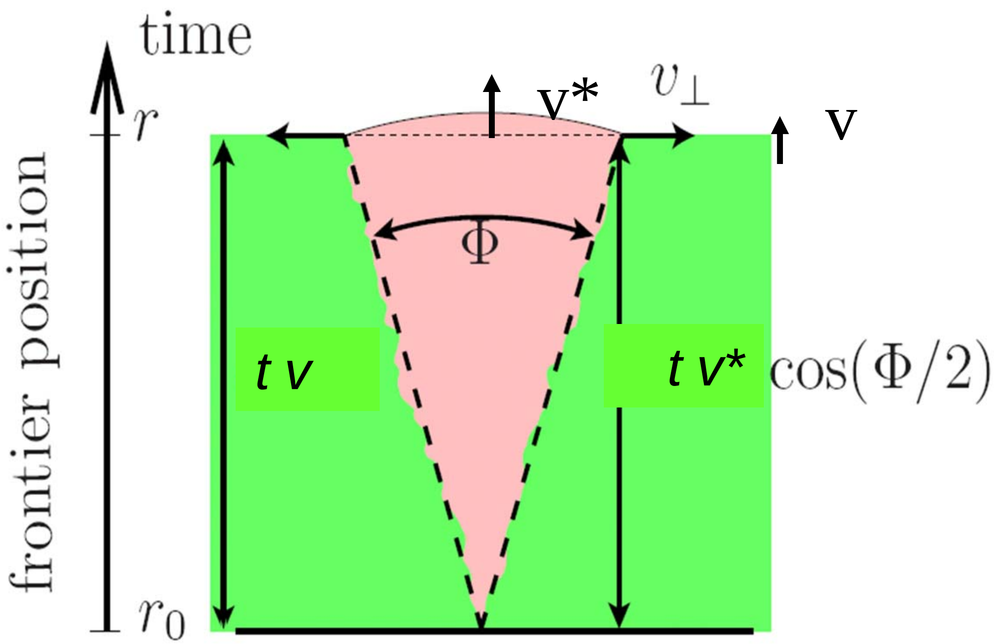
R is favorable mutant strain
growth velocity $v^* = v(1 + s)$

R' is unfavorable mutant strain
growth rate $v^* = v(1 - s)$



Sector angles and selective advantage (O. Hallatschek)

Consider a front advancing for a time t…



$v =$ growth velocity of wild type

$v^* =$ growth velocity of mutant

$v^* = (1+s)v$

$$t\ v = t\ v^* \cos(\Phi/2)$$

$$\rightarrow \frac{1}{1+s} = \cos(\Phi/2)$$

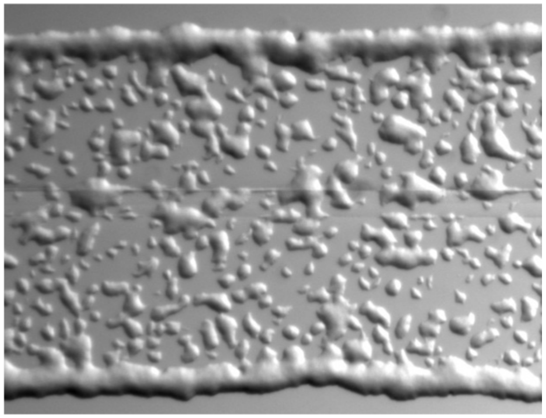
$$\Phi = 2\arccos[1/(1+s)]$$

$$\Phi \approx 2\sqrt{2s}, \quad s << 1$$

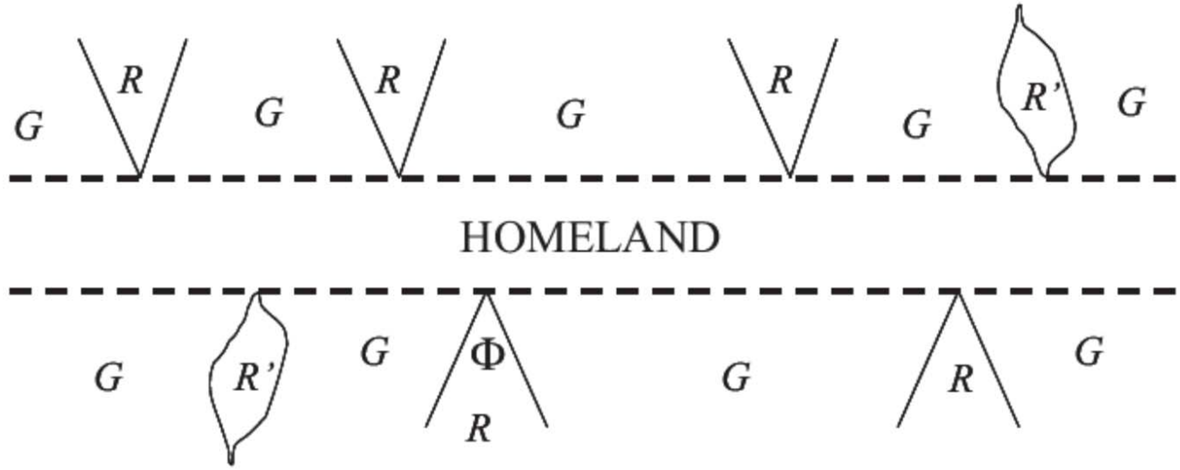
$v_\perp = v_g \approx \sqrt{2s}\ v, \quad s << 1$

$v_g =$ velocity of Fisher genetic wave at frontier

Selective advantages from trigonometry



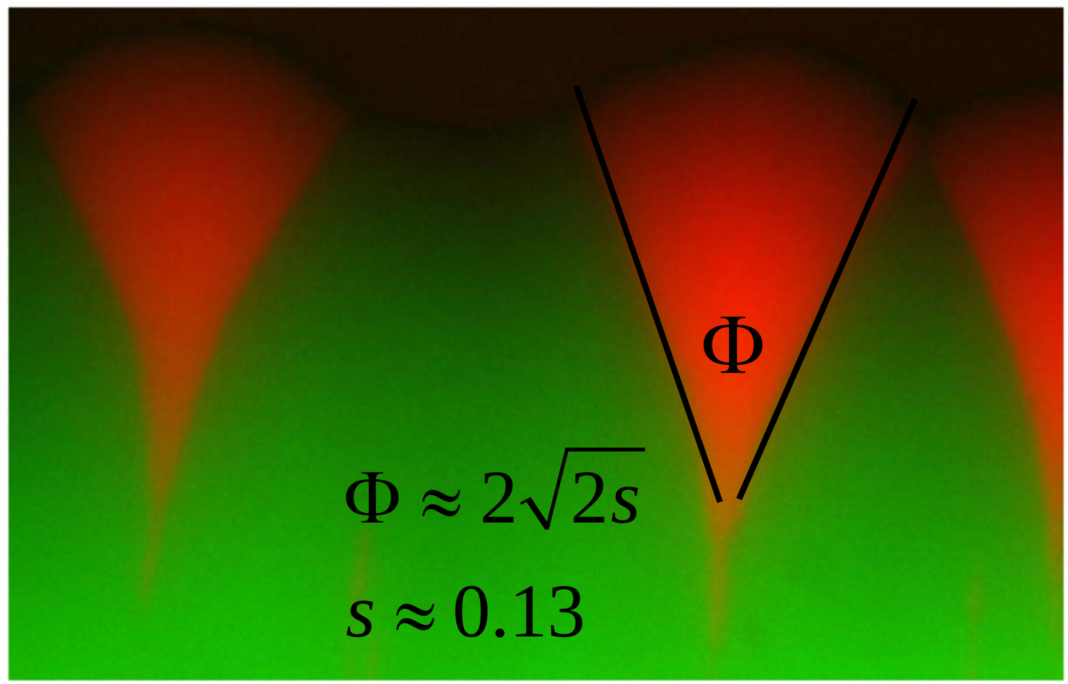
Razor blade innoculation



G is wildtype "indicator strain"
growth rate *a*

R is favorable mutant strain
growth velocity *a(1 + s)*

R' is unfavorable mutant strain
growth velocity *a(1 – s)*



Favorable Mutation in a Radial Expansion

s = 0.25

Favorable Mutation in a Radial Expansion

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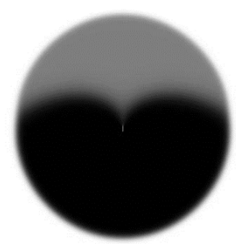

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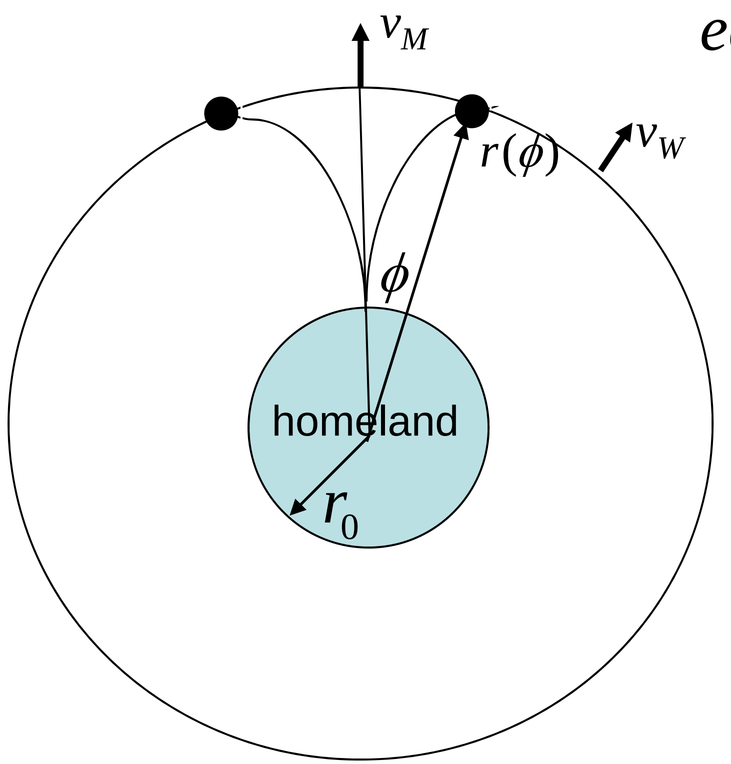
Favorable Mutation in a Radial Expansion

Fisher genetic waves trace out a logarithmic spiral in radial inoculations (K. Korolev)

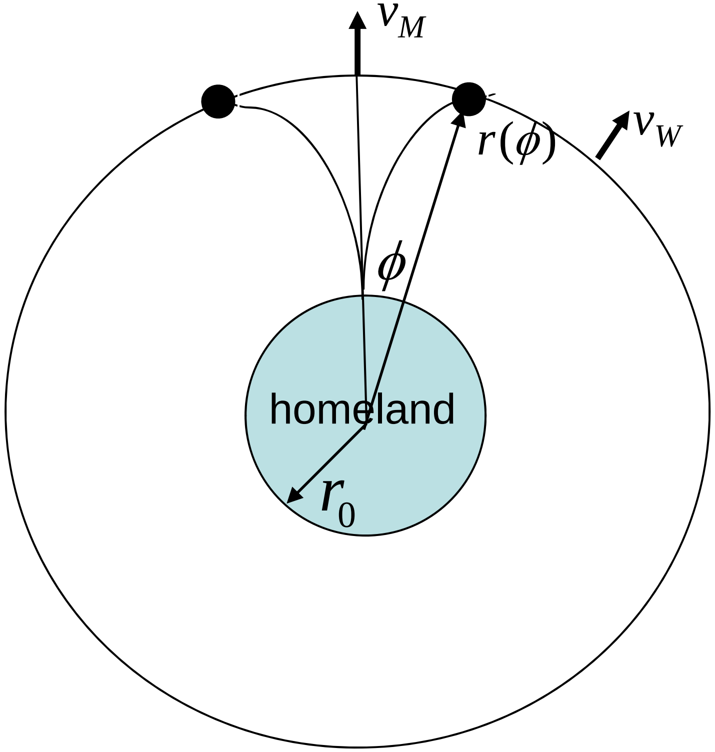


equate time increments at the front…..

$$\sqrt{d^2 r + r^2 d^2 \phi} / v_M = dr / v_W$$

$$\phi(r) = \sqrt{\frac{v_M^2}{v_W^2} - 1} \;\; \ln(r / r_0) = \sqrt{s(2+s)} \ln(r / r_0)$$

Fisher genetic waves trace out a logarithmic spiral in radial inoculations (K. Korolev)

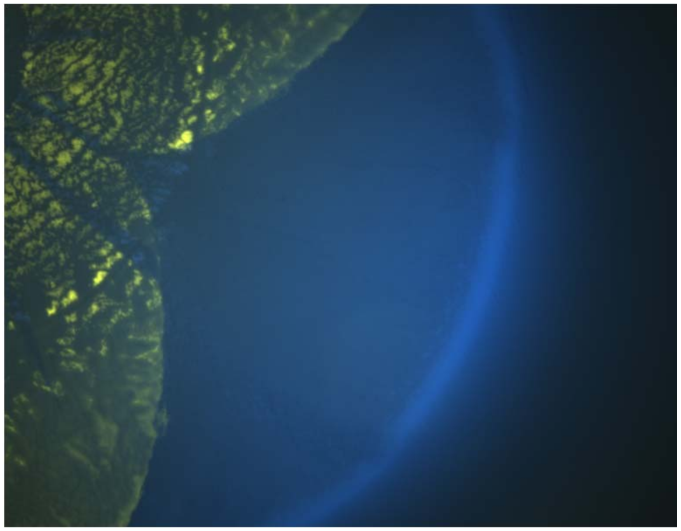


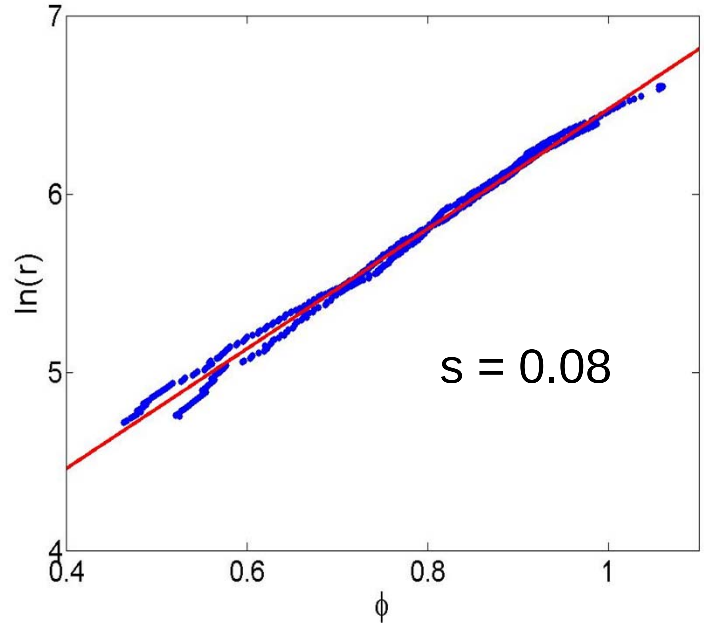
$$\phi(r)=\sqrt{\frac{v_M^2}{v_W^2}-1}\;\ln(r/r_0)=\sqrt{s(2+s)}\,\ln(r/r_0)$$

Genetic boundary is a logarithmic (equiangular) spiral…. c. f.

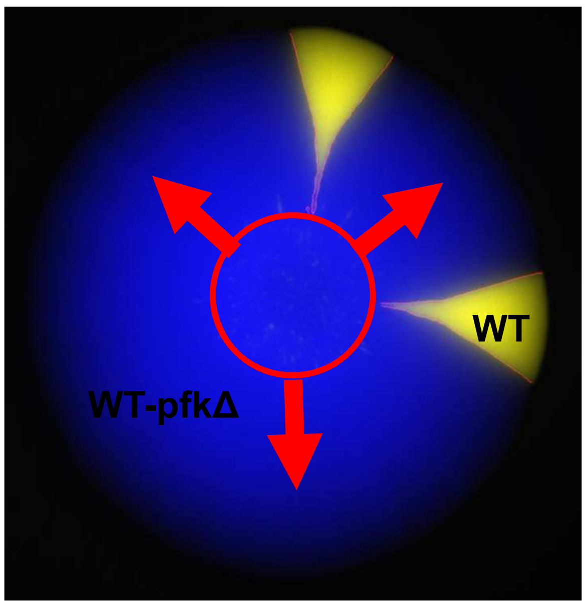
1. Jakob Bernoulli
2. Insect flight trajectories…
3. Nautilus shell….
4. Sectors in microorganisms

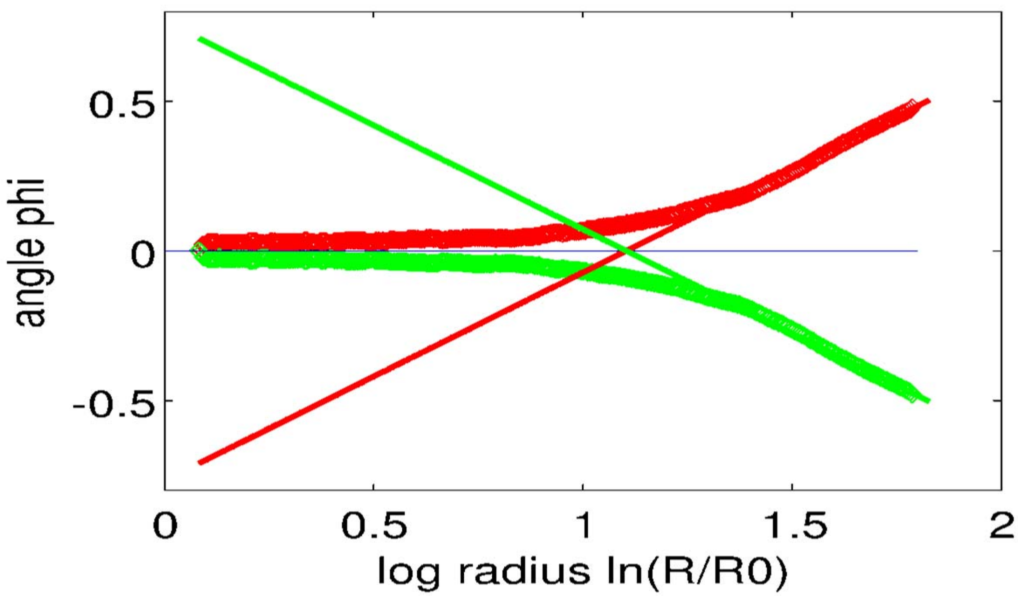
P. aeruginosa (K. Korolev)





S. cerevisiae (M. Mueller)





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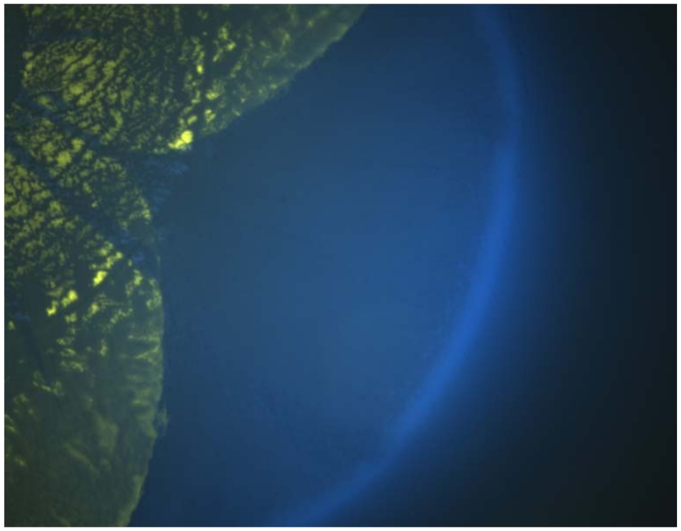

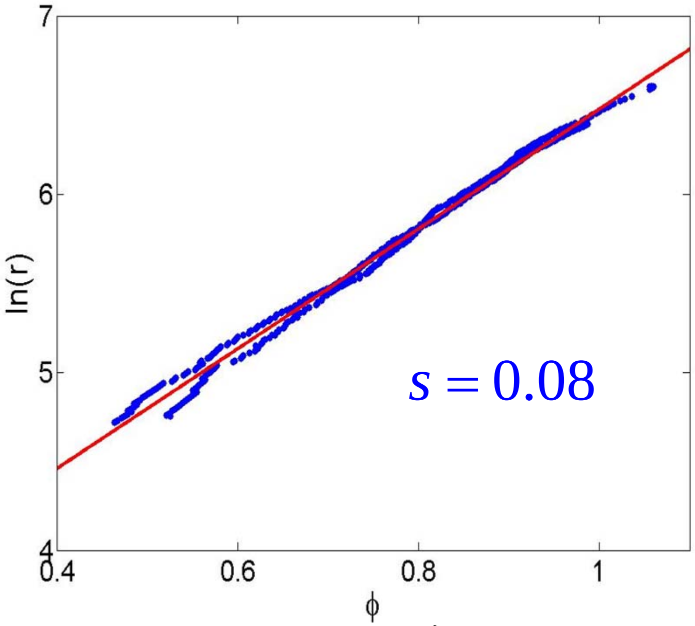

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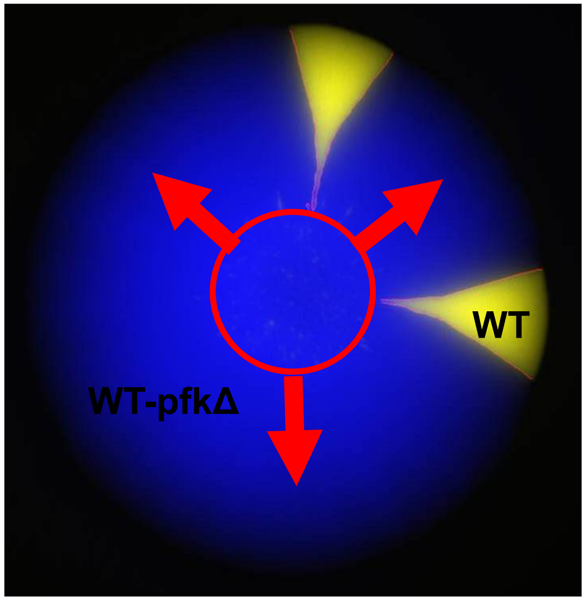
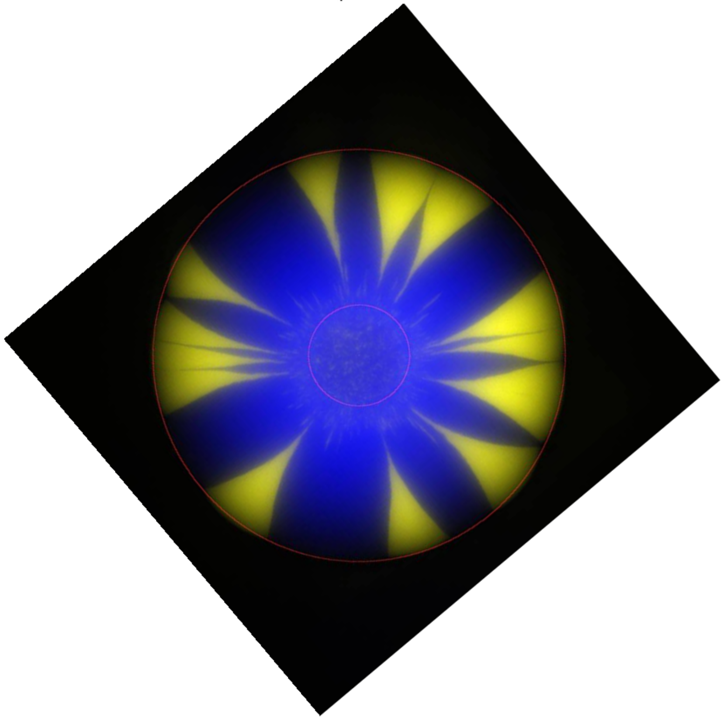
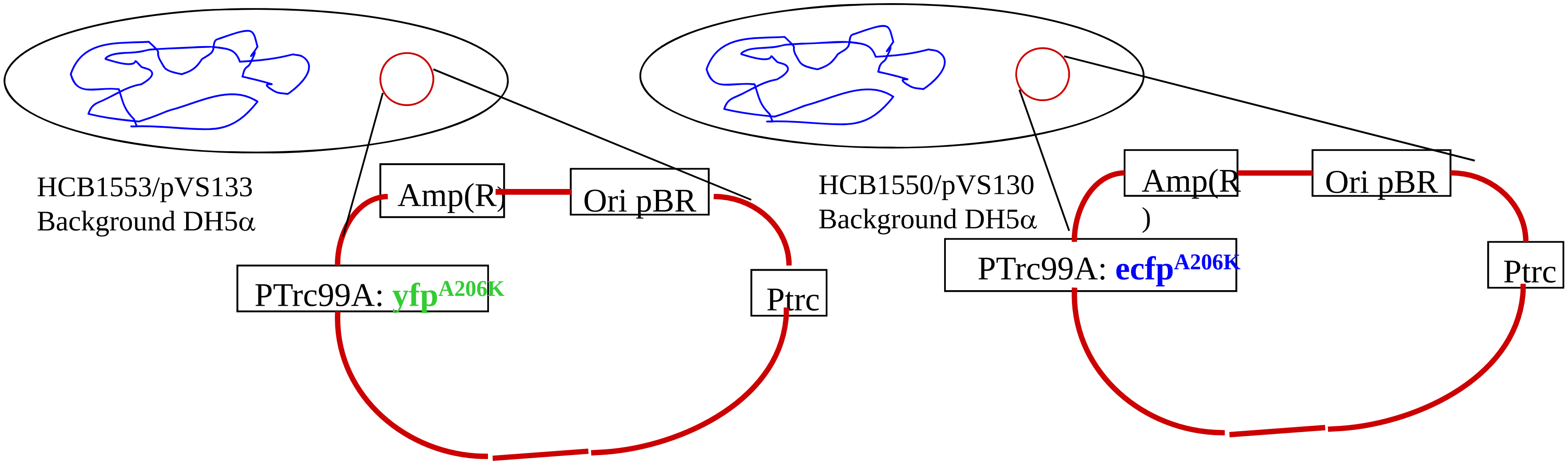




Genetic Demixing of Escherichia coli



Linear inoculants (razor blade inculation) 50%-50% mixtures

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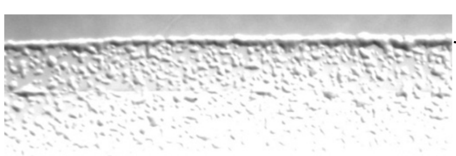
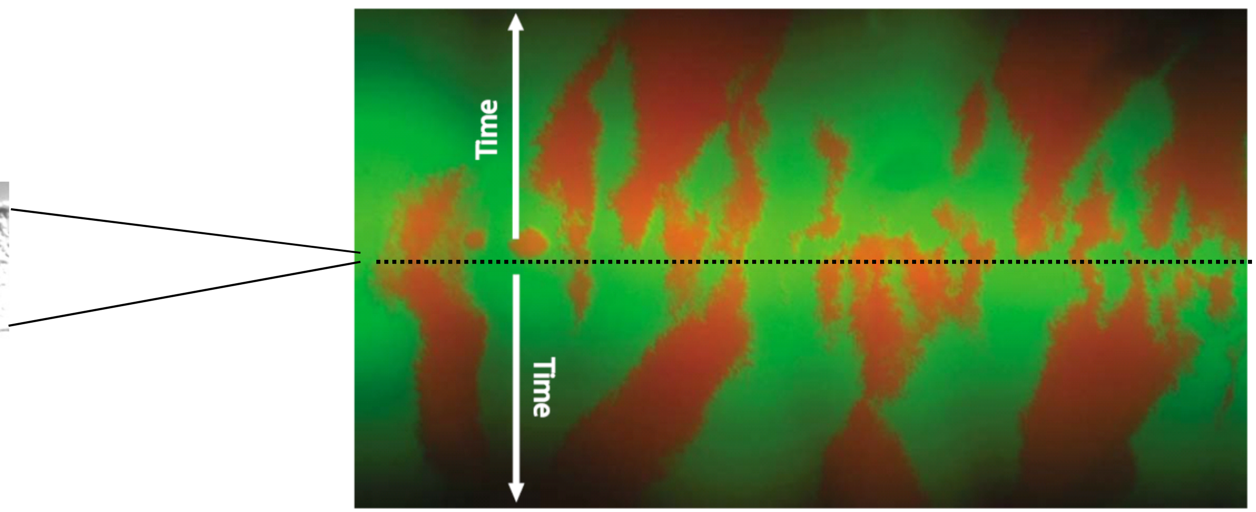
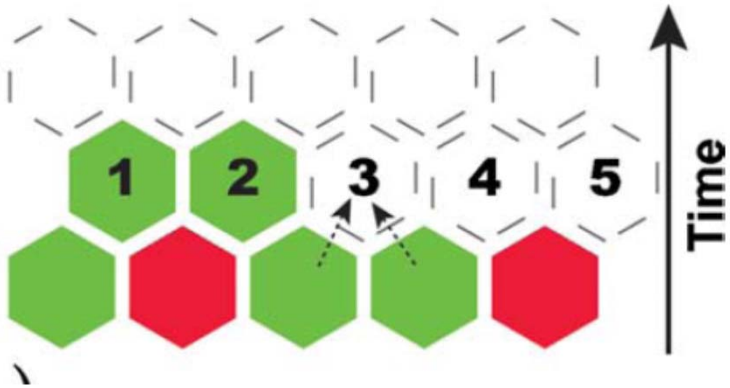
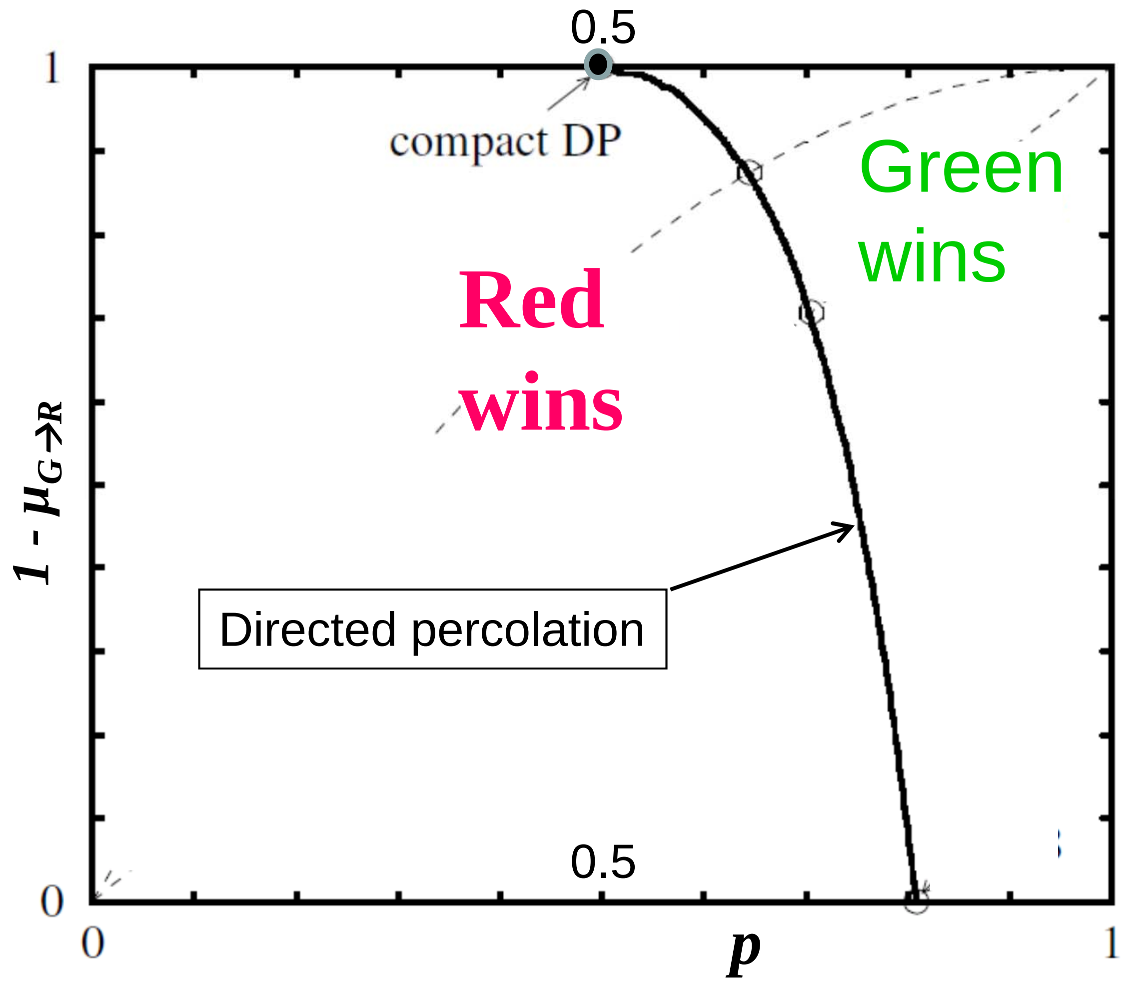






Phase transitions at frontiers…

(mutation-selection balance)



$P(G\,|\,RG) = p = 0.5 + s$

$s =$ selective advantage

$P(R\,|\,RG) = 1 - p$

$P(R\,|\,GG) = \mu_{G\to R} > 0$

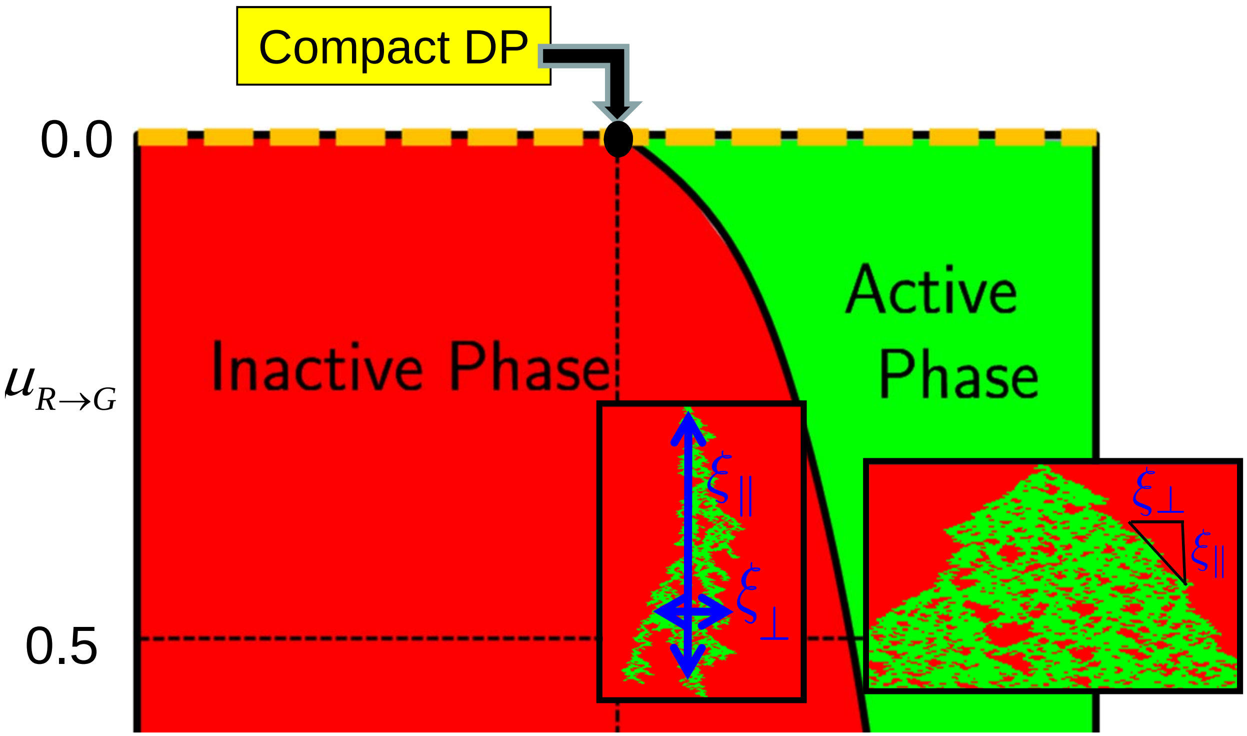
$\mu_{G\to R}$ is the rate of *deleterious* mutations

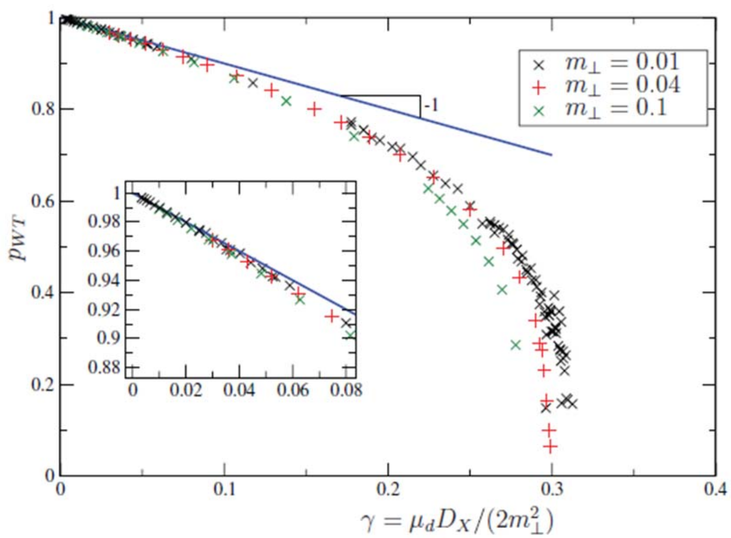
$P(G\,|\,RR) = 0,\ P(R\,|\,RR) = 1$

(no back mutations, $R \to G$)

Directed Percolation Phase Transition

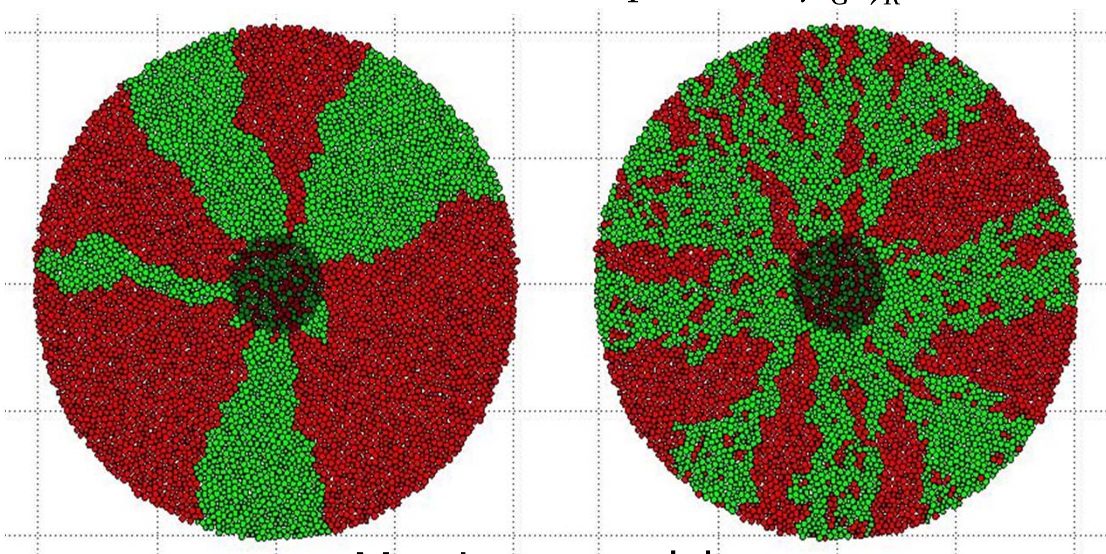
(mutation-selection balance)





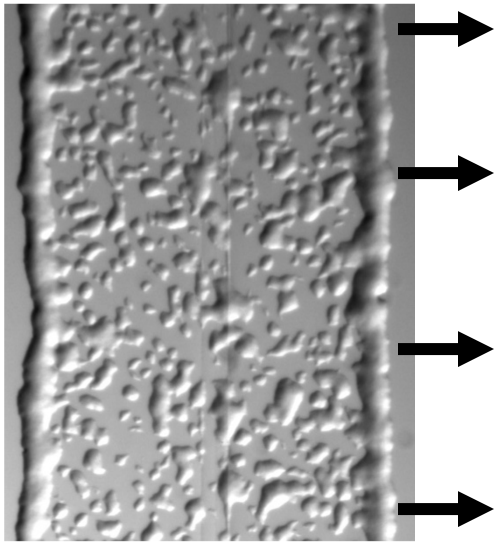
Oskar Hallatschek

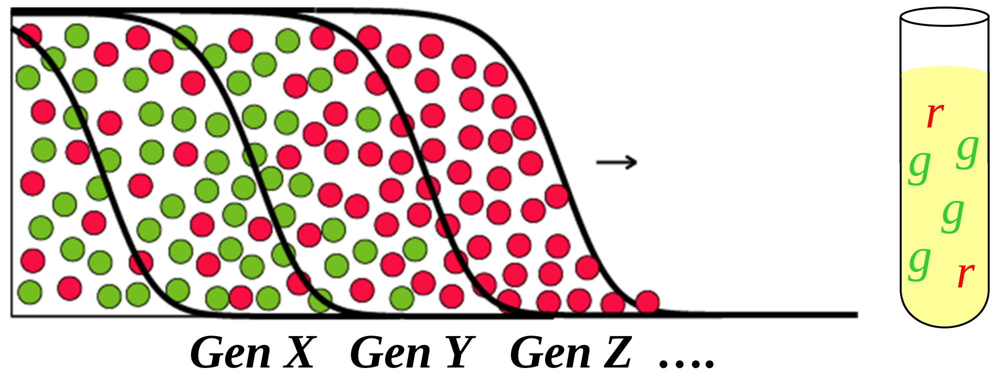
$p = 0.5,\ \mu_{G\to R} = 0$ $p = 0.7,\ \mu_{G\to R} = 0.1$



Max Lavrentovich

Razor blade inoculations are like massively parallel serial dilution experiments…



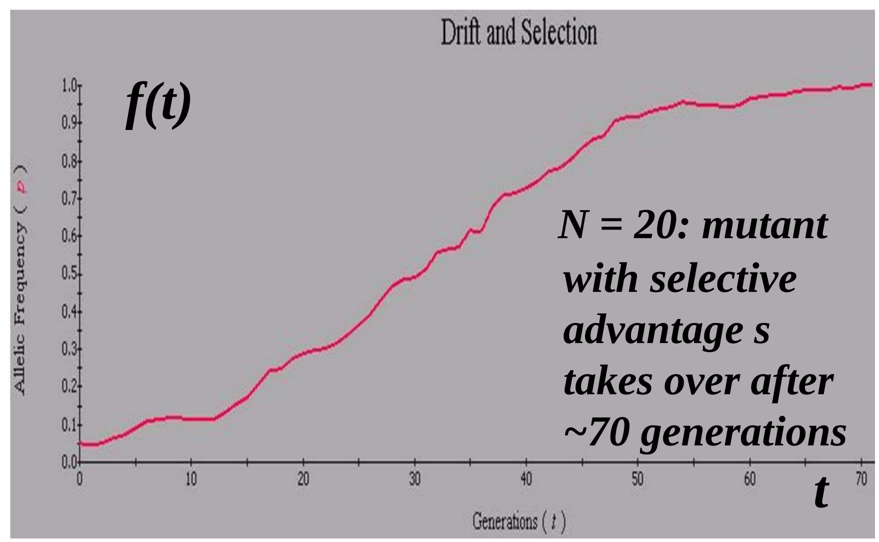


➲In effect, a moving population front is a serial dilution experiment in a well mixed test tube

For a "zero-dimensional" frontier, *f(t),* the fraction of red cells with selective advantage *s* at time *t obeys*

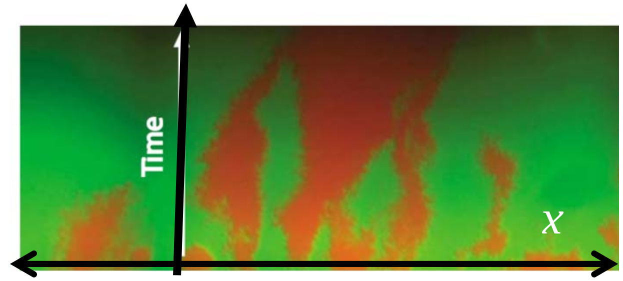
$$\frac{df(t)}{dt} = sf(1-f) + \sqrt{\frac{f(1-f)}{N}}\,\Gamma(t)$$

$$<\Gamma(t)\Gamma(t')> = \delta(t-t') \text{ (Ito calculus)}$$

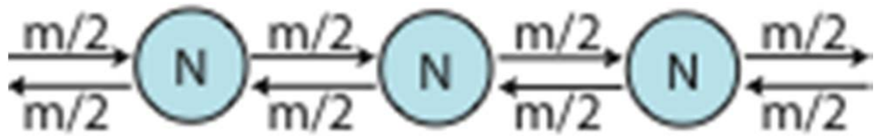


Assume
(1) the interface remains flat &
(2) cells stop growing behind the frontier

Then invoke "dimensional reduction" and the….



One Dimensional Stepping Stone Model of Population Genetics



$N =$ population size on an island

$f(x,t) =$ red fraction at position x, time t

$1-f(x,t) =$ green fraction at position x, time t

$D \propto$ m, spatial diffusion constant

$$\frac{\partial f(x,t)}{\partial t} = D\frac{\partial^2 f}{\partial x^2} + sf(1-f) + \sqrt{f(1-f)/2N}\;\Gamma(x,t)$$

$$<\Gamma(x,t)\Gamma(x',t')>= 2\delta(t-t')\delta(x-x')$$

Describes number fluctuations (i.e., genetic drift) on each island

Frequency-dependent selection

$$\frac{\partial f(x,t)}{\partial t} = D\frac{\partial^2 f(x,t)}{\partial x^2} + s(f)\,f(1-f) + \sqrt{f(1-f)/2N}\;\Gamma(x,t)$$

Let w_R and w_G be the offspring produced during one generation at a given point on the frontier...

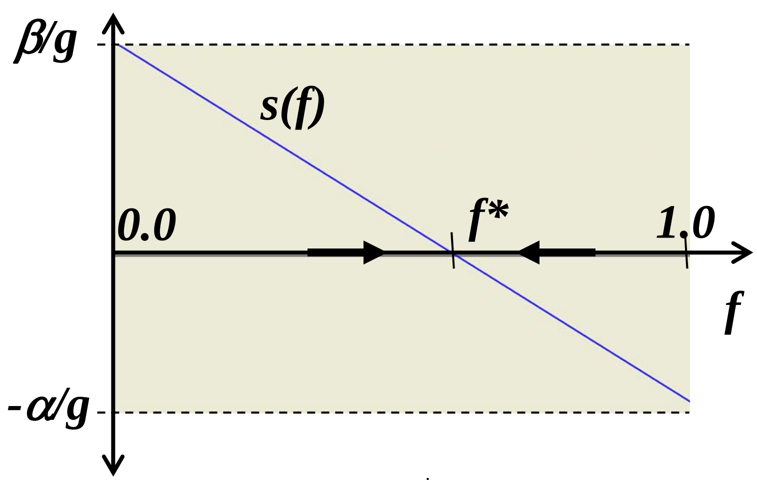
then, $s(f) \approx 2\dfrac{w_R - w_G}{w_R + w_G}$

Describe mutualism by...

$$w_R(x,t) = g + \beta(1 - f(x,t))$$

$$w_G(x,t) = g + \alpha f(x,t)$$

assume $\alpha, \beta << g$



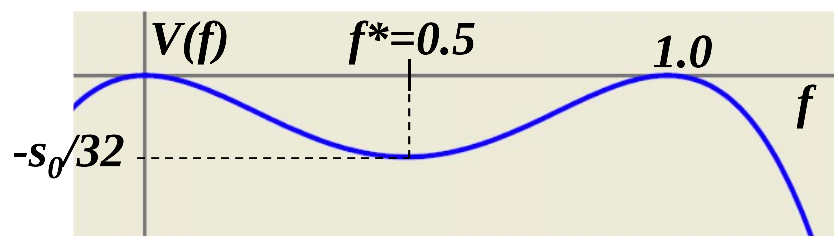
$$s(f) \approx s_0(f^* - f)$$

$$s_0 = (\alpha + \beta)/g$$

$$f^* = \beta/(\alpha + \beta)$$

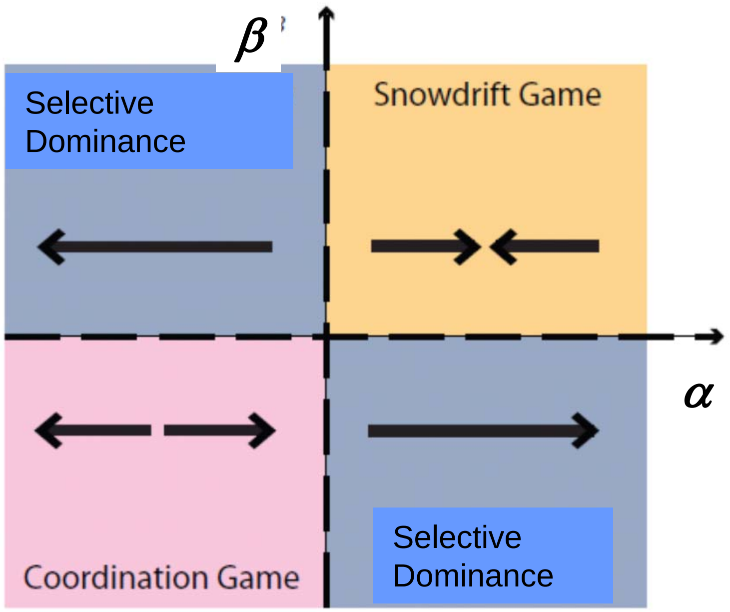
$$\frac{\partial f(x,t)}{\partial t} = D\frac{\partial^2 f(x,t)}{\partial x^2} - \frac{dV(f)}{df} + \sqrt{f(1-f)/2N}\;\Gamma(x,t)$$

$$\frac{dV(f)}{df} = -s_0(f^* - f)\,f(1-f)$$



Connection with game theory (g = 1, zero dimensions)

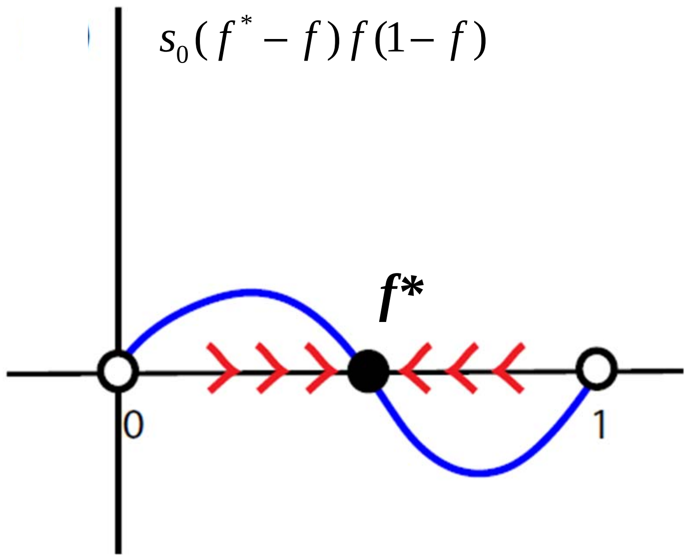
$$\frac{\partial f(x,t)}{\partial t} = \cancel{D\frac{\partial^2 f(x,t)}{\partial x^2}} + s_0(f^* - f)f(1-f) + \sqrt{f(1-f)/2N}\,\Gamma(x,t)$$



$$s_0 = (\alpha + \beta)$$

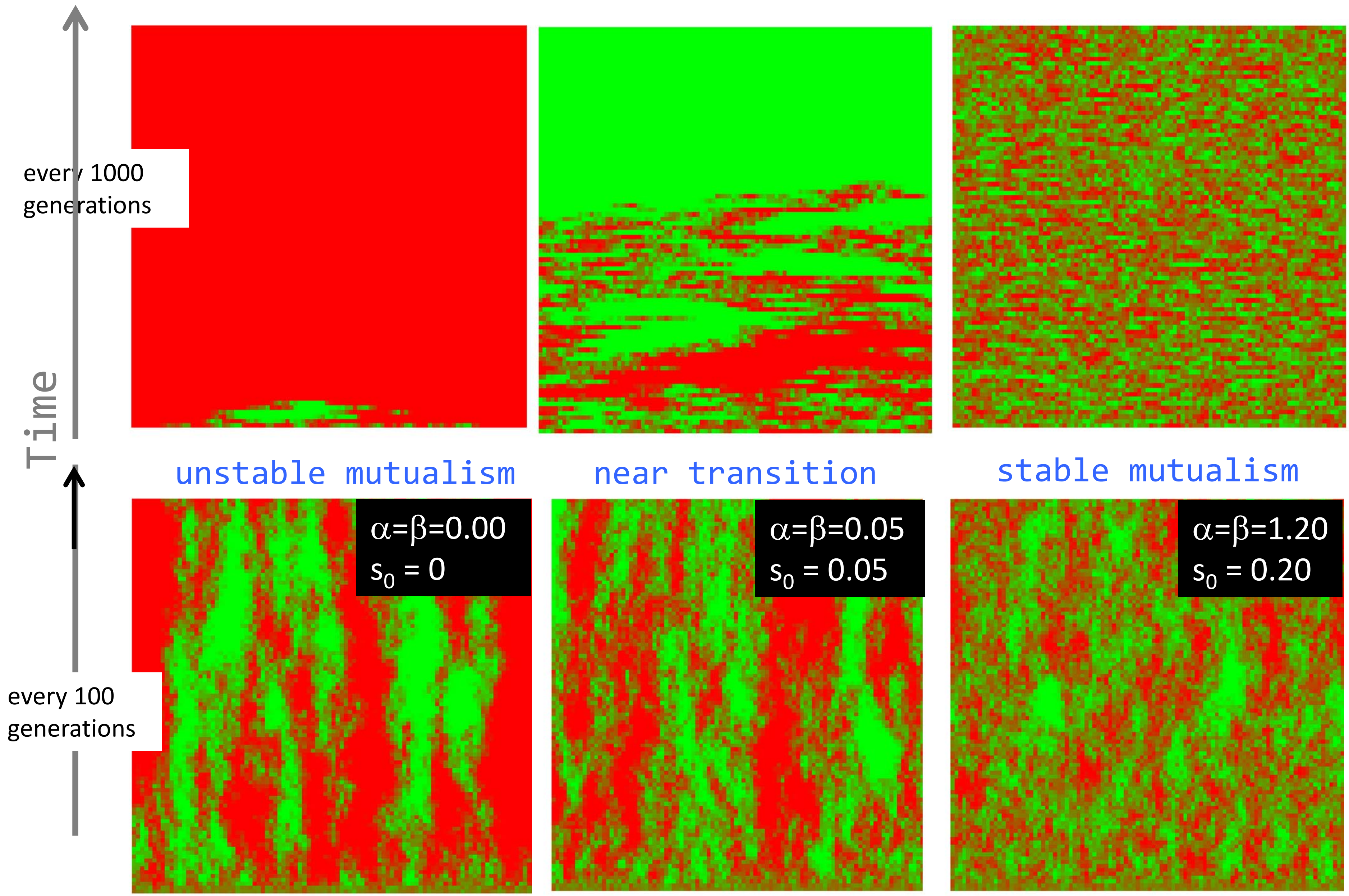
$$f^* = \beta / (\alpha + \beta)$$

In a well-mixed culture, the evolutionarily stable strategy (ESS) for mutualists leads to (transient) mixing….



M. Nowak et al., Nature ***428****, 646 (2004)*
J. Gore et al. Nature ***459****, 253 (2009)*
E. Frey et al., Phys. Rev. Lett. ***105****, 178101 (2010)*

Computer simulations: Can mutalism prevent genetic demixing?



$\alpha=\beta$, L=100, N=30, mN=2

Null Model: No selective advantage for mutualism $(s_0 = 0)$

$$\frac{\partial f(x,t)}{\partial t} = D\frac{\partial^2 f(x,t)}{\partial x^2} + \sqrt{f(1-f)/2N}\,\Gamma(x,t)$$

$$<\Gamma(x,t)\Gamma(x',t')>=2\delta(t-t')\delta(x-x')$$

$H(x,t) =$ heterozygosity correlation function

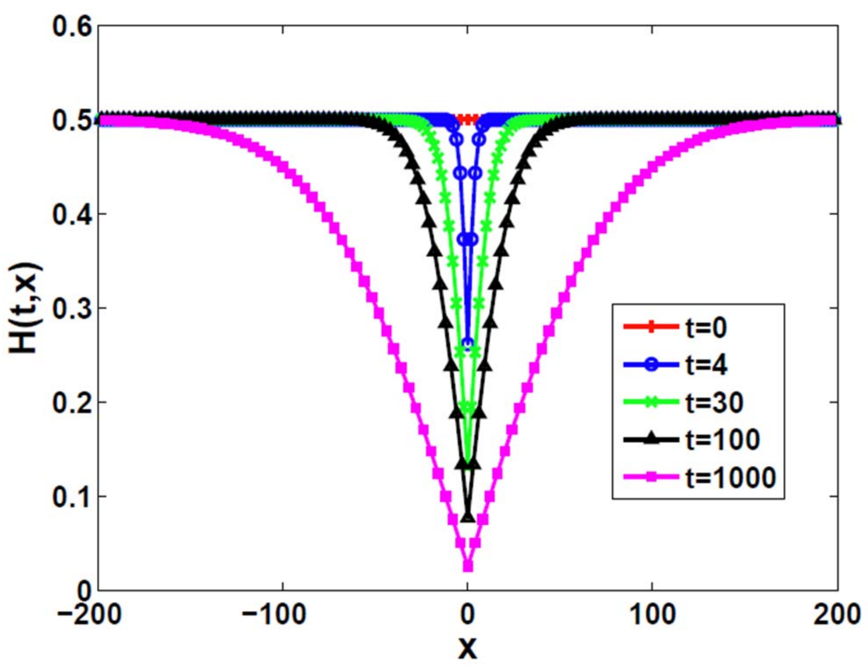
$= 2 < f(y,t)[1-f(y+x,t)] >$ = probability of different colors at separation x

$$\frac{\partial H(x,t)}{\partial t} = 2D_s\frac{\partial^2 H(x,t)}{\partial x^2} - \frac{1}{2N}H(0,t)\delta(x)$$

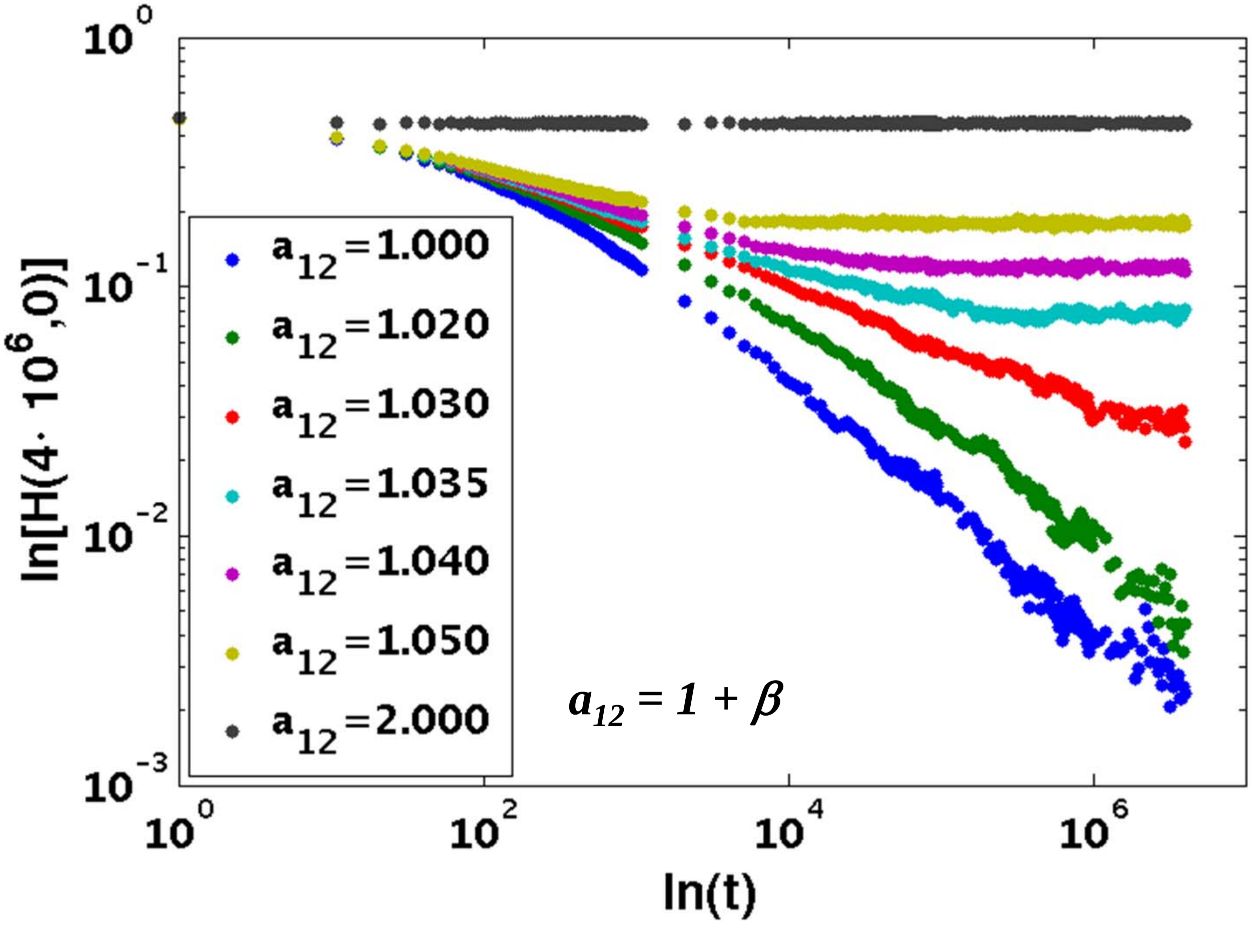
$H(x,0) \equiv H_0 = 1/2,$ for 50-50 random initial conditions

$$\lim_{t\to\infty} H(x=0,t) \approx (t_f/t)^{1/2}$$

one color dominates locally

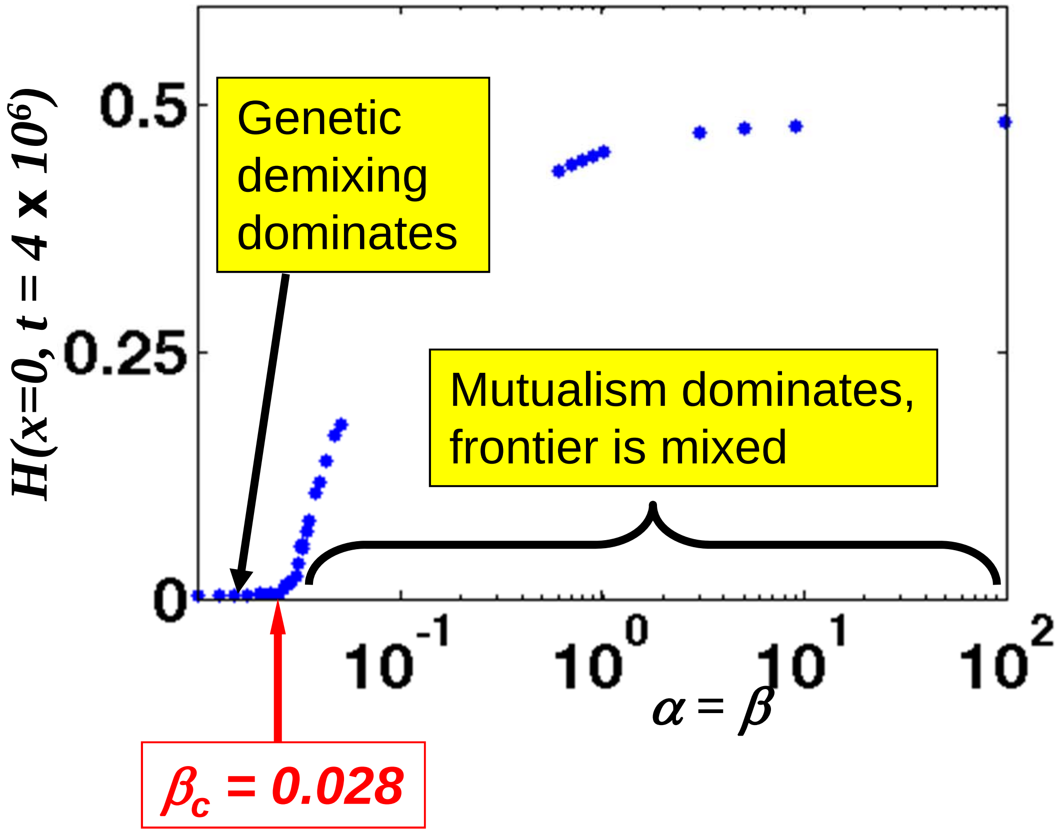


Local heterozygosity reaches a steady state value for large $\beta=\alpha$



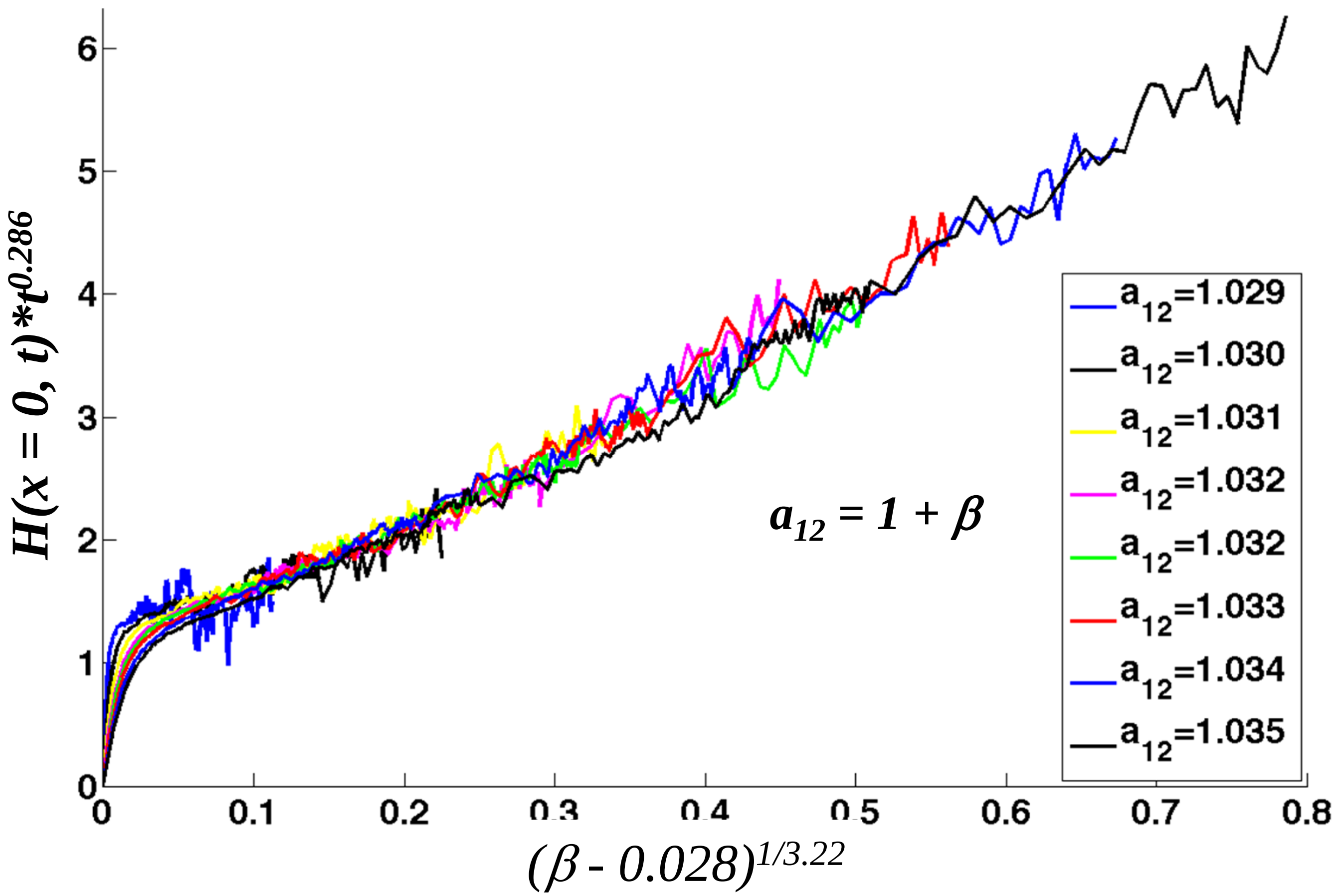
$\alpha = \beta$, L=10000, N=30, mN=2

Mutualism is unstable for small β, but stable for large β



$\alpha = \beta$, L=10000, N=30, mN=2

H(x,t) data for collapse for $\alpha = \beta$ suggest a nonequilbrium phase transiton at a critical value of the "cooperativity" $s_0 = 2\beta$…

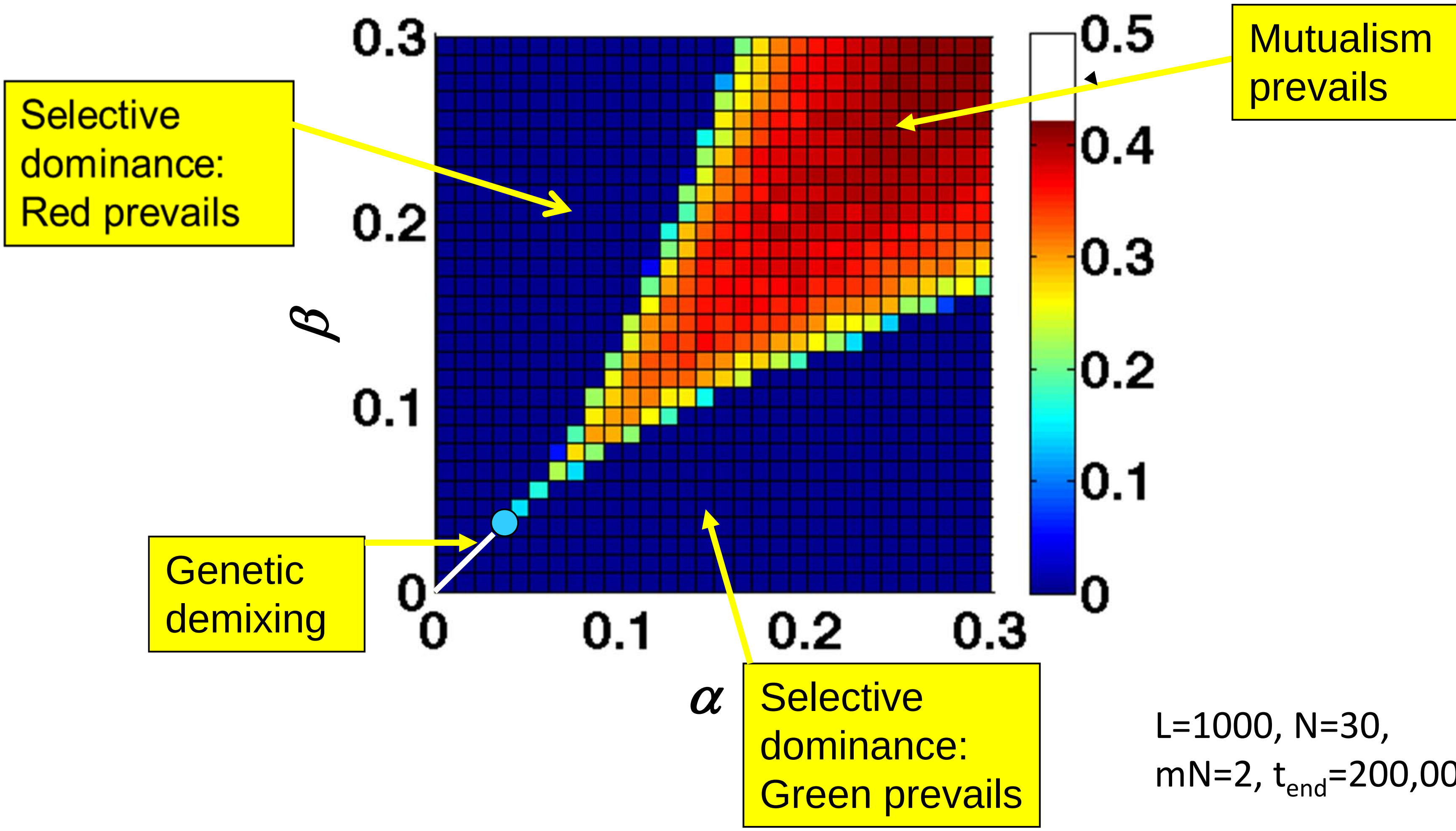


Conjecture that these are the critical exponents of the "DP2 model": H. Hinrichsen, Adv. Phys. ***49,*** *815 (2000)*

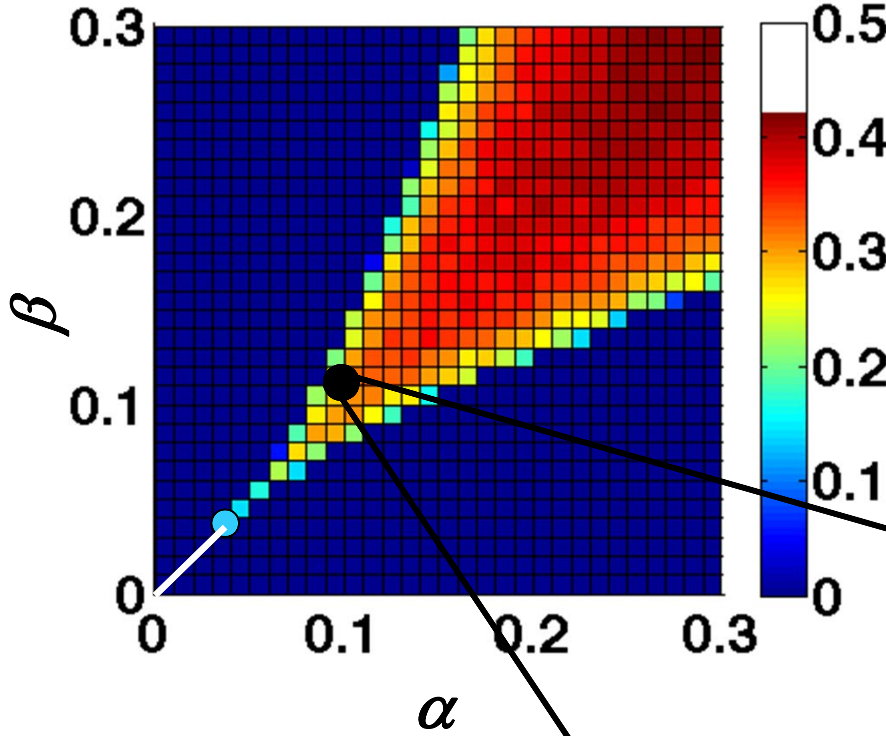
$\alpha = \beta$, L=10000, N=30, mN=2

Transition between genetic demixing/fixation and mutualism at the frontier persists for $\alpha \neq \beta$ & $f^* \neq 0.5$

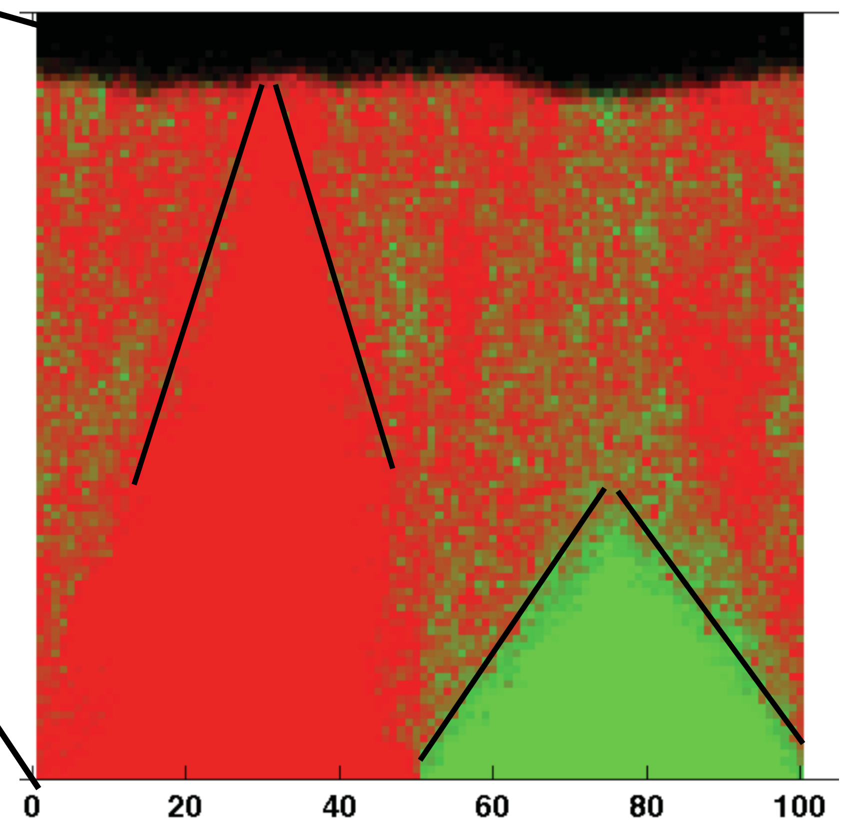
Transitions are in either the "DP2" ($\alpha = \beta$ & $f^* = 0.5$) or "directed percolation" ($\alpha \neq \beta$ & $f^* \neq 0.5$) universality classes.



L=1000, N=30, mN=2, t_{end}=200,000



Mutualistic “Smoke”



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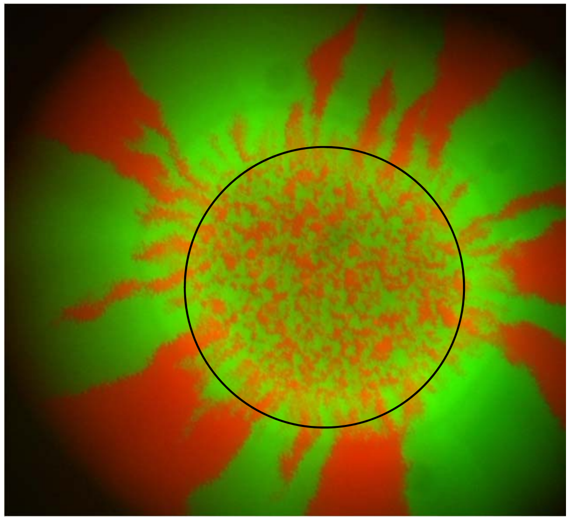

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