Kavli Institute for Theoretical Physics
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Quasi-linear wave-mean flow interactions in large-scale planetary circulations

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The quasi-linear approximation

Quasi-linear (QL) model
(for quadratic non-linearities, for example in barotropic, QG, primitive equations….)

Retained:

- Wave-mean flow interaction (mean flow = zonal average)
- Interaction of waves of opposite zonal wavenumber (Reynolds stress)

Neglected:

- All other eddy-eddy interactions

The quasi-linear approximation

Advection of a quantity $a = \overline{a} + a'$ by a meridional flow $v = \overline{v} + v'$ (zonal average/eddy decomposition):

$$\frac{\partial a}{\partial t} = -\overline{v}\frac{\partial \overline{a}}{\partial y} - \overline{v}\frac{\partial a'}{\partial y} - v'\frac{\partial \overline{a}}{\partial y} - v'\frac{\partial a'}{\partial y}$$

Equation for the mean flow unchanged

$$\frac{\partial \overline{a}}{\partial t} = -\overline{v}\frac{\partial \overline{a}}{\partial y} - \overline{v'\frac{\partial a'}{\partial y}}$$

Equation for the eddies linearized around the mean flow

$$\frac{\partial a'}{\partial t} = -\overline{v}\frac{\partial a'}{\partial y} - v'\frac{\partial \overline{a}}{\partial y} - \left(v'\frac{\partial a'}{\partial y} - \overline{v'\frac{\partial a'}{\partial y}}\right)$$

0

Quasi-Linear Approximation

Why a quasi-linear approximation?

- Truncation that captures wave mean flow interactions while conserving 1st and 2nd order inviscid invariants (see *Wanming Qi poster*)

- Linear framework useful for **analytical theories** (*SSST of Farrell and Ioannou; Zonostrophic instability of Srinivasan and Young, 2012; Kinetic theory of Bouchet and coauthors, 2013, see Cesare Nardini poster)*

- First step to develop **statistical closures** (QL equivalent to a second order closure, *CE2 of Marston and Tobias*)

Why a quasi-linear approximation?

- QL works in **barotropic flows** when there is **time scale separation** between mean flow and eddies and **no unstable modes** (e.g. *Bouchet and coauthors, see Cesare Nardini poster*)

- "Quasi-linear reasoning" ubiquitous in the large scale dynamics literature. Large-scale atmospheric flows look linear over a wide range of parameters (Schneider and Walker, 2008). Some theoretical explanation: Schneider and Walker, 2006.

- **How does QL work in large-scale planetary baroclinic flows with large-scale forcing and "strong" surface friction?**

(Whitaker and Sardeshmuck, 1997; Zhang and Held, 1999; Delsole, 2001)

An idealized dry GCM

- FMS GFDL pseudospectral dynamical core

- **Radiation**: Newtonian relaxation toward temperature profile
- **Convection**: relaxation of vertical lapse rate toward 0.7 × (dry adiabatic)
- Diffusion of momentum and dry static energy in the **boundary layer**

OR

- **Radiation and convection**: Newtonian relaxation to a statically stable temperature profile, (Held-Suarez)
- Rayleigh drag in the **boundary layer**

- Uniform surface, no seasonal cycle
- Run at T85 (256 x 128) with 30 vertical sigma-levels
- 600 days average after 1400 days spin-up

(Held and Suarez, 1994; Schneider and Walker, 2006)

Removal of the eddy-eddy interactions in the GCMs

At each time step:

Advection of a quantity $a = \overline{a} + a'$ by a meridional flow $v = \overline{v} + v'$ (zonal average/eddy decomposition):

$$\frac{\partial a}{\partial t} = -v\frac{\partial a}{\partial y} = -\overline{v}\frac{\partial \overline{a}}{\partial y} - \overline{v}\frac{\partial a'}{\partial y} - v'\frac{\partial \overline{a}}{\partial y} - \boxed{v'\frac{\partial a'}{\partial y}}$$

transformed into

$$\frac{\partial a}{\partial t} = -v\frac{\partial a}{\partial y} = -\overline{v}\frac{\partial \overline{a}}{\partial y} - \overline{v}\frac{\partial a'}{\partial y} - v'\frac{\partial \overline{a}}{\partial y} - \boxed{\overline{v'\frac{\partial a'}{\partial y}}}$$

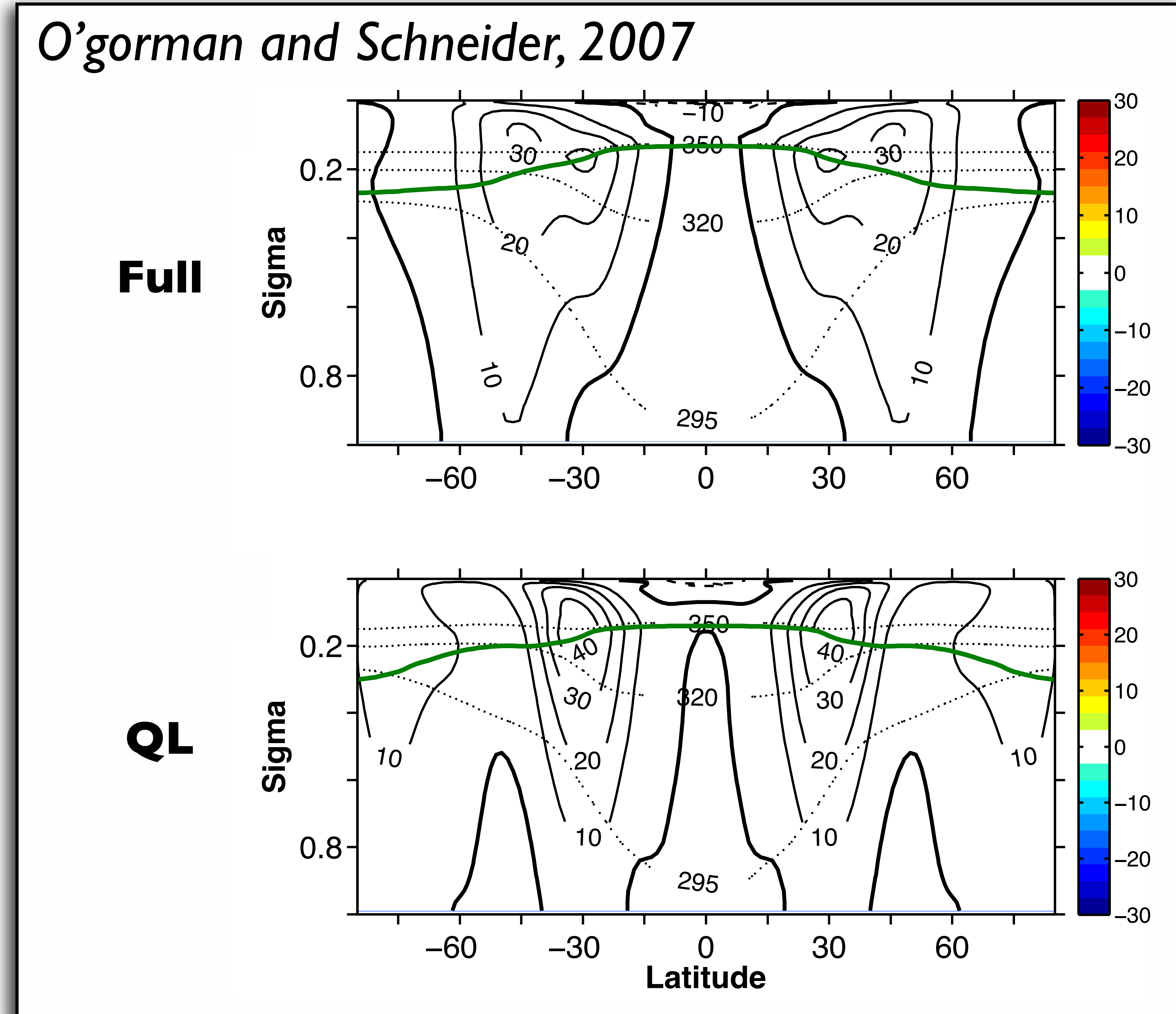
O'gorman and Schneider, 2007

Mean Zonal circulation

Contours: zonal mean flow (m/s)

Dotted lines: potential temperature (K)

Grey line: tropopause



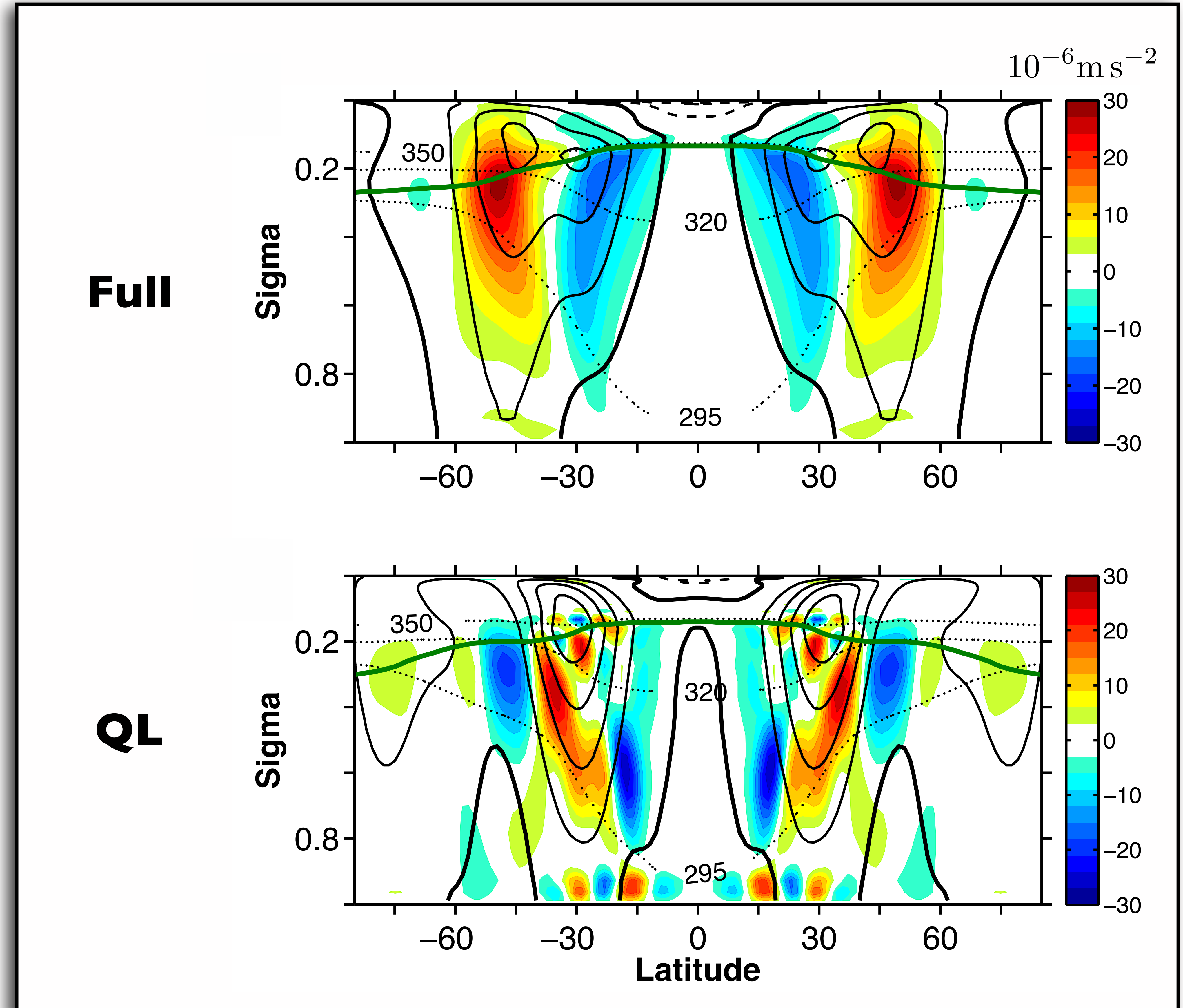
Eddy momentum flux convergence

Colors: convergence of eddy momentum flux $\overline{u'v'} \cos \phi$

Contours: zonal mean flow (m/s)

Dotted lines: potential temperature (K)

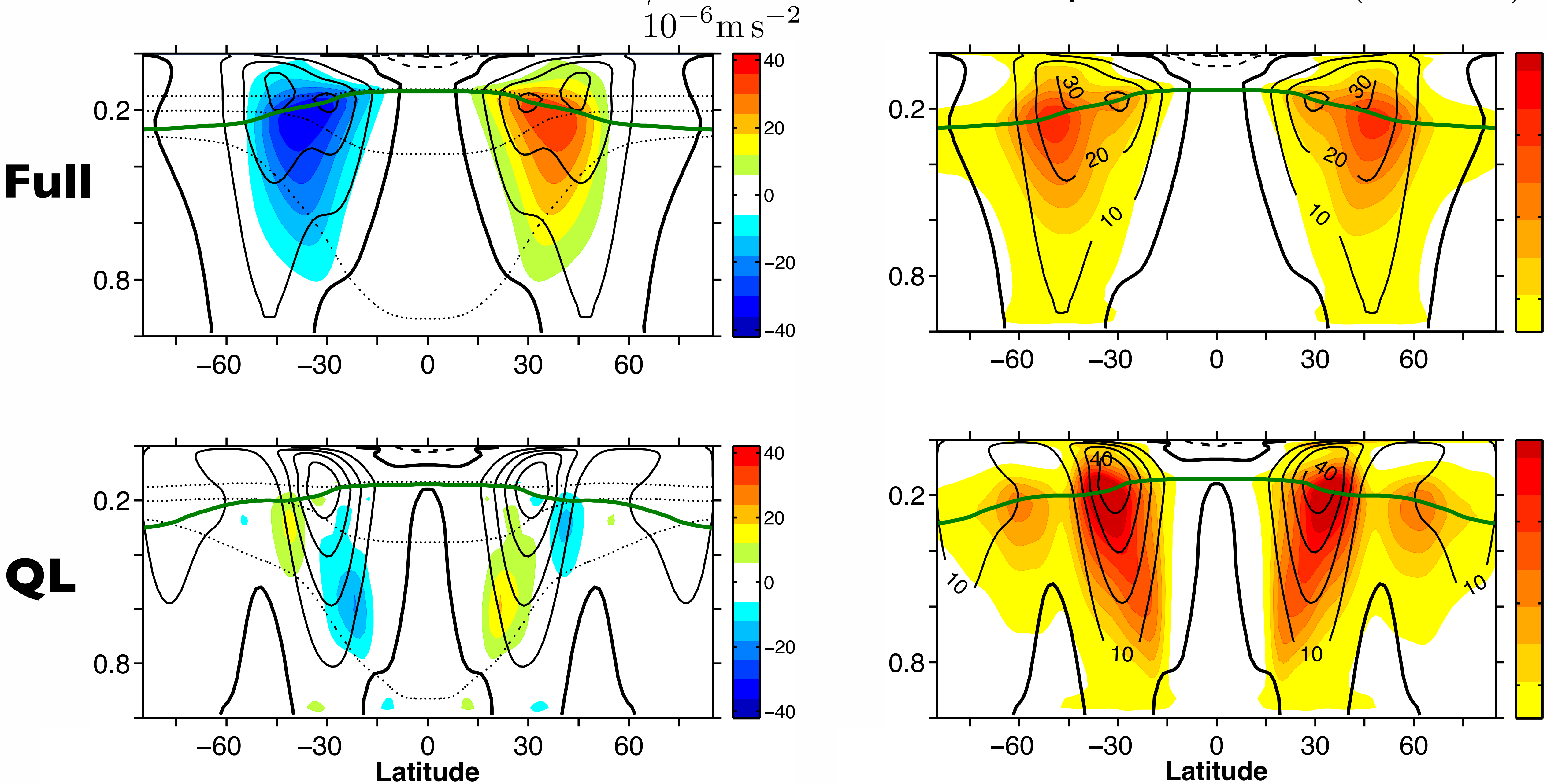
Green line: tropopause



Momentum flux and eddy kinetic energy

Contours: Zonal wind $\overline{u}$ in m/s
Colors: momentum flux $\overline{u'v'}\cos\phi$

Contours: Zonal wind $\overline{u}$ in m/s
Colors: Square root of EKE $(\overline{u'^2}+\overline{v'^2})^{\frac{1}{2}}$



- Momentum flux not enhanced in the upper troposphere
- Weak momentum flux associated with strong EKE in the upper troposphere

QL model

- Narrow, strong jets
- Momentum flux not enhanced in the upper troposphere
- Weak momentum flux associated with strong EKE in the upper troposphere

Dynamics ???

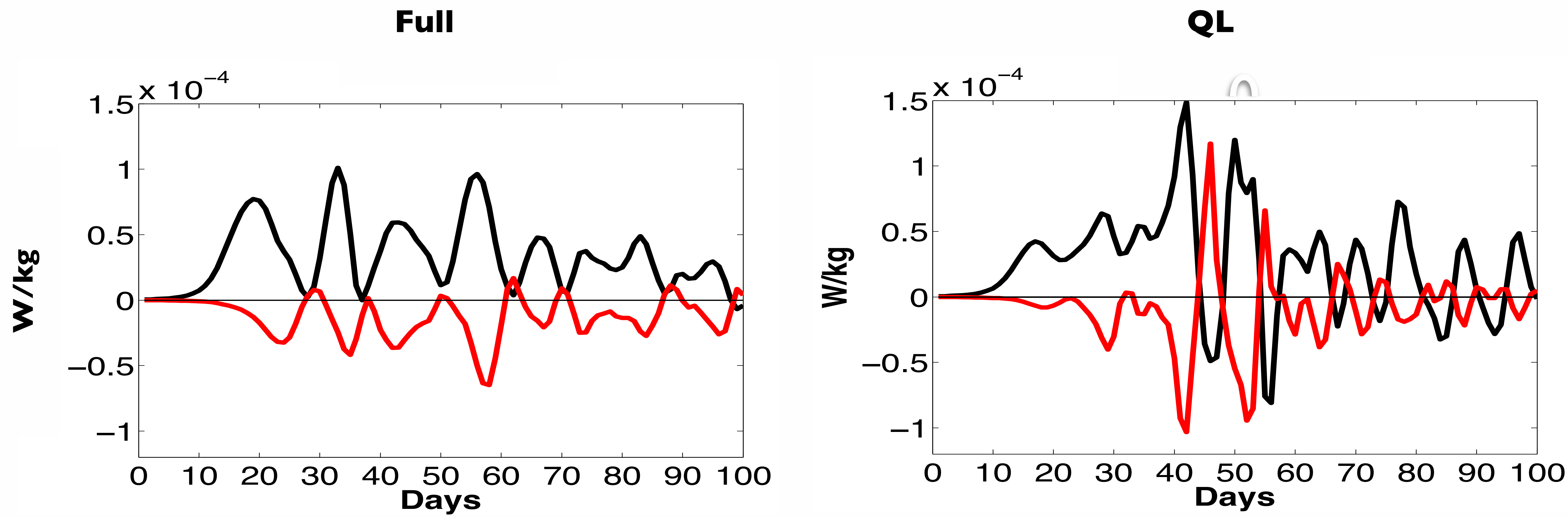
→ **Baroclinic wave lifecycle experiments**

- Initialize a zonal wavenumber 6 perturbation in the zonally averaged circulation (fully non-linear model)
- Experiments run with full model and the no eddy-eddy model

Lifecycle experiments

Baroclinic conversion. Eddy available potential energy to eddy kinetic energy

Barotropic conversion. Zonal kinetic energy to eddy kinetic energy



- Barotropic conversion large and positive at the end of some lifecycles
- Baroclinic decay significant

EP flux and QG potential vorticity flux

Arrows:
EP flux

$$\mathbf{F} = R\cos\phi \begin{pmatrix} -\overline{u'v'} \\ f\,\overline{v'\theta'}/\frac{\partial\bar{\theta}}{\partial p} \end{pmatrix}$$

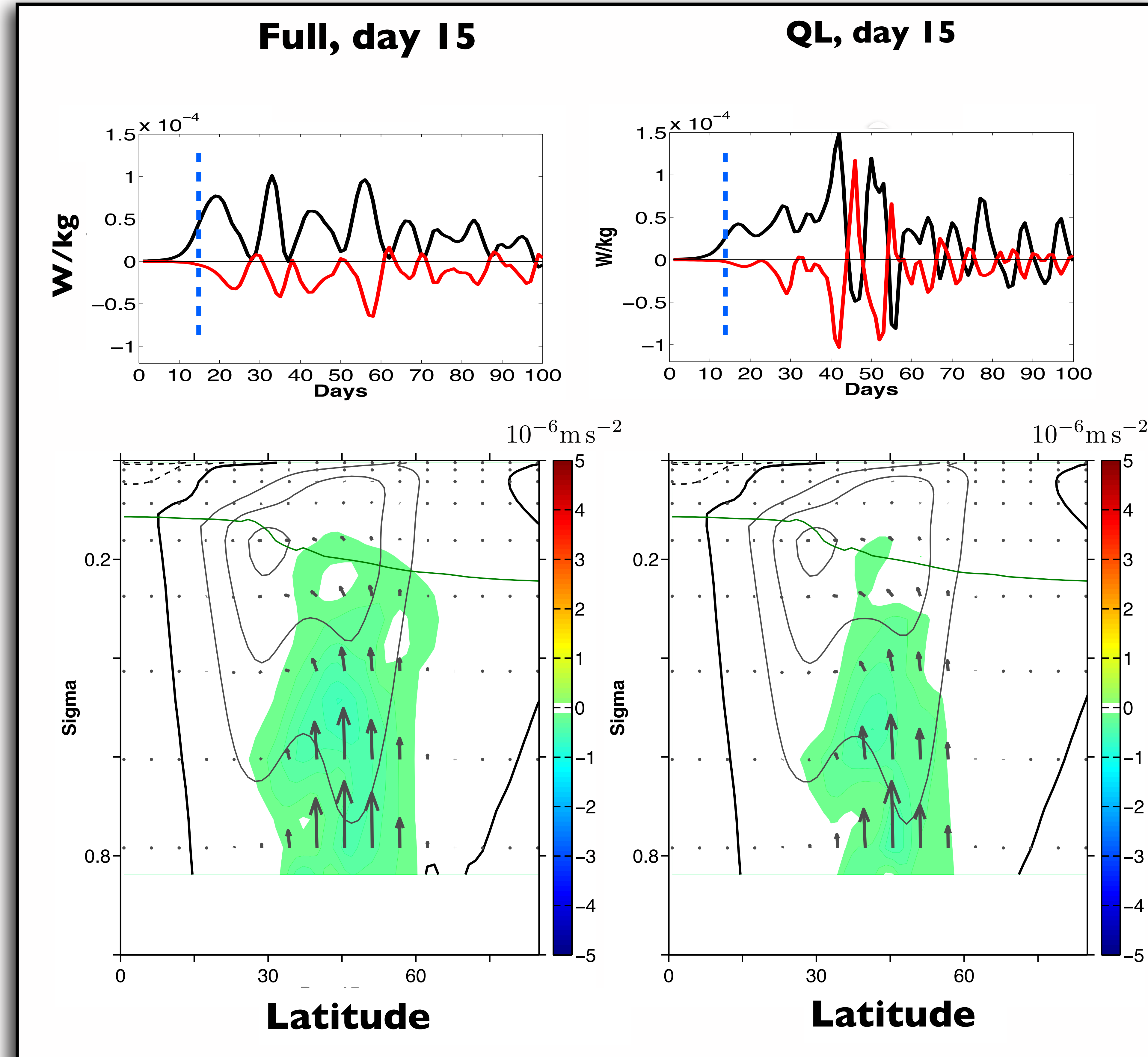
Colors:
EP flux divergence,
or QG PV flux,

$$\nabla\cdot\mathbf{F} = R\cos\phi\,\overline{v'q'}$$

Wave activity equation:

$$\frac{\partial A}{\partial t} + \nabla\cdot\mathbf{F} = \mathcal{D}$$

(Nakamura and Zhu, 2010)



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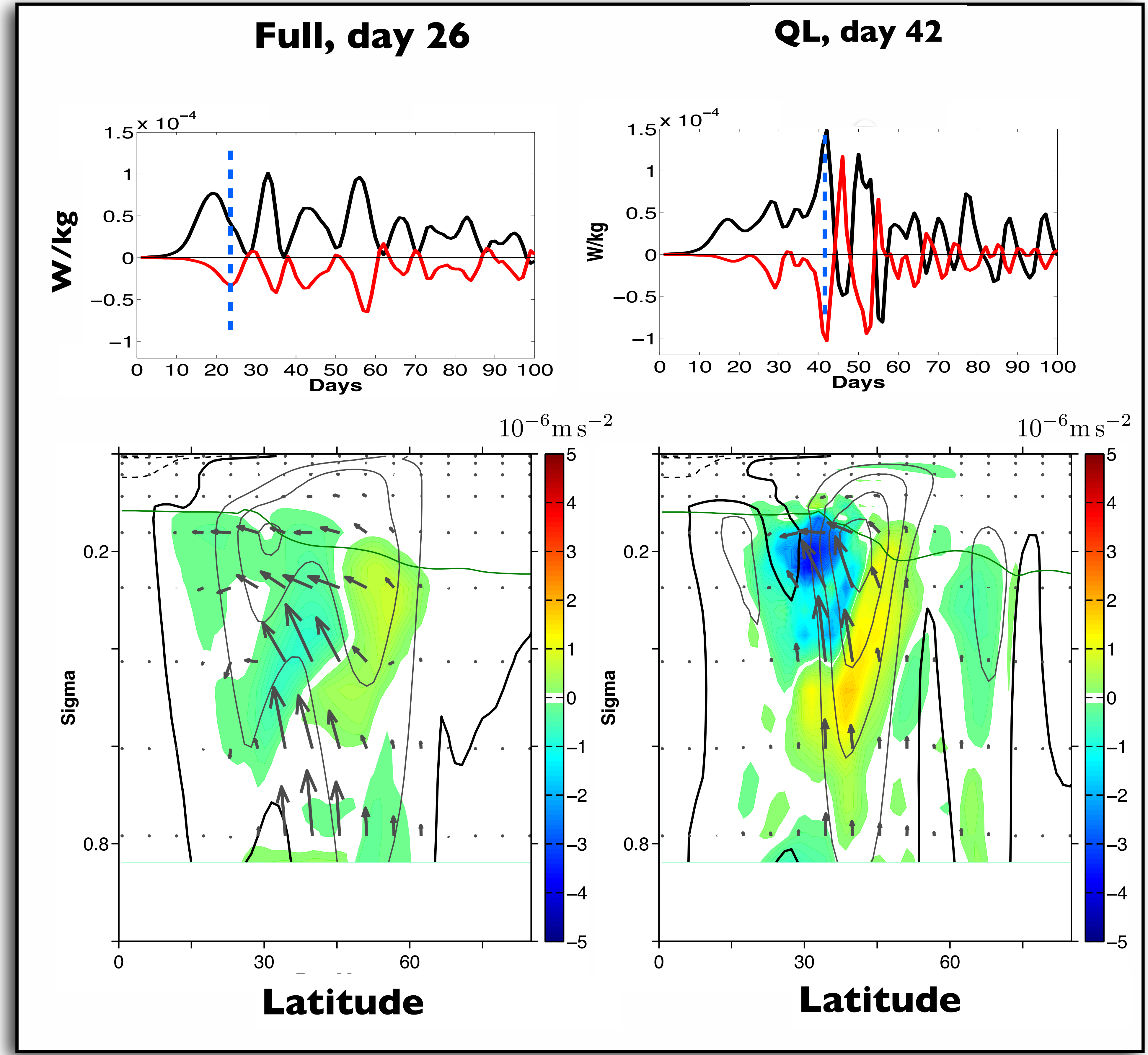

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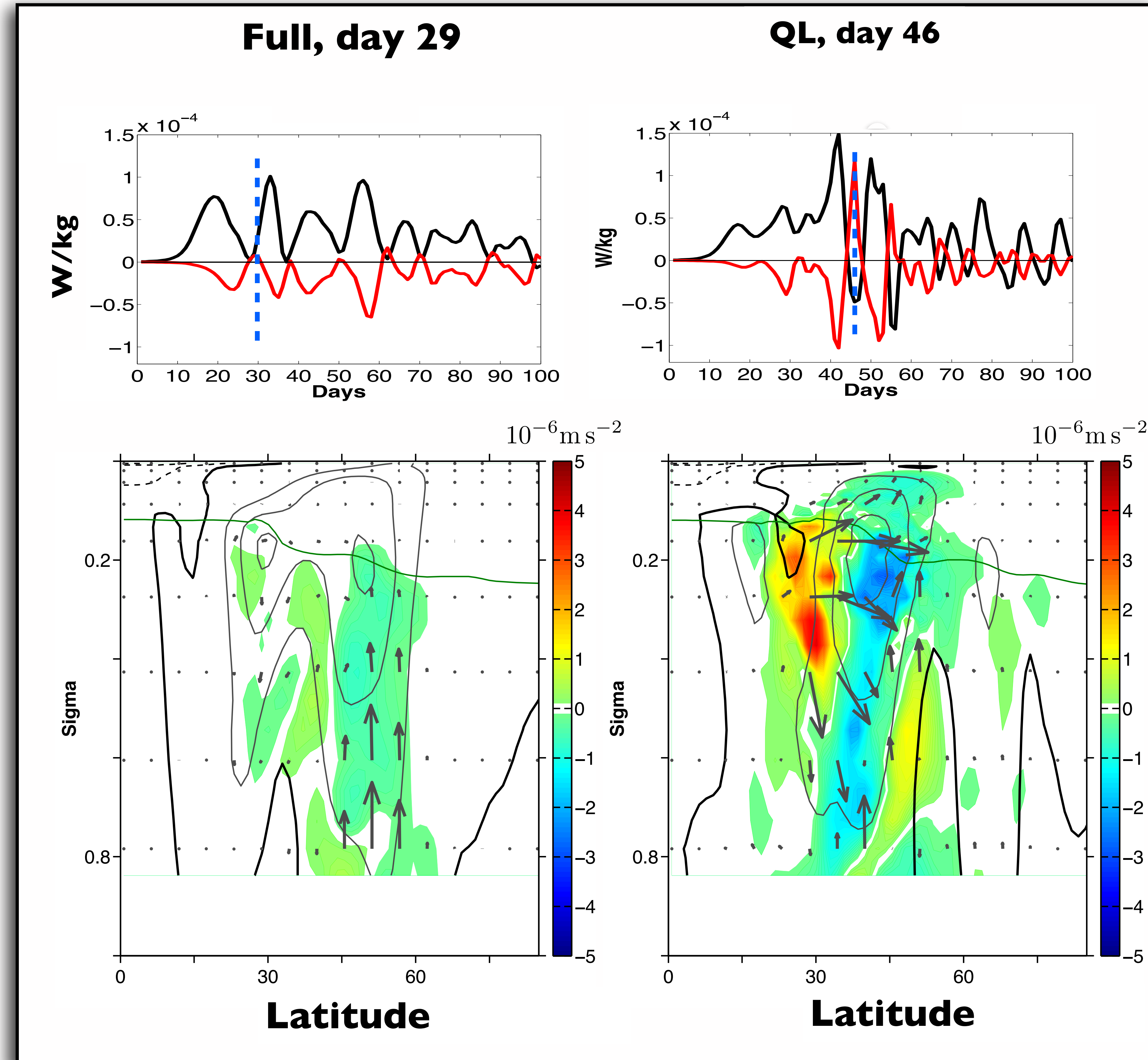
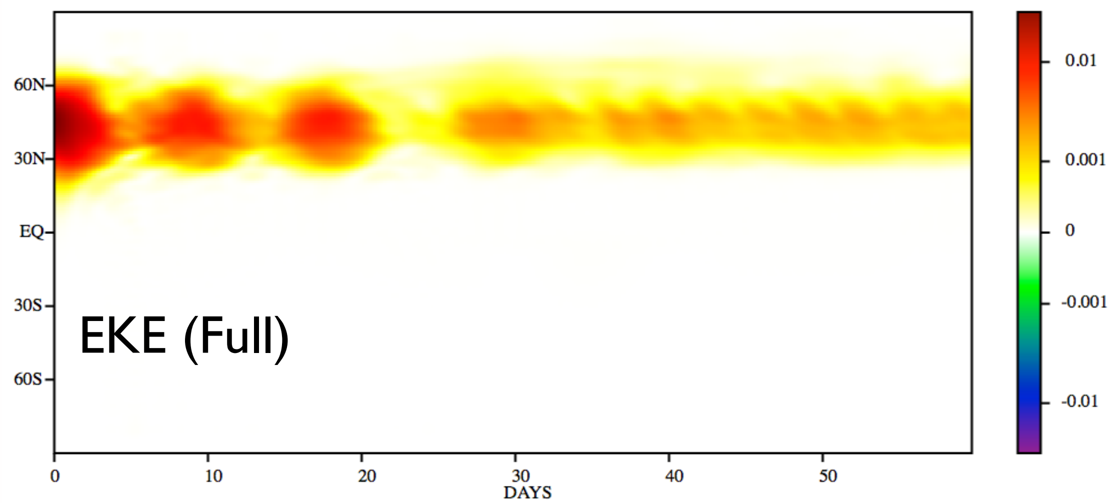


QL Lifecycle experiments

- Absence of eddy-eddy interaction prevents wave activity dissipation
- Wave activity reemitted from critical layer
- “Reflected” wave packet can decay baroclinically

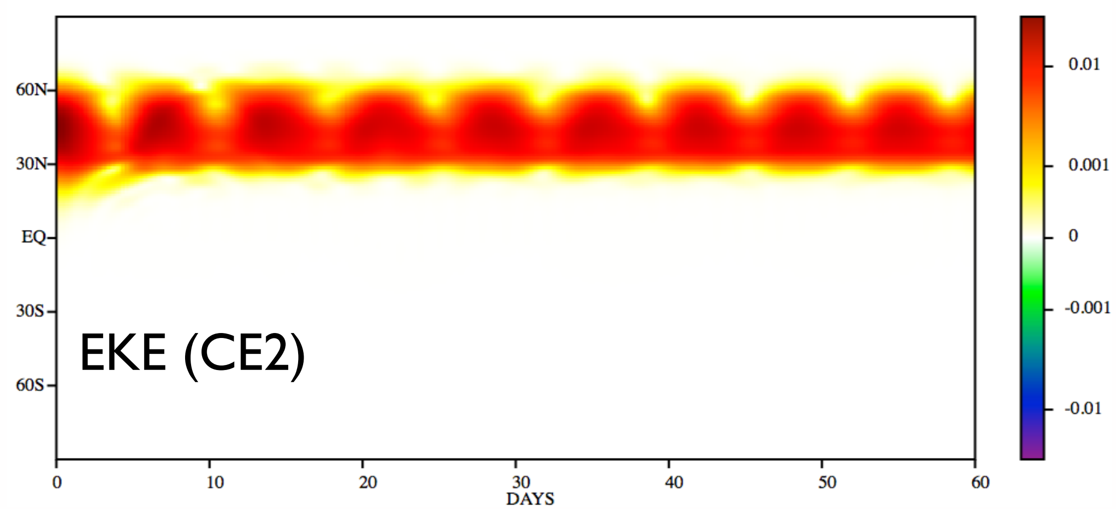
Wave breaking in a barotropic model

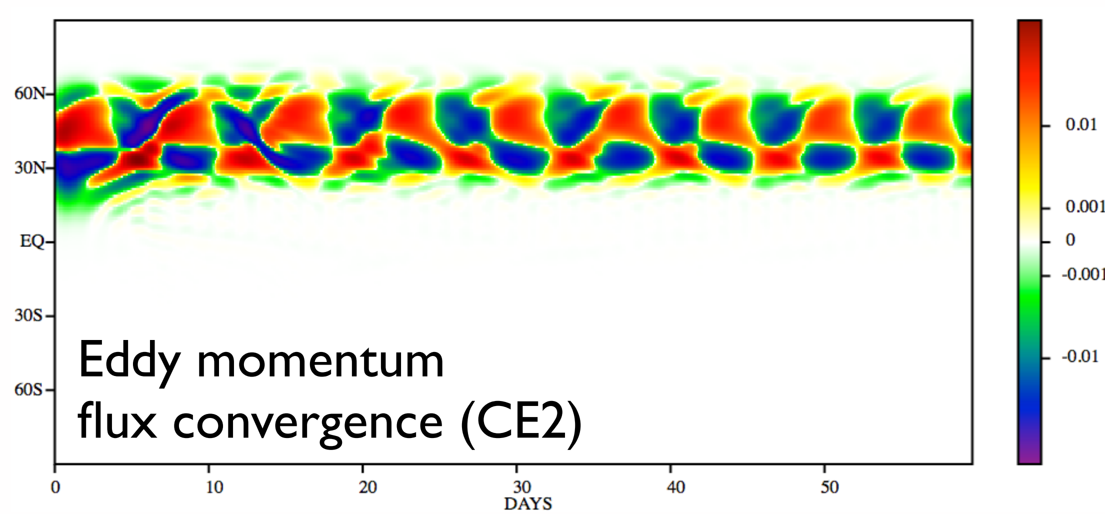
Initial wave at mid-latitudes. The only dissipation is hyperviscosity

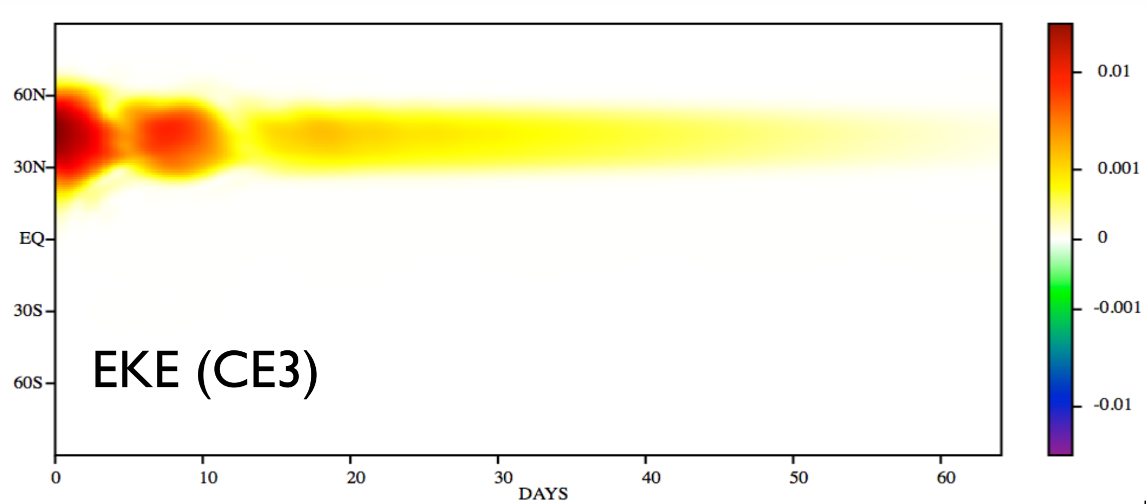


CE2 ~ quasi-linear approx

CE3: third order closure. Negative eigenvalues of the second order cumulant are projected out for realizability.







- Rossby wave "resonance" in CE2
- More realistic Rossby wave absorption in CE3

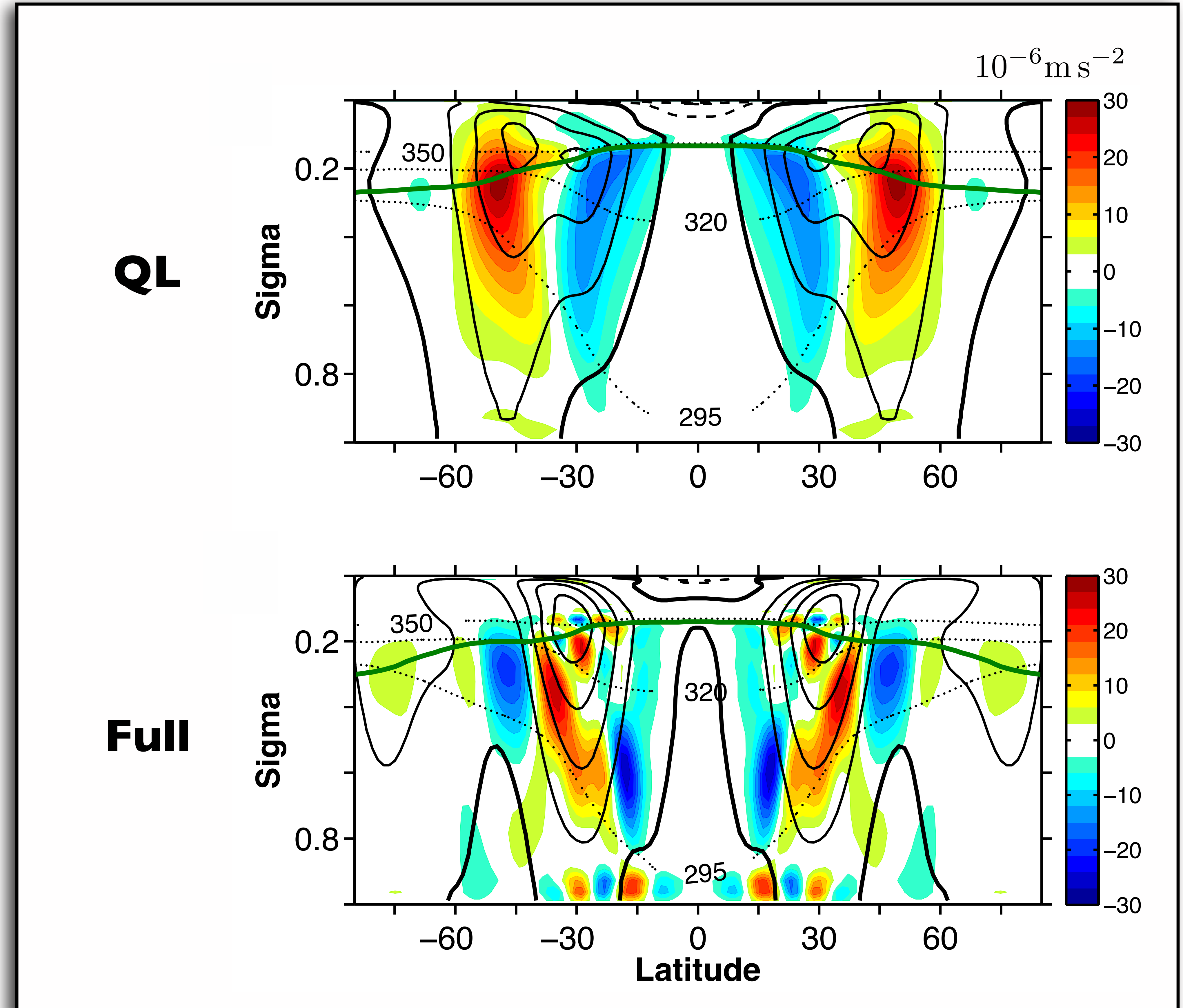
Eddy momentum flux convergence

Colors:
Eddy momentum flux convergence

Contours:
zonal mean flow (m/s)

Dotted lines:
potential temperature (K)

Green line:
tropopause



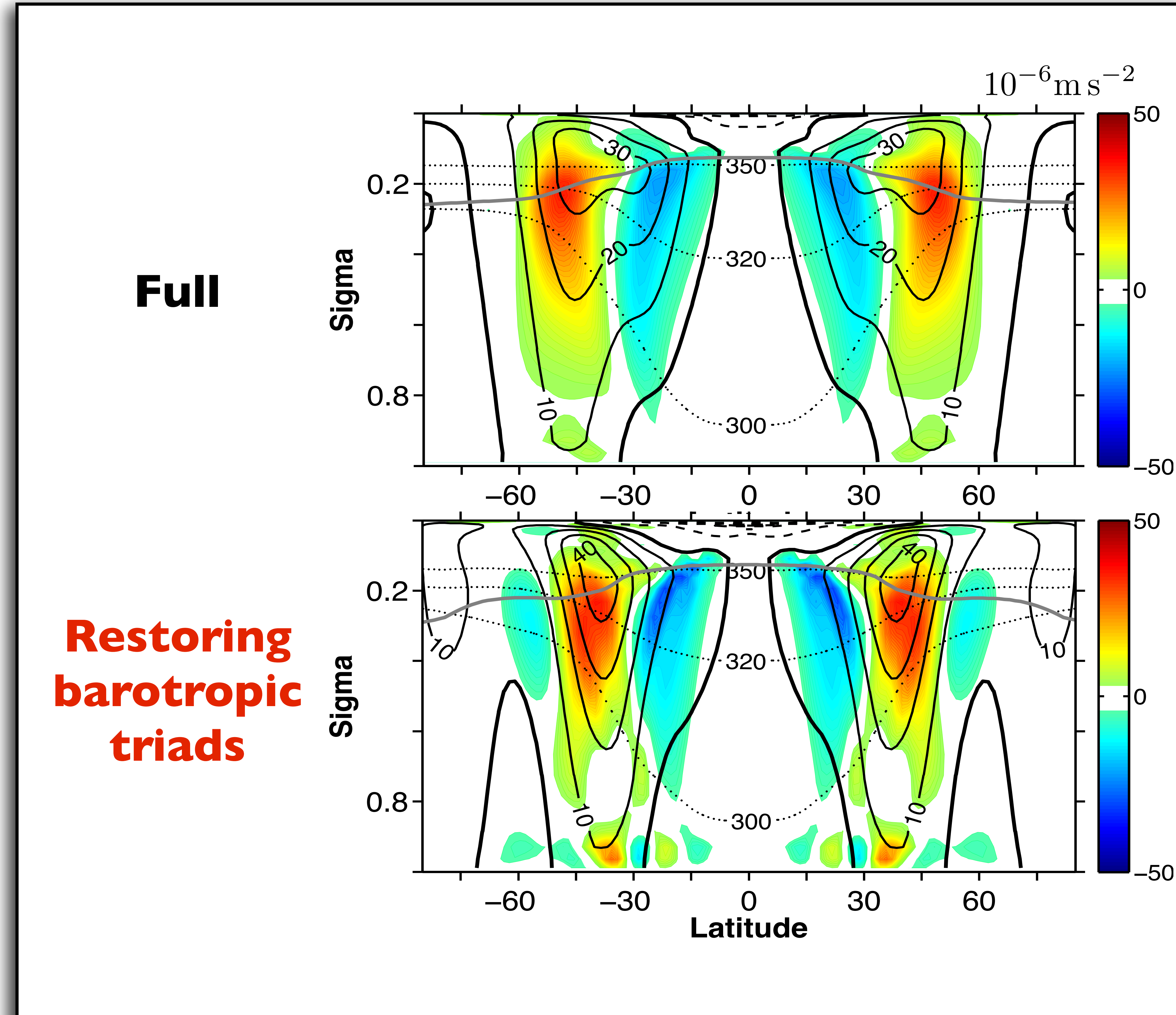
Restoring barotropic triads

Colors: eddy momentum flux convergence

Contours: zonal mean flow (m/s)

Dotted lines: potential temperature (K)

Grey line: tropopause

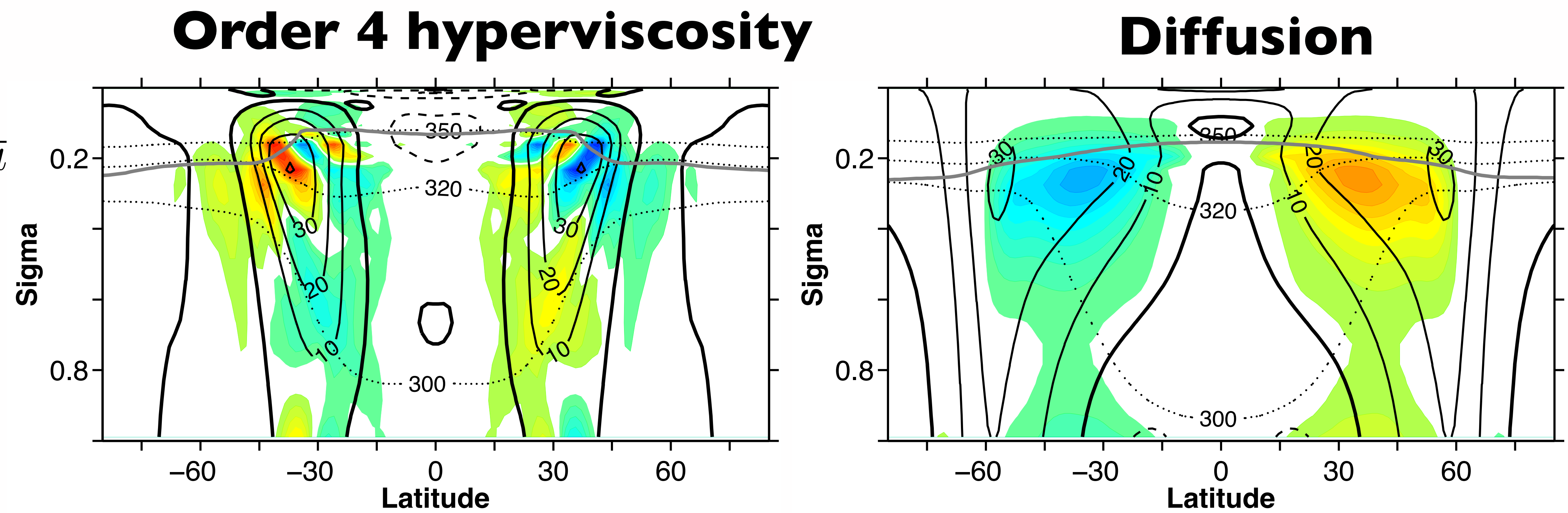


Viscous critical layer in the QL approximation

Held and suarez GCM, QL

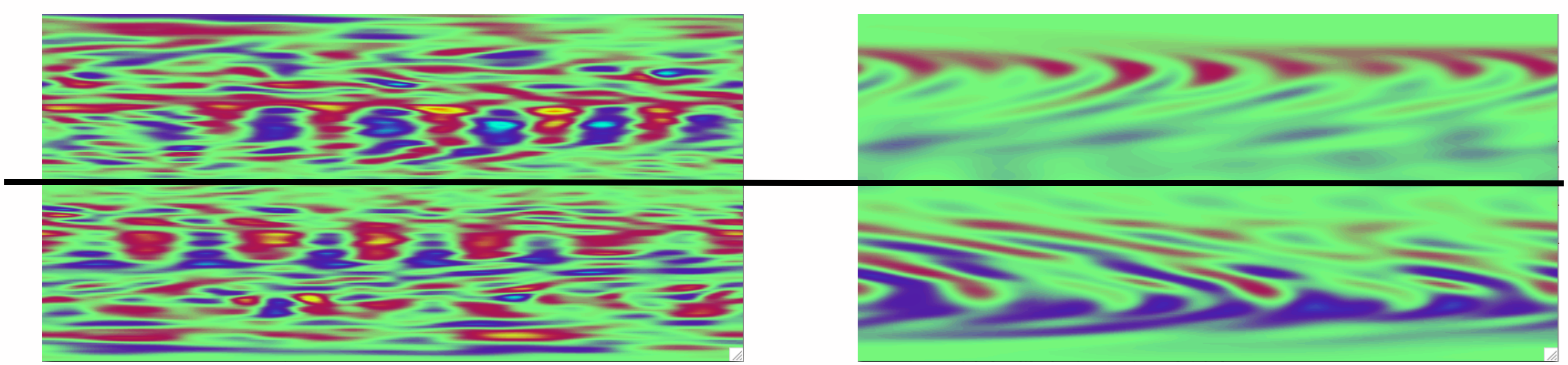
Contours: Zonal wind $\overline{u}$ in m/s

Colors: momentum flux $\overline{u'v'}\cos\phi$



Relative vorticity, 274 hpa

Eq



Upper-tropospheric wave absorption and EMF enhancement captured by a viscous critical layer

Varying surface friction

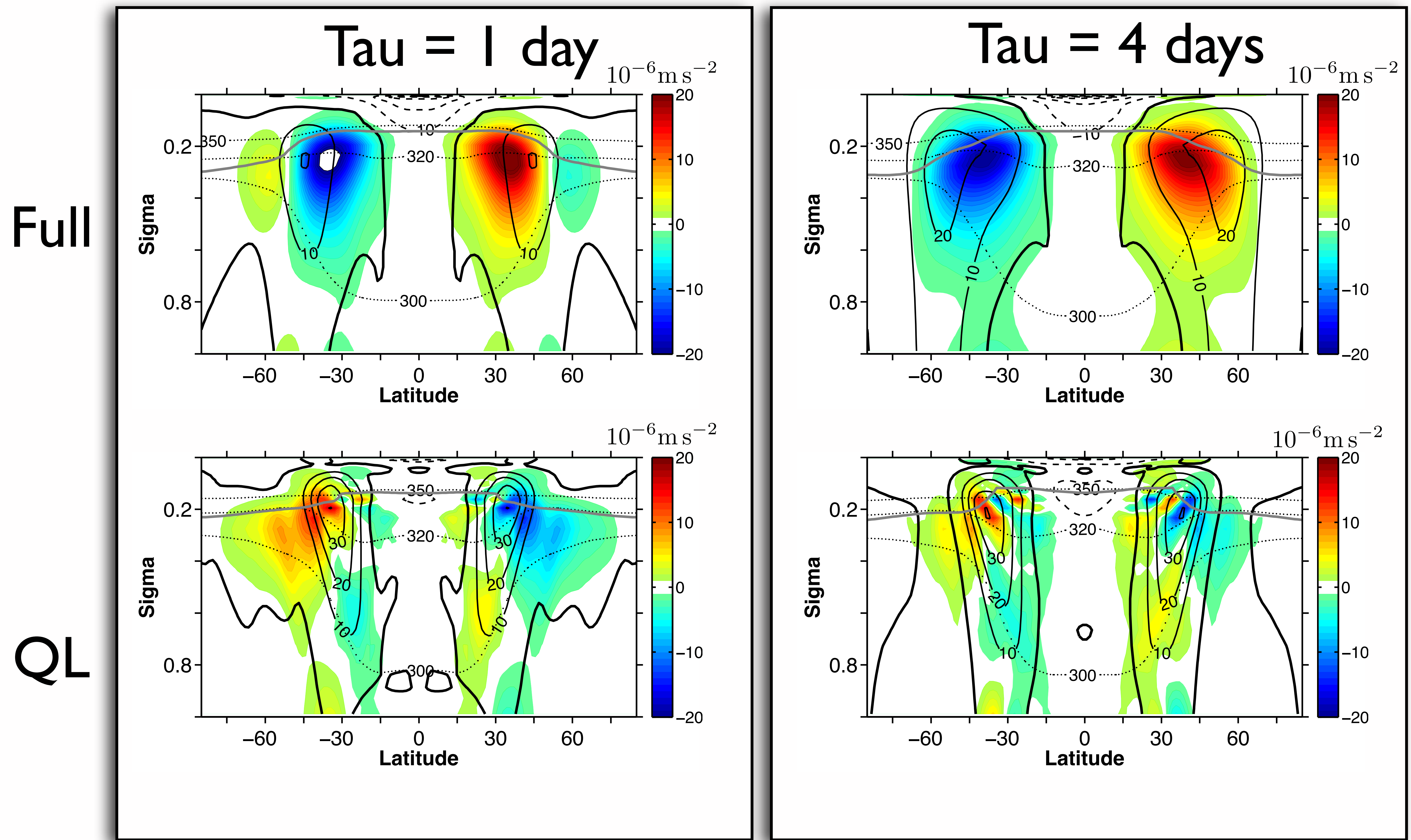
- Surface friction essential in the general circulation for momentum and energy budget.

- Complex effect on midlatitudes eddies amplitude, jet streams strength and location, energy conversions (James, 1987; Robinson 1997; Chen et al., 2007; Liu and Schneider, 2014, etc...)

- How does QL captures general circulation when varying surface friction? **We change Tau, the time scale of Rayleigh drag in the planetary boundary layer.**

Varying surface friction: momentum fluxes

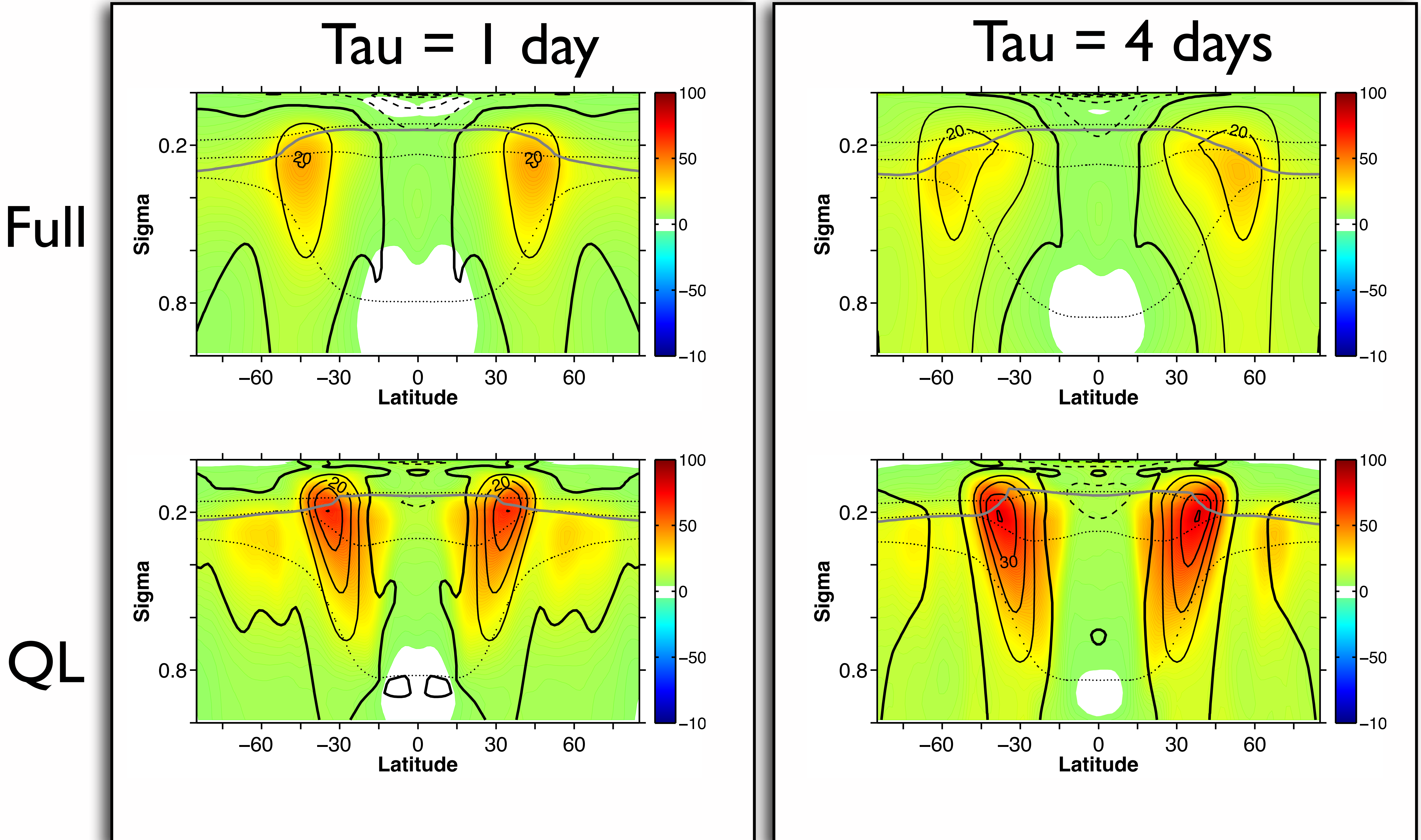
Colors: momentum flux $\overline{u'v'}\cos\phi$ in m^2/s^2 **Contours**: Zonal wind $\overline{u}$ in m/s



Varying surface friction: zonal wind and EKE

Colors: Eddy kinetic energy $(\overline{u'^2} + \overline{v'^2})^{\frac{1}{2}}$ in m/s

Contours: Zonal wind $\overline{u}$ in m/s



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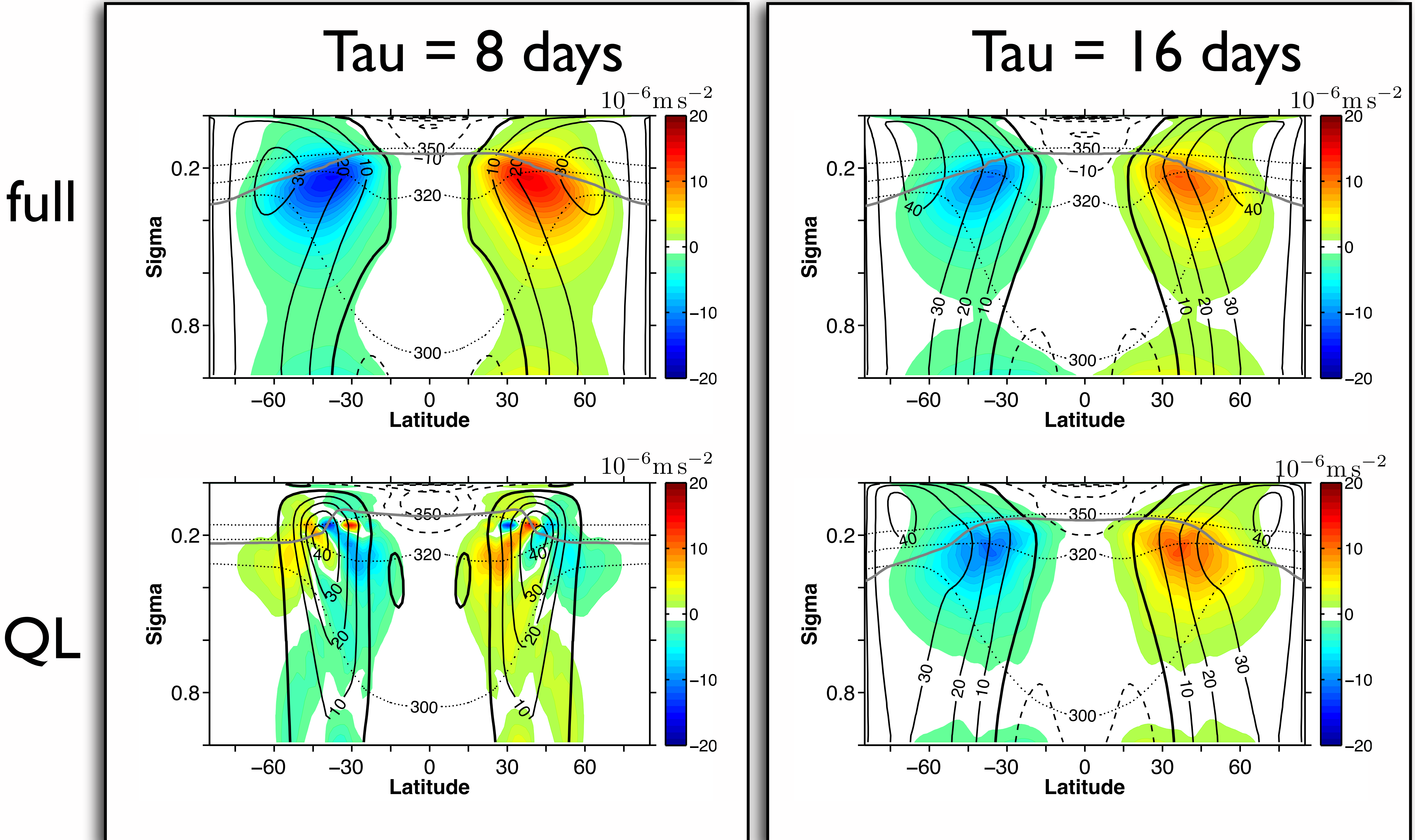

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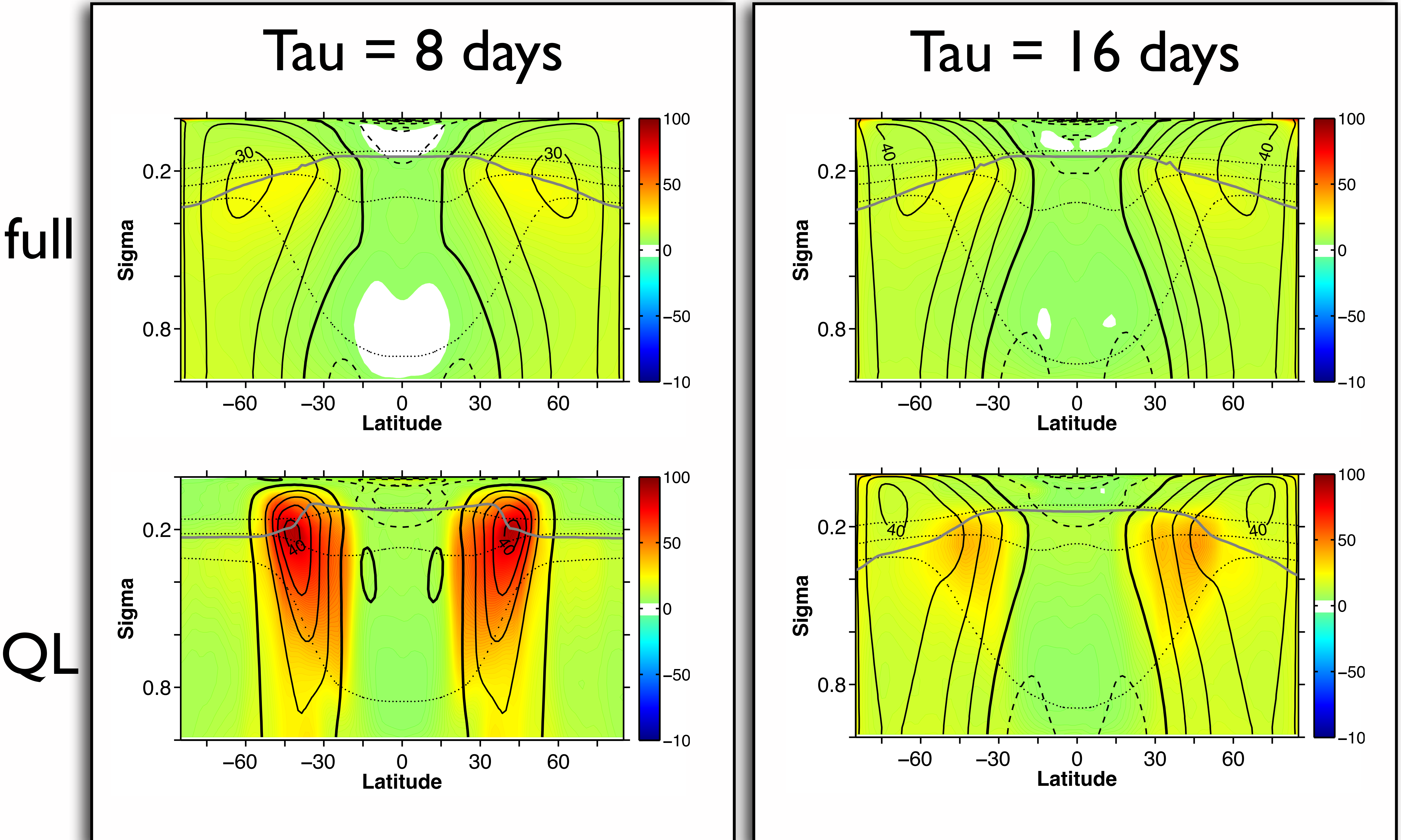
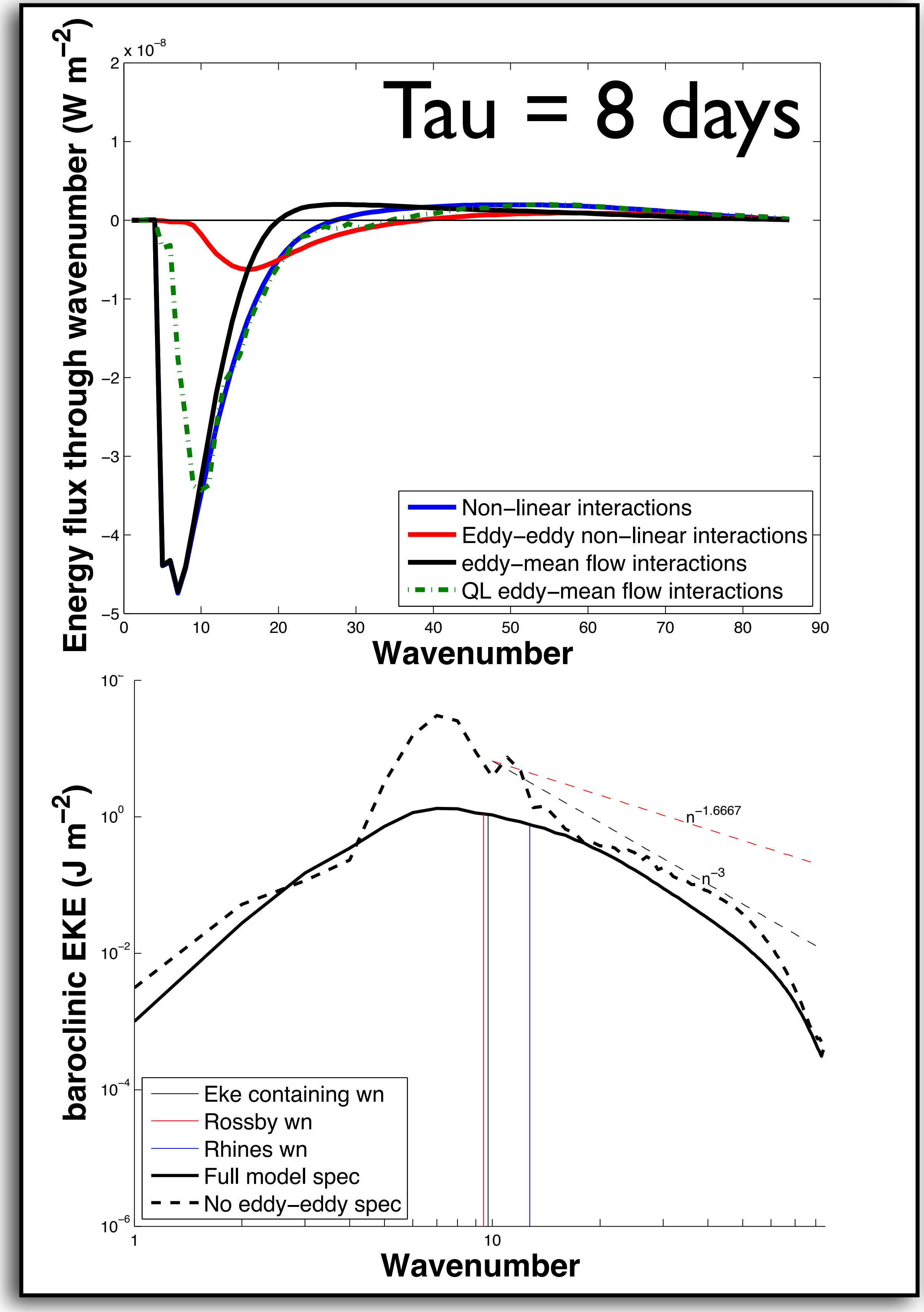
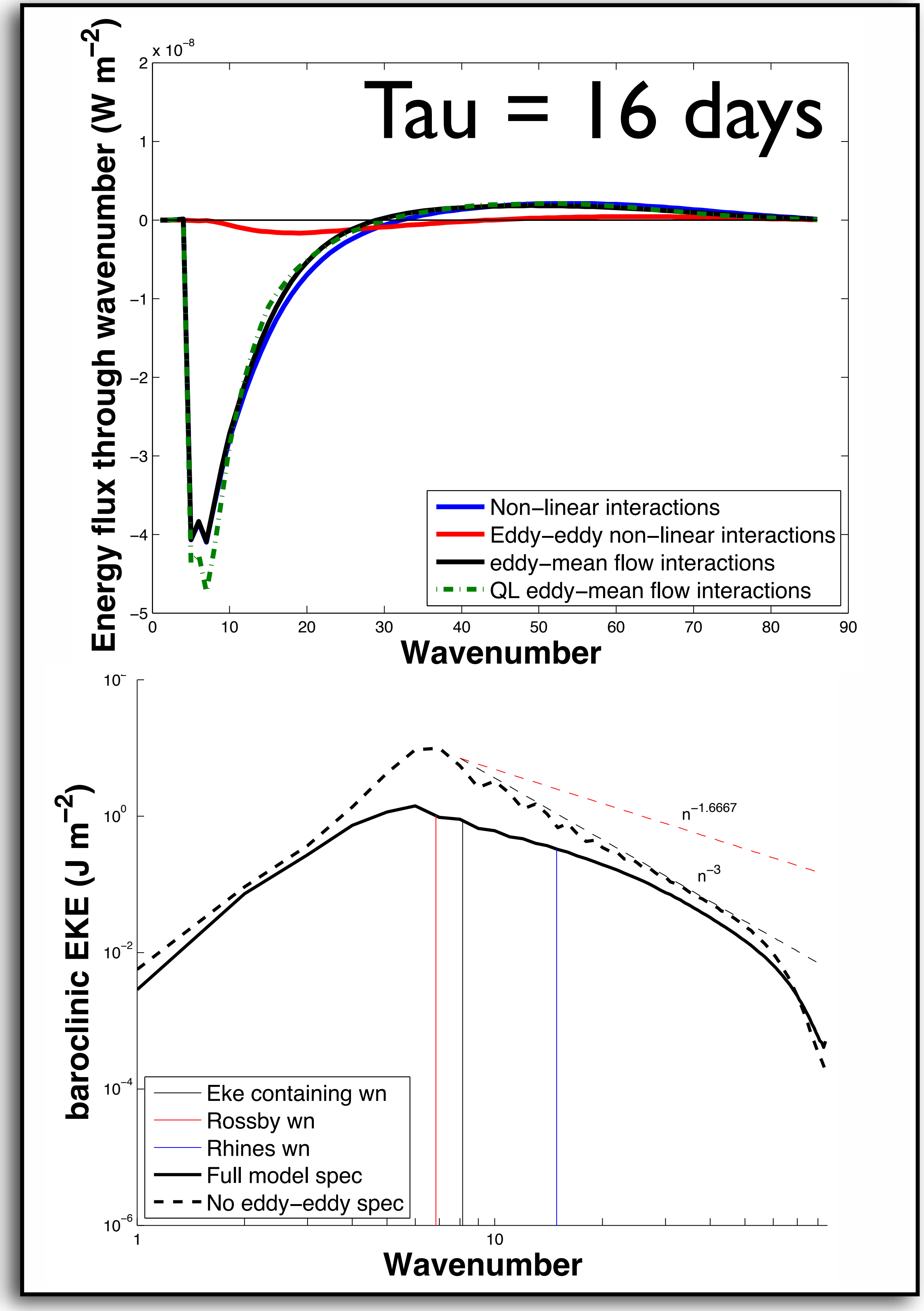


Varying surface friction: spectral budget

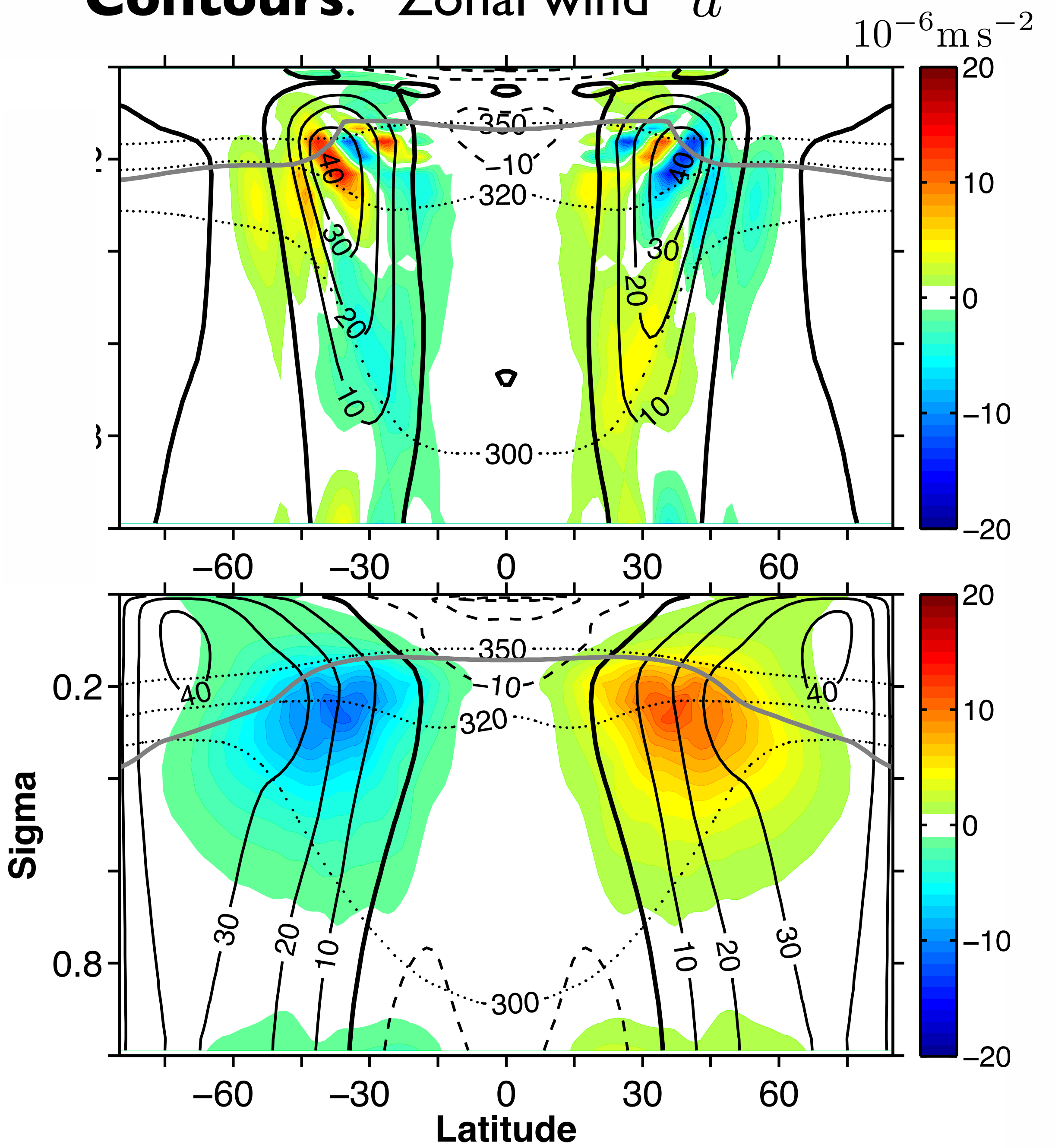




Multiple equilibria at "intermediate" frictions

Tau = 12 days, quasi-linear approximation, two different initial conditions

Colors: momentum flux $\overline{u'v'} \cos\phi$ in m²/s²
Contours: Zonal wind $\overline{u}$



Varying surface friction in QL

- Jet structure and poleward migration of the jet with weaker friction not captured below a threshold ($\sim 10^1$ days)

- Abrupt transition to a configuration where momentum flux and zonal winds (but not EKE) are well captured (linear decay of *Held and Philipps, 1987*). Scale separation between waves and NL critical layer?

- Multiple equilibria possible for some intermediate values.

Conclusions

- Relevant irreversible process for wave activity dissipation lies in eddy-eddy interactions. No length scale separation between eddies and surf zone (SWW theory: even if weak, eddy-eddy interactions matter at leading order for waves outside the critical layer).

- Upper tropospheric EMF enhancement not fully explained by a “more linear” upper-troposphere and “more turbulent” lower troposphere, as proposed by linear theories (*Held 2000, 2007*)

- For weak enough friction, QL captures mean flows and eddy forcing, but not eddy statistics. Why? we are still far from time scale separation....

- Third order closures (parametrizing eddy fluxes of potential enstrophy and capturing NL wave breaking) seem necessary to develop direct statistical simulation of geophysical flows.