Evolutionary dynamics in heterogeneous environments: three baby steps

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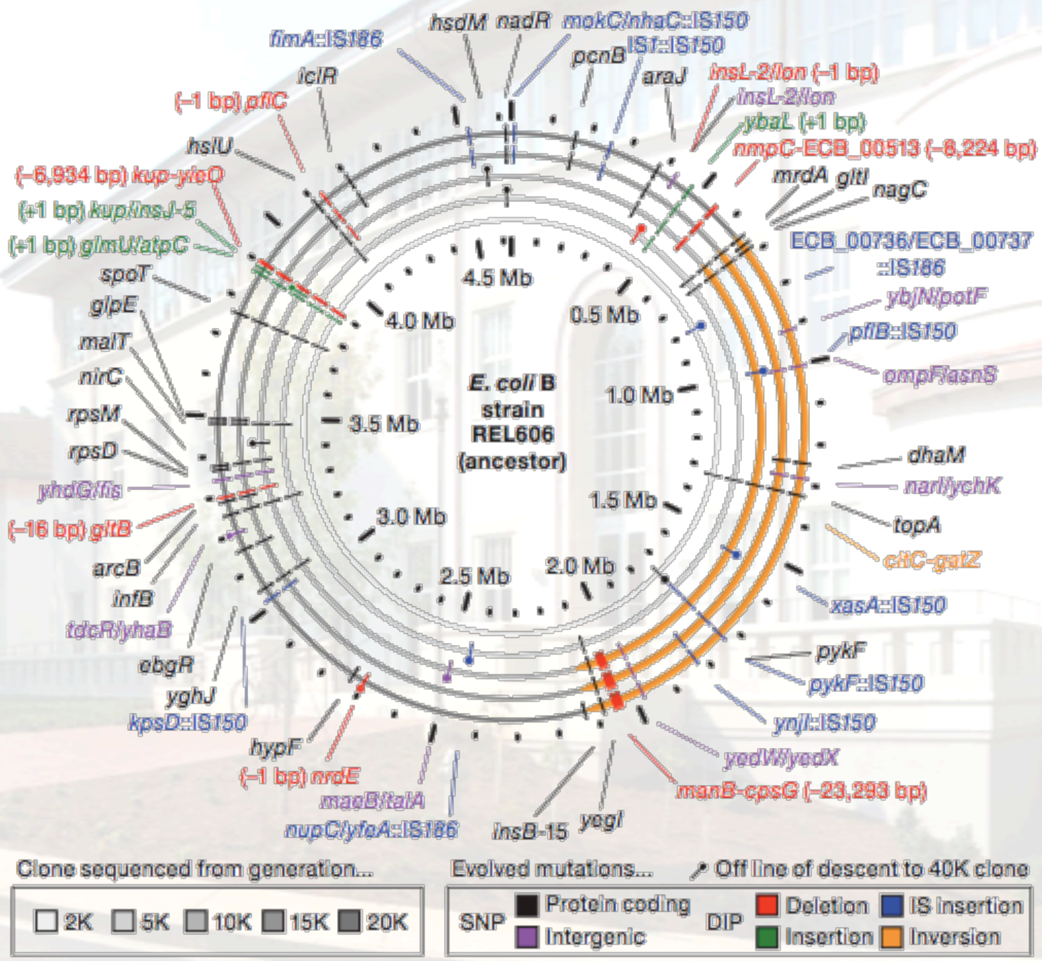
Long term *E. coli* evolution experiment by Lenski et al.



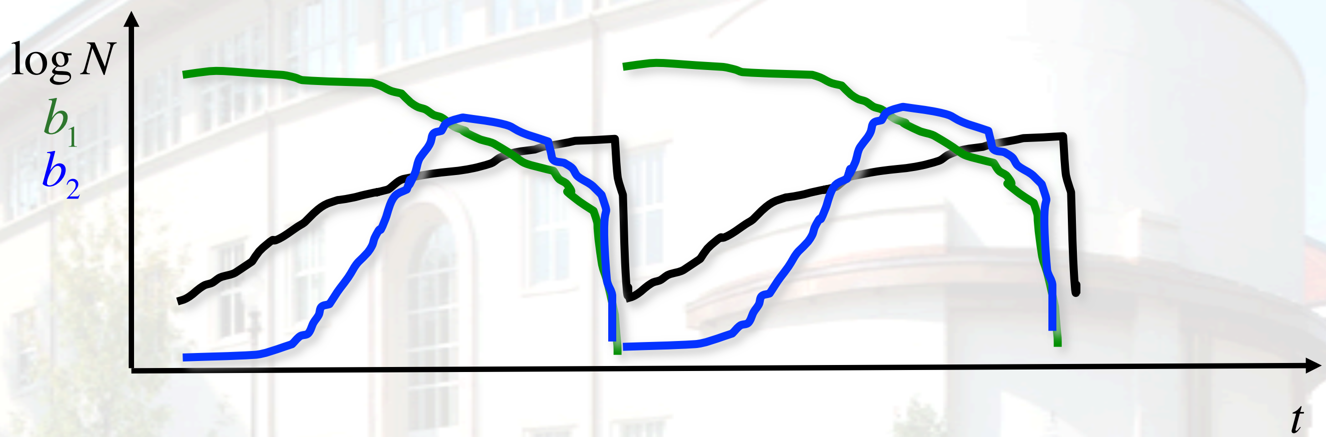
- Only 45 surviving mutations in 20k generations

Credits: Barrick et al., 2009

Possible solution: heterogeneity

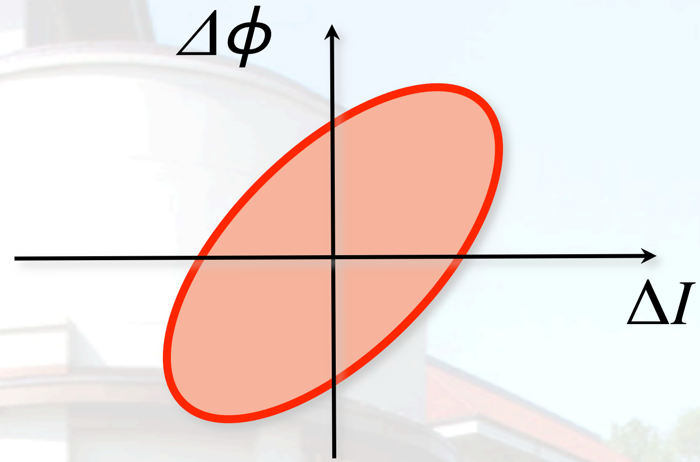
- Heterogeneous time: adiabatic variation of selective pressures
  - Annealed time scales: generation, environment, fixation (Problem 1)
  - Quenched time scales: generation, fixation, environment (Problem 2)
- Heterogeneous space: evolution in structured environments
  - Why move? Go West young man!
  - Motion through self-created inhomogeneous environment creates temporally inhomogeneous signal.

Long term *E. coli* evolution experiment by Lenski et al.



generation time < environment time < fixation time

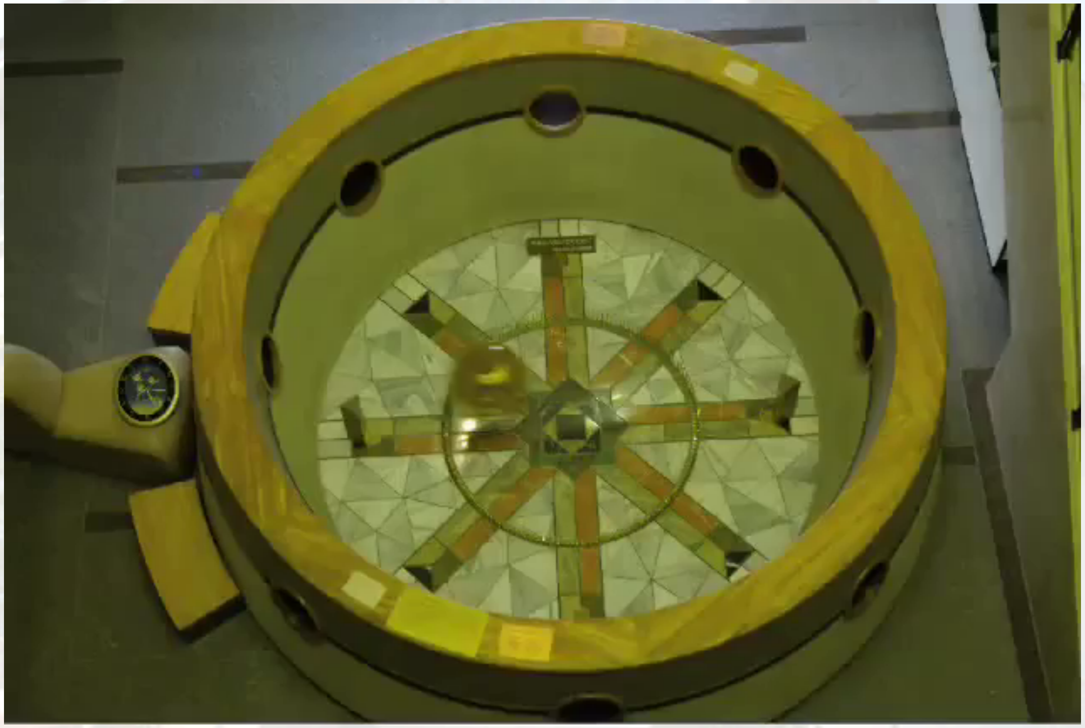
Everyday geometric effects



Useful rotation = ωt +
Surface integral in parameter space

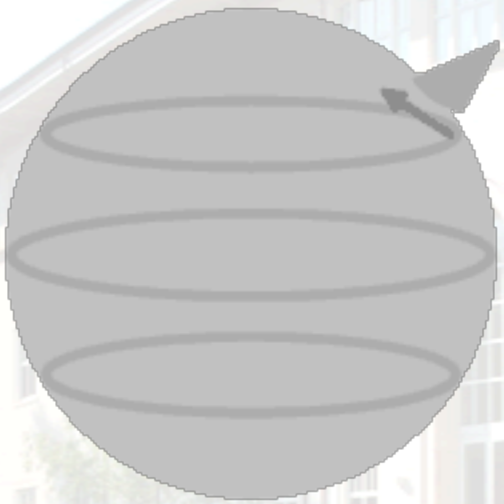
http://www.youtube.com/watch?v=t84a0L76ju4

Adiabatic geometric effects: mechanics

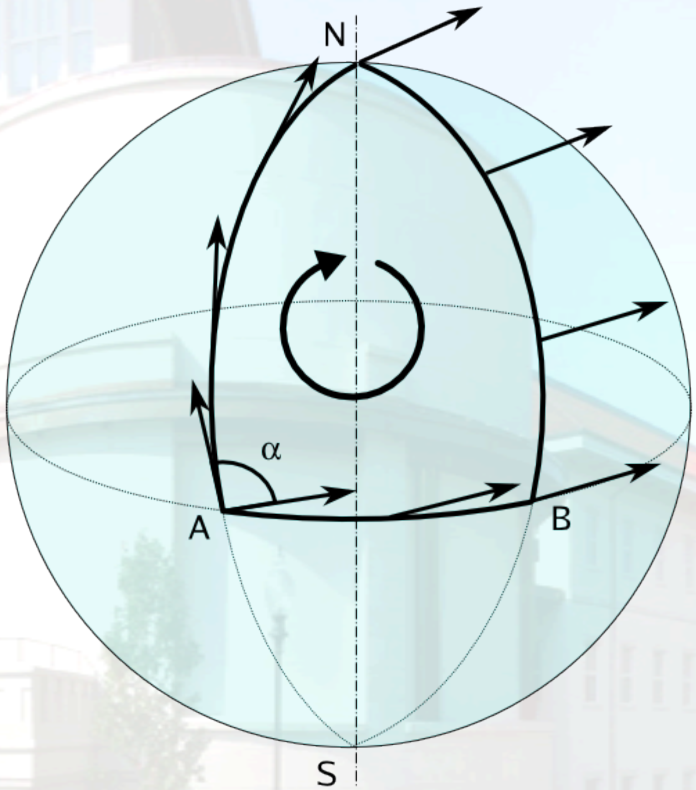


http://www.youtube.com/watch?v=b14I3-A8iUQ

Adiabatic geometric effects: mechanics



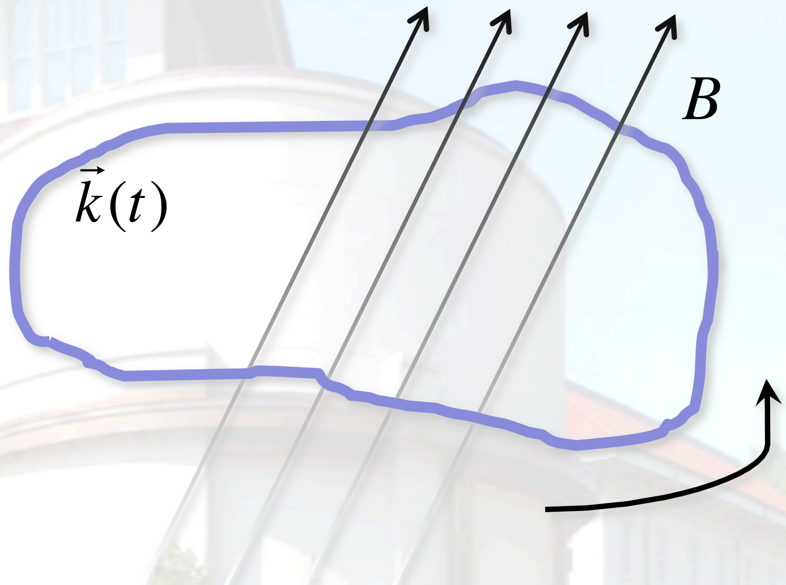
http://en.wikipedia.org/wiki/Foucault_pendulum



Plane rotation =
Area in the parameter space

Adiabatic geometric effects: quantum mechanics

- **Adiabatic theorem:** A physical system remains in its instantaneous eigenstate if a perturbation is acting slowly enough and there is a gap between the eigenvalue and the rest of the spectrum.
- Therefore, when parameters undergo periodic driving, the wave function can change by a phase.
- The phase is an integral of *Berry curvature* over the covered area in the parameter space.



$$\left|s(T)\right\rangle = e^{i\phi}\left|s(0)\right\rangle$$

$$\phi = \int dA \cdot B$$

Property of adiabatic geometric effects

- Proportional to *area* in the parameter space (i.e., at least two out of phase parameters needed)
- Depends on the geometry of the contour (*sequence* of environment states), but not on how fast it is traversed
- Reverses sign under time reversal
- Of order of 1/T
- System dynamics must be non-Markovian

Geometric effects exist in statistical physics

- Dynamics of P(w) is Markovian – no geometric effects
- Dynamics of fluxes is not Markovian – fluxes will have geometric corrections

Small for small fitness differences

$\log Z_{flux}$ = [off-equilibrium, qst]*t +
Surface integral in parameter space

Model of competition for resource

(Mutant with small, maybe zero averaged, fitness advantage)

$$X_1 \xrightarrow{b_1 X_1} 2X_1$$

$$X_1 \xrightarrow{d_1 X_1 (X_1 + X_2)} \varnothing$$

$$X_2 \xrightarrow{b_2 X_2} 2X_2$$

$$X_2 \xrightarrow{d_2 X_2 (X_1 + X_2)} \varnothing$$

$$\frac{dX_1}{dt} = \left[b_1 - d_1 \left(X_1 + X_2 \right) \right] X_1$$

$$\frac{dX_2}{dt} = \left[b_2 - d_2 \left(X_1 + X_2 \right) \right] X_2$$

$$b_i = b_0 \left[1 + \delta b_i \cos\left(\omega t + \varphi_i \right) \right]$$

$$d_i = d_0 \left[1 + \delta d_i \cos\left(\omega t + \phi_i \right) \right]$$

Growth of X_1 over long times = (quasi steady state growth)*t + geometric term.

Adiabatic population dynamics with out of phase parameter changes

- A better parameterization:

$$p = x_2 / (x_1 + x_2) \qquad K = X_1 + X_2$$

$$b = (b_1 + b_2)/2$$
$$d = (d_1 + d_2)/2$$
$$r = b - dK$$

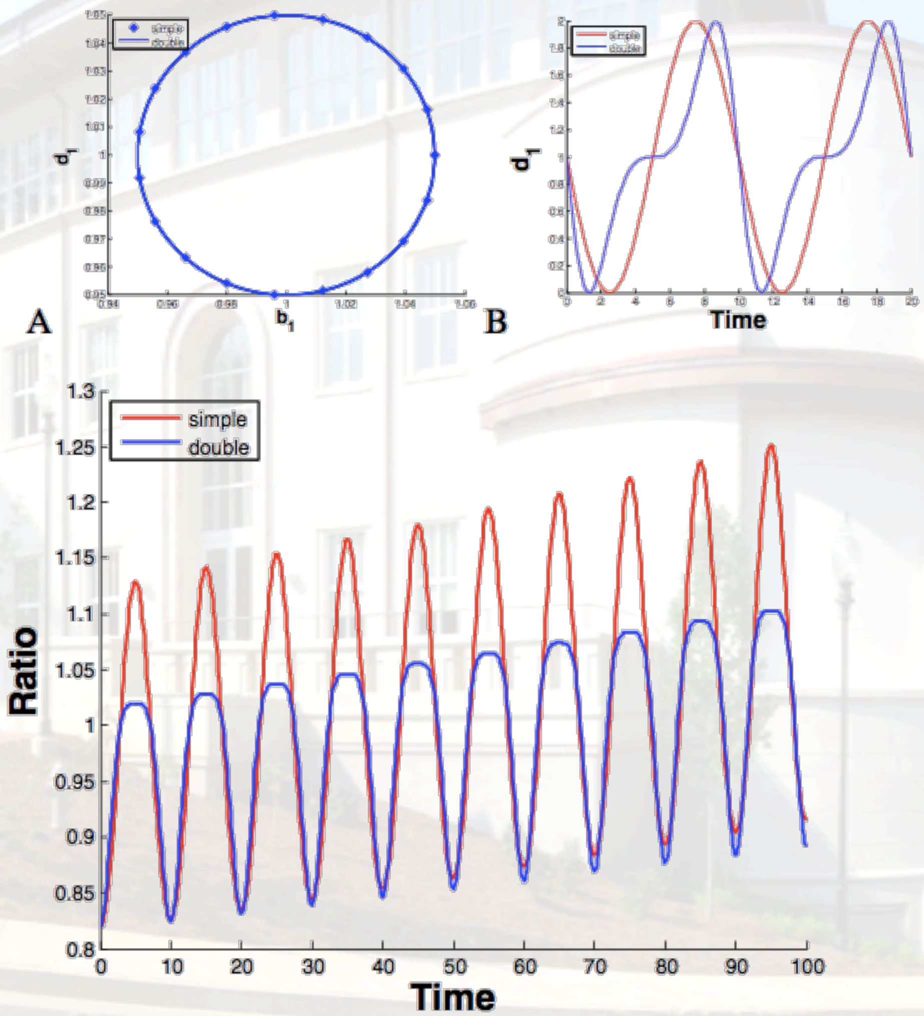
- Results:

$$\log \frac{p(t)}{1-p(t)} = \log \frac{p(0)}{1-p(0)} + s_{qst} t + I_{geom}$$

$$I_{geom}(\mathcal{T}) = \int_0^{\Lambda(\mathcal{T})} \left[\frac{\delta r(\lambda)}{r(\lambda)} - \frac{\delta K(\lambda)}{K(\lambda)} \right] \frac{K'(\lambda)}{K(\lambda)} d\lambda$$

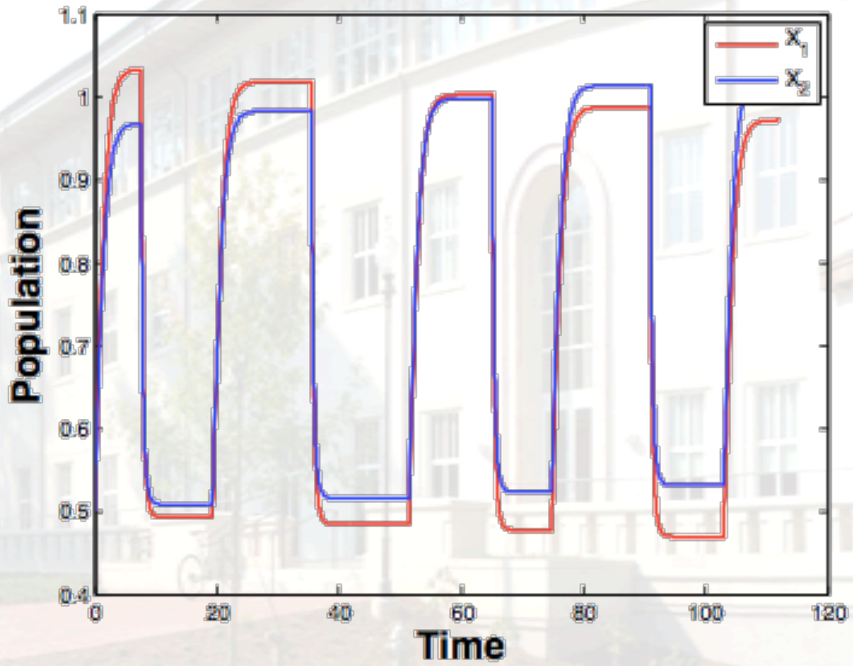
Similar results for all cumulants

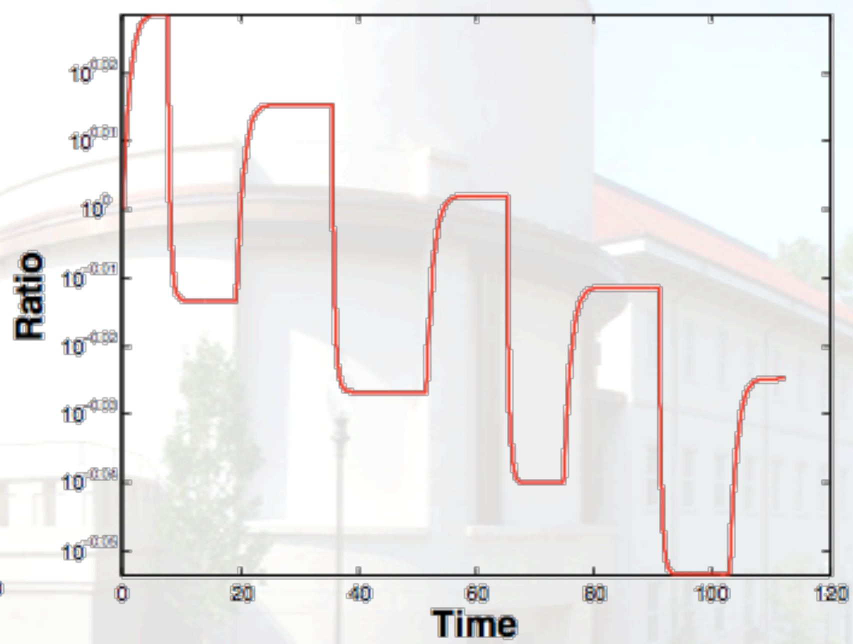
Adiabatic population dynamics with out of phase parameter changes (log-average fitness difference = 0)



Population dynamics with infrequent switching

(growth/death rates and population size are still out of phase)

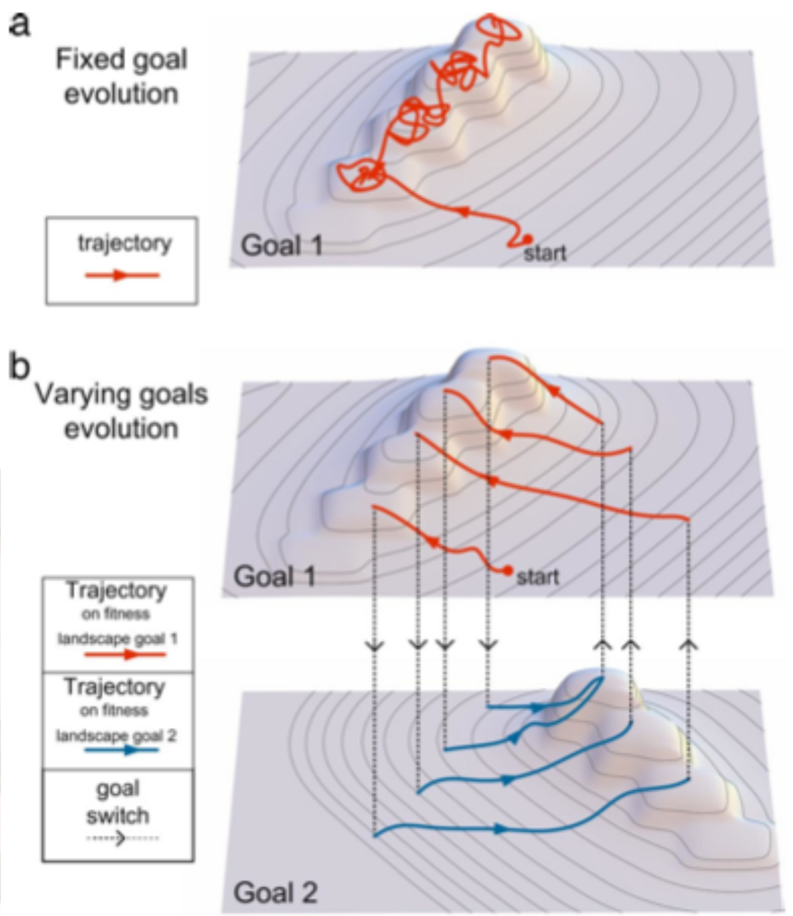




Long term *E. coli* evolution experiment by Lenski et al.

- The sequence of states matters, not just which states
- May be the dominant effect if average fitnesses are close
- Likely can improve fixation speed with different protocols
- Other things being equal, it's *better to be more fit when the population grows*, then the other way around

Alon, Deem and others: switching evolutionary goal speeds up evolution

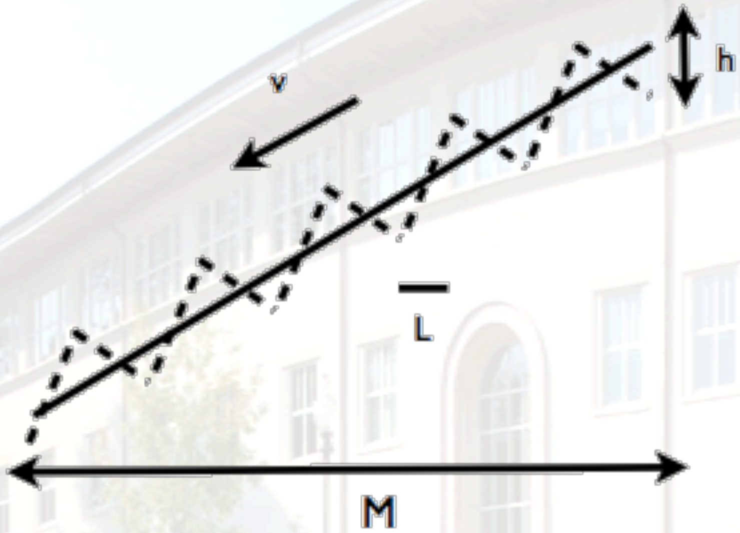


- Crossing a fitness valley is exponentially suppressed by the population size
- Often it is shorter to wait till a valley goes away than to cross it
- How large should perturbations be to allow this? *Should the valleys really disappear?*

generation time < fixation time < environment time

Credits: Kashtan and Alon, 2007; Sun and Deem 2007

Small highly epistatic fluctuations can allow to cross barriers (valley→barrier; fitness→potential)



- Very large population (small genetic drift)

$$\frac{dx}{dt} = -\frac{1}{\gamma}\frac{\partial U(x,t)}{\partial x} + \eta,$$

$$\frac{1}{\gamma}\frac{\partial U}{\partial x} = -v + \phi(x)s(t),$$

$$\phi(x) \equiv \frac{h}{L} \times \mathrm{sign}\left[\sin\frac{\pi x}{L}\right],$$

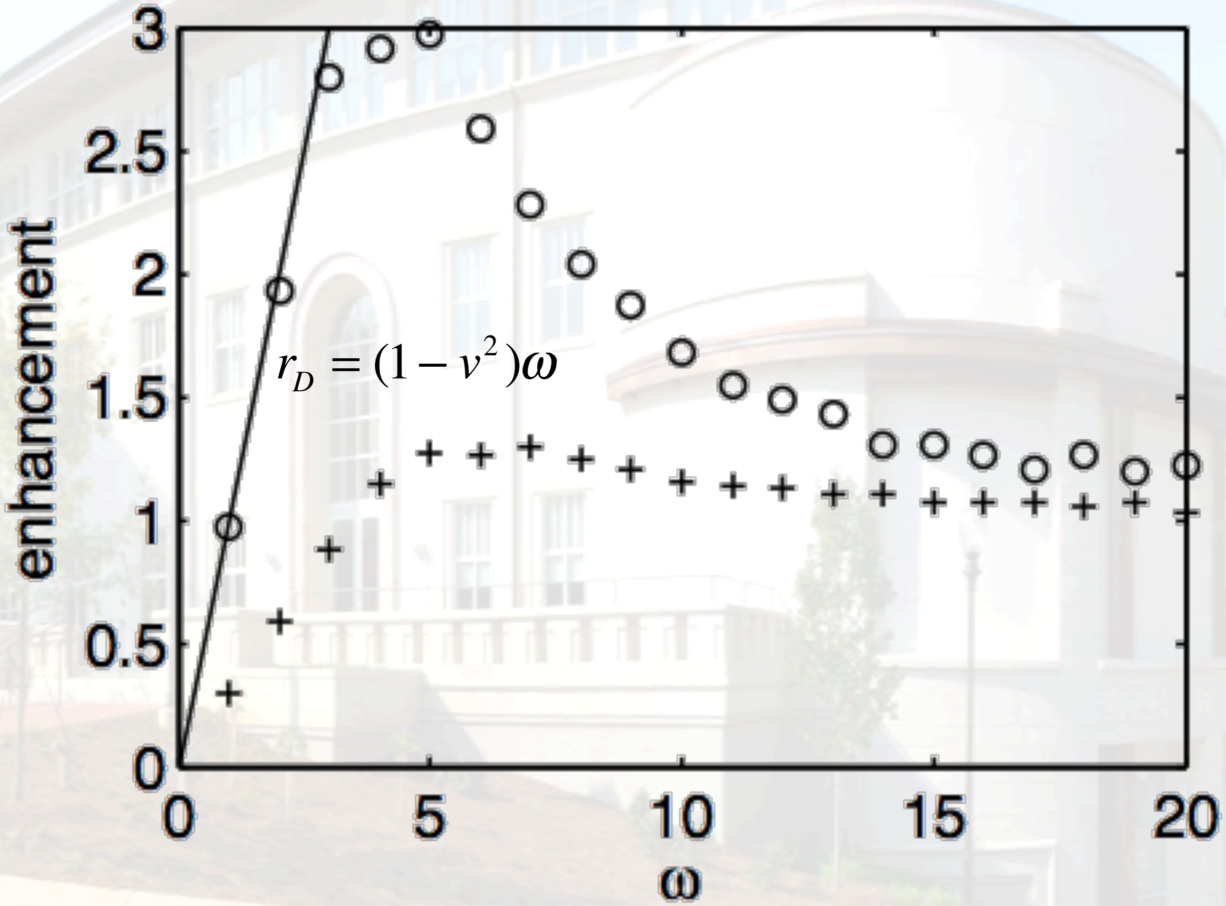
$$s(t) \equiv \mathrm{sign}\left[\sin\frac{\pi t}{T}\right],$$

$\omega = \frac{L^2}{2DT}$ – period of fluctuations in diffusivity units

$\beta = \frac{h}{D}$ – heigh of fluctuations in diffusivity units

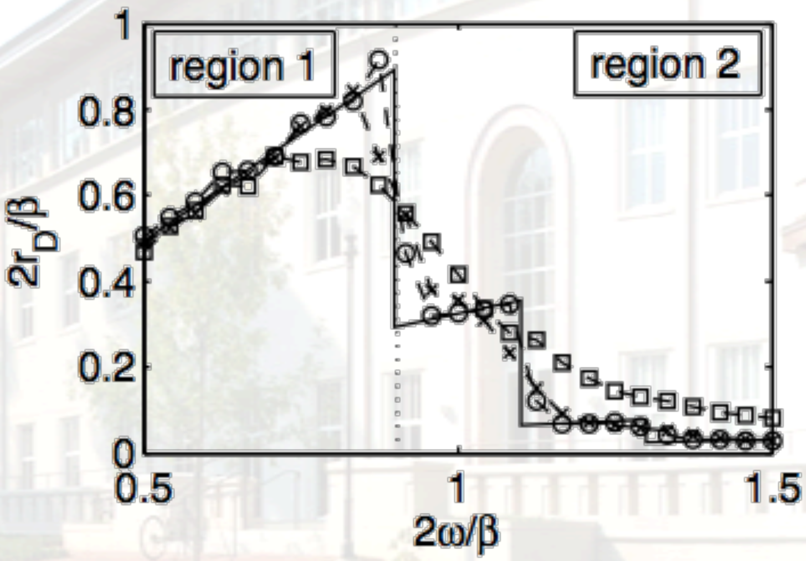
$$\frac{dx}{dt} = \frac{\beta}{2\omega}\left[-\phi(x)s(t) + v\right] + \sqrt{\frac{1}{\omega}}\eta$$

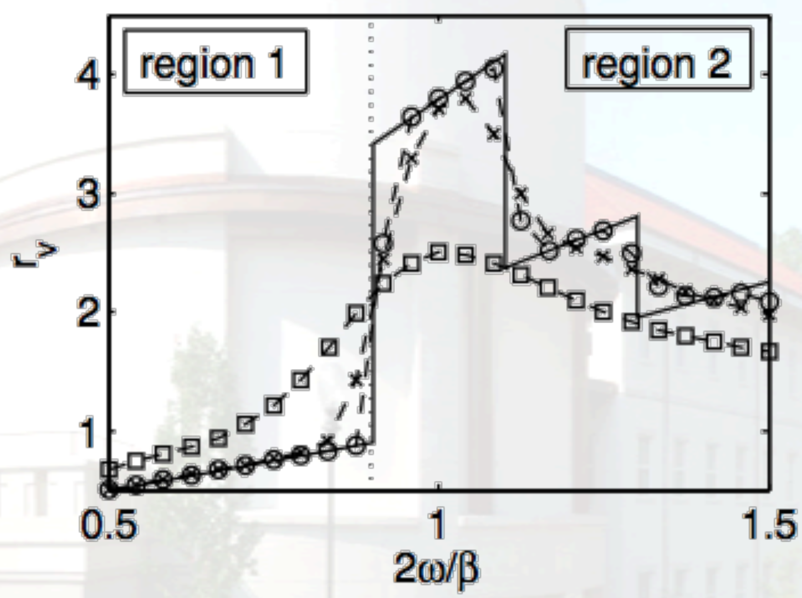
Ratcheting up diffusion: D_{eff}/D (renormalization of the population size)



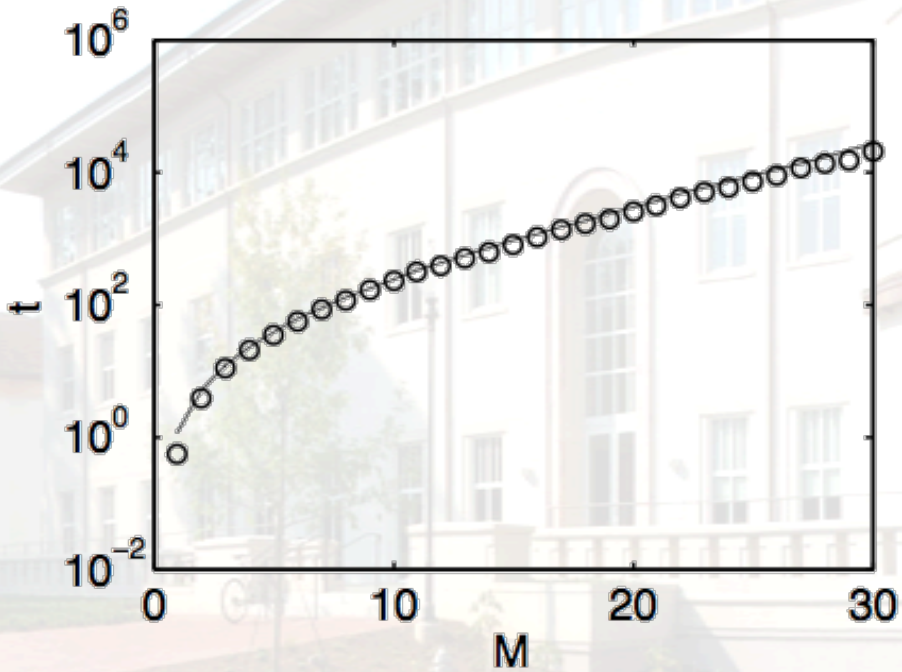
See also: Vergassola and Avellaneda, 1997

Effective diffusion and effective drift





Crossing a barrier of width M: first passage time

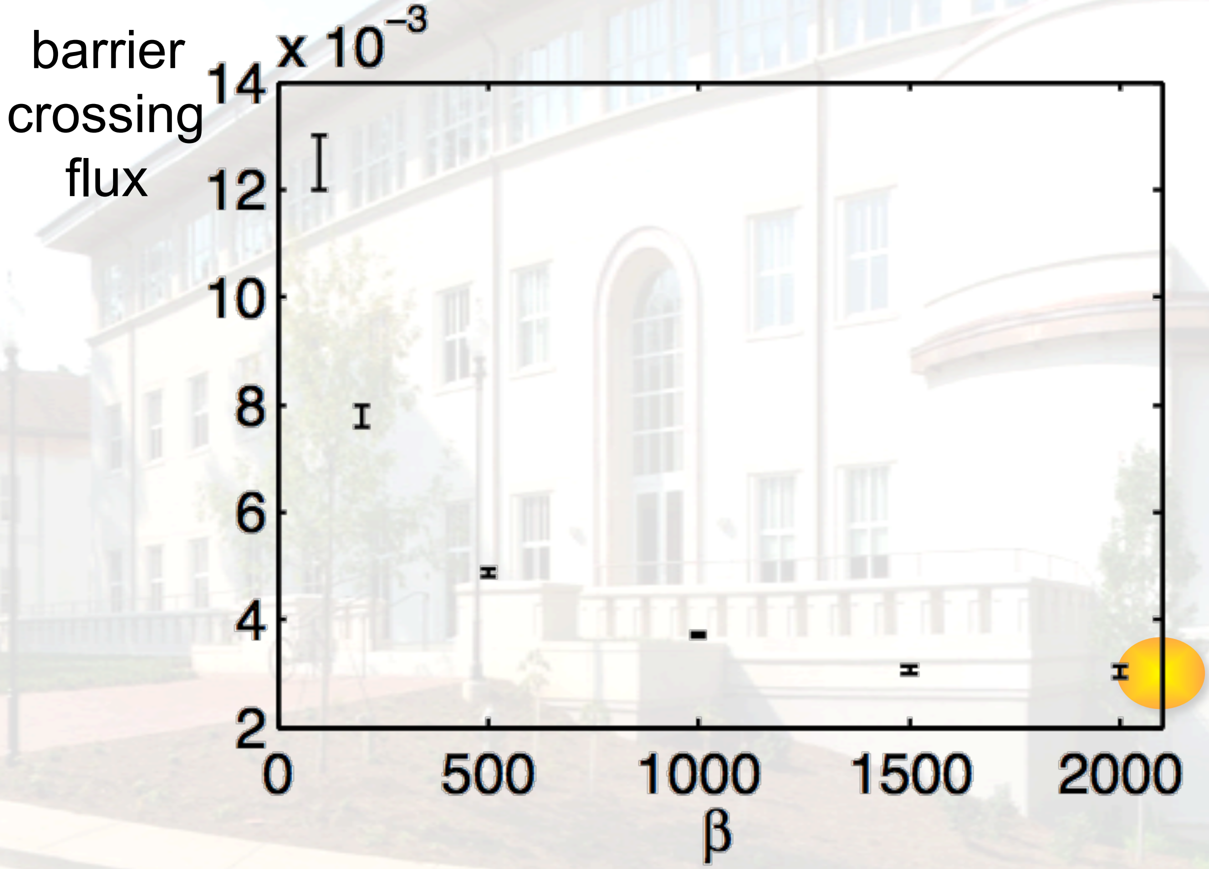


$$\langle t \rangle_D = \frac{\mathcal{M}^2}{D}\left[\frac{1}{2Pe} - \frac{1}{4Pe^2}\left(1 - e^{-2Pe}\right)\right]$$

$$Pe = \left|\frac{vL\mathcal{M}}{2D}\right|$$

crossing with fluctuating perturbations =
crossing without them but with much larger D (smaller N)

Enhanced long-range diffusion: nonzero barrier crossing flux (Peclet number)



non-zero:
peak crossing transport exists for small intrinsic noise

Slow environmental fluctuations

- Enhanced diffusion (genetic drift), renormalize the population size
- Small fluctuations are sufficient if rugged (epistatic)
- Allow barrier crossing with little intrinsic noise
- Relations to optimization literature

How do time-dependent environments get created?

- For example, need motion across spatial inhomegeneities
- Why would motility evolve?
  - To escape predators?
    - But motility enhances probability of meeting a predator
  - To get more food
    - But chemotactic genes are younger than flagellar genes

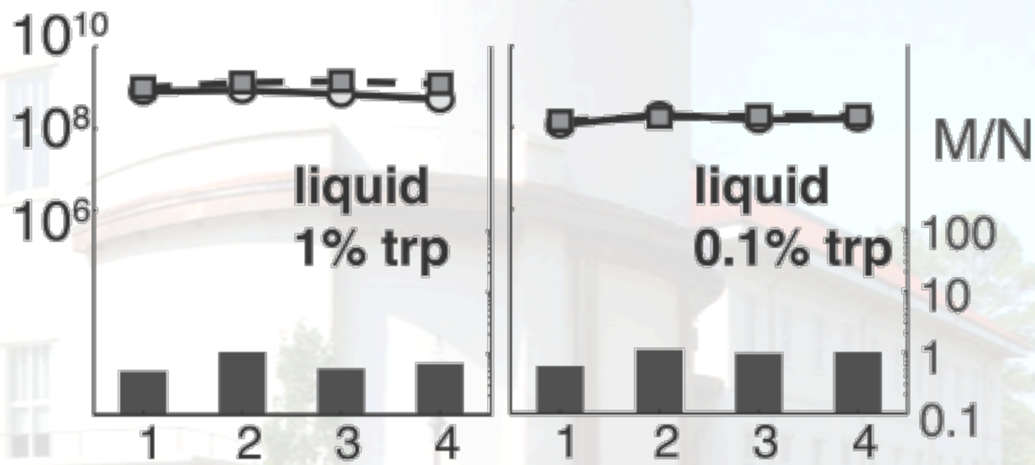
Levin and Weiss labs

No selective advantage for motile bacteria in well-mixed environments

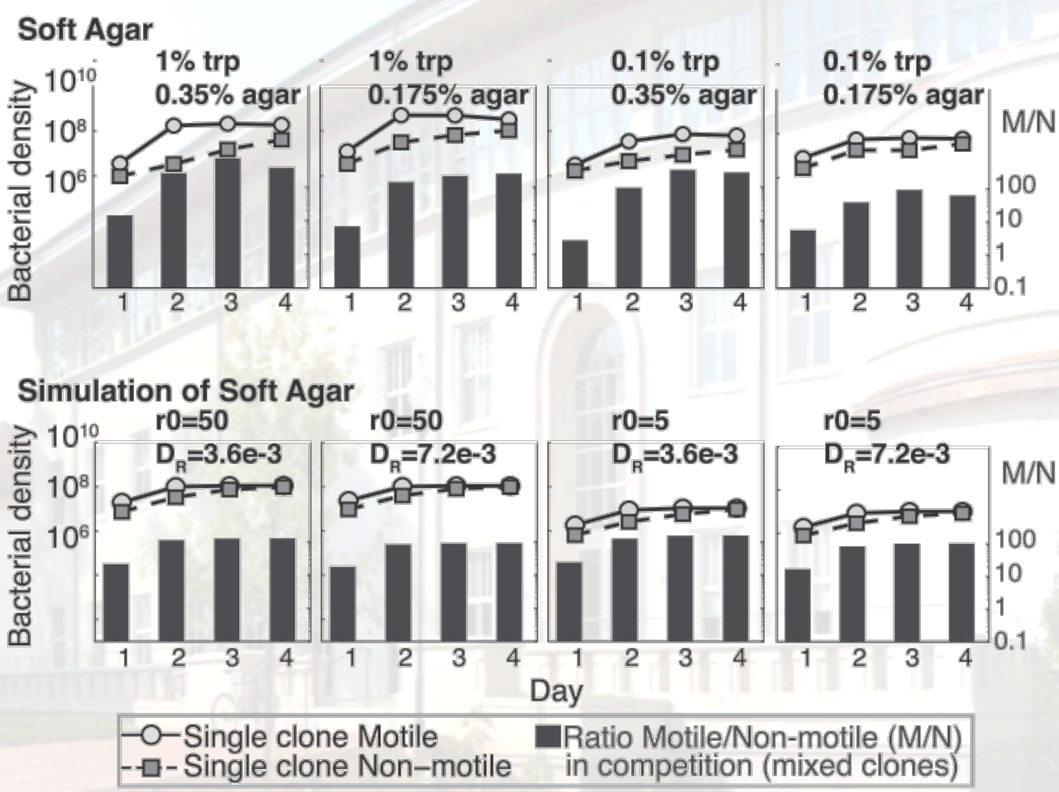
$$\frac{\partial b_M}{\partial t} = D_M\left(\frac{\partial^2 b_M}{\partial x^2} + \frac{\partial^2 b_M}{\partial y^2}\right) + \frac{\alpha r}{r+k} b_M$$

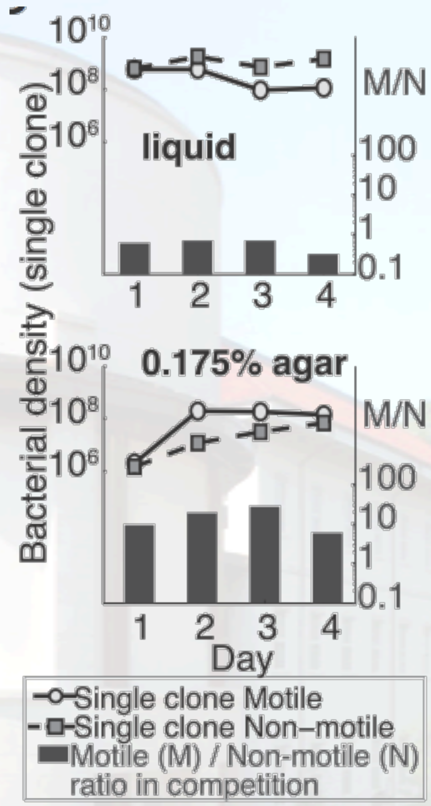
$$\frac{\partial b_N}{\partial t} = D_N\left(\frac{\partial^2 b_N}{\partial x^2} + \frac{\partial^2 b_N}{\partial y^2}\right) + \frac{\alpha r}{r+k} b_N$$

$$\frac{\partial r}{\partial t} = D_r\left(\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2}\right) - \frac{\nu r}{r+k}(b_M + b_N),$$



But large advantage in semi-soft, structured agar





Che+

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Motility is favored without chemotaxis

- Same story should hold for various source-sink time-dependent models
- Caused by asymmetry in gain/loss from going to different places
- Taking risks, exploring is evolutionary advantageous

Conclusions

- Medium time scale fluctuations – sequence of the environments matters, geometric effects emerge
- Slow time scale fluctuations – ratcheting can help cross fitness valleys
- Motility can evolve in structured environments, and then will generate temporally variable environments for individuals

The End