Asymptotically-Motivated Models of Langmuir Circulation Dynamics on Ocean Submesoscales

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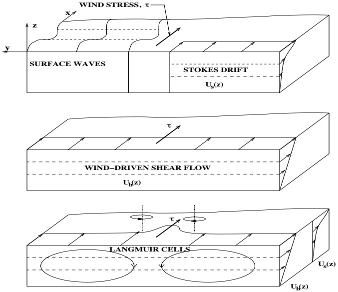
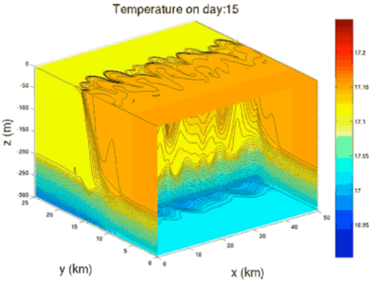
Langmuir Circulation (LC)

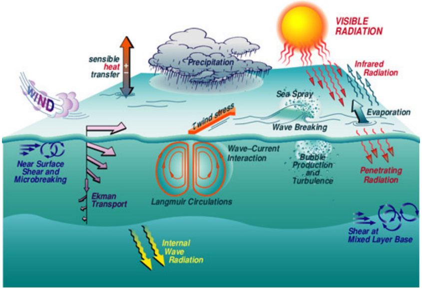


Craik–Leibovich (CL) Theory

- Dominant upper ocean motion due to irrotational surface waves: $O(\mathrm{m/s})$
- Wind-driven shear flow, LC, other rotational (turbulent) motions weaker: $O(\mathrm{cm/s})$
- Rectified effects of filtered surface waves arise from Stokes drift velocity $\mathbf{u}_s(z)\hat{\mathbf{x}}$
- CL vortex force $\mathbf{u}_s(z)\hat{\mathbf{x}} \times \omega$ and Stokes–Coriolis force $2\boldsymbol{\Omega}_e \times \mathbf{u}_s(z)\hat{\mathbf{x}}$ and additional contribution to tracer (ϕ) advection $\mathbf{u}_s(z)\hat{\mathbf{x}} \cdot \nabla\phi$

The Ocean Surface Boundary Layer (BL): Submeso- to BL Scales



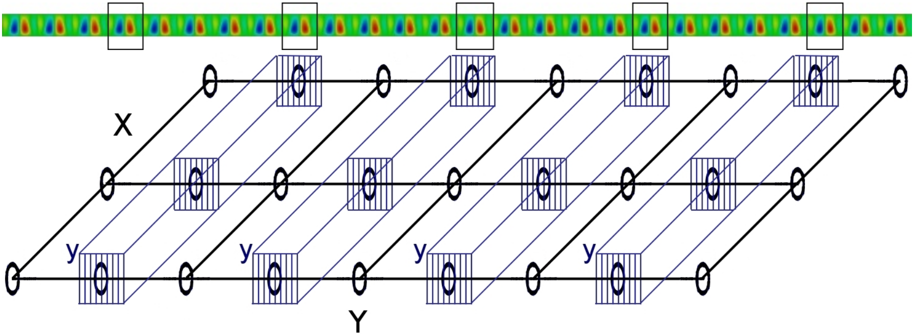


Efficient multiscale framework to study coupling b/w 3D $O(50)$-m non-hydrostatic BL turbulence and $O(5)$-km largely-hydrostatic, rotationally influenced submesoscale flows:

- DNS.. LES[a]. . . Multiscale Models. . . . . . Hydrostatic Primitive Equation Models
- Identify and understand new physical phenomena

[a] Hamlington *et al.*, *JPO* (2014, in revision) "Langmuir–Submesoscale Interactions. . .":
LES on 1024-core supercomputer required $\simeq$ 1 month of wall-clock time to simulate $\simeq$ 2 weeks physical time in a 20 km$\times$20 km$\times$160 m domain with 5 m$\times$5 m$\times$1.25 m spatial resolution for $La_t \equiv \sqrt{u_*/u_{s_0}} = 0.29$.

Asymptotic Reduction



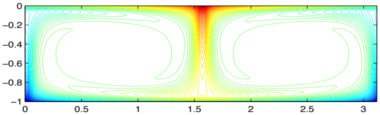
Asymptotic Limits

1. Hydrostatic (small aspect-ratio) submesoscale (subMS) dynamics: disparate-scale coupling between LC and subMS flows $\Rightarrow$ Long cross-wind scales ($\delta \to 0$)
2. Strong CL vortex force: "pure" Langmuir turbulence regime, in which turbulence *constrained* by vortex force $\Rightarrow$ Long downwind scales ($La_t \to 0$)

Secondary Stability Analysis of Fully 3D CL Equations: Trends as $La_t \to 0$

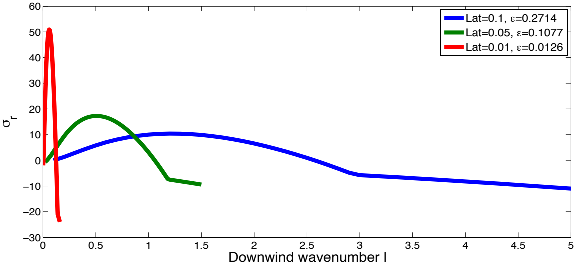
- Linearize about **fully nonlinear 2D** flow: $U(x,y,z,T) = U_{2D}(y,z) + u(x,y,z,T)$

$U_{2D}(y,z)$

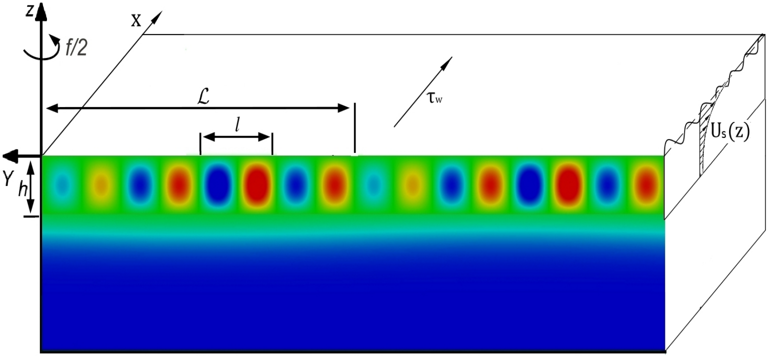


- Employ spatial **Floquet theory**, e.g.

$$u(x,y,z,T) = e^{i\gamma y}\left[\sum_{n=-\infty}^{\infty} \hat{u}_n(z) e^{i(nky)}\right] e^{ilx} e^{\sigma t} + \text{c.c.}$$



Long Crosswind Scales: Multiscale PDEs for 2D–3C LC



Small aspect-ratio (hydrostatic) approximation: $\delta \equiv h/\mathcal{L} \to 0$.

Master PDEs: Rotating, Stratified CL Equations

- Downwind invariant (2D–3C)
- f-plane approximation
- Boussinesq approximation

PDEs

$$D_t u - \delta Ro^{-1} v = La\left(\partial_y^2 + \partial_z^2\right) u$$

$$D_t v + \delta Ro^{-1}\left(u + S\,U_s\right) = -\partial_y p + S\,U_s \partial_y u + La\left(\partial_y^2 + \partial_z^2\right) v$$

$$D_t w = -\partial_z p + S\,U_s \partial_z u + Ri\, b + La\left(\partial_y^2 + \partial_z^2\right) w$$

$$D_t b = Pr^{-1} La\left(\partial_y^2 + \partial_z^2\right) b$$

$$\partial_y v + \partial_z w = 0. \quad D_t \equiv \partial_t + v\partial_y + w\partial_z. \text{ Wind-Stress BC: } \partial_z u = 1$$

Parameters

$$\delta \equiv h/\mathcal{L}. \quad Ro \equiv U/f\mathcal{L}. \quad Ri \equiv Bh/U^2. \quad La \equiv \nu_{sgs}/Uh. \quad S = 0 \text{ or } 1.$$

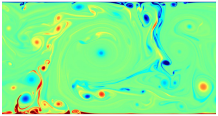
Derivation of Multiscale PDEs

- Introduction of slow horizontal length and slow time scales:
  $Y = \delta\, y$ and $T = \delta\, t$
- Multi-scale differentiation: $\partial_y \to \partial_y + \delta \partial_Y$ and $\partial_t \to \partial_t + \delta \partial_T$
- Mean–fluctuation decomposition and asymptotic expansion:

$$f(y,z,t) \sim \bar{f}(Y,z,T) + \delta \bar{f}_1(Y,z,T) + \ldots + f'(y,z,t;Y,T) + \delta f_1'(y,z,t;Y,T) + \ldots$$

where $\bar{f}$ is the (leading-order) fast time, fast horizontal average of f

Note: $\bar{w} \sim \delta \bar{w}_1(Y,z,T) + \ldots \equiv \delta \overline{W}(Y,z,T) + \ldots$



–Johnston & Doering, *PRL* (2009)

- Distinguished limit, as $\delta \to 0$:

$$\boxed{\overline{La} \equiv La/\delta = O(1). \quad [Ri, Ro] = O(1). \quad \overline{w'(u',v',b')} = O(\delta).}$$

Leading-Order Mean System: Submesoscale (Coarse-Scale) Dynamics

Mean Equations

$$\overline{D}_T\overline{u} + \delta^{-1}\partial_z\left(\overline{w'u'}\right) - Ro^{-1}\overline{v} = \overline{La}\,\partial_z^2\overline{u}$$

$$\overline{D}_T\overline{v} + \partial_Y\left(\overline{v'v'}\right) + \delta^{-1}\partial_z\left(\overline{w'v'}\right) + Ro^{-1}\left(\overline{u} + SU_s\right) = -\partial_Y\overline{p} + SU_s\partial_Y\overline{u} + \overline{La}\,\partial_z^2\overline{v}$$

$$0 = -\partial_z\overline{p} + Ri\,\overline{b} + SU_s\partial_z\overline{u} - \partial_z\left(\overline{w'w'}\right)$$

$$\overline{D}_T\overline{b} + \delta^{-1}\partial_z\left(\overline{w'b'}\right) = Pr^{-1}\overline{La}\;\partial_z^2\overline{b}$$

$\partial_Y\overline{v} + \partial_z\overline{W} = 0.$ $\quad\overline{D}_T \equiv \partial_T + \overline{v}\partial_Y + \overline{W}\partial_z.$ Wind-Stress BC: $\partial_z\overline{u} = 1$

- The blue terms represent eddy fluxes, which must be calculated by solving the fluctuation system
- In the absence of these terms, the hydrostatic primitive equations are recovered

Leading-Order Fluctuation System: Langmuir Circulation (Fine-Scale) Dynamics

Fluctuation Equations

$$D_t' u' + w' \partial_z \overline{u} = La \left(\partial_y^2 + \partial_z^2 \right) u'$$

$$D_t' v' + w' \partial_z \overline{v} = -\partial_y p' + SU_s \partial_y u' + La \left(\partial_y^2 + \partial_z^2 \right) v'$$

$$D_t' w' - \partial_z \left(\overline{w' w'} \right) = -\partial_z p' + SU_s \partial_z u' + Ri\, b' + La \left(\partial_y^2 + \partial_z^2 \right) w'$$

$$D_t' b' + w' \partial_z \overline{b} = Pr^{-1}\, La\ \left(\partial_y^2 + \partial_z^2 \right) b'$$

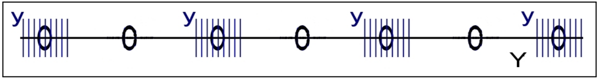
$$\partial_y v' + \partial_z w' = 0. \quad D_t' \equiv \partial_t + \left(\overline{v} + v' \right) \partial_y + w' \partial_z. \text{ Traction BC: } \partial_z u' = 0$$

- The blue terms show that the fluctuations "feed" on vertical gradients of the mean fields and are advected by the mean horizontal flow
- Coriolis forces are sub-dominant (as expected) on fine scales, only "appearing" indirectly through profiles of mean fields

Multiscale Numerical Algorithm

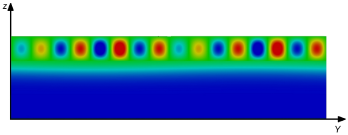
The integration of the multi-scale system is performed as follows:

1. Perform $M \leq M_{max}$ fine-scale time steps Δt of fluctuation system on N fine (y) *l-periodic* grids during which time mean fields are frozen
2. Perform one coarse-scale time step ΔT of mean system on coarse (Y) grid of length L to update mean fields using fine-scale fluxes



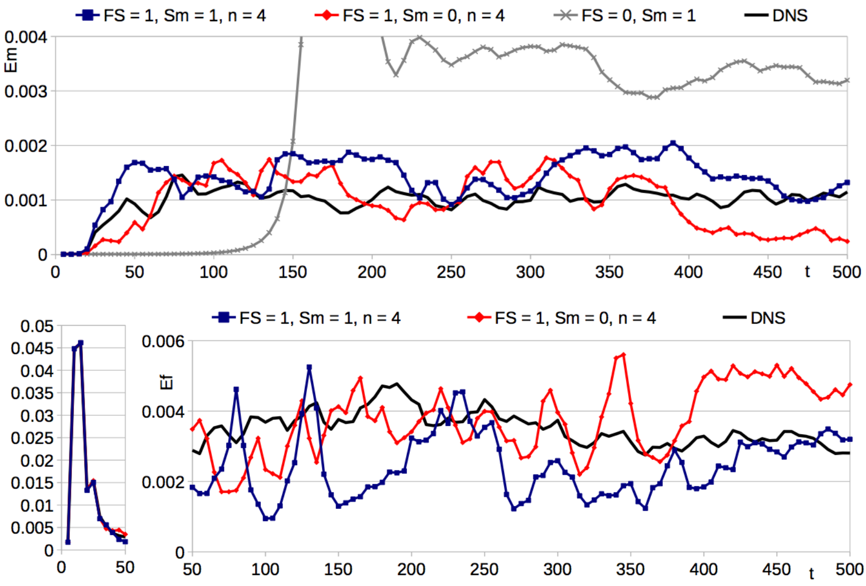
Computational Advantage: No communication b/w embedded fine-scale domains $\Rightarrow$ "embarrassingly" parallelizable

LC Interaction with Submesoscale IW ($Ro \to \infty$, $Ri = 1$)



Mode	$\|\bar{v}\|_{MAX}$	$\|\bar{W}\|_{MAX}$
Pure Traveling IW	0.005	0.05
Pure LC	0.17	0
LC–IW	0.5	2.3

Time-Evolution of Mean and Fluctuation Area-Averaged Cross-Wind KE

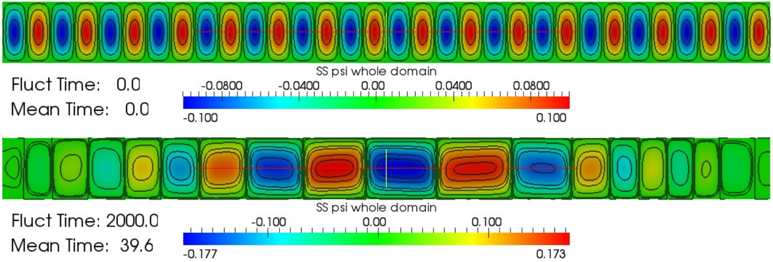


Computational Parameters for Domain Size: $H = \pi$, $l = \pi$, $\mathcal{L} = 16\pi$

Model	N_z	N_y	N_Y	# CPUs	Wall Clock for $t = 25$
DNS	128	16×128	—	1	22 hours
MS	128	4×128	32	4	2 hours

Computational Acceleration: MS simulations 11 times faster than DNS

Challenge: The Need for a Phase-Variable Formalism?



- Desire adaptivity in location and width of embedded fine scale domains
- Employ **WKBJ-like** approximation by replacing fine scale y coordinate

with $\eta \equiv \frac{\Theta(Y,T)}{\delta}$

Challenge: The Need for a Phase-Variable Formalism? (Cont'd)

- **Phase–Advection Equation:** $\boxed{\partial_T \Theta + \mathcal{V} \partial_Y \Theta = \overline{La} \delta^2 \partial_Y^2 \Theta; \quad k(Y,T) \equiv \partial_Y \Theta(Y,T)}$

- Rational procedure needed for choosing $\mathcal{V}(Y,T)$; e.g.,

  i) $\mathcal{V}(Y,T) \equiv \frac{1}{H_{ML}} \int_{-H_{ML}}^{0} \bar{v}(Y,z,T)dz$

  ii) Solvability conditions arising in the fluctuation equations at $O(\delta)$?

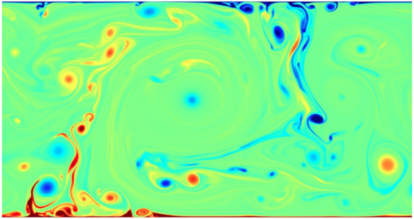
- **Fluctuation equations** modified by "additional" slowly-varying coefficients:

$$D_t^{\eta} v' + w' \partial_z \overline{v} = -\partial_Y \Theta \partial_\eta p' + SU_s \partial_Y \Theta \partial_\eta u' + La \left[(\partial_Y \Theta)^2 \partial_\eta^2 + \partial_z^2 \right] v'$$
$$D_t^{\eta} \equiv \partial_t + (\overline{v} - \mathcal{V} + v')(\partial_Y \Theta) \partial_\eta + w' \partial_z$$

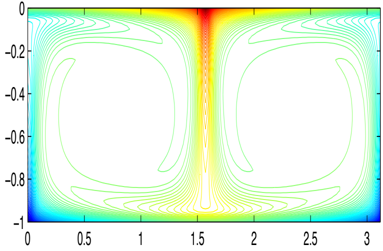
- **Mean equations** formally unchanged. . . but evaluation of flux divergences modified since embedded domains advected by $\mathcal{V}(Y,T)$ $\Rightarrow$ Hybrid E–L approach

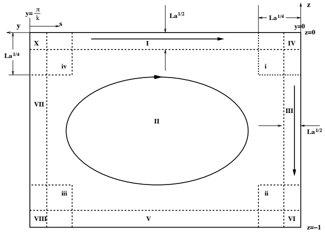
Challenge: Mode Reduction for Embedded Domain Computations

Turbulent RBC (Johnston *PRL*, 2009)



Steady LC





- Even turbulent (2D) RBC organizes into thin, well-separated plumes
- Asymptotic analysis of *steady* LC as $La \to 0$ shows that $\psi'(y, z)$ smooth: well-represented with a few Fourier modes in y and z
- **Goal:** Modified **mean-field** model, accounting for *localized* $u'(y, z)$

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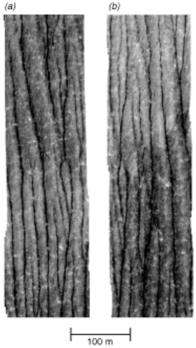
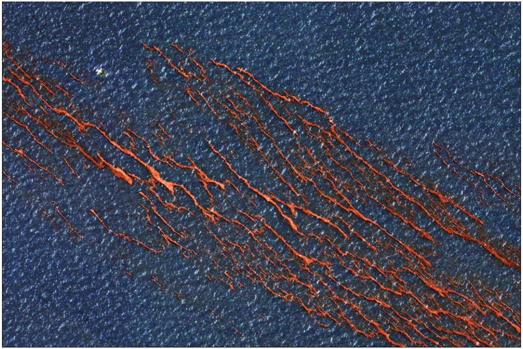
Long Downwind Scales:
Asymptotically Reduced PDEs for Strongly Anisotropic LC





Emergence of Anisotropic Structure

Isotropically Scaled (Non-Rotating, Constant-Density) CL Equations

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = -\nabla p + \frac{1}{La_t^2}(\mathbf{U}_s \times \boldsymbol{\omega}) + \frac{1}{R_*}\nabla^2\mathbf{u}$$

where, in 3D, $La_t = \sqrt{u_*/u_{s_0}}$ and $R_* \equiv u_* H/\nu_e$ replace single parameter ("laminar Langmuir number") $La \equiv La_t R_*^{-3/2}$ which governs 2D dynamics

Ocean Surface BL Turbulence Regimes

Shear flow turbulence regime: $La_t \gg 1$ with $R_* \gg 1$

Langmuir turbulence regime: $La_t = O(0.1)$ with $La \ll 1$

▷ As $La_t \to 0$, vorticity aligns with Stokes drift

▷ Motivates consideration of formal limit $La_t \to 0$

Implications of Leading-Order Physics: Rapid Distortion and Secular Growth

- As $La_t \to 0$, there is a "rapid-distortion" (strong Stokes "shear") transient that drives strong cellular (v,w) 2D–3C velocity fluctuations. Upon rescaling time,

$$\boxed{\Omega(y,z,\tau) \;=\; -\left[U_s'(z)\partial_y U_0(y,z)\right]\tau + \Omega_0(y,z)}$$

- Upon rescaling pressure and seeking a leading-order ("geostrophic-like") balance:

$$\boxed{\nabla\mathcal{P} = \mathbf{U}_s \times \boldsymbol{\Omega} \quad \Rightarrow \quad \mathbf{U}_s \cdot \nabla\boldsymbol{\Omega} = \boldsymbol{\Omega}\cdot\nabla\mathbf{U}_s}$$

- From this balance, the following deductions can be made:

(i) $\partial_x\mathcal{P} = 0$ (ii) $U_s\partial_x\Omega_x = \Omega_z U_s'(z)$

(iii) $U_s\partial_x\Omega_y = 0$

(iv) $U_s\partial_x\Omega_z = 0$

- $\Omega_x \neq 0$, $\partial_x\Omega_x = 0$, $\boxed{u\text{–fluctuations} \ll (v,w)\text{–fluctuations}}$

Rescaled CL Equations in Strong CL Vortex-Force Limit

Anisotropically Scaled CL Equations

$$\partial_t u + \varepsilon\, u \partial_x u + (\mathbf{v}_\perp \cdot \nabla_\perp)\, u \;=\; -\varepsilon^{-1} \partial_x P + La \left[\partial_x^2 + \nabla_\perp^2\right] u$$

$$\partial_t \mathbf{v}_\perp + \varepsilon\, u \partial_x \mathbf{v}_\perp + (\mathbf{v}_\perp \cdot \nabla_\perp)\, \mathbf{v}_\perp \;=\; -\nabla_\perp P + La \left[\partial_x^2 + \nabla_\perp^2\right] \mathbf{v}_\perp + U_s \left(\nabla_\perp u - \varepsilon^{-1} \partial_x \mathbf{v}_\perp\right)$$

$$\varepsilon\, \partial_x u + \nabla_\perp \cdot \mathbf{v}_\perp \;=\; 0$$

- Wind stress BC: $\partial_z u = 1$ along $z = 0, -1$
- x-invariance at leading-order: $\partial_x P = \partial_x v = \partial_x w = 0$ and $\nabla_\perp \cdot \mathbf{v}_\perp = 0$

Multiple Scale Expansion

1. Limit process: $\varepsilon \propto La_t \to 0$ with $R_* = o(La_t^{-2})$
2. Introduce slow x scale: $X \equiv \varepsilon x$ so that $\partial_x \to \partial_x + \varepsilon \partial_X$
3. Expand fields:

$$
\begin{aligned}
u(x,y,z,t) &= u_0(x,X,y,z,t) + \varepsilon u_1(x,X,y,z,t) + \ldots \\
\mathbf{v}_\perp(x,y,z,t) &= \mathbf{v}_{0\perp}(X,y,z,t) + \varepsilon \mathbf{v}_{1\perp}(x,X,y,z,t) + \ldots \\
P(x,y,z,t) &= P_0(X,y,z,t) + \varepsilon P_1(x,X,y,z,t) + \ldots
\end{aligned}
$$

4. Substitute into PDEs, collect terms of like order and **average** over fast x
5. Obtain closed set of equations for $\bar{u}_0 \equiv U(X,y,z,t)$, $\mathbf{v}_{0\perp} \equiv \mathbf{V}_\perp(X,y,z,t)$ and $P_0 \equiv \Pi(X,y,z,t)$

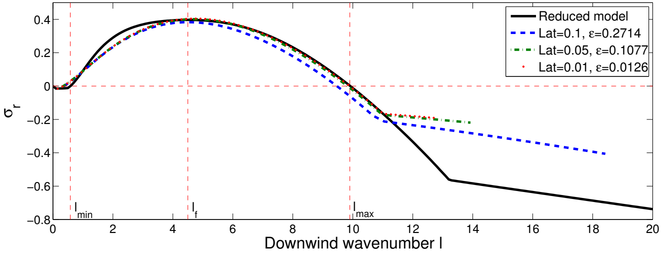
Asymptotically Reduced PDEs

Reduced CL (rCL) Equations

$$
\begin{aligned}
D_t^{\perp} U &= -\partial_X \Pi + La \nabla_{\perp}^2 U \\
D_t^{\perp} \Omega + U_s(z) \partial_X \Omega &= U_s'(z) \Big(\partial_X V - \partial_y U \Big) + La \nabla_{\perp}^2 \Omega \\
\nabla_{\perp}^2 \Pi &= 2J[\partial_y \psi, \partial_z \psi] + \nabla_{\perp} \cdot \Big(U_s(z) \nabla_{\perp} U \Big) + U_s'(z) \partial_X (\partial_y \psi) \\
\nabla_{\perp}^2 \psi &= -\Omega, \quad \mathbf{V}_{\perp} \equiv \nabla_{\perp} \times \psi \hat{\imath}
\end{aligned}
$$

- $D_t^{\perp}(\cdot) \equiv \partial_t(\cdot) + (\mathbf{V}_{\perp} \cdot \nabla_{\perp})(\cdot) \equiv \partial_t(\cdot) + J[(\cdot), \psi]$,
  where $J[(\cdot), \psi] = \partial_z \psi \partial_y(\cdot) - \partial_y \psi \partial_z(\cdot)$
- Fast x averaged BCs along $z = 0, -1$: $\partial_z U = 1, \;\; \Omega = 0, \;\; \psi = 0$
- *Advection by U and stretching of Ω are subdominant processes*

Secondary Stability Results: Comparison of rCL and CL Equations



La_t	ε	σ_m (rescaled)	l_f (rescaled)
0.10	0.2714	10.4101 (0.3835)	1.20 (4.4208)
0.05	0.1077	17.2789 (0.4010)	0.50 (4.6416)
0.01	0.0126	50.9579 (0.4045)	0.06 (4.7622)
Reduced model	$\varepsilon \to 0$	0.3968	4.5

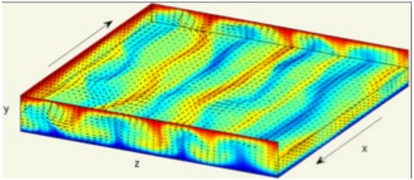
Numerical Simulations of rCL Equations: Phenomenology

Numerical Simulations of rCL Equations: “Turbulent” Dynamics

rCL Equations Summary

- Derived reduced PDEs for anisotropic turbulent Langmuir circulation in strong vortex-force limit
- rCL equations provide nonlinear rectification of RDT theory of Langmuir turbulence by Teixeira & Belcher (*Ocean Modelling*, 2010)
- Reduced PDEs capture dominant linear and secondary instabilities
- Reduced PDEs offer several analytical and computational advantages: Filter rapid-distortion transients and fine x-scale variability $\Rightarrow$ larger Δx, Δt
- Limiting procedure suppresses "self-sustaining process" proposed by Waleffe for wall-bounded shear-flow turbulence

Plane Couette Flow (PCF)



(Courtesy J. Gibson)

- Applying methodology to PCF turbulence. . .

$$\mathcal{U} = U_w, \quad (\mathcal{V}, \mathcal{W}) = \frac{1}{Re} U_w$$

SubMS and BL Scales

Dimensional Scales

Scale	Submesocale	Value	BL	Value
Horizontal Length	$\mathcal{L}$	1–10 km	$l = h$	50-100 m
BL Depth	h	50-100 m	h	50-100 m
Horizontal Velocity	U	0.1 m/s	U	0.05–0.1 m/s
Vertical Velocity	$Uh/\mathcal{L}$	$<$0.01 m/s	U	0.05–0.1 m/s
Advection Time	$\mathcal{L}/U$	5–10 hr	h/U	0.5 hr
Wind Stress	u_*	0.01 m/s	u_*	0.01 m/s
Stokes Drift Velocity	u_{s0}	0.1 m/s	u_{s0}	0.1 m/s
Dynamic Pressure	$\rho_0 U^2$	10 Pa	$\rho_0 U^2$	10 Pa
Buoyancy Anomaly	$B = g\|\Delta\rho\|/\rho_0$	0.001 m/s^2	B	0.001 m/s^2