What drives the weather changes

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“The answer is blowing in the wind”

Hydrostatics

Only normal forces

$$\nabla p = \rho \mathbf{f}$$

$$\mathbf{f} = -\nabla \phi$$

$$\nabla \rho \times \nabla \phi = 0$$

$$\phi = gz$$

$$\partial p/\partial z = -\rho g$$

Horizontal temperature gradient causes wind

$$T(z) = T_0 - \alpha z$$

$$p(z) = p(0)(1 - \alpha z/T_0)^{mg/\alpha}$$

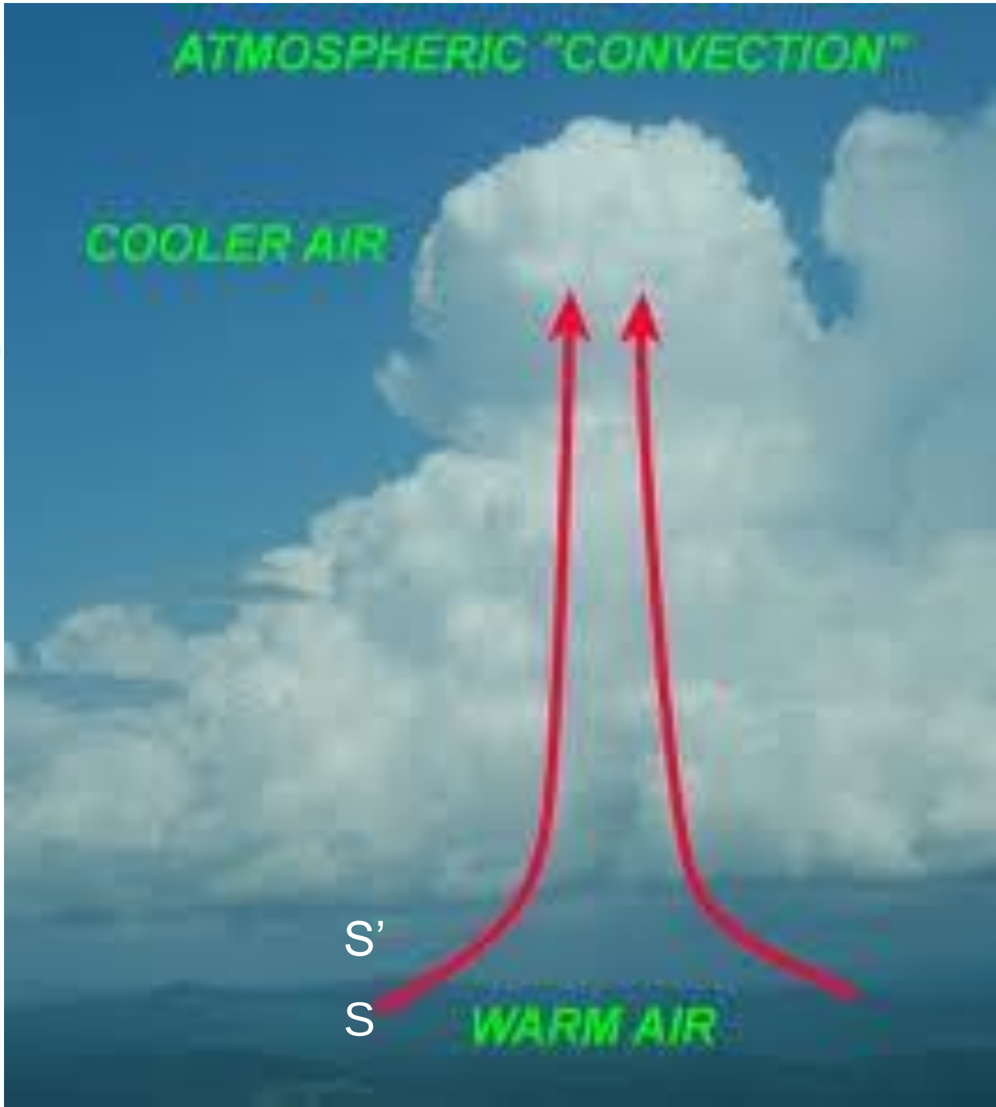
$$\alpha \simeq 6.5^\circ/km$$

$$\rho(p', s) > \rho(p', s') \;\Rightarrow\; \left(\frac{\partial \rho}{\partial s}\right)_p \frac{ds}{dz} < 0$$

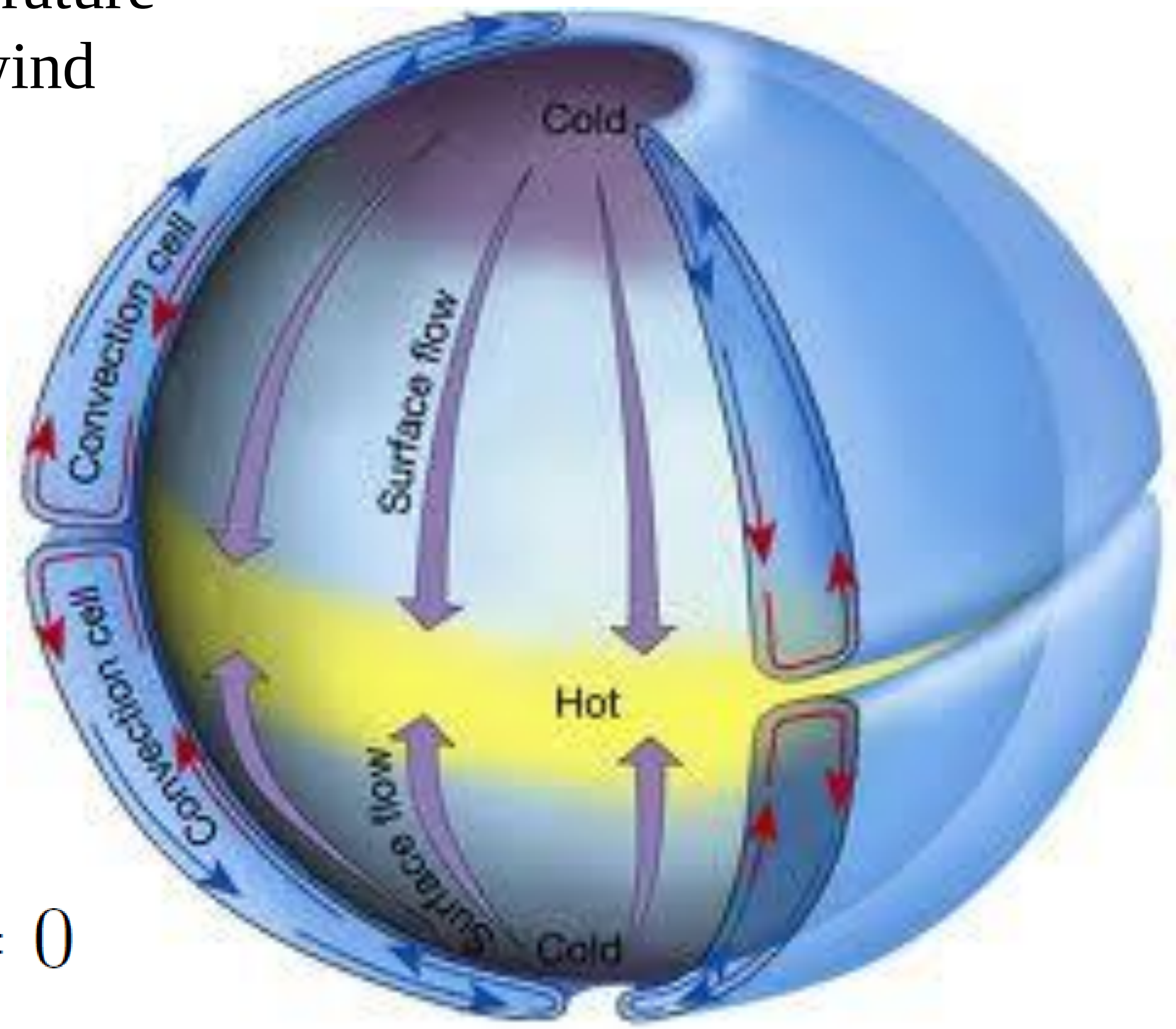
$$\frac{ds}{dz} = \left(\frac{\partial s}{\partial T}\right)_p \frac{dT}{dz} + \left(\frac{\partial s}{\partial p}\right)_T \frac{dp}{dz}$$

$$= \frac{c_p}{T}\frac{dT}{dz} - \left(\frac{\partial V}{\partial T}\right)_p \frac{g}{V} > 0$$

$$-\frac{dT}{dz} < \frac{g}{c_p} \sim 10^\circ/km$$

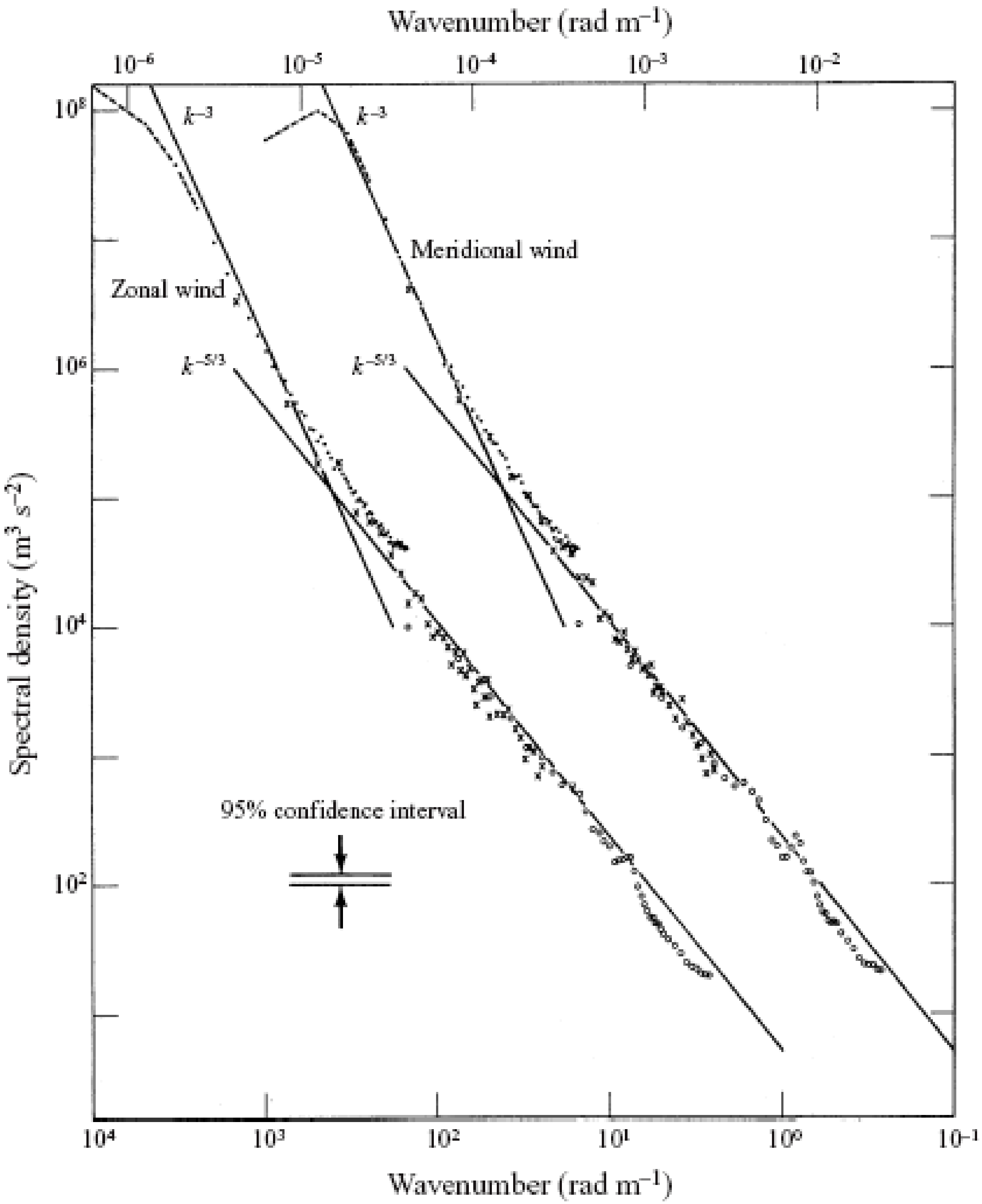


Horizontal temperature gradient causes wind



$$\nabla \rho \times \nabla \phi \neq 0$$

Atmospheric spectrum



Nastrom, Gage,
J. Atmosph. Sci. 1985

Atmospheric flows are driven by the gradients of solar heating.

Vertical gradients cause thermal convection on the scale of the troposphere depth (less than 10 km).

Horizontal gradients excite motions on a planetary (10000 km) and smaller scales.

Weather is mostly determined by the flows at intermediate scale (hundreds of kilometers).

Where these flows get their energy from?

The puzzle is that three-dimensional small-scale motions cannot transfer energy to larger scales while large-scale planar motions cannot transfer energy to smaller scales.

Euler equation in 2d

$$\frac{d\mathbf{u}}{dt} = \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\nabla)\mathbf{u} = -\nabla p$$

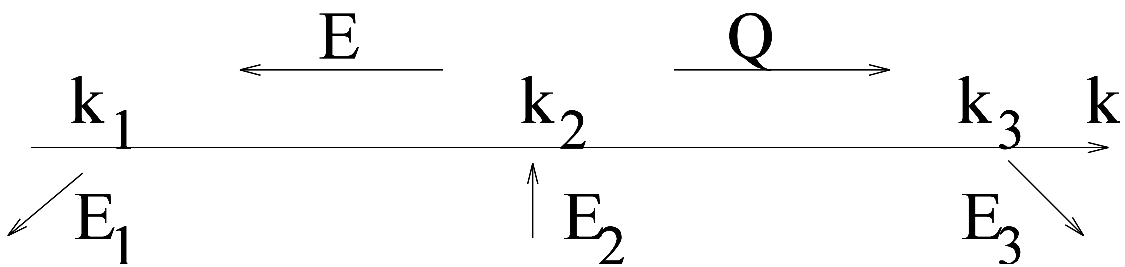
$$\omega = \nabla \times \mathbf{u}$$

$$\frac{d\omega}{dt} = \frac{\partial \omega}{\partial t} + (\mathbf{u}\nabla)\omega = 0$$

Two cascades in two dimensions

vorticity $\omega = \nabla \times \mathbf{u}$ is conserved

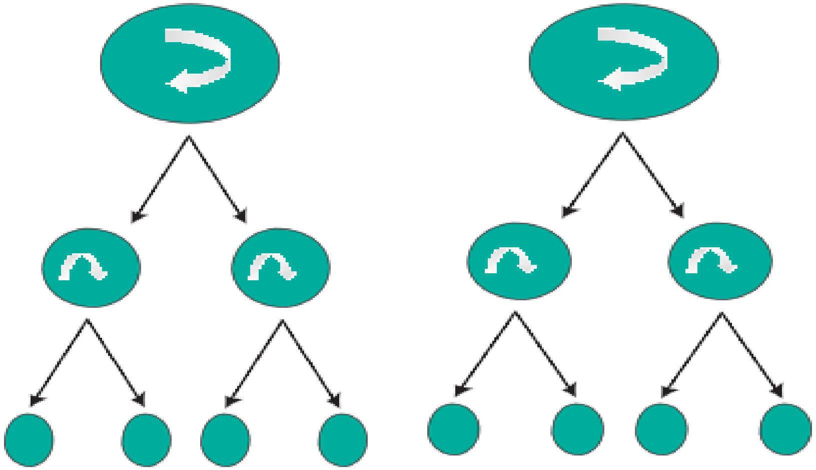
$$E = \int |\mathbf{v_k}|^2 d\mathbf{k} \text{ and } \Omega = \int |\mathbf{k} \times \mathbf{v_k}|^2 d\mathbf{k}$$

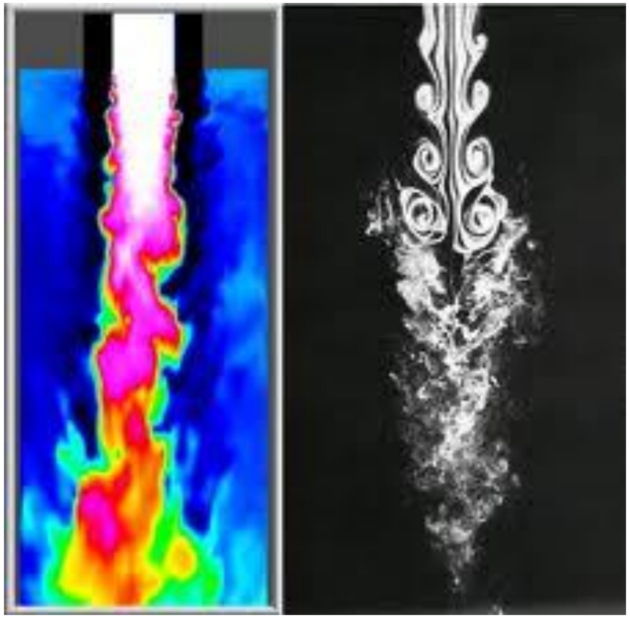


$$E_1 + E_3 = E_2 \,, \quad k_1^2 E_1 + k_3^2 E_3 = k_2^2 E_2$$

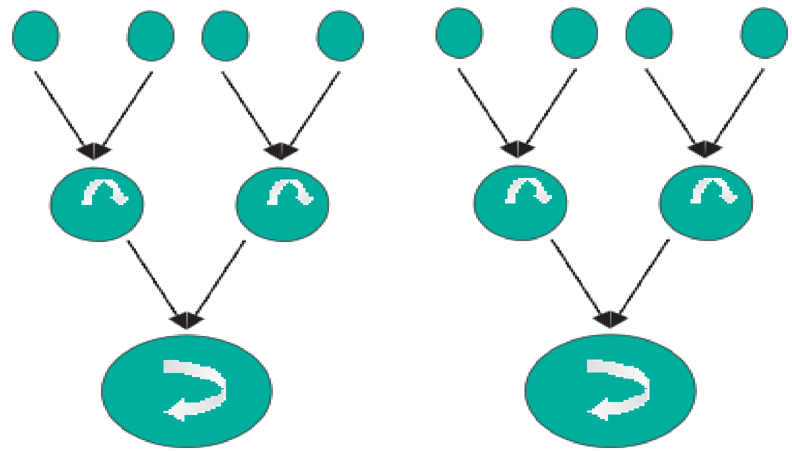
$$E_1 = E_2 \frac{k_3^2 - k_2^2}{k_3^2 - k_1^2} \,, \qquad E_3 = E_2 \frac{k_2^2 - k_1^2}{k_3^2 - k_1^2}$$

Direct cascade





Inverse cascade



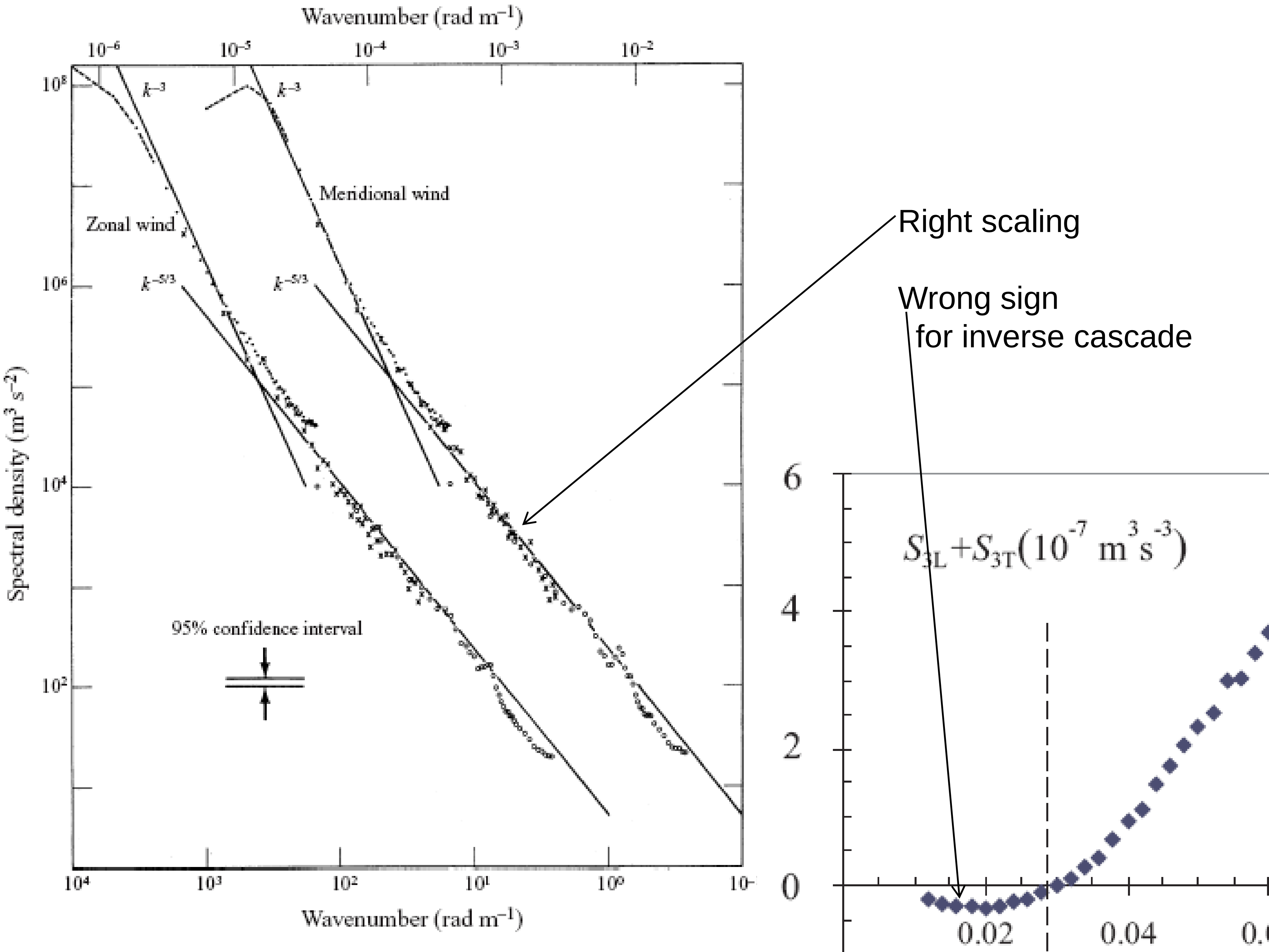
Kolmogorov energy cascade

$$\frac{\text{kinetic energy } (\delta v_r)^2}{\text{time } r/\delta v_r} = \text{energy flux } \epsilon$$

$$\epsilon = \nu \langle |\nabla v|^2 \rangle \simeq v_{rms}^3 / L$$

$$(\delta v_r)^3 \sim \epsilon r$$

$$S_3 = \langle (\delta v_r)^3 \rangle = -\frac{12 \epsilon r}{d(d+2)}$$


Wavenumber (rad m^{-1})
Spectral density (m^3 s^{-2})
k^{-3}
$k^{-5/3}$
Meridional wind
Zonal wind
95% confidence interval
Right scaling
Wrong sign
for inverse cascade
$S_{3L}+S_{3T}(10^{-7}\ m^3 s^{-3})$

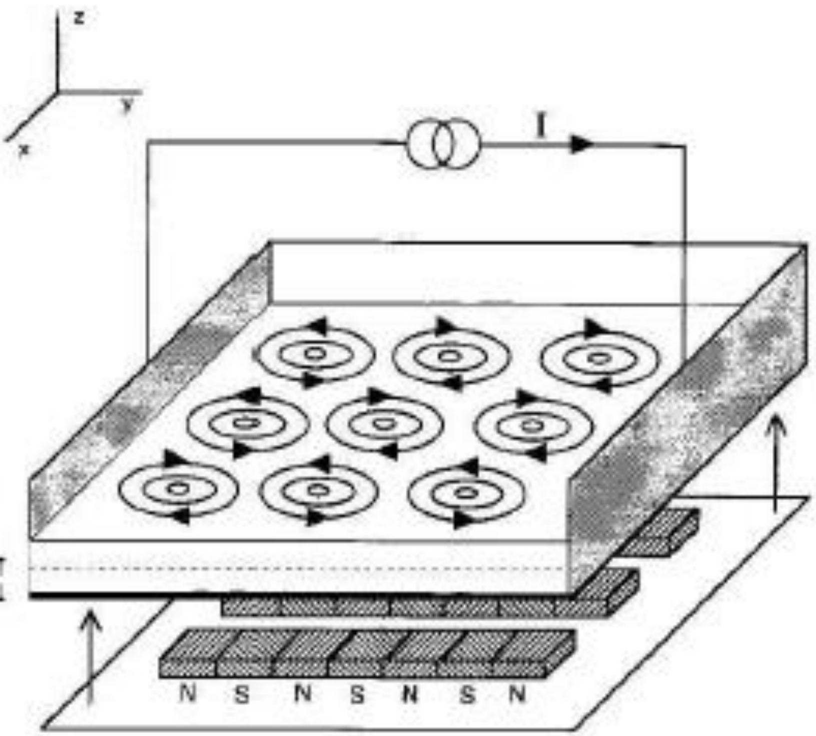
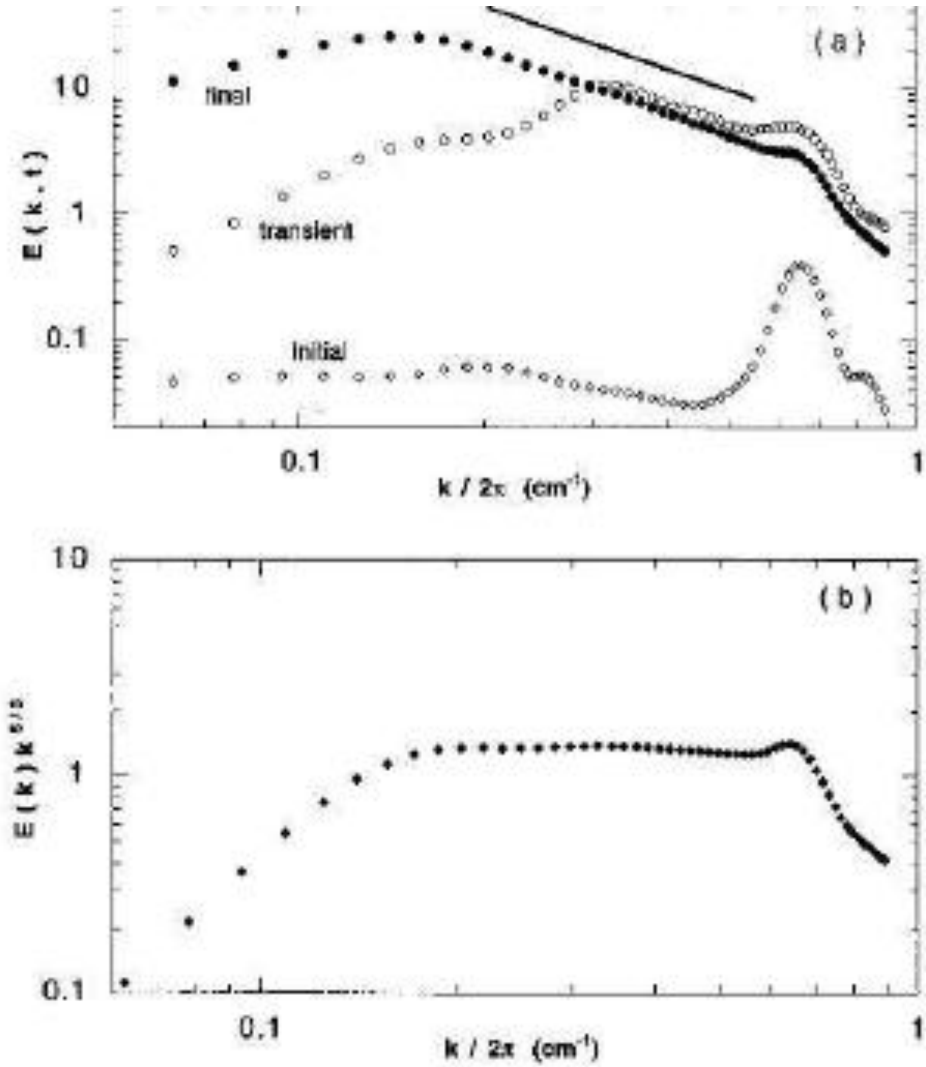


FIG. 1. The experimental set-up.



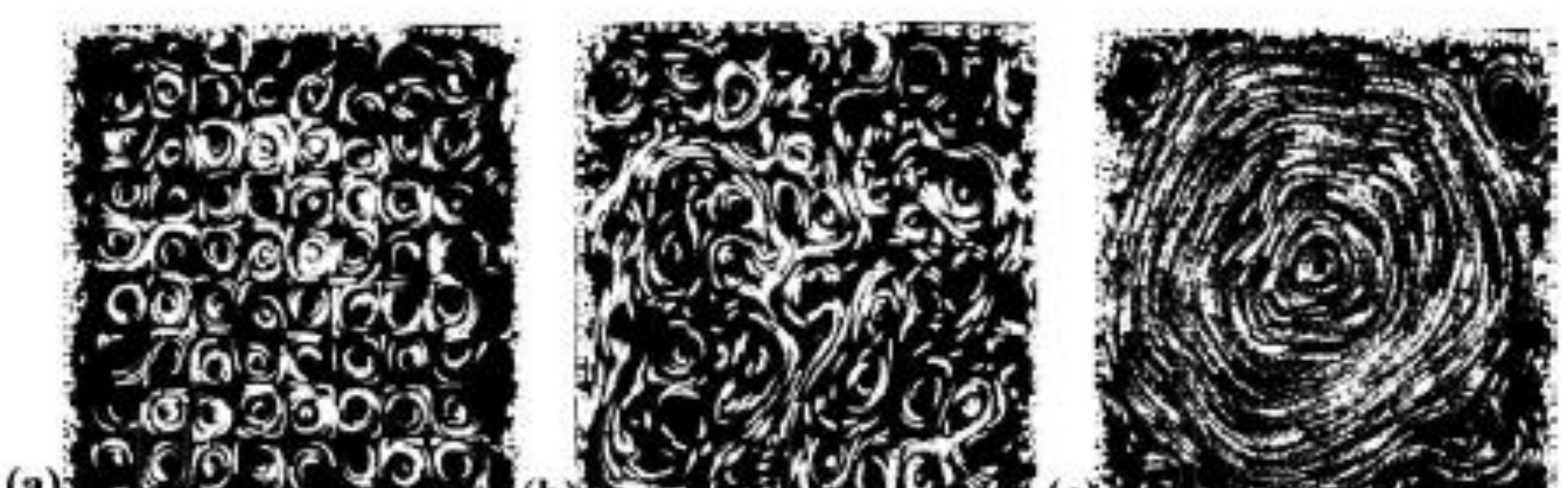
We expect from turbulence
fragmentation, **mixing** and **loss of coherence**.

However,
an inverse turbulent cascade proceeds from small to large scales and brings some self-organization and eventually appearance of
a **coherent system-size condensate**.

M. G. Shats, H. Xia, H. Punzmann & G. Falkovich , Phys Rev Let **99**, 164502 (2007);

Thin layer
Condensation in two-dimensional turbulence

Temporal development of turbulence in a thin layer



$$E = E_0 e^{-at}$$

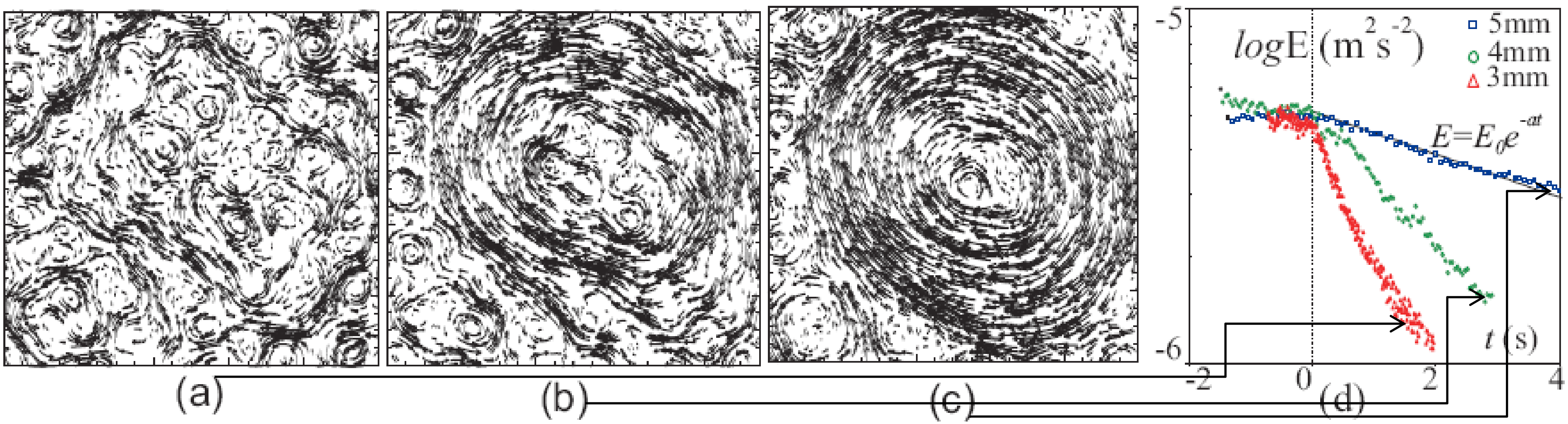
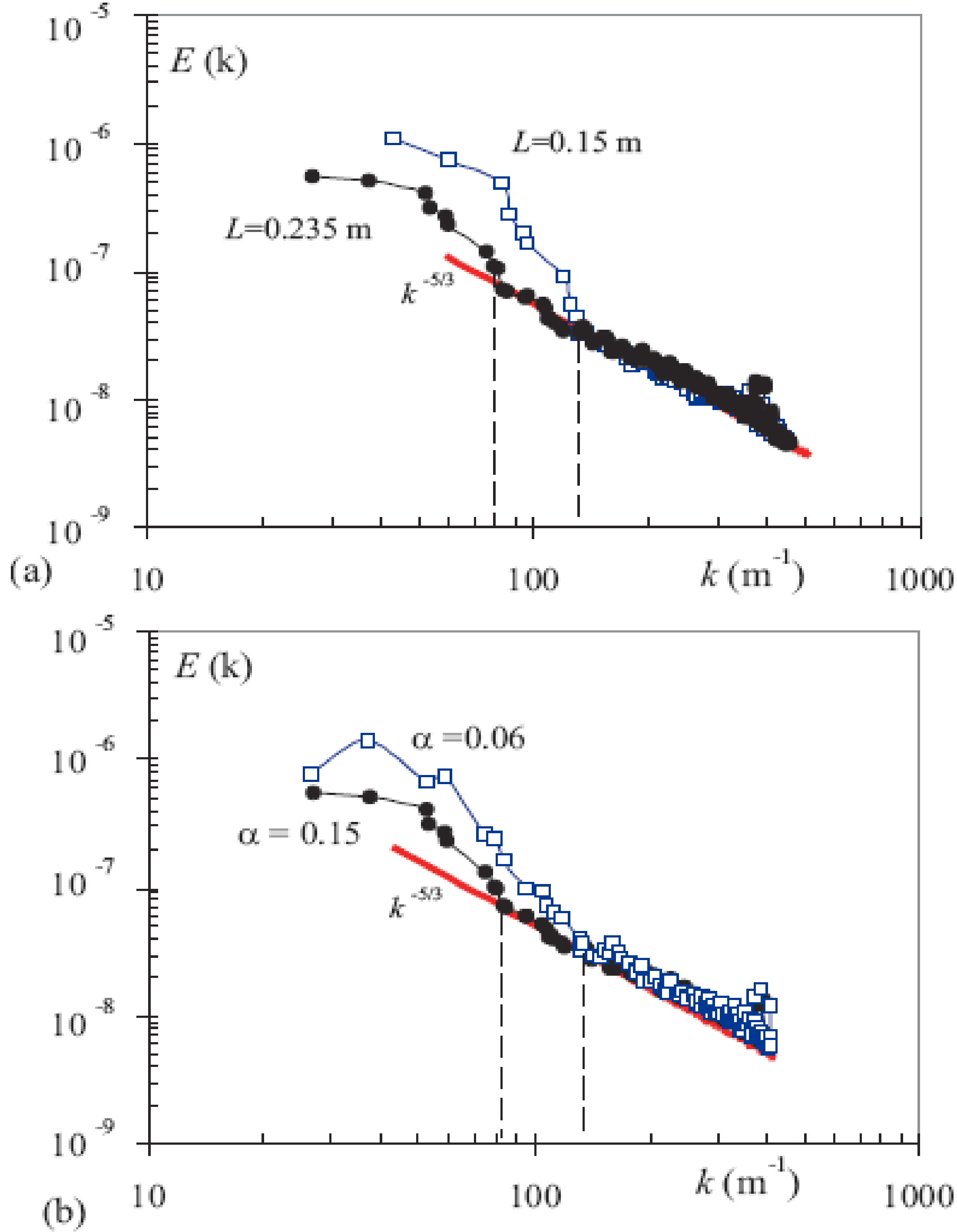
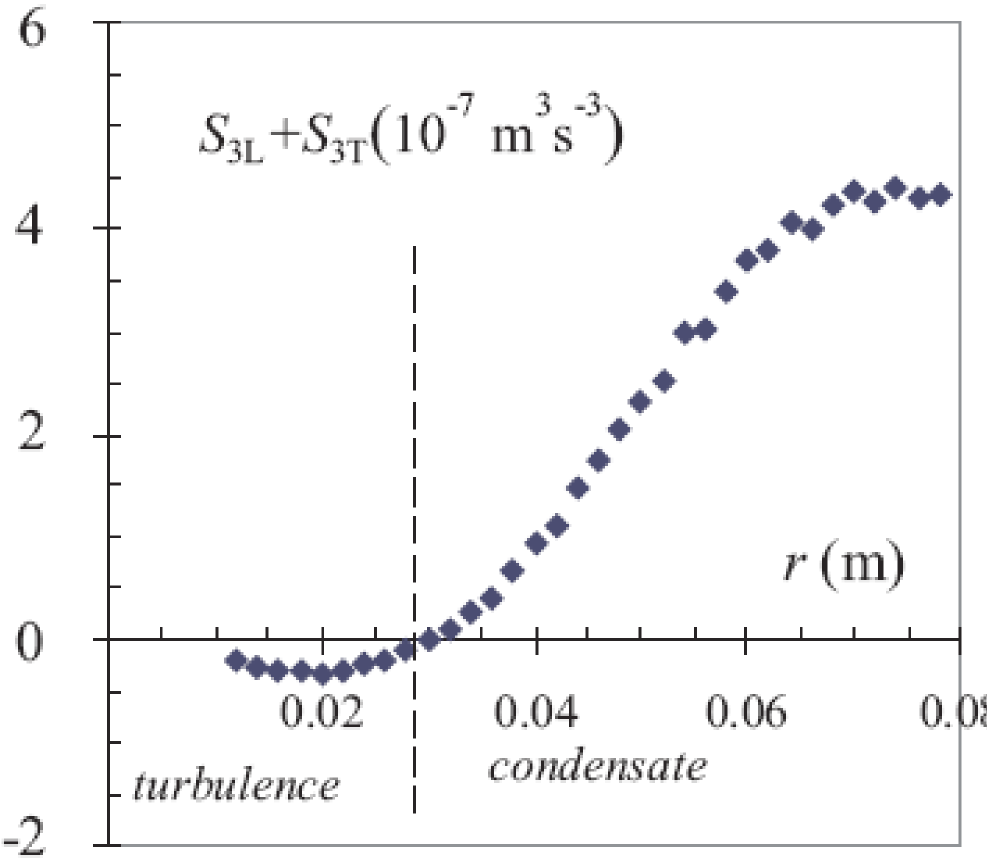


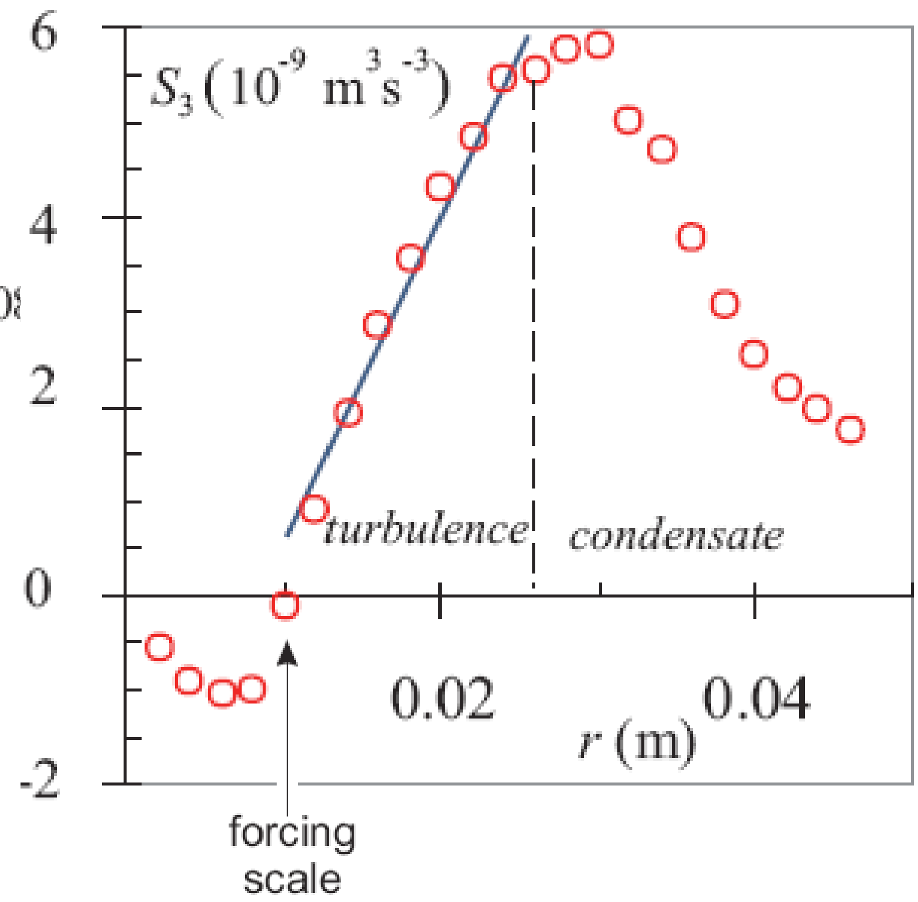
FIGURE 1. Time-averaged velocity field of the condensate in the square box of $L \approx 0.10$ m at different thicknesses of the bottom fluid (Fluorinert FC-77): (a) $\Delta h_b = 3$ mm, $\alpha = 0.3$ s^{-1}, (b) $\Delta h_b = 4$ mm, $\alpha = 0.15$ s^{-1} and (c) $\Delta h_b = 5$ mm, $\alpha = 0.05$ s^{-1}. (d) Decay of the total kinetic energy for cases (a-c).



E (k)
L=0.15 m
L=0.235 m
k^-5/3
10^-5
10^-6
10^-7
10^-8
10^-9
10
100
1000
k (m^-1)
(a)
E (k)
α =0.06
α = 0.15
k^-5/3
10^-5
10^-6
10^-7
10^-8
10^-9
10
100
1000
k (m^-1)
(b)



Strong condensate changes sign of the third moment in the turbulence interval of scales

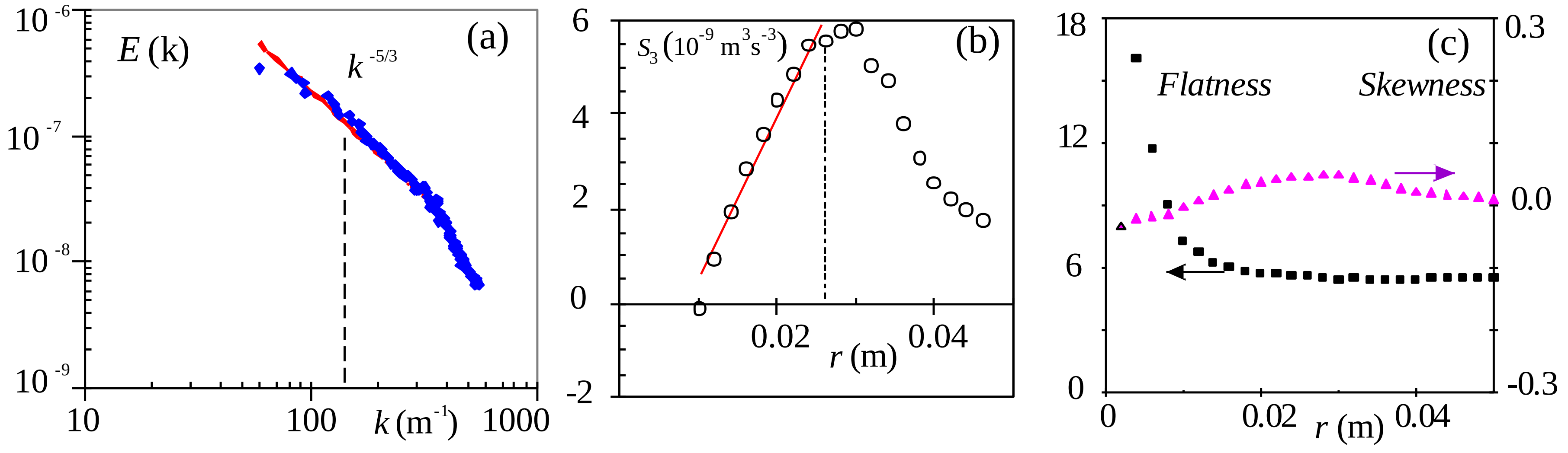


Subtracting the mean flow restores the sign

Mean subtraction recovers isotropic turbulence

1.Compute time-average velocity field (400 snapshots)

2. Subtract from 400 instantaneous velocity fields

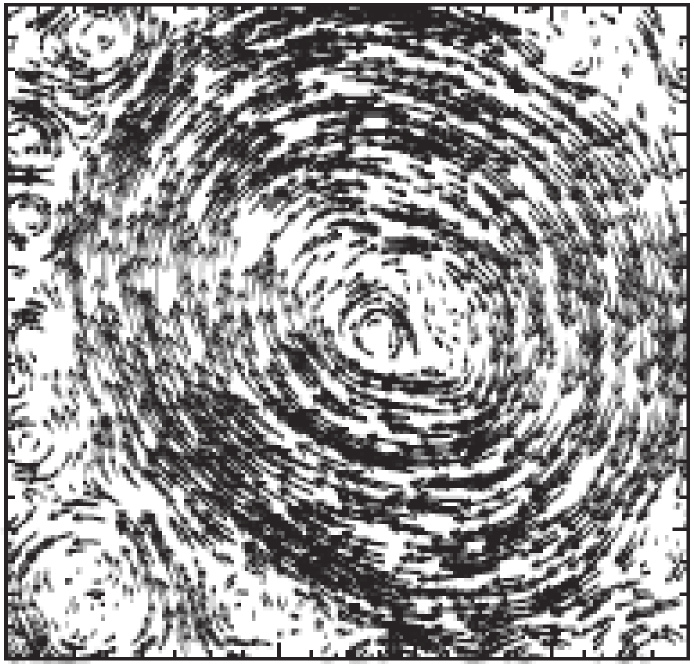


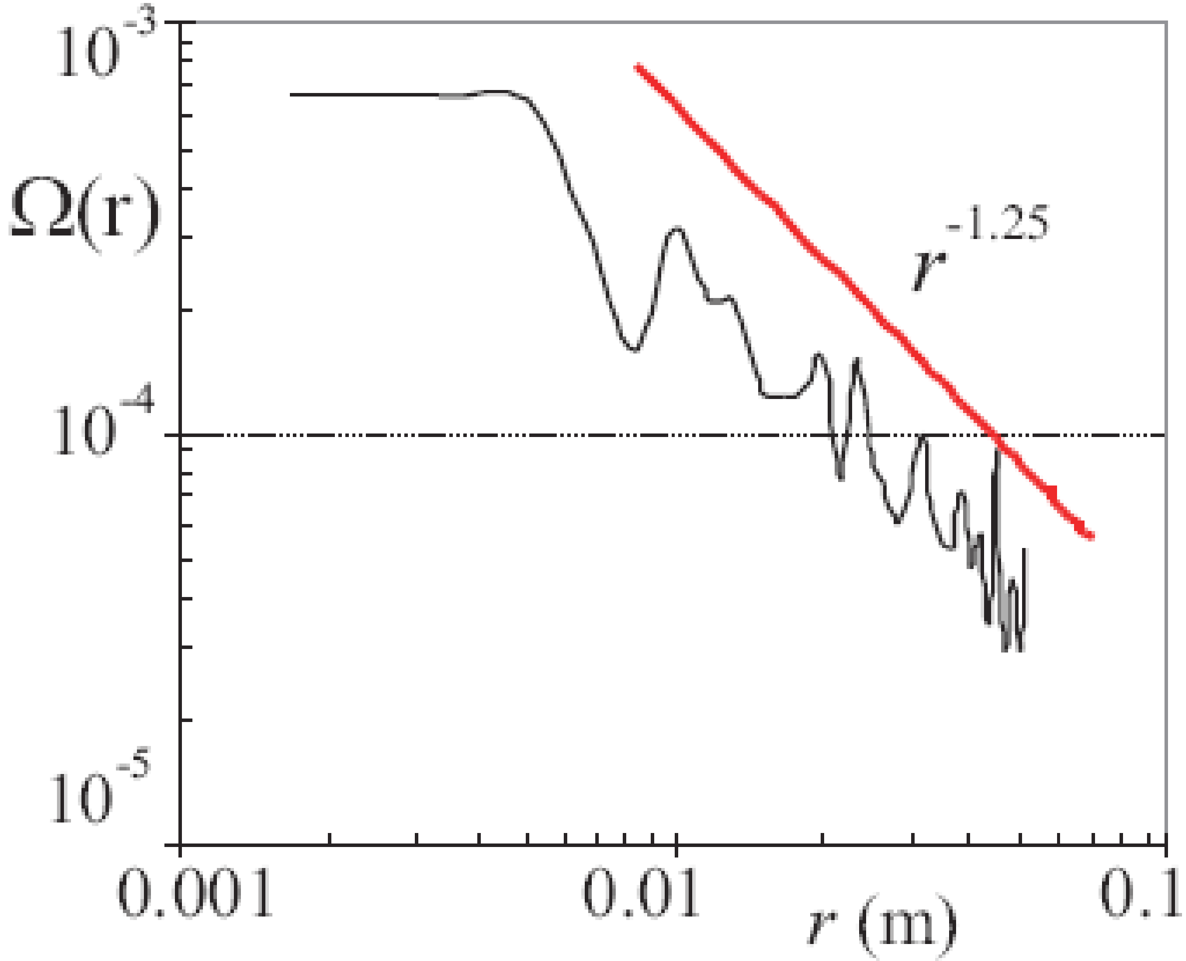
Recover ~ $k^{-5/3}$ spectrum in the energy range

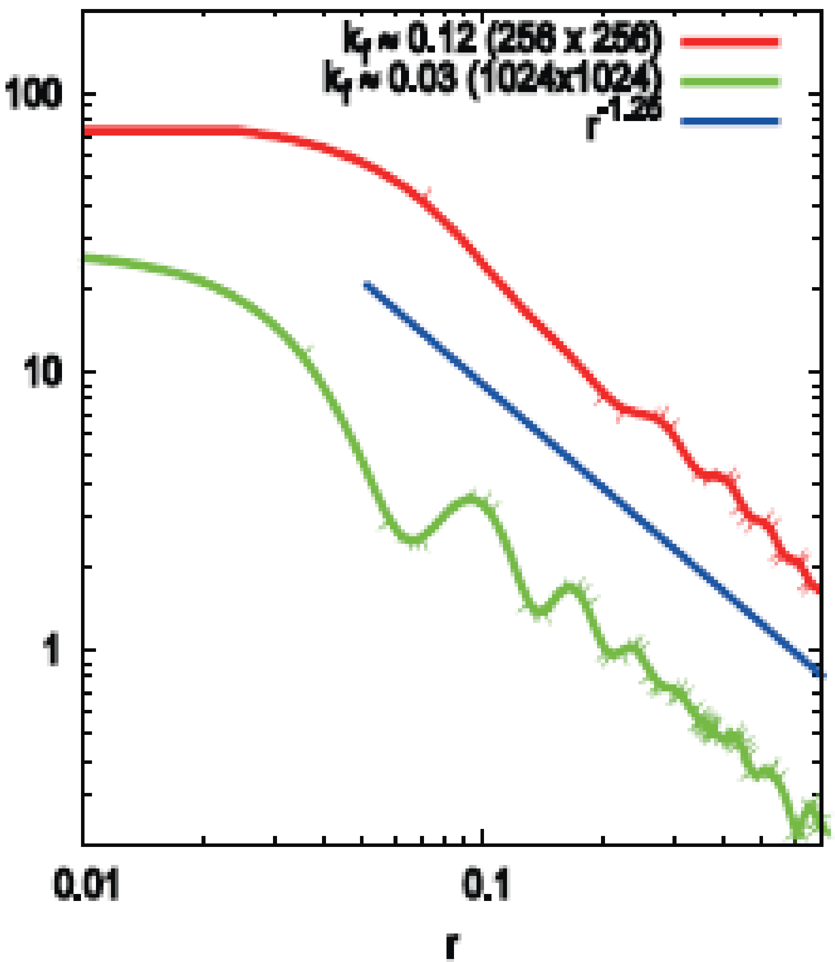
Kolmogorov law – linear S3 (r) dependence in the “turbulence range”;

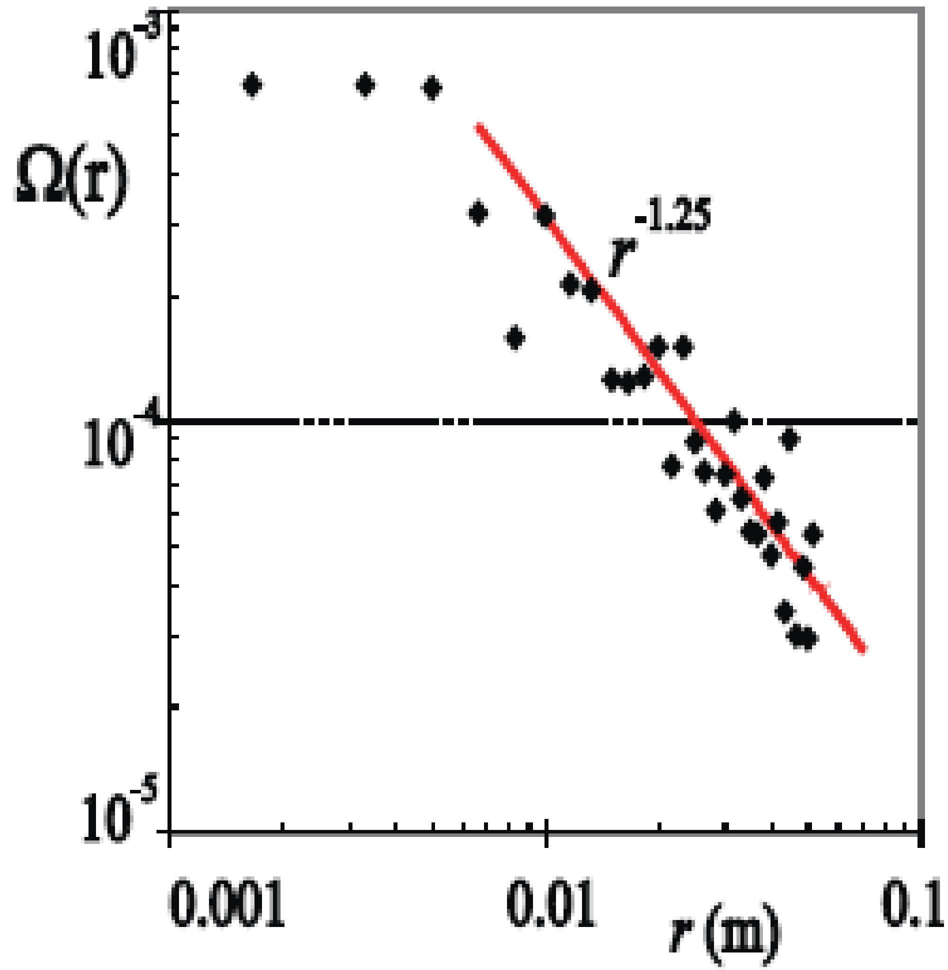
Kolmogorov constant C≈7

Universal profile of a coherent vortex

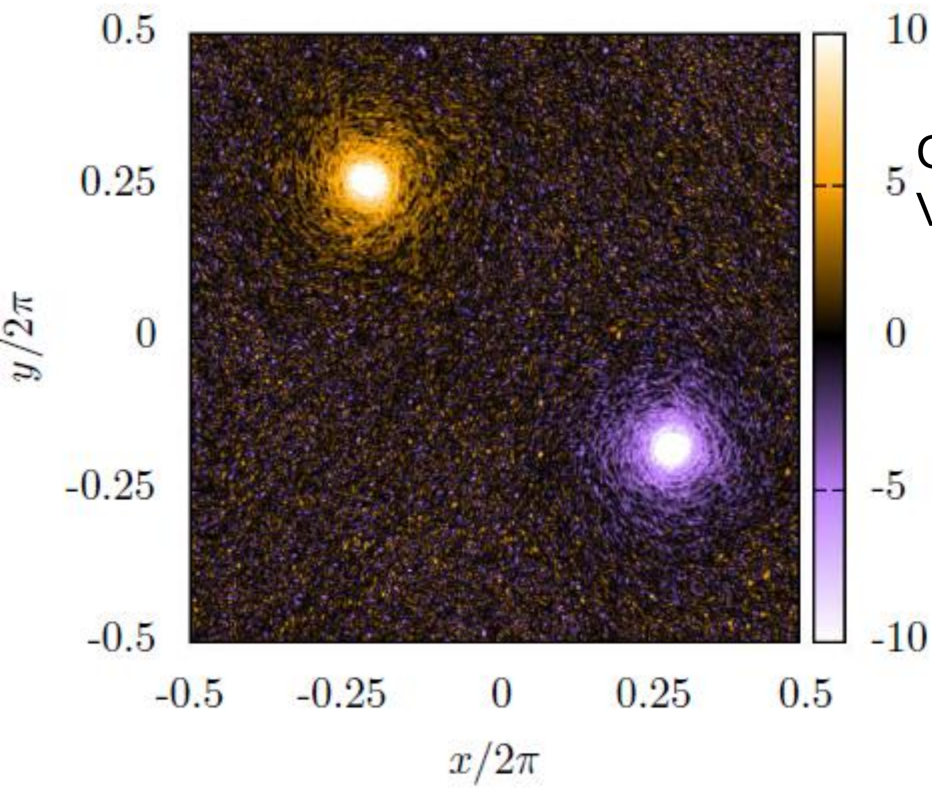




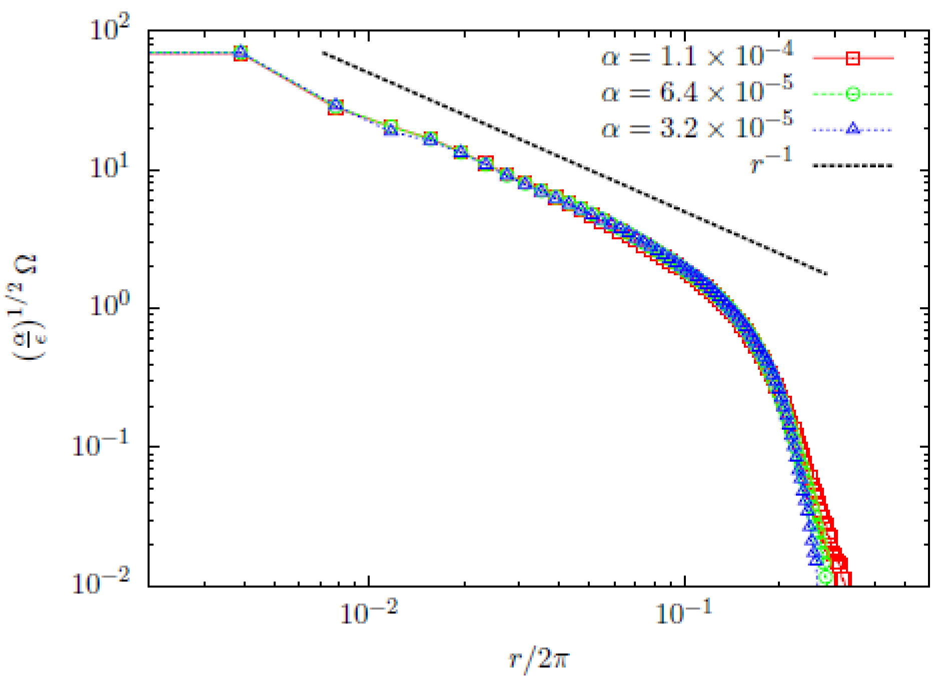




Connaughton, Chertkov, Lebedev, Kolokolov, Xia, Shats, Falkovich



G. Boffetta, J. Laurie, I. Kolokolov,
V. Lebedev, GF, 2014



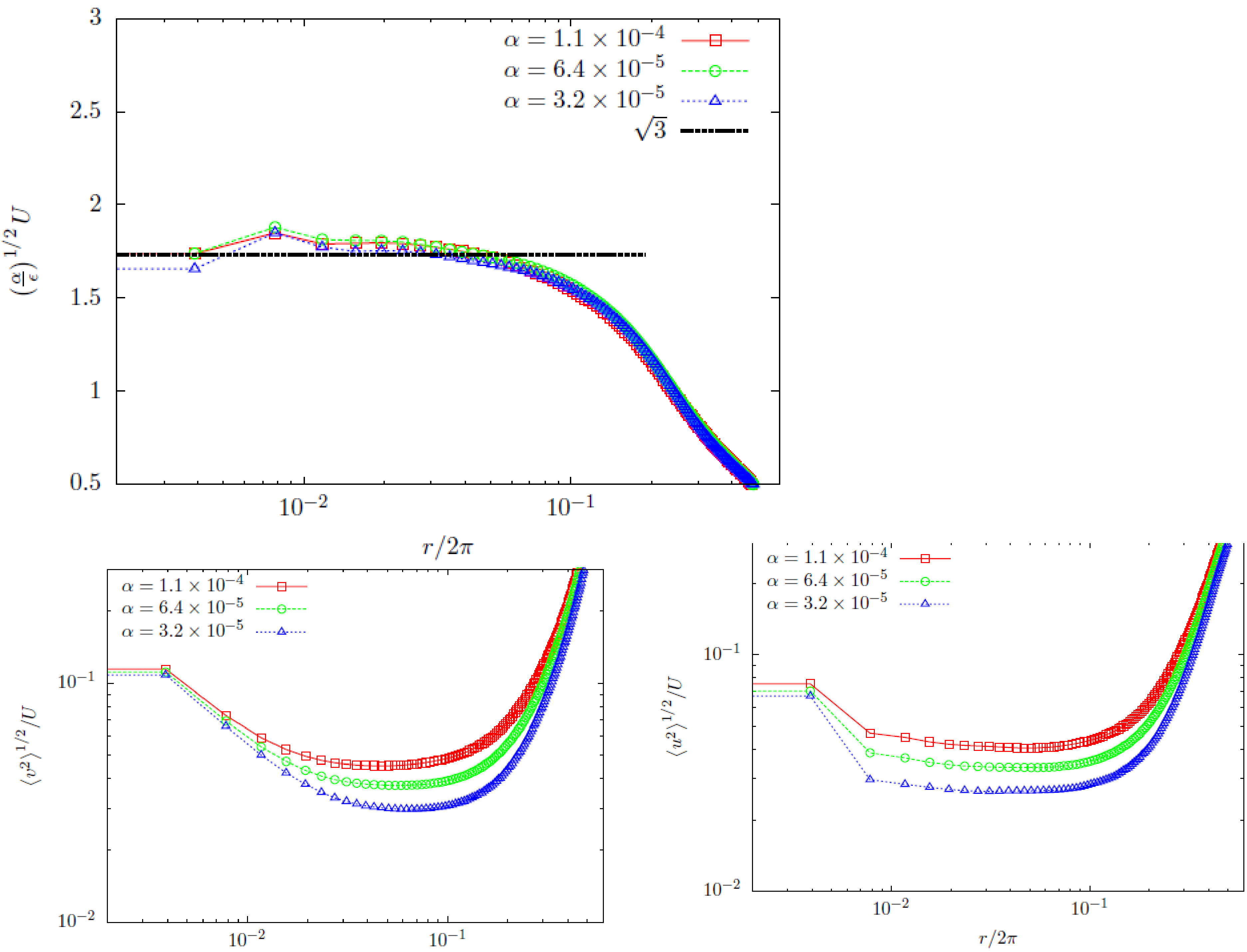
$$\mathbf{v} = (v_\phi, v_r) = (U + u, v) \text{ with } U(r), v(r, \phi, t), u(r, \phi, t)$$

$$\partial_t \mathbf{v} + \alpha \mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v} = \mathbf{f} - \nabla p$$

$$r^{-2} \partial_r r^2 \langle uv \rangle = -\alpha U$$

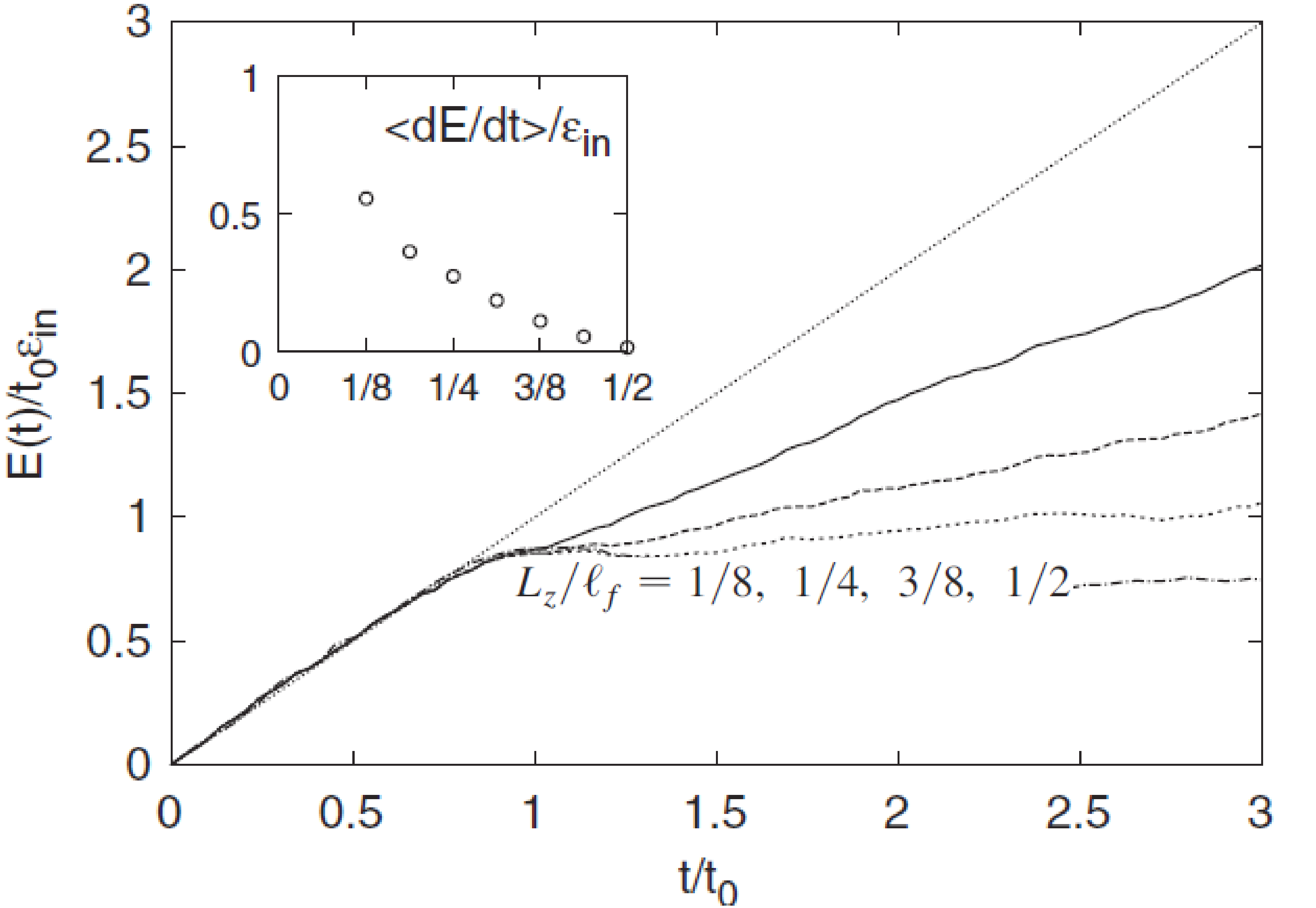
$$\epsilon = \frac{1}{r} \partial_r \left(rU \langle uv \rangle \right) + \alpha U^2$$

$$U^2 = 3\epsilon / \alpha$$

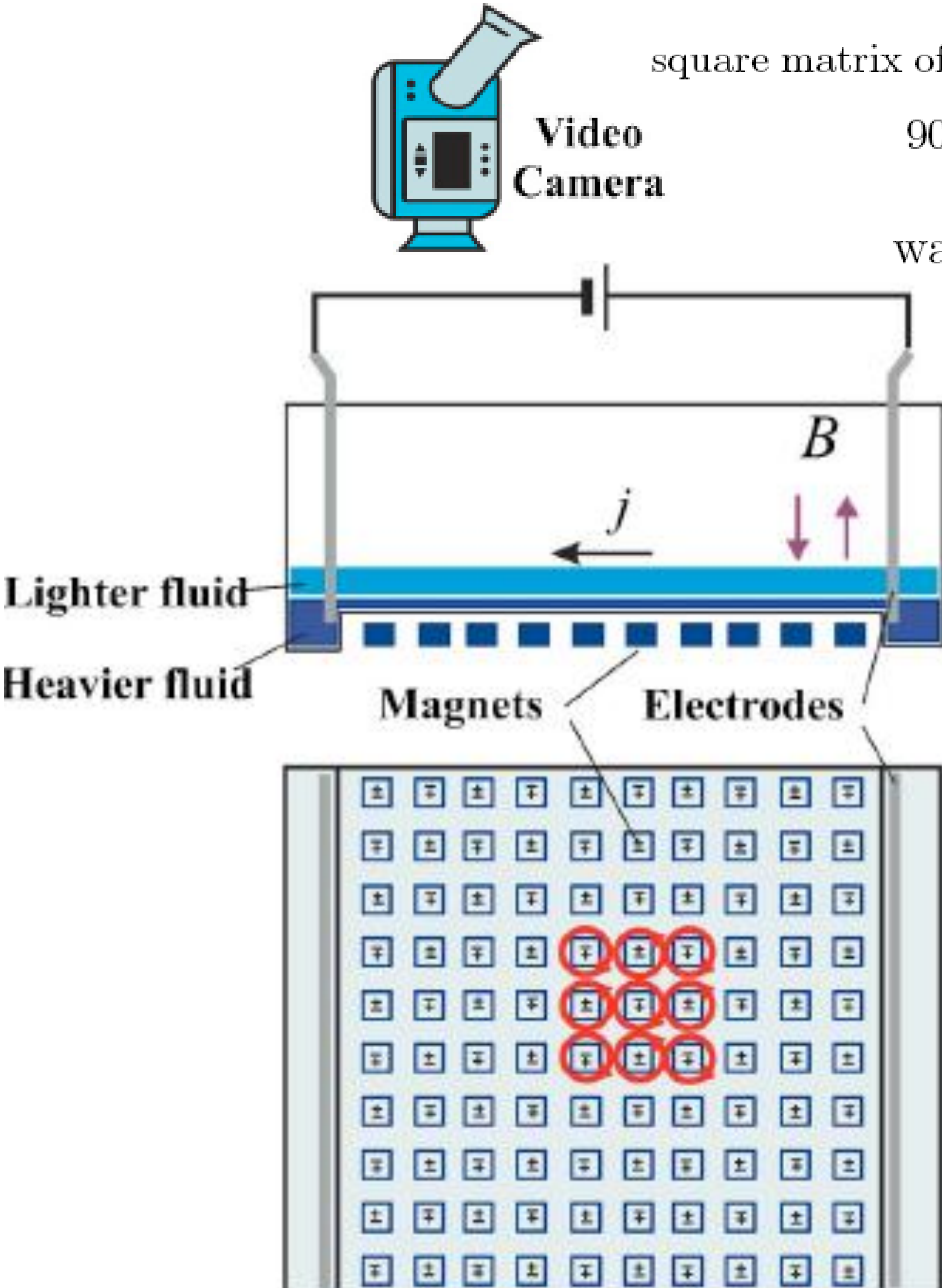


α = 1.1 × 10^−4
α = 6.4 × 10^−5
α = 3.2 × 10^−5
√3
(α/ε)^{1/2} U
r/2π
⟨v²⟩^{1/2}/U
⟨u²⟩^{1/2}/U
10^−2
10^−1

To understand atmosphere one needs to move from thin to **thick layers**



Antonio Celani,[1] Stefano Musacchio,[2,3] and Dario Vincenzi[3] PRL **104,** 184506 (2010)



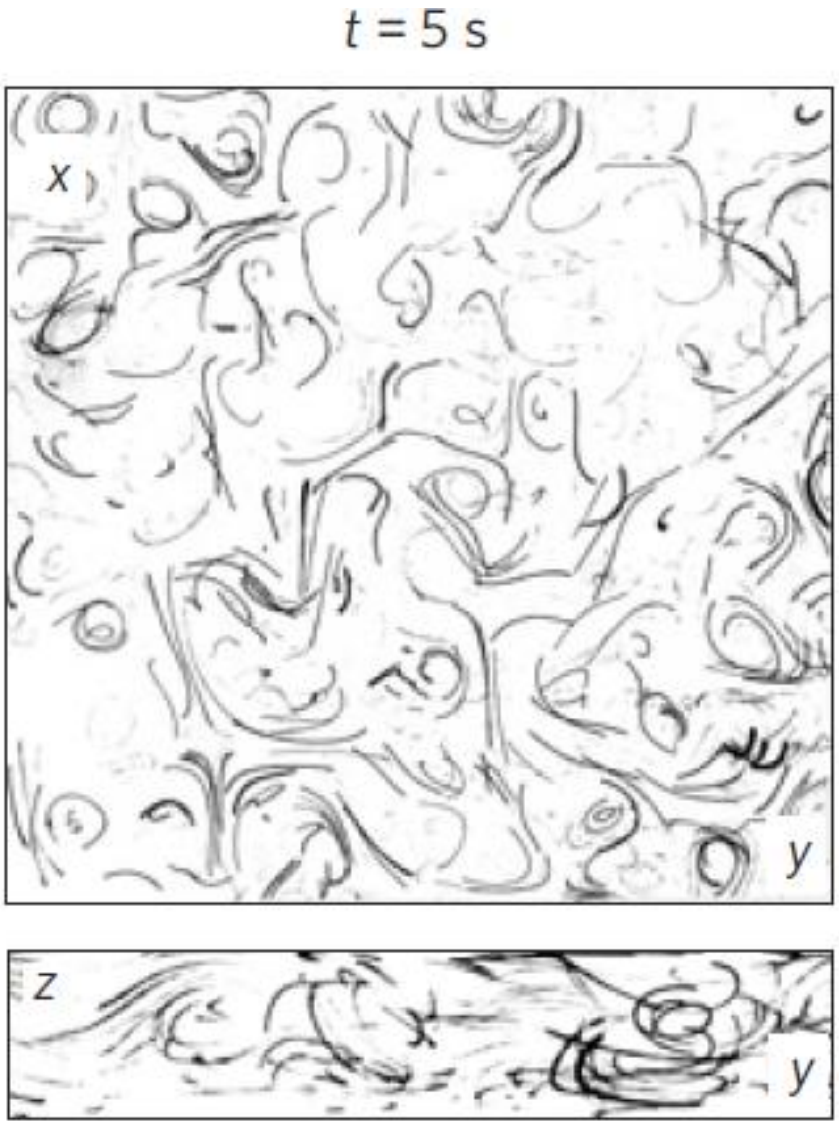
square matrix of 30×30 permanent magnets (10 mm apart)

900 vortices (9 mm in diameter)

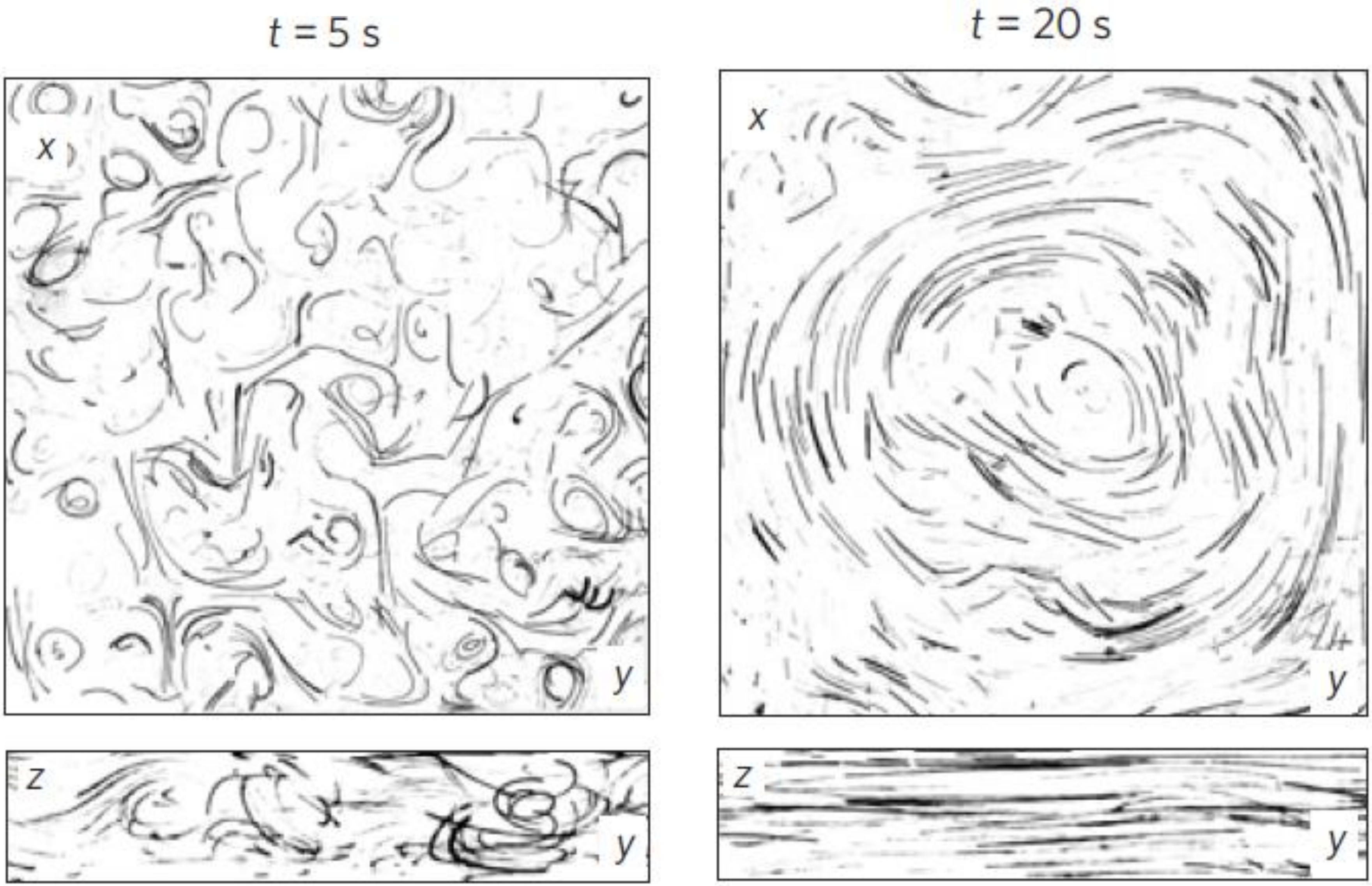
walls of the fluid cell (0.3×0.3 m^2)

insertable square boundaries

$$L = (0.09 - 0.24) \text{ m}$$

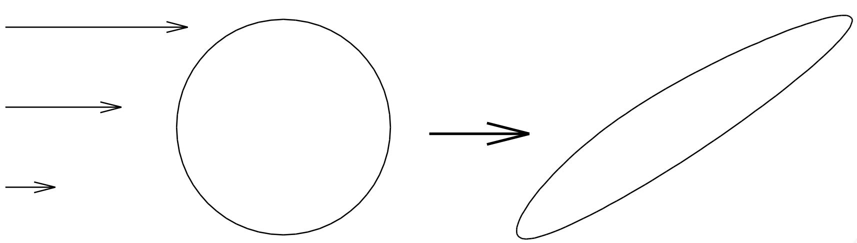


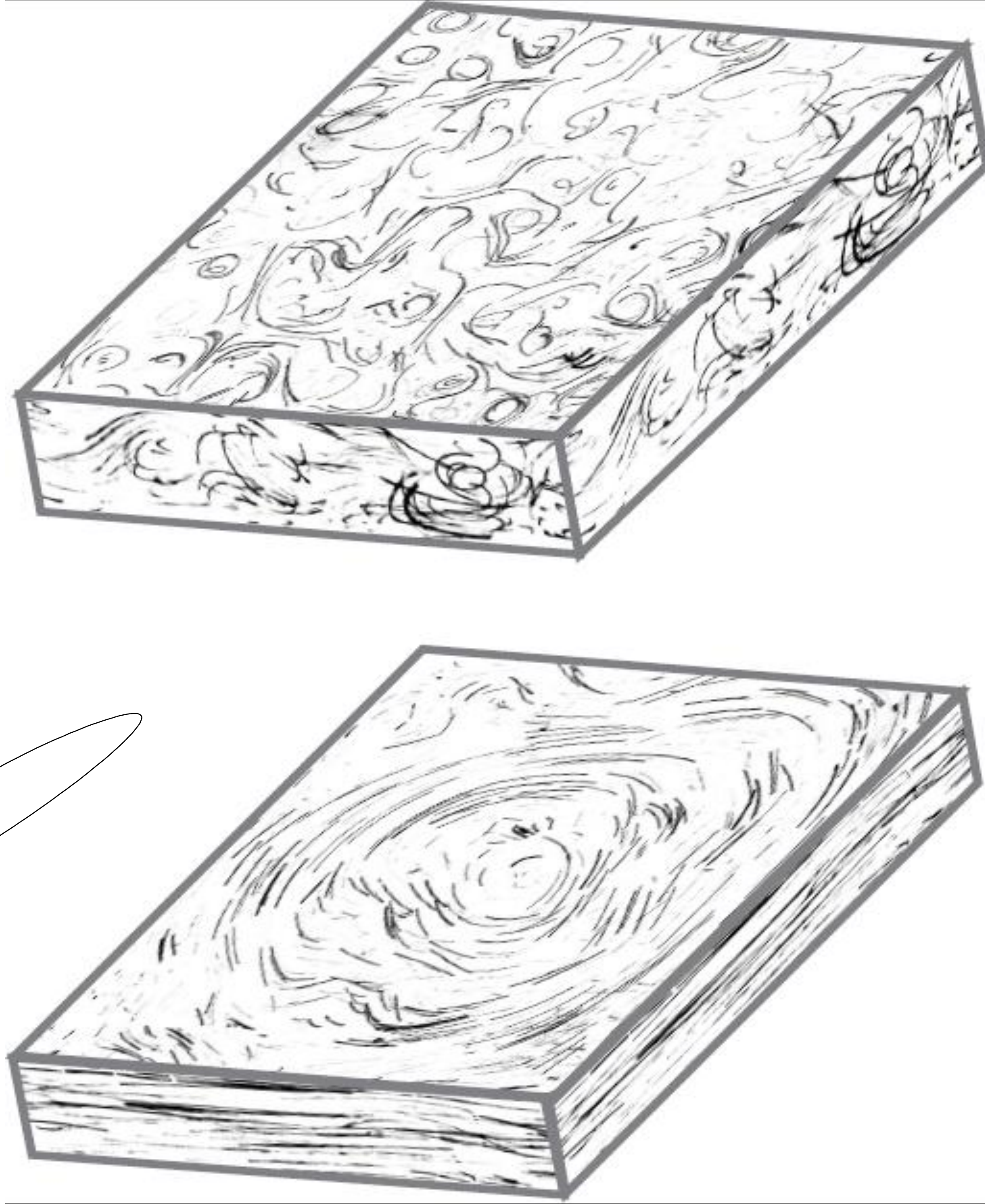
t = 5 s
x
y
z
y

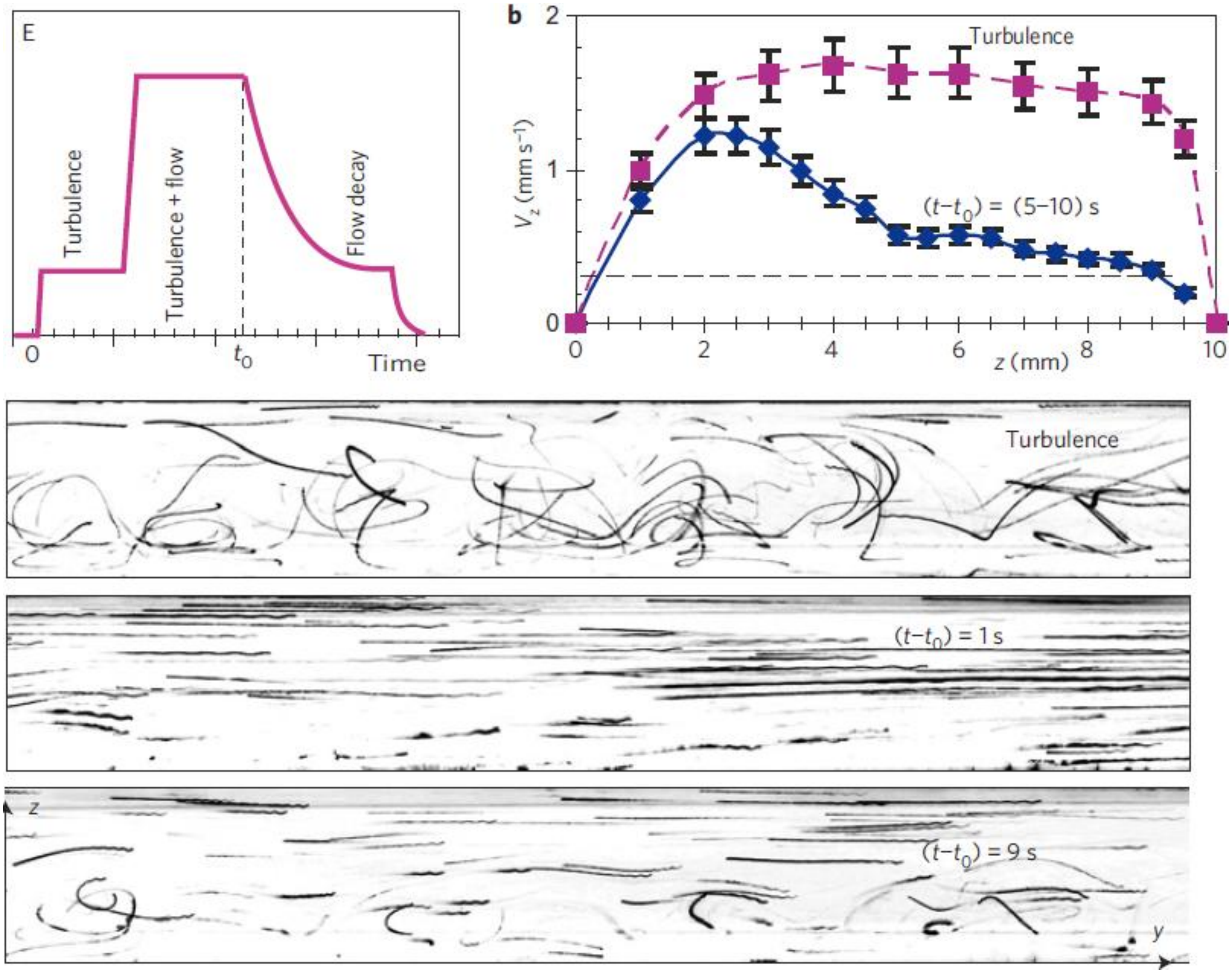


H. Xia[1], D. Byrne[1], G. Falkovich[2] and M. Shats[1] NATURE PHYSICS, April 1, 2011

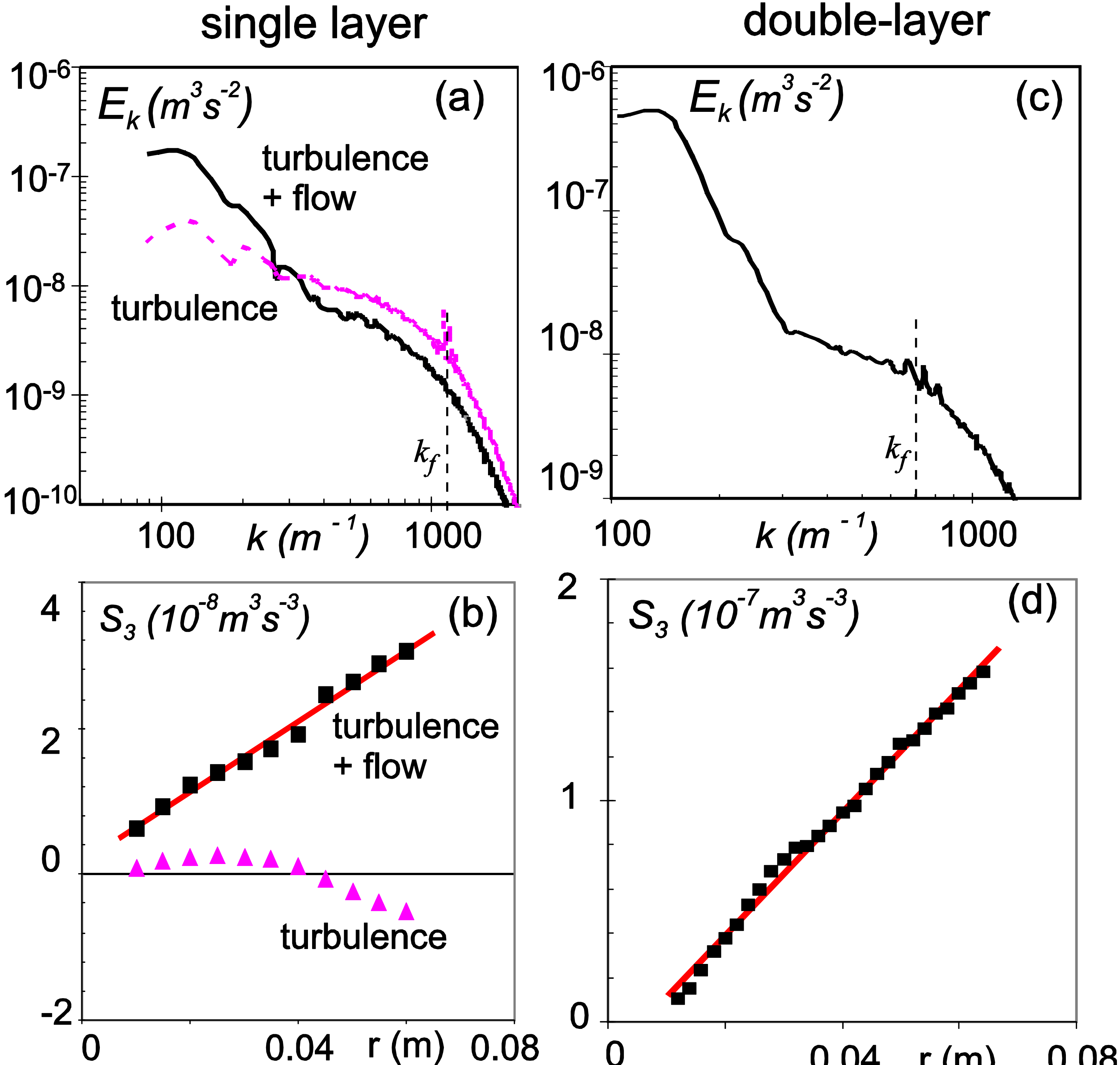
Vertical shear suppresses
vertical vortices

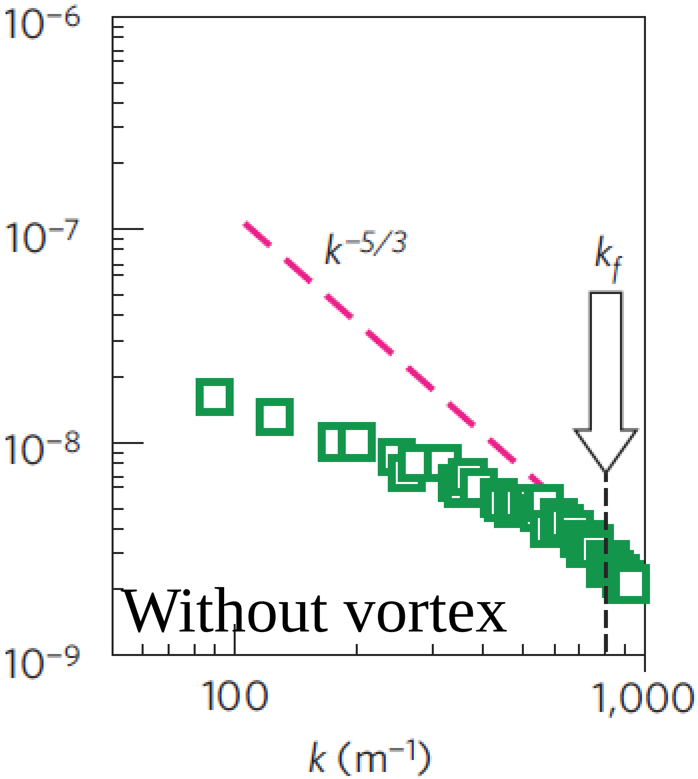


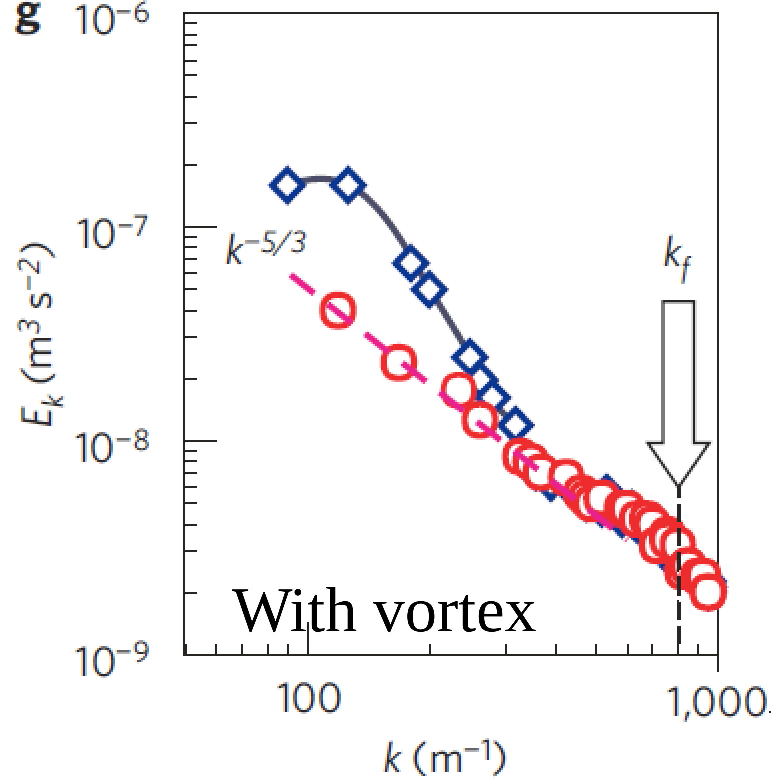


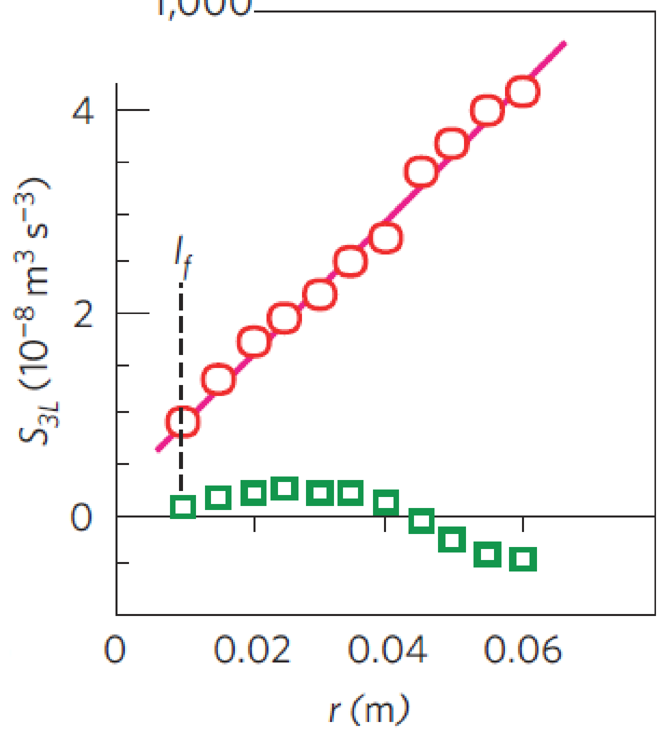


E
Turbulence
Turbulence + flow
Flow decay
0
t_0
Time
b
2
1
0
Turbulence
V_z (mm s^{-1})
$(t-t_0)$ = (5–10) s
0
2
4
6
8
10
z (mm)
Turbulence
$(t-t_0)$ = 1 s
$(t-t_0)$ = 9 s
z
y


single layer
double-layer
$E_k (m^3s^{-2})$
(a)
turbulence
+ flow
turbulence
k_f
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
100
$k (m^{-1})$
1000
$E_k (m^3s^{-2})$
(c)
k_f
100
$k (m^{-1})$
1000
$S_3 (10^{-8}m^3s^{-3})$
(b)
turbulence
+ flow
turbulence
4
2
0
-2
0
0.04
r (m)
0.08
$S_3 (10^{-7}m^3s^{-3})$
(d)
2
1
0
0
0.04
r (m)
0.08







$$S_{3\mathrm{L}} = \langle (\delta V_{\mathrm{L}})^3 \rangle$$

$$\varepsilon = -(2/3)\, S_{3\mathrm{L}}/r$$

Moral

- A strong large-scale flow effectively suppresses fluctuations in the vertical velocity.
- The resulting flow is planar even at small scales yet it is three-dimensional as it depends strongly on the vertical coordinate.
- Turbulence in such flows transfers energy towards large scales.

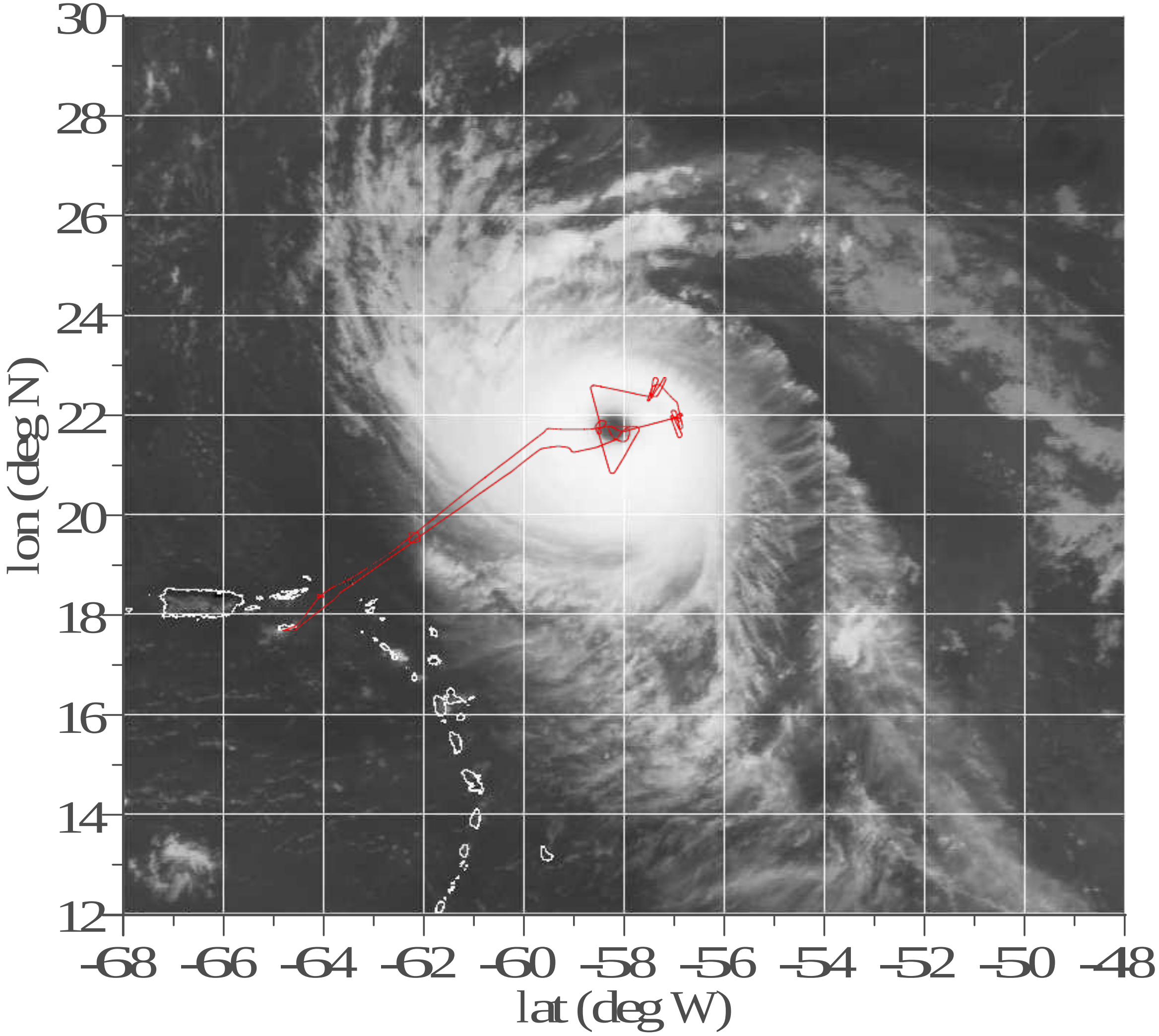
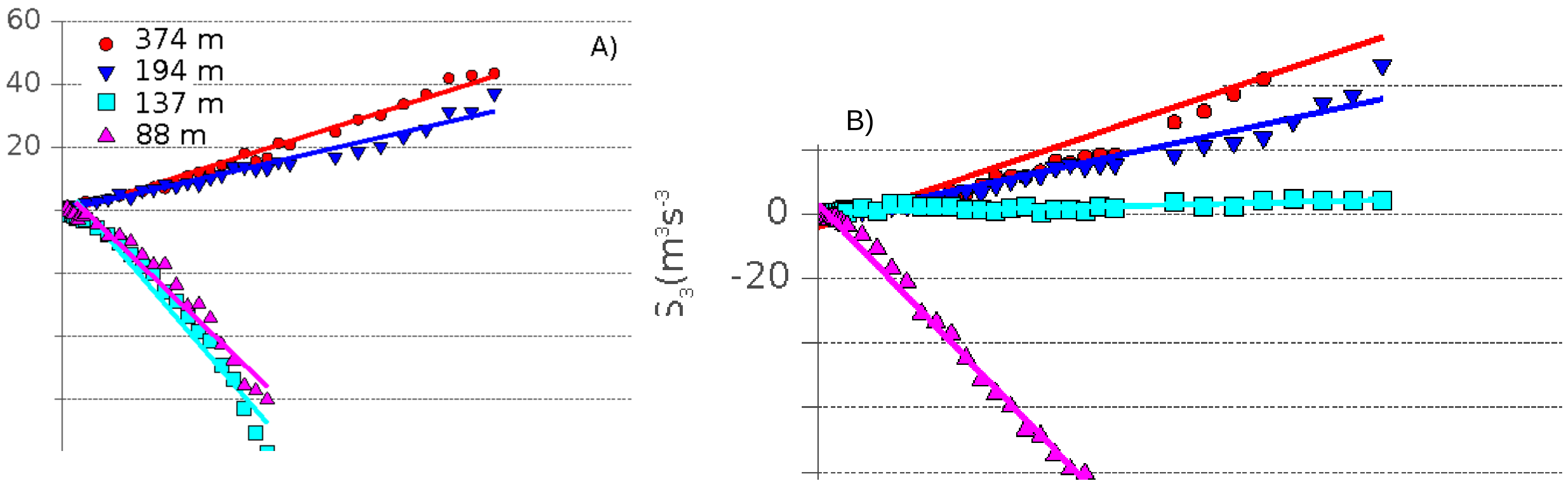


Figure 1. Satellite image of hurricane Isabel, 12th September 2003 with the flight track of the WP-3D Orion aircraft (N43RF) overlaid. Stepped decent measurements of the boundary layer performed between outer rain bands.

Three- to two-dimensional turbulence transition in the hurricane boundary layer
D. Byrne and A. Zhang, 2013

A transition from 3d to 2d turbulence from in-situ aircraft measurements in the hurricane boundary layer



Third order structure function of horizontal velocities for different flight-leg heights in hurricane A) Isabel and B) Fabian.

These results represent the first measurement of the 2D upscale energy flux in the atmosphere and also the first to characterize the transition from 3D to 2D. It is shown that the large-scale parent vortex may gain energy directly from small-scales in tropical cyclones.

Summary

Inverse cascades seems to be scale invariant (and
at least partially conformal invariant).

Condensation into a system-size coherent mode
breaks symmetries of inverse cascades.

Condensates can enhance and suppress fluctuations in different systems. **Spectral condensates** of universal forms can coexist with turbulence.

Small-scale turbulence and large-scale vortex can conspire to provide for an inverse energy cascade.

Fluid Mechanics

The multi-disciplinary field of fluid mechanics is one of the most actively developing fields of physics, mathematics and engineering. In this book, the fundamental ideas of fluid mechanics are presented from a physics perspective.

Using examples taken from everyday life, from hydraulic jumps in a kitchen sink to Kelvin–Helmholtz instabilities in clouds, the book provides readers with a better understanding of the world around them. It teaches the art of fluid-mechanical estimates and shows how the ideas and methods developed to study the mechanics of fluids are used to analyse other systems with many degrees of freedom in statistical physics and field theory.

Aimed at undergraduate and graduate students, the book assumes no prior knowledge of the subject and only a basic understanding of vector calculus and analysis. It contains 32 exercises of varying difficulties, from simple estimates to elaborate calculations, with detailed solutions to help readers understand fluid mechanics.

Gregory Falkovich is a Professor in the Department of Physics of Complex Systems, Weizmann Institute of Science. He has researched in plasma, condensed matter, fluid mechanics, statistical and mathematical physics and cloud physics and meteorology, and has won several awards for his work.

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FALKOVICH **Fluid Mechanics**

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Fluid Mechanics

A Short Course for Physicists

GREGORY FALKOVICH

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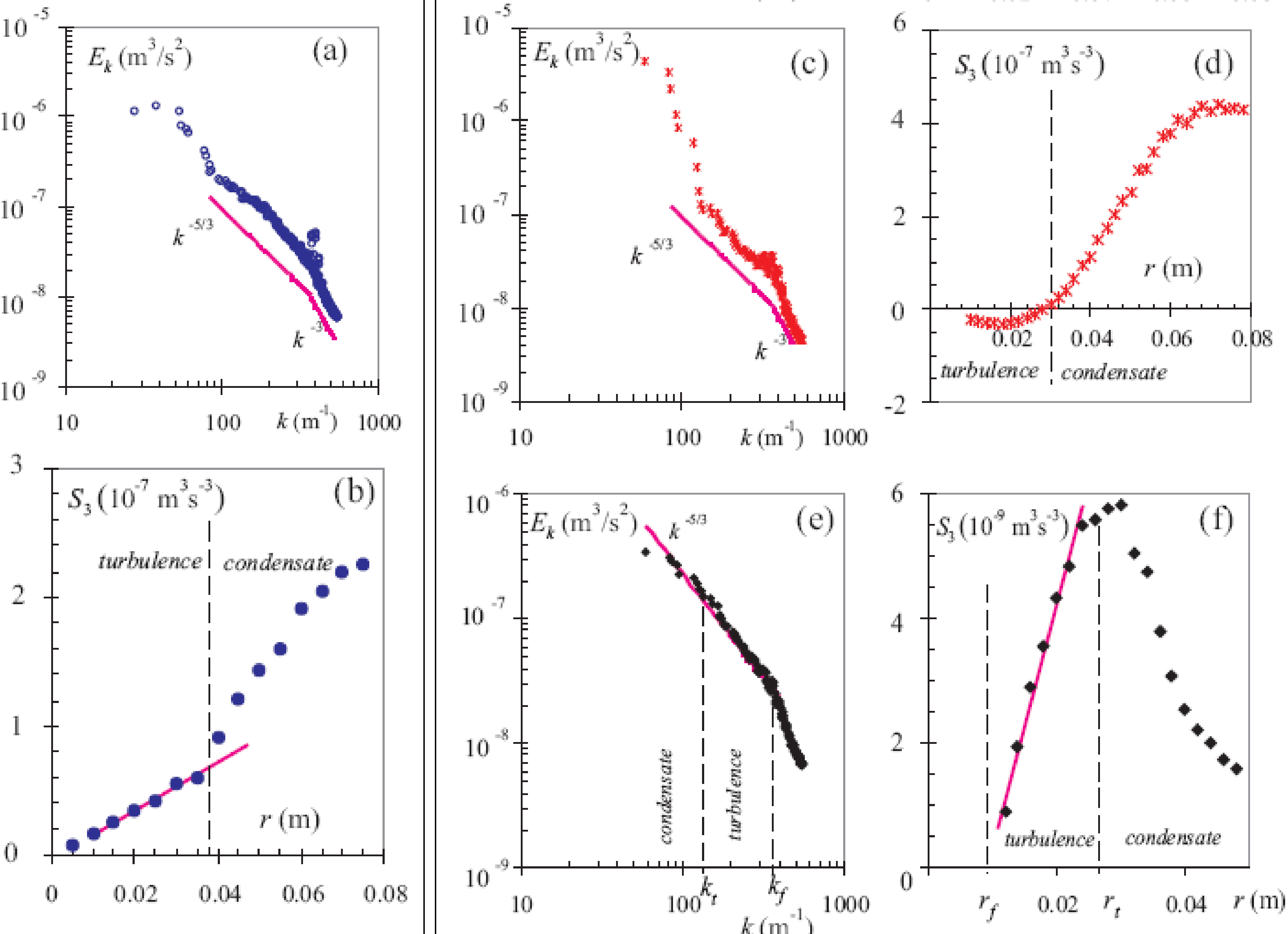
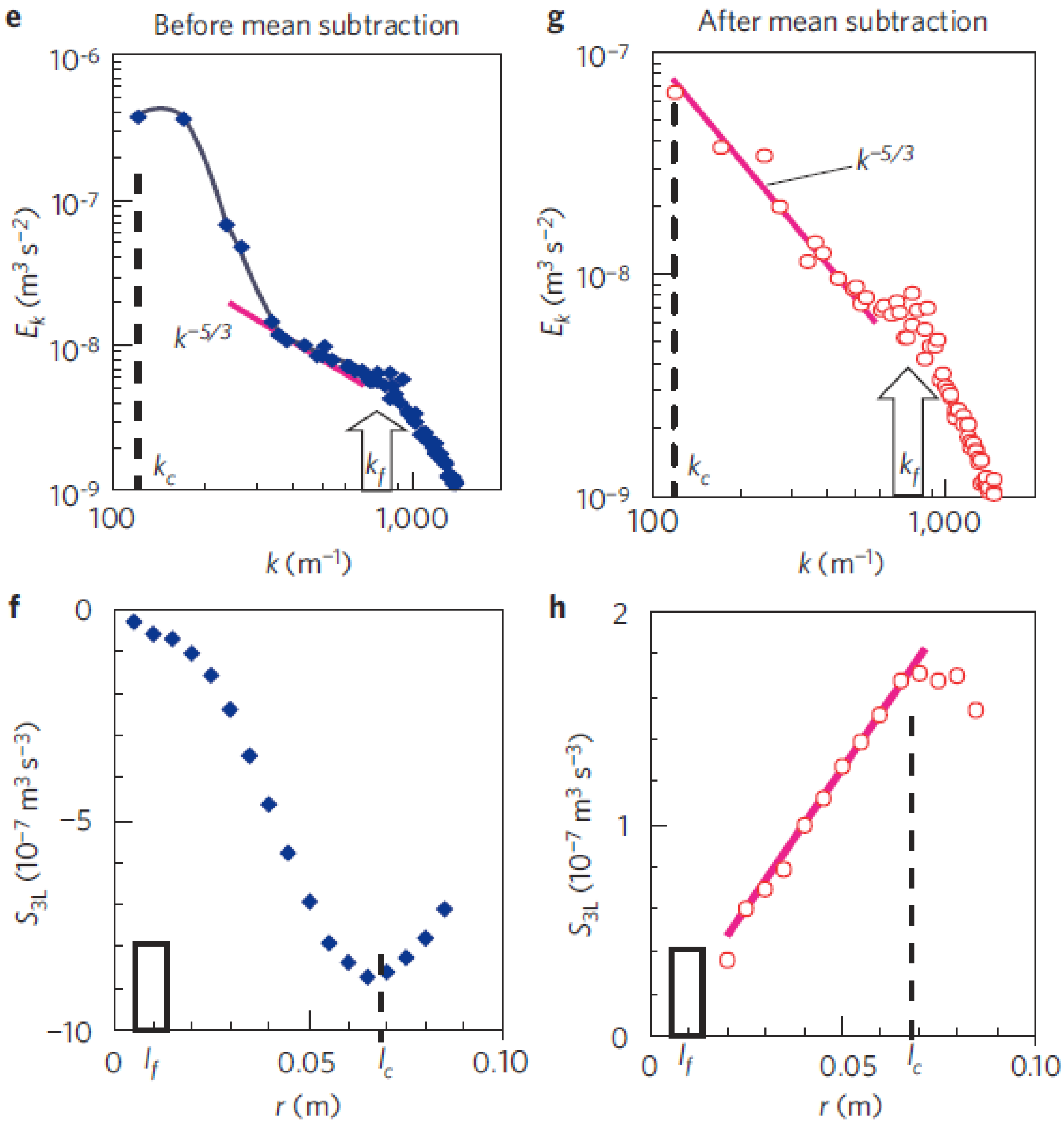
flow coherent over the system size (the condensate) appears [2,5,6,17–19] with the velocity estimated from the energy balance, $\alpha V^2 \approx 2\epsilon$, which gives $s \cong V/L_s \cong L_s^{-1}\sqrt{2\epsilon/\alpha}$ and

$$k_t = \pi/l_t \cong \pi L_s^{-3/2}(C\alpha/2)^{-3/4}\epsilon^{1/4}. \quad (1)$$



Weak condensate

Strong condensate


e
Before mean subtraction
10^{-6}
10^{-7}
10^{-8}
10^{-9}
E_k (m^3 s^{-2})
$k^{-5/3}$
k_c
k_f
100
1,000
k (m^{-1})
g
After mean subtraction
10^{-7}
10^{-8}
10^{-9}
E_k (m^3 s^{-2})
$k^{-5/3}$
k_c
k_f
100
1,000
k (m^{-1})
f
0
−5
−10
S_{3L} (10^{-7} m^3 s^{-3})
l_f
l_c
0
0.05
0.10
r (m)
h
2
1
0
S_{3L} (10^{-7} m^3 s^{-3})
l_f
l_c
0
0.05
0.10
r (m)