Multiscale Approach to the Direct Statistical Simulation of Flows

Brad Marston

with Greg Chini (University of New Hampshire)

and Steve Tobias (University of Leeds)

IN DEO SPERAMUS

NSF

Second-order Cumulant Expansions and Unhappiness

"An alternative procedure which does not suffer this disadvantage consists of deriving a new system of equations whose unknowns are the statistics themselves...."

Edward Lorenz, The Nature and Theory of the General Circulation of the Atmosphere (1967)

“Direct Statistical Simulation” (DSS)

DSS vs. DNS

Low-order statistics are smoother in space than the instantaneous flow.

Statistics evolve slowly in time, or not at all, and hence may be described by a fixed point, or at least a slow manifold.

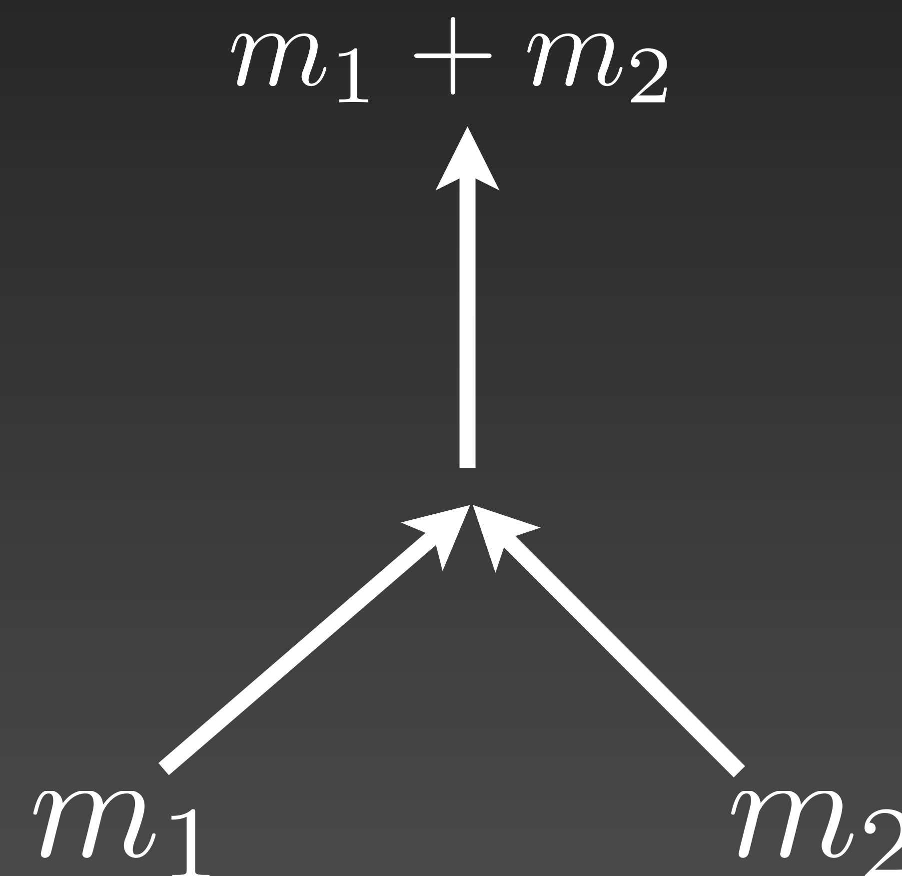
Correlations are *non-local* and highly anisotropic and inhomogeneous. Statistical formulations must respect this.

Barotropic Toy Model of Jets

$$\partial_t \zeta + \vec{v} \cdot \vec{\nabla}(\zeta + f) = -\kappa\zeta - \nu_2 \nabla^4 \zeta + \eta$$

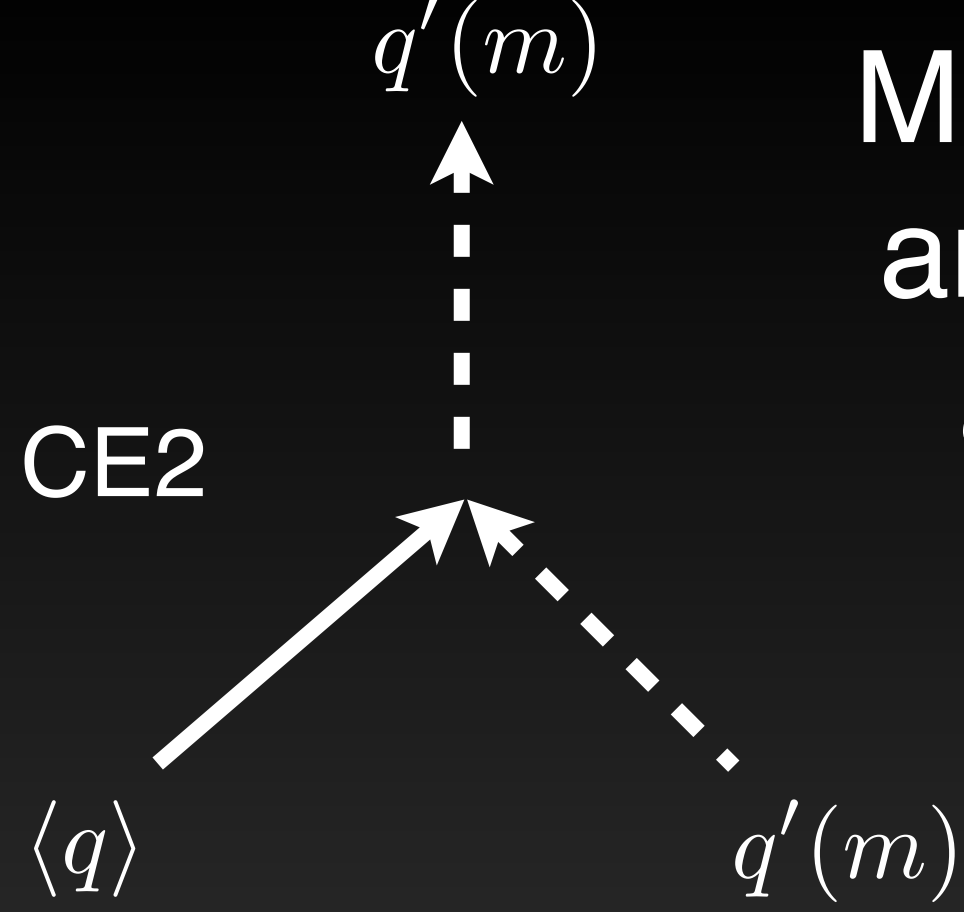
$$\zeta = \hat{r} \cdot \vec{\nabla} \times \vec{v} \qquad f = 2\Omega \sin(\phi)$$

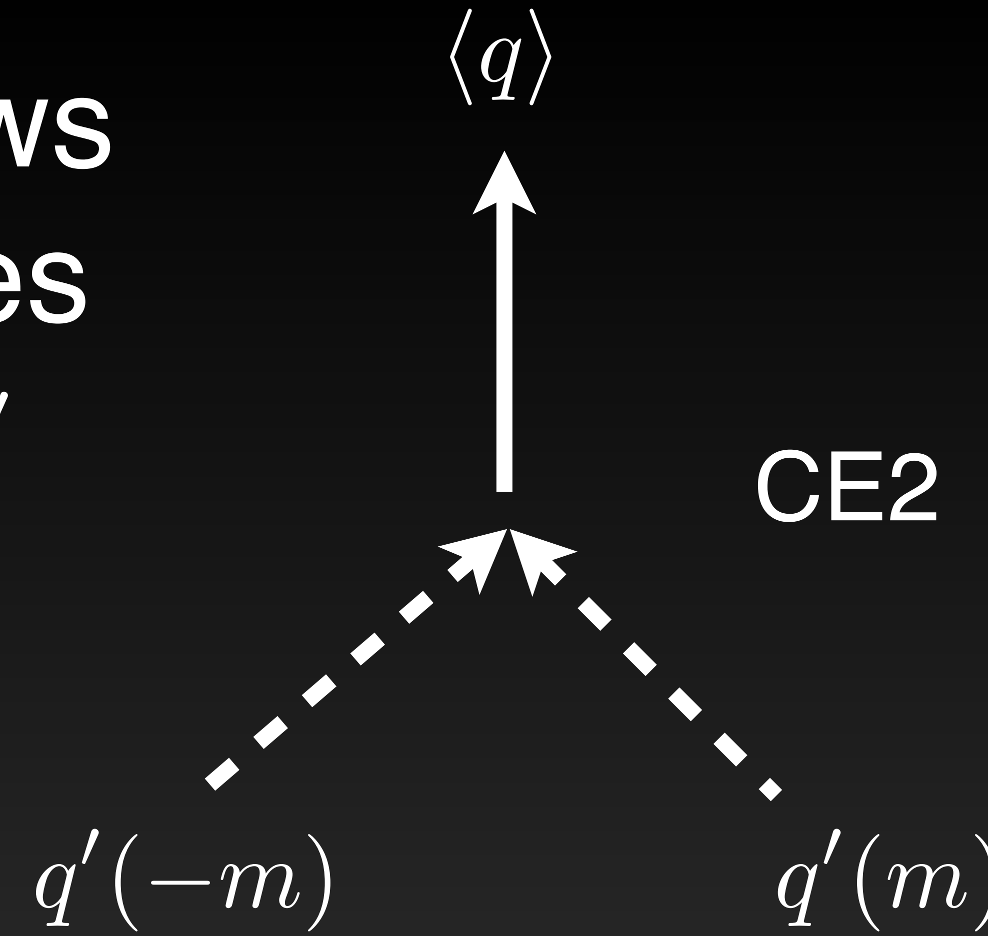
$$\zeta(\theta, \phi) = \sum_{\ell=0}^{L} \sum_{m=-\ell}^{\ell} \zeta_{\ell,m} Y_\ell^m(\theta, \phi)$$

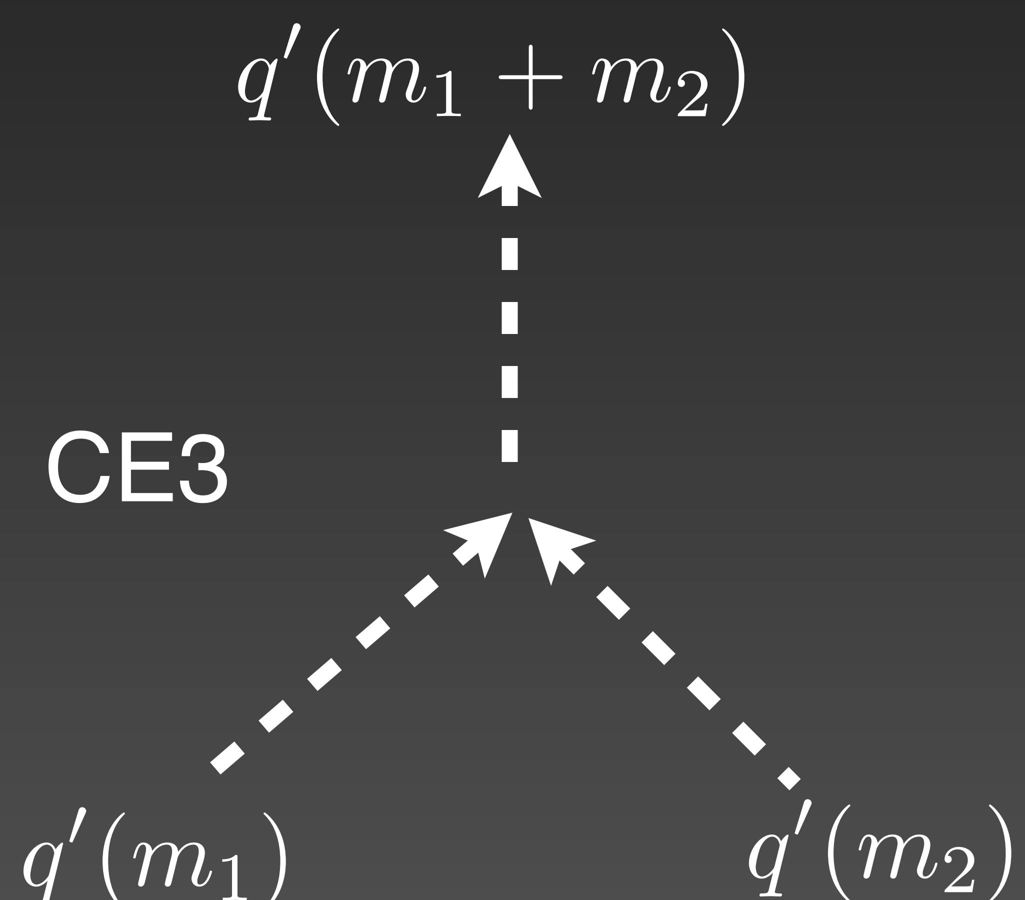


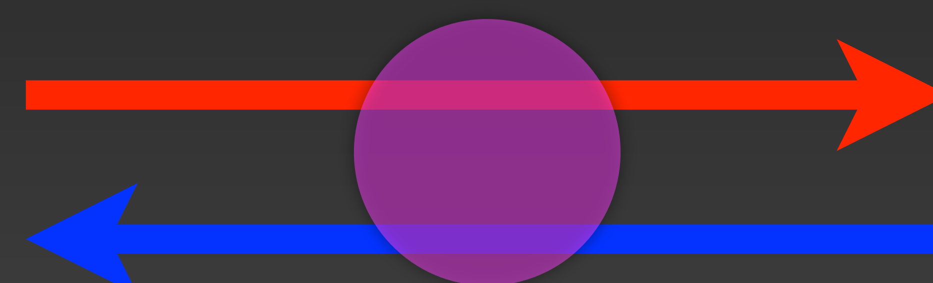
Mean Flows and Eddies

$$q = \langle q \rangle + q'$$









Eddies sheared apart by mean flows

Zonal Averages

$$\langle q_{\ell_1 m_1}\ q_{\ell_2 m_2}\ q_{\ell_3 m_3} \rangle$$

$$\langle q_{\ell_1 m}\ q^*_{\ell_2 m}\ q_{\ell_3, 0} \rangle = \langle q_{\ell_1 m}\ q^*_{\ell_2 m} \rangle\ \langle q_{\ell_3, 0} \rangle = c_{\ell_1 \ell_2 m}\ c_{\ell_3}$$

$$\mathrm{CE2} : \langle q_{\ell_1 m_1}\ q_{\ell_2 m_2}\ q^*_{\ell_3 m_1 + m_2} \rangle = 0 \text{ if } m_1 > 0 \text{ and } m_2 > 0$$

$$\begin{aligned}
\dot{c}_\ell &= A_\ell + B_{\ell;\ell_1 0}\ c_{\ell_1} + C^{(-)}_{\ell;\ell_1 m;\ell_2 m}\ c_{\ell_1 \ell_2 m} \\
\dot{c}_{\ell_1 \ell_2 m} &= 2\Gamma_{\ell_1 m} \delta_{\ell_1 \ell_2} + B_{\ell_1;\ell m}\ c_{\ell \ell_2 m} + B_{\ell_2;\ell m}\ c_{\ell_1 \ell m} \\
&+ C^{(+)}_{\ell_1;\ell 0;\ell' m}\ c_\ell\ c_{\ell' \ell_2 m} + C^{(+)}_{\ell_2;\ell 0;\ell' m}\ c_\ell\ c_{\ell_1 \ell' m}
\end{aligned}$$

$CE2 \approx SSST$ (Brian Farrell & Petros Ioannou)

Srinivasan & Young; Parker & Krommes; Tobias & JBM;
Bouchet, Nardini & Tangarife; Ait-Chaalal, Schneider & Sabou; …

CE3 & CE2.5

CE3

$$\dot{c}_1 = F + L[c_1] + Q[c_1 c_1 + c_2]$$

$$\dot{c}_2 = L[c_2] + Q[c_1 c_2 + c_3] + \Gamma$$

$$\dot{c}_3 = L[c_3] + Q[c_1 c_3 + c_2 c_2 + c_4]$$

Remove eigenvectors of c2 with negative eigenvalues

$$\dot{c}_3 = L[c_3] + Q[c_1 c_3 + c_2 c_2] - c_3/\tau$$

CE2.5:

$$c_3 = \tau Q[c_2 c_2]$$

Demonstration at end of talk (if time)

A Conservative Approximation

CE2 conserves total angular momentum, energy, and enstrophy in absence of forcing and damping.

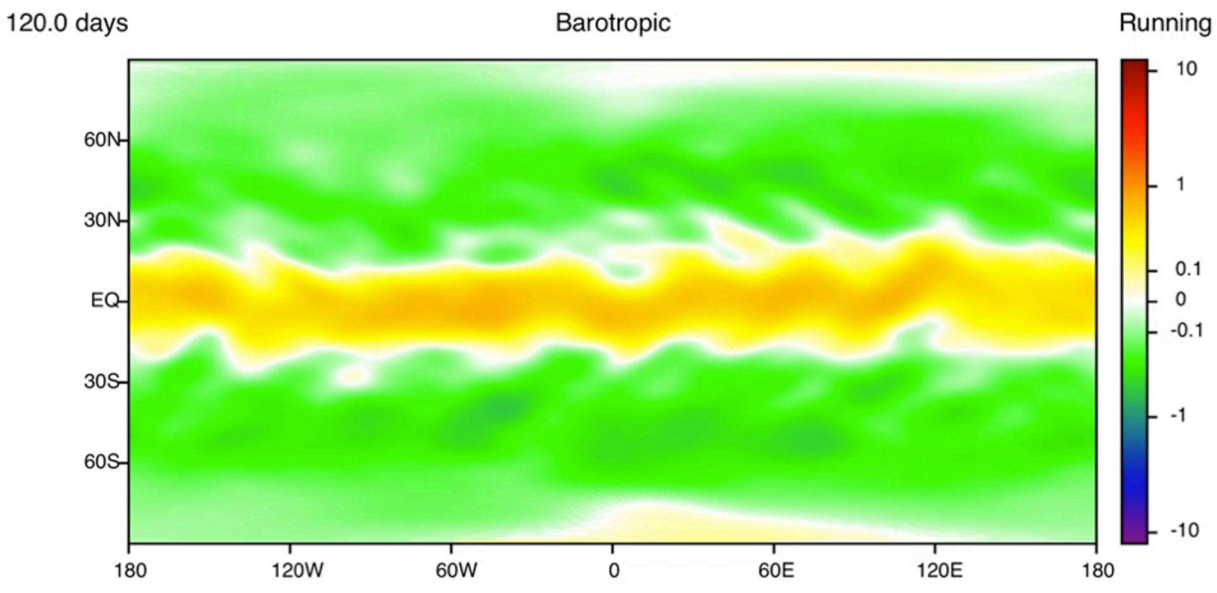
CE3 additionally conserves the 3rd Casimir prior to projection (see poster by Wanming Qi).

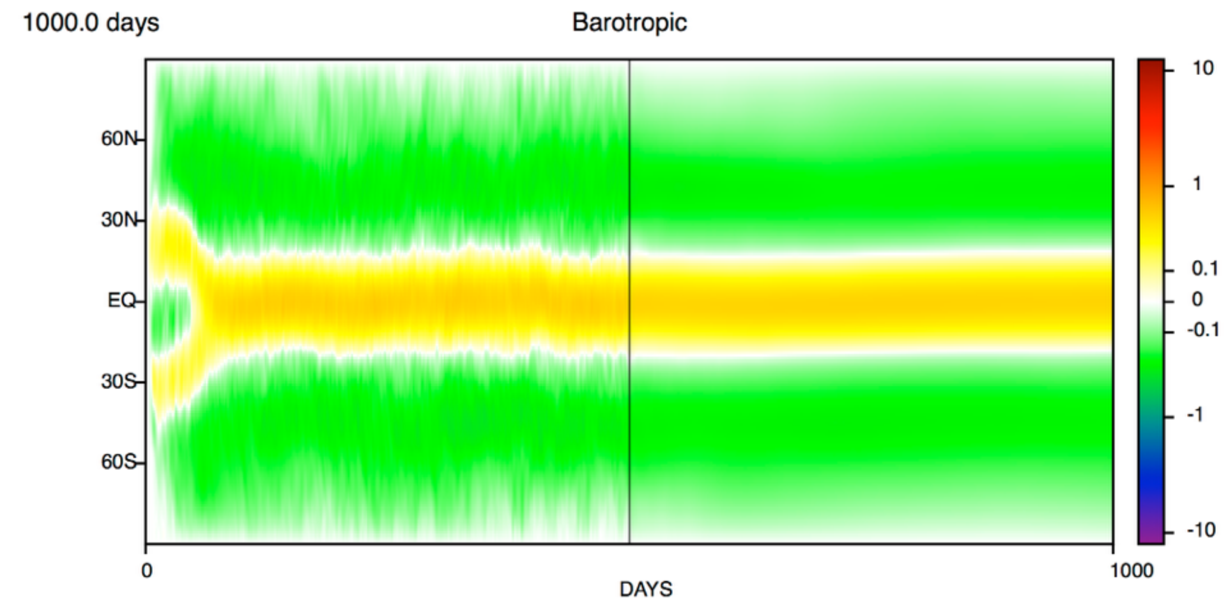
Realizability

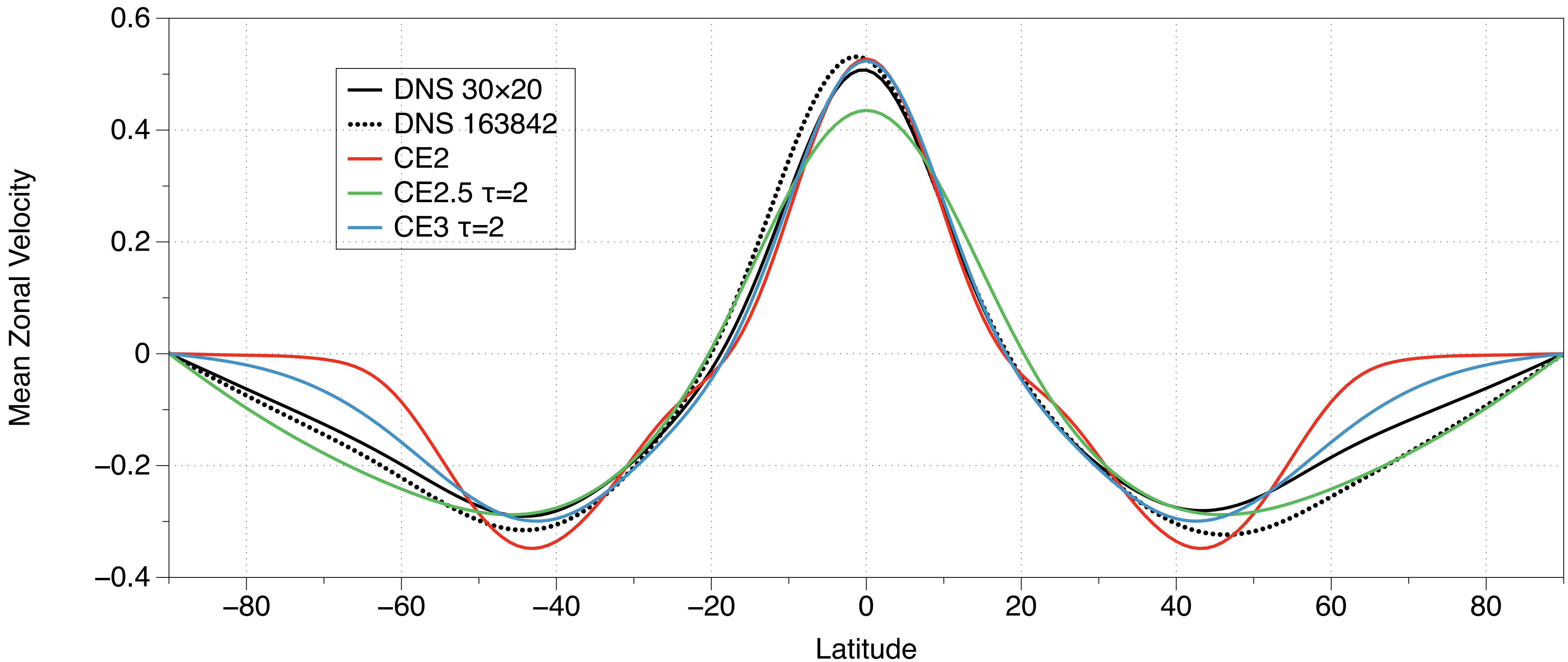
CE2 = Quasilinear = Gaussian PDF = Realizable

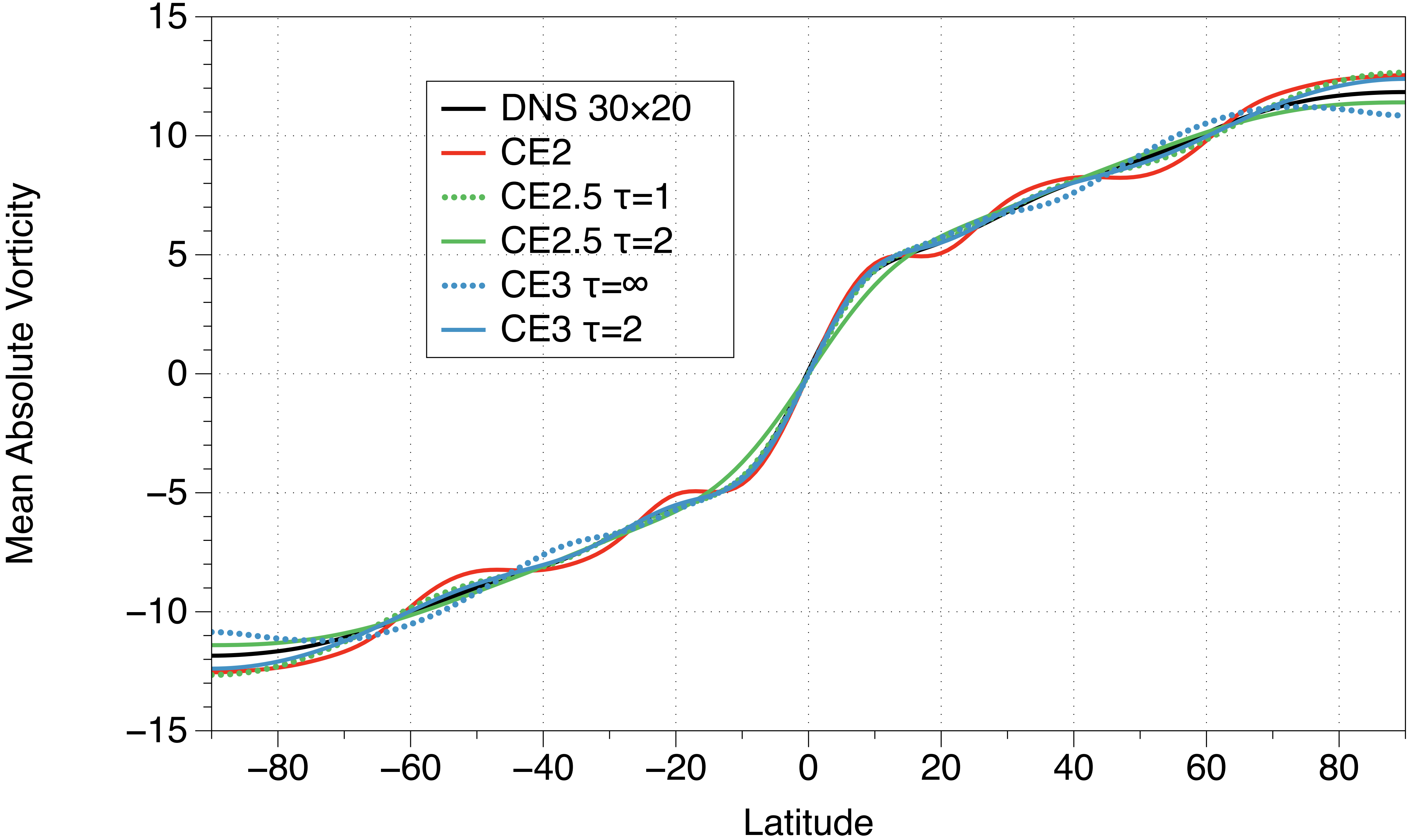
CE2.5 = generalized EDQNM = realizable?

CE3: Not realizable [Kraichnan (1980)]. Fixed this by projecting out offending eigenvectors.

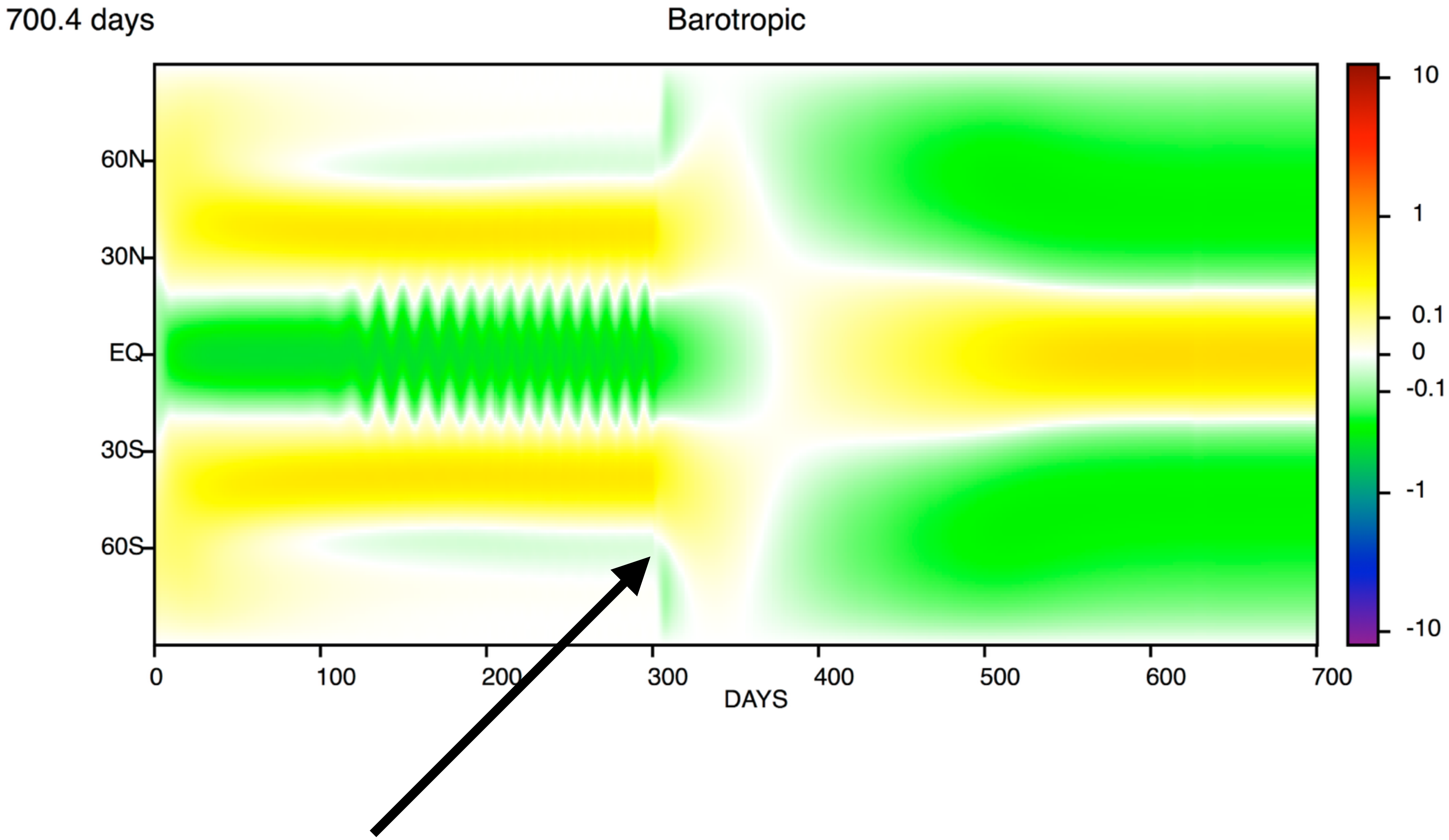

120.0 days
Barotropic
Running
60N
30N
EQ
30S
60S
180
120W
60W
0
60E
120E
180
10
1
0.1
0
-0.1
-1
-10


1000.0 days
Barotropic
60N
30N
EQ
30S
60S
0
DAYS
1000
10
1
0.1
0
-0.1
-1
-10


Mean Zonal Velocity
Latitude
DNS 30×20
DNS 163842
CE2
CE2.5 τ=2
CE3 τ=2
0.6
0.4
0.2
0
−0.2
−0.4
−80
−60
−40
−20
0
20
40
60
80


DNS 30×20
CE2
CE2.5 τ=1
CE2.5 τ=2
CE3 τ=∞
CE3 τ=2
Mean Absolute Vorticity
Latitude
15
10
5
0
−5
−10
−15
−80
−60
−40
−20
0
20
40
60
80

Backwards Jet



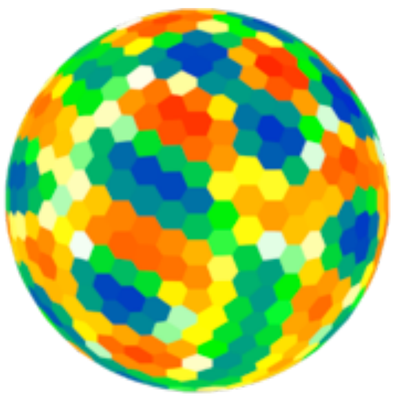
Turn on CE2.5

Mac App Store Preview

GCM

By Brad Marston

Open the Mac App Store to buy and download apps.



Description

Idealized General Circulation Models (GCMs) of planetary atmospheres, solved by a variety of methods.

GCM Support

What's New in Version 1.0.4

New wave lifecycle model, better organized menu. Bug fixes to CE3 (now conserves 3rd Casimir) and the calculation of the eddy diffusivity.

Free

Category: Education
Updated: May 23, 2013
Version: 1.0.4
Size: 1.4 MB
Language: English
Seller: Brad Marston

Rated 4+

Requirements: OS X 10.8.3 or later, 64-bit processor

Customer Ratings

We have not received enough ratings to display an average for the current version of this application.

All Versions:
8 Ratings

Screenshots

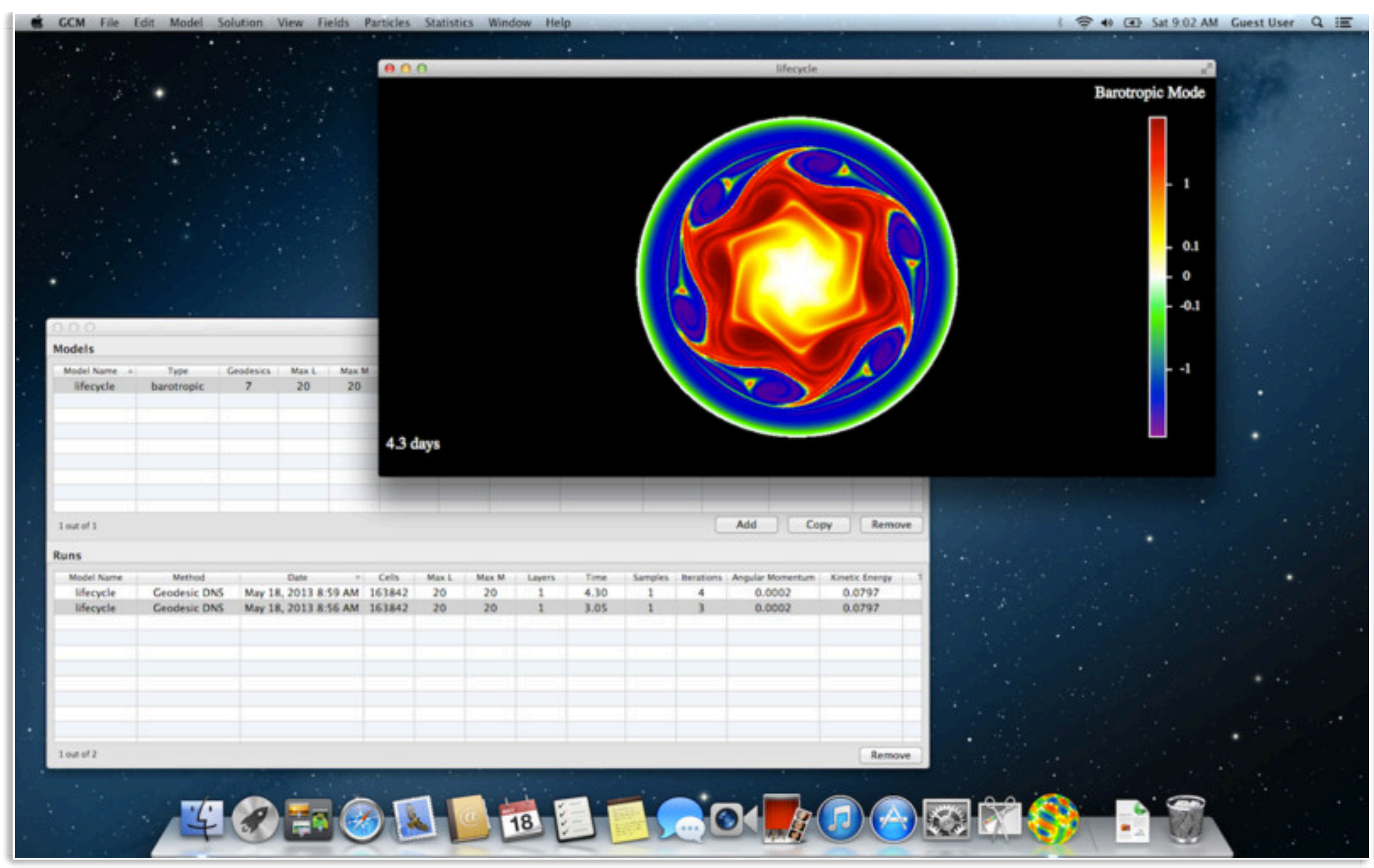
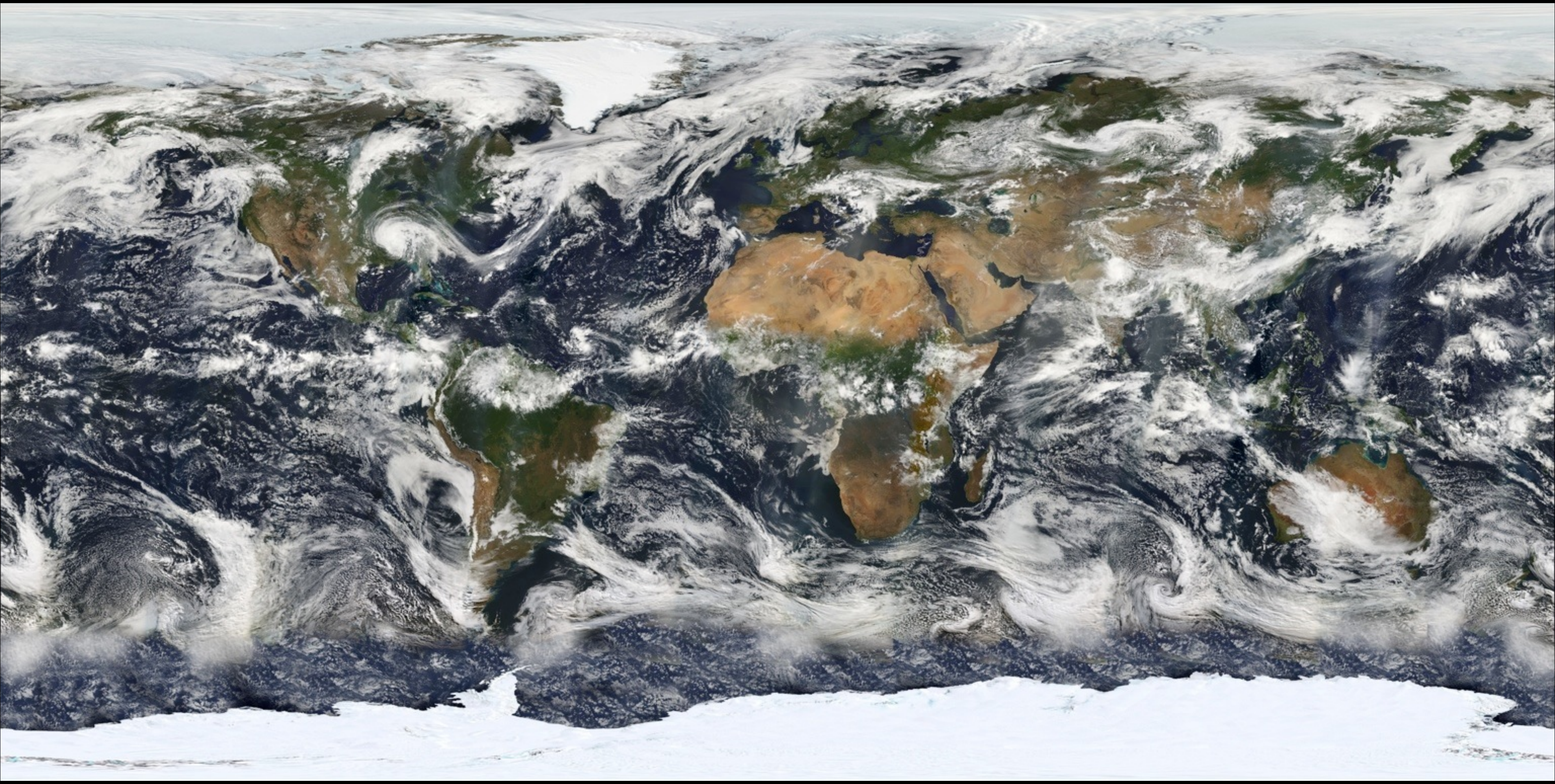


Image: NASA



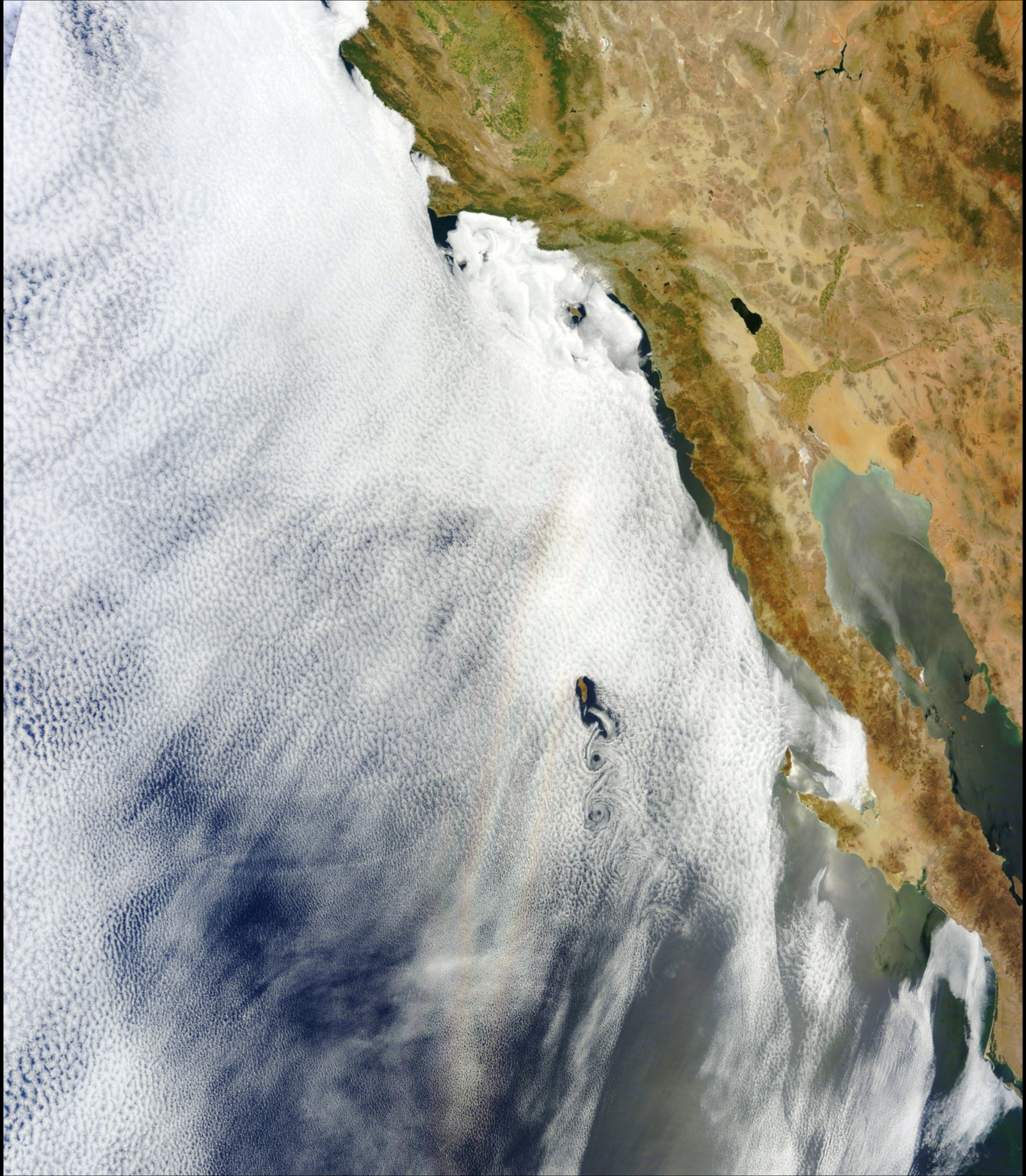


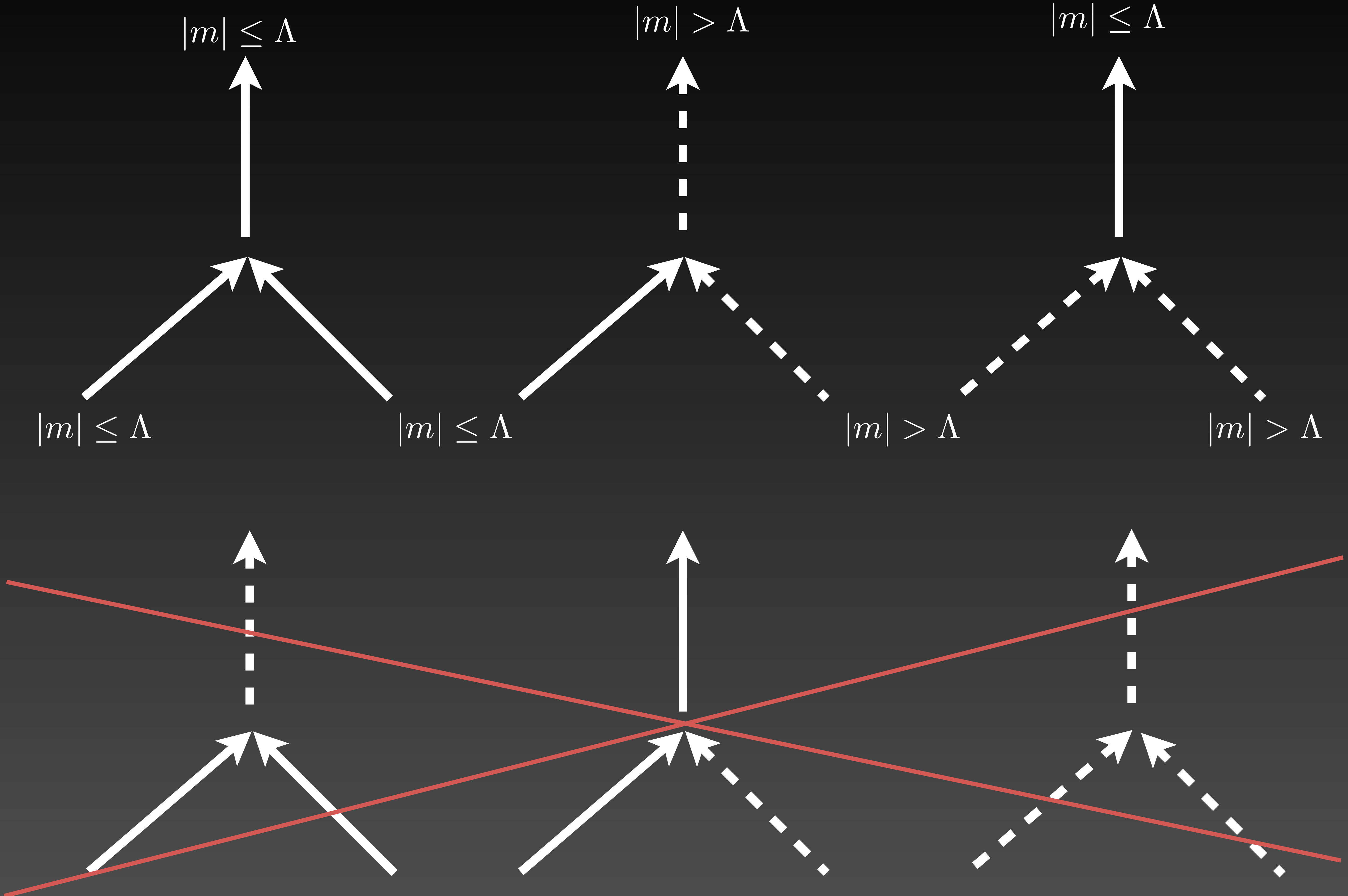
Image:
NASA

Image:
NASA

See Poster by Bettina Meyer

Image: Wikipedia

Separate Triads Into Long and Short Scales



Generalized 2nd Order Cumulant Expansion (GCE2)

$$\frac{\partial}{\partial t} q = L[q] + Q[q,\ q]$$

$$q = \ell + h$$

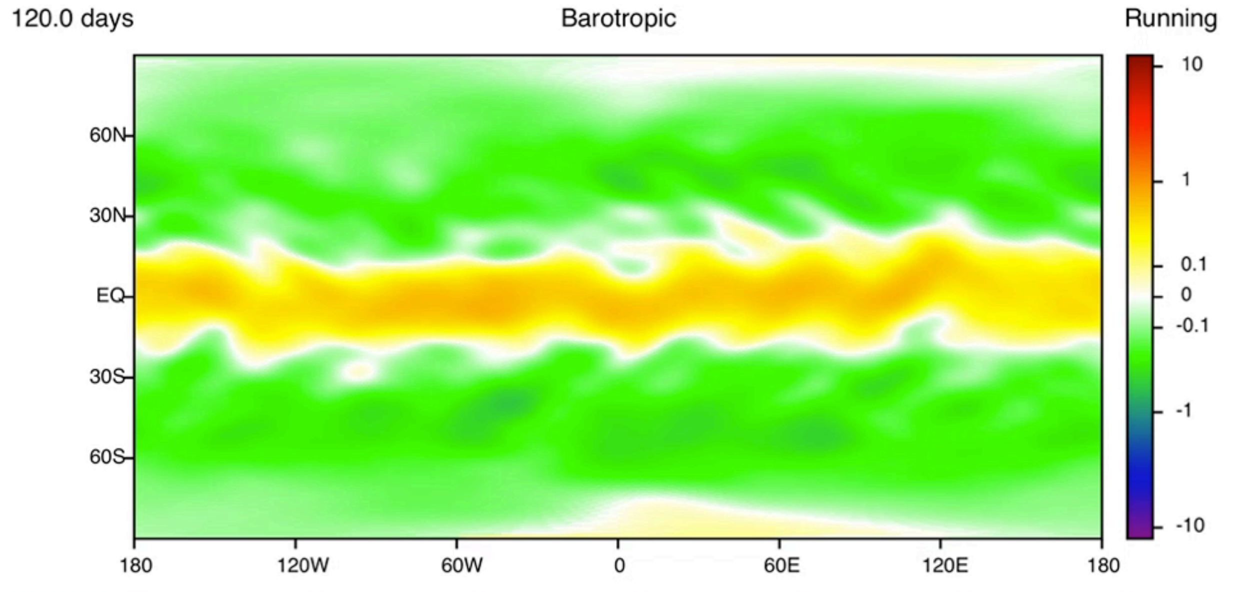
$$\frac{\partial}{\partial t} \ell = Q[\ell,\ \ell] + Q[(h,\ h)]$$

$$\frac{\partial}{\partial t} h = Q[\ell,\ h]$$

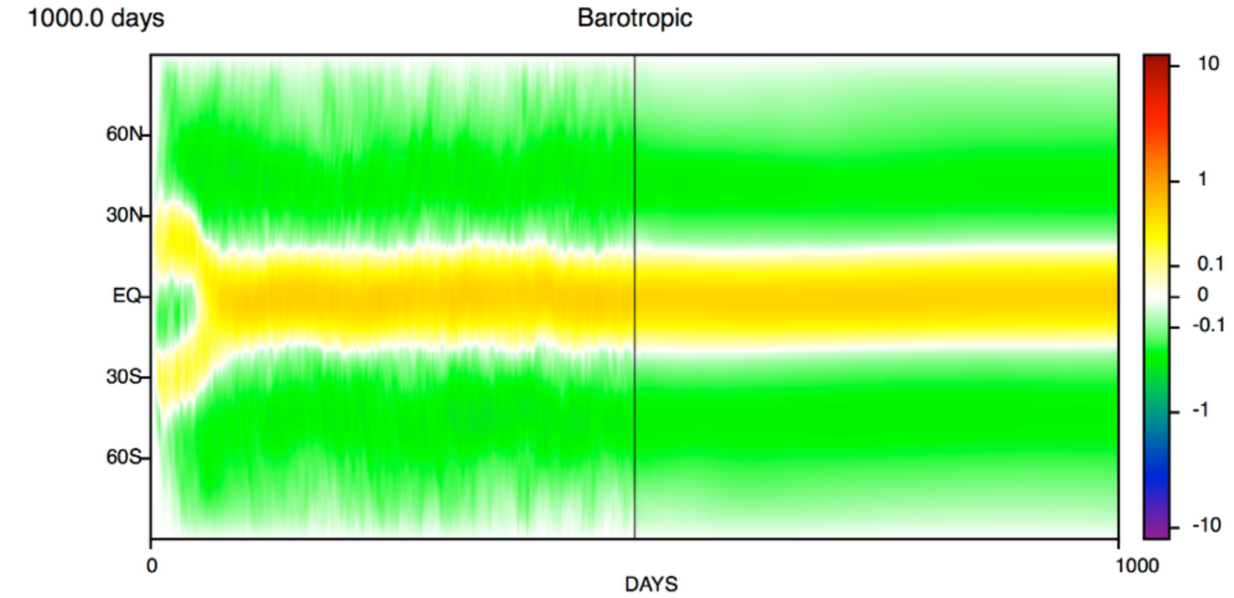
$$\frac{\partial}{\partial t} (h\ h) = 2Q[\ell,\ (h]\ h)$$

Closure

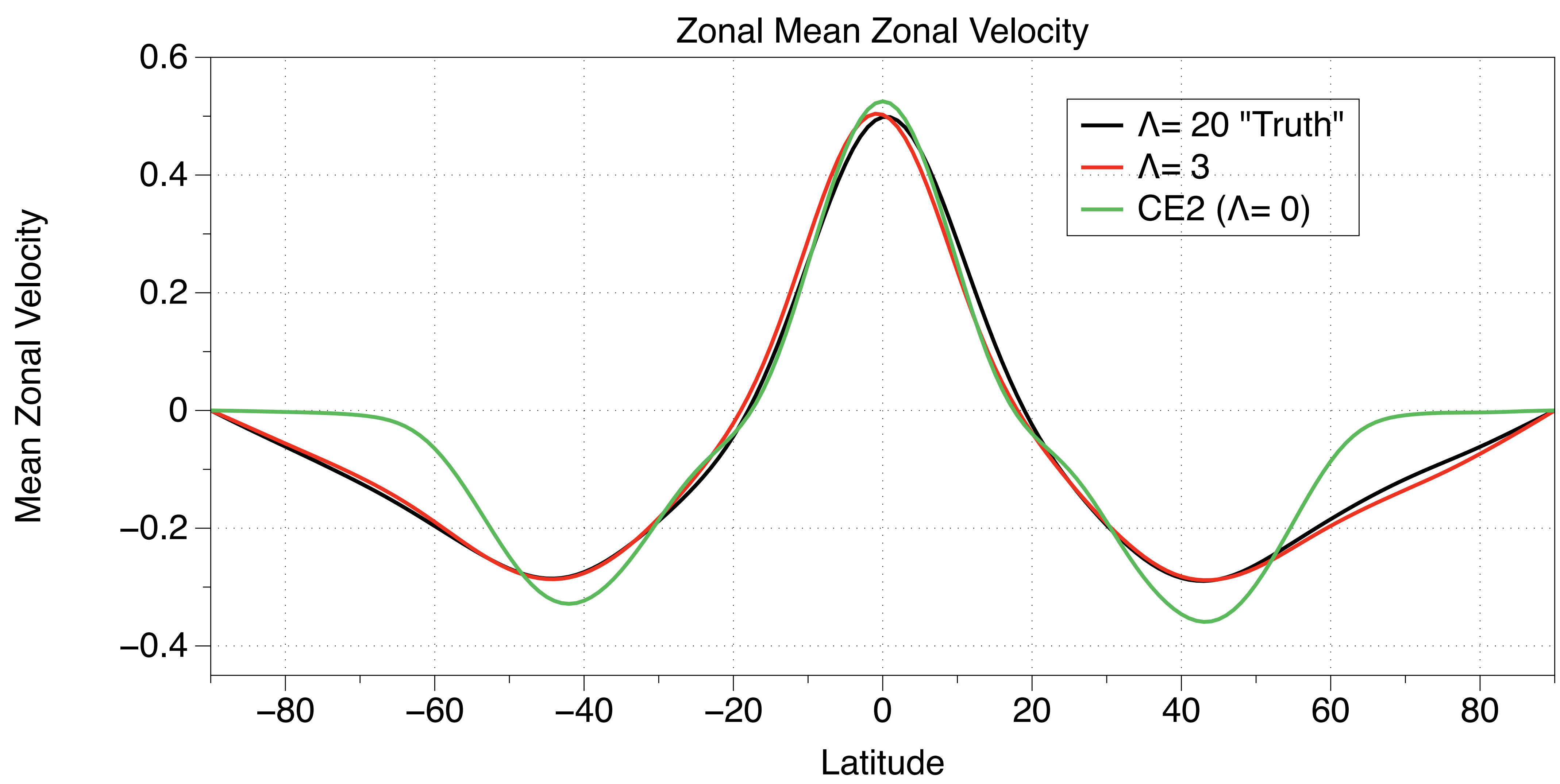
Malecha, Chini, and Julien, J. Comp. Phys. (2013);
Bakas and Ioannou, PRL **110**, 224501 (2013)



120.0 days
Barotropic
Running
60N
30N
EQ
30S
60S
180
120W
60W
0
60E
120E
180
10
1
0.1
0
-0.1
-1
-10

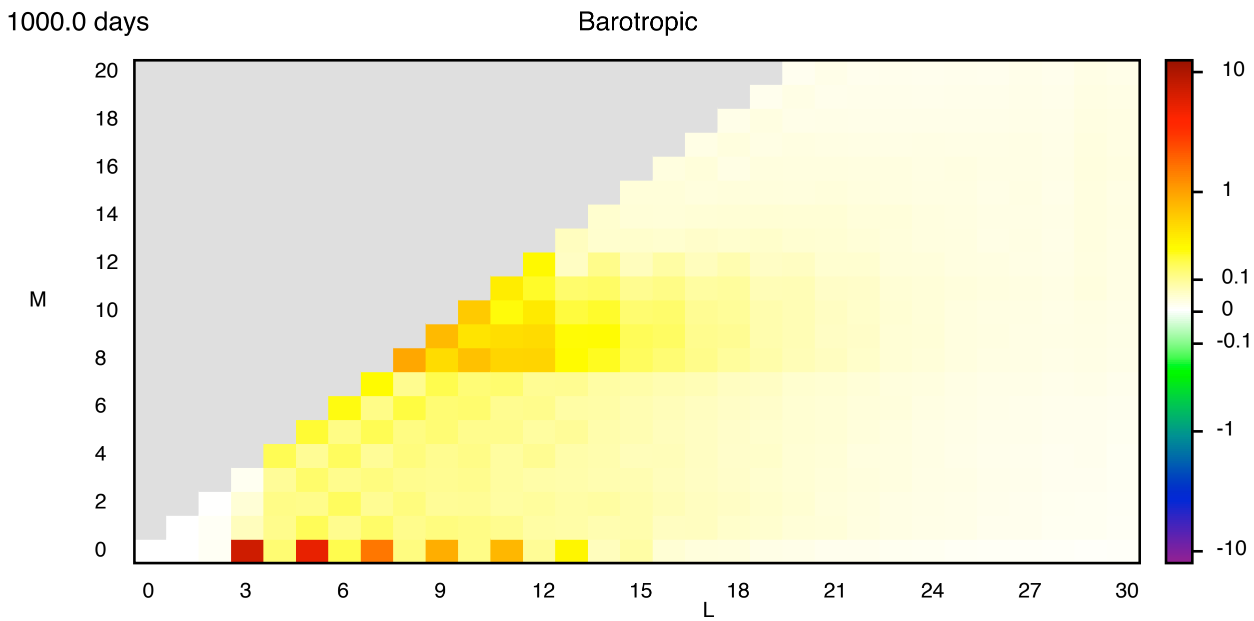


1000.0 days
Barotropic
60N
30N
EQ
30S
60S
0
DAYS
1000
10
1
0.1
0
-0.1
-1
-10

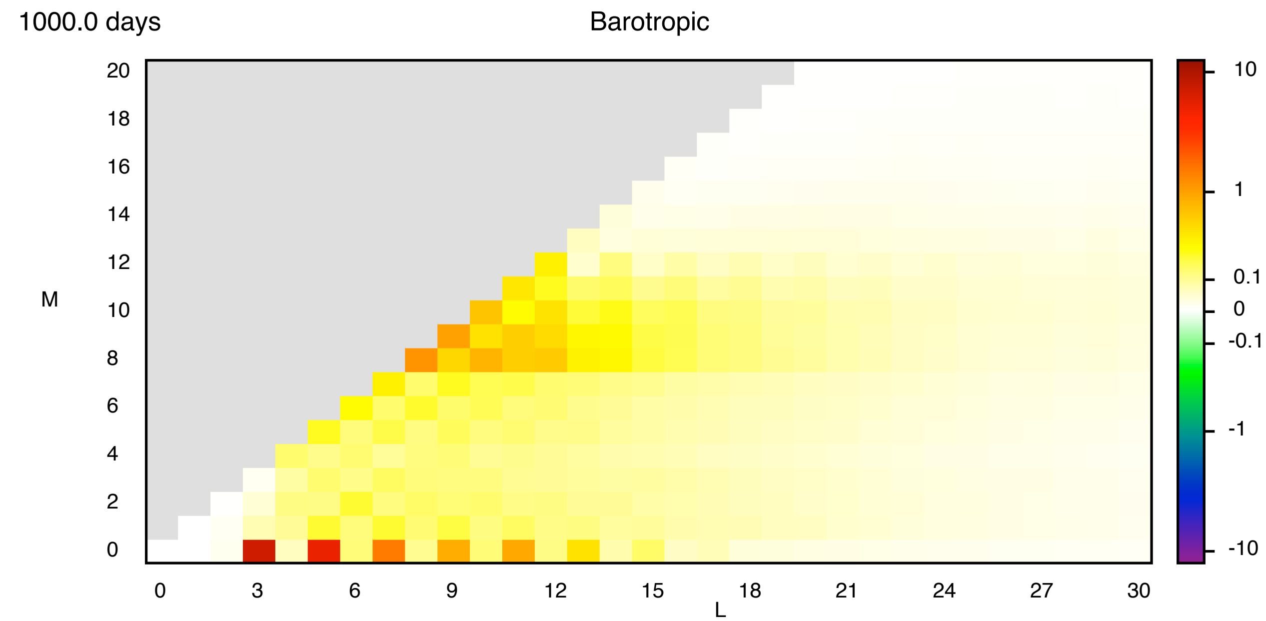


Zonal Mean Zonal Velocity
Λ= 20 "Truth"
Λ= 3
CE2 (Λ= 0)
Mean Zonal Velocity
0.6
0.4
0.2
0
−0.2
−0.4
−80
−60
−40
−20
0
20
40
60
80
Latitude

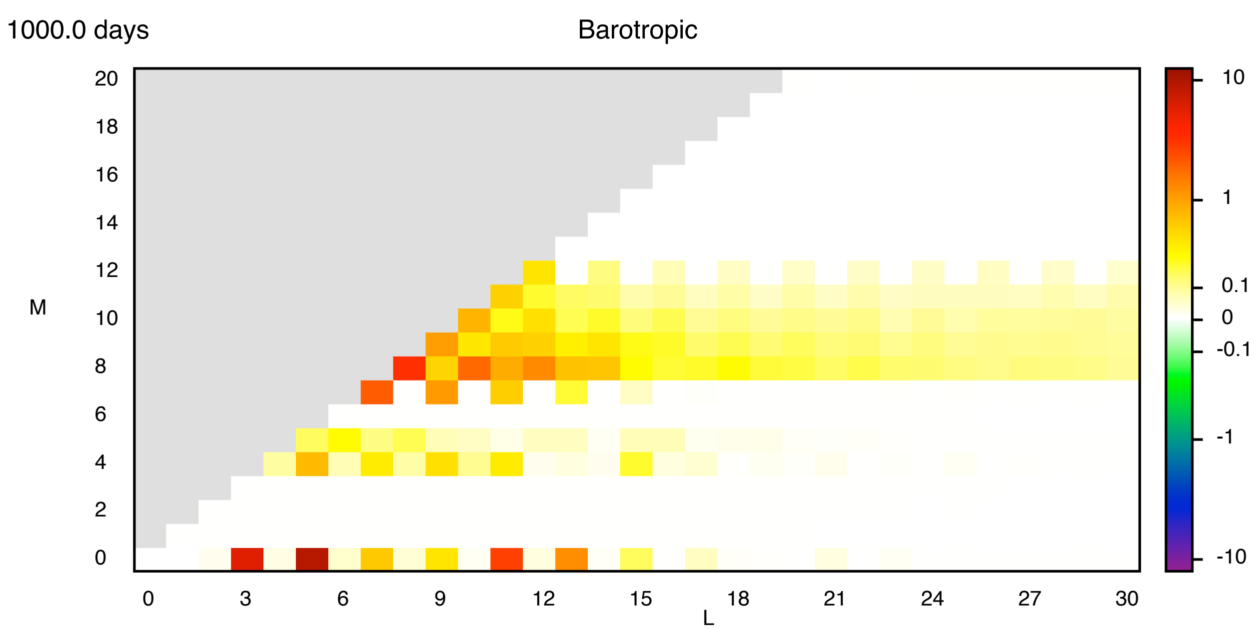
Vorticity Power Spectra



$\Lambda = 20$ “Truth”

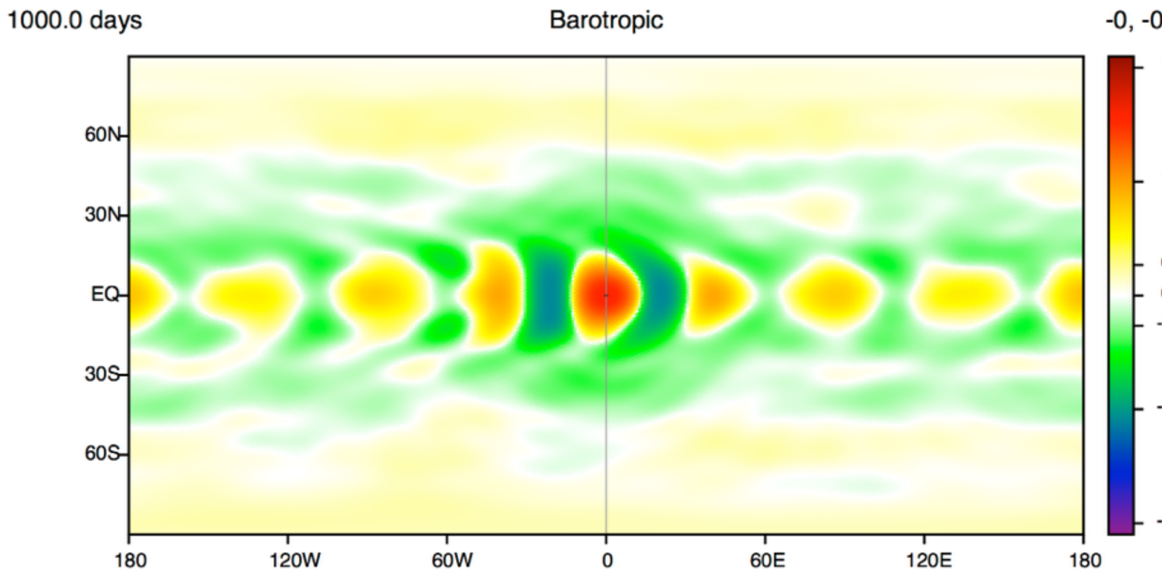


$\Lambda = 3$

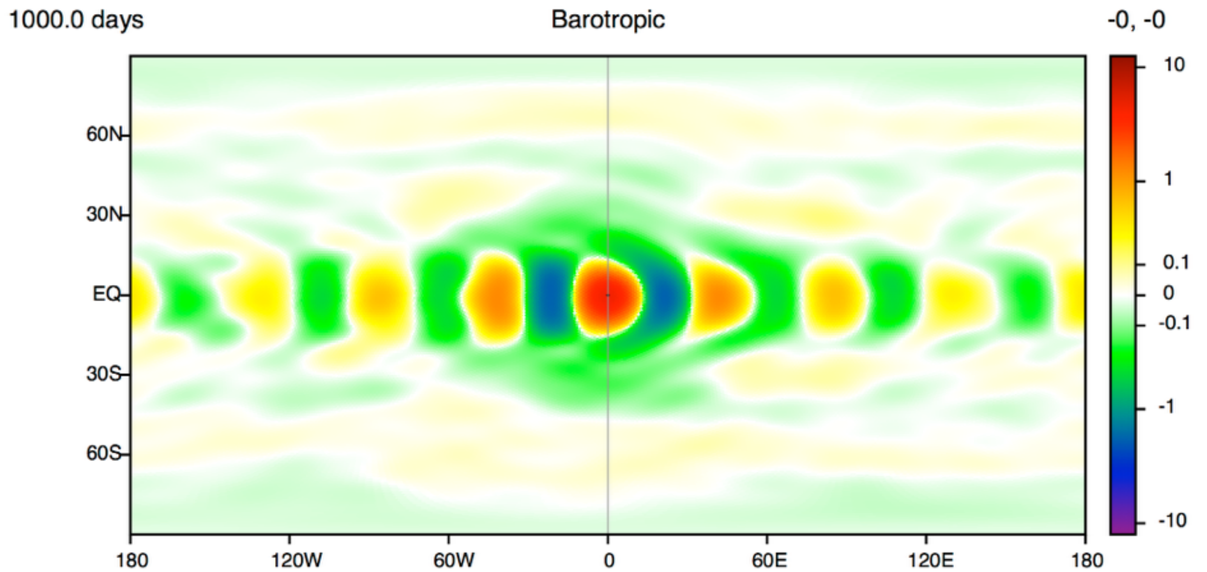


$\Lambda = 0$

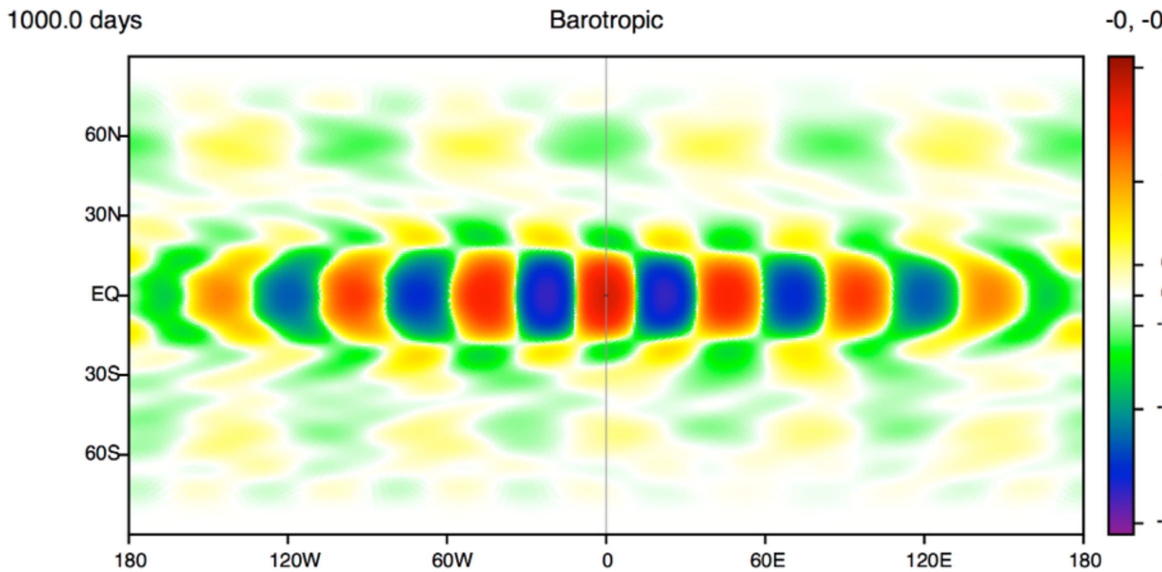
Two-Point Vorticity Correlations



$\Lambda = 20$ "Truth"

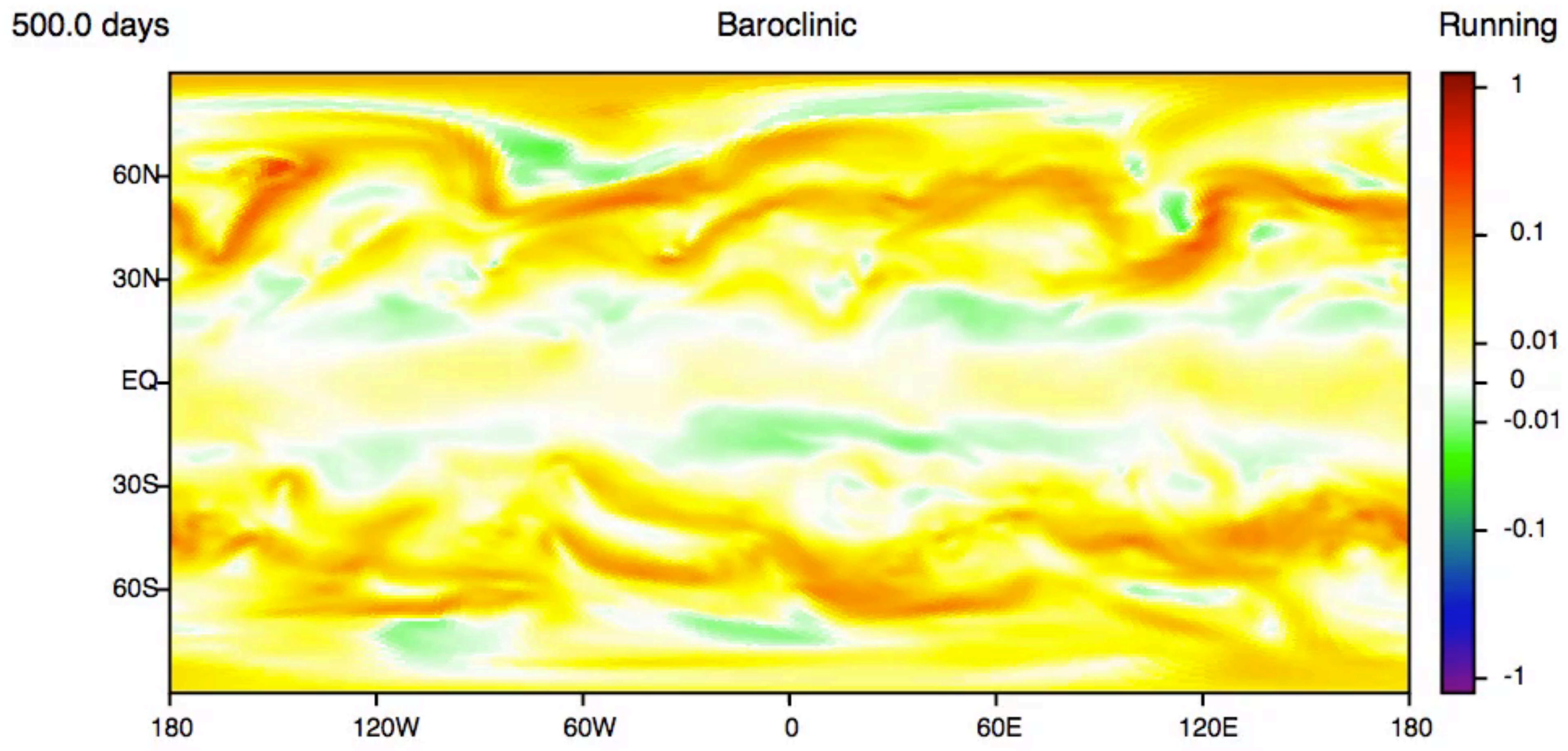


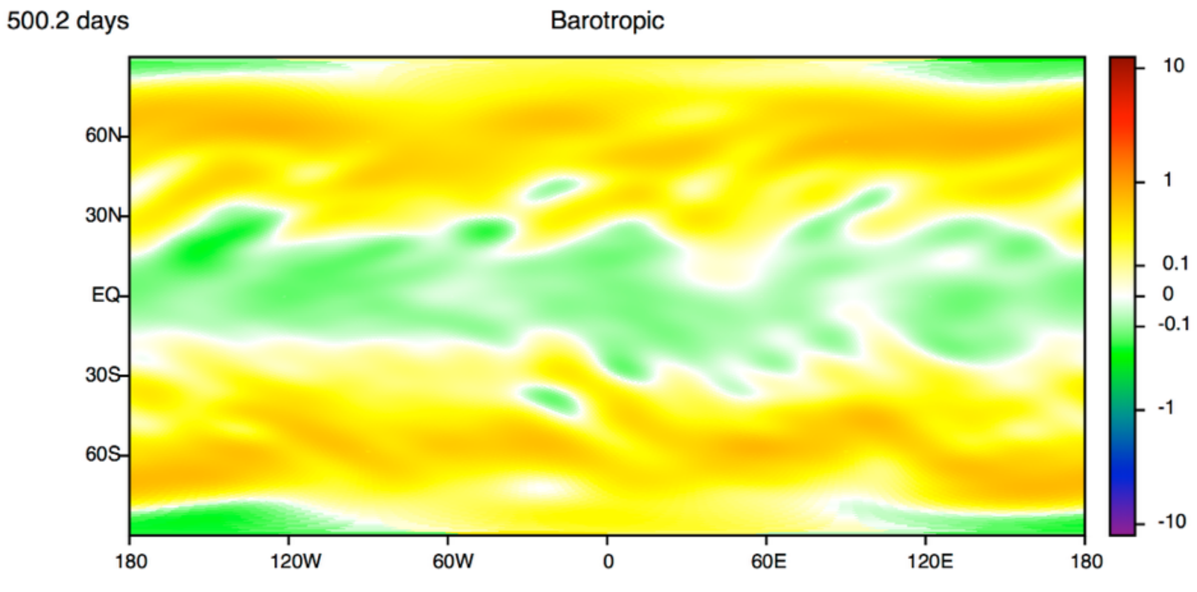
$\Lambda = 3$

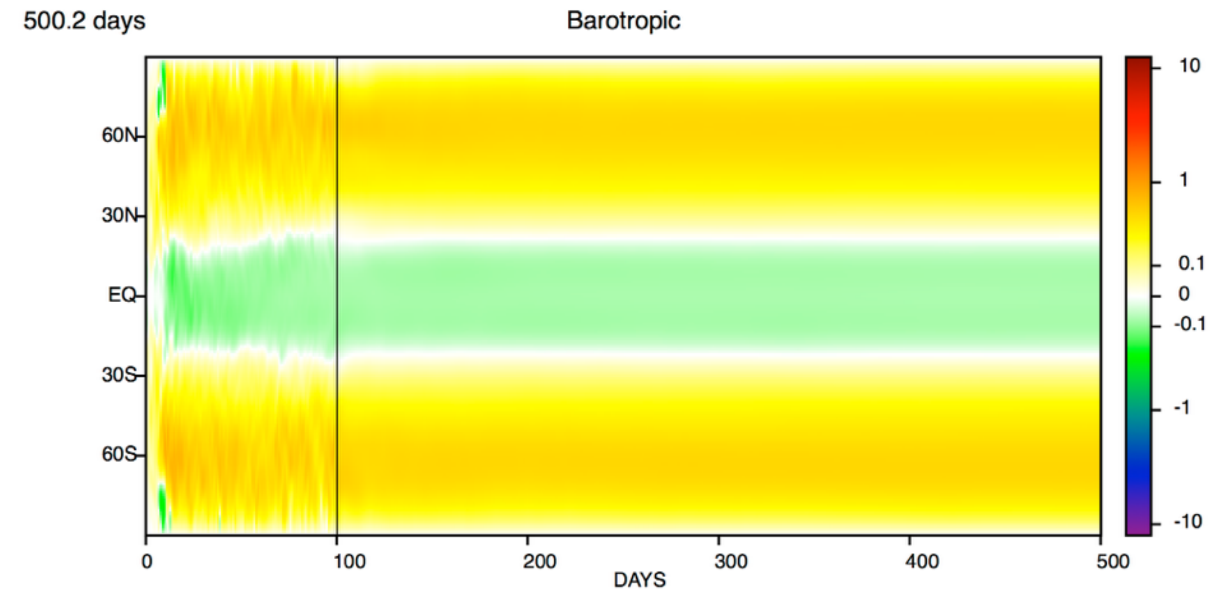


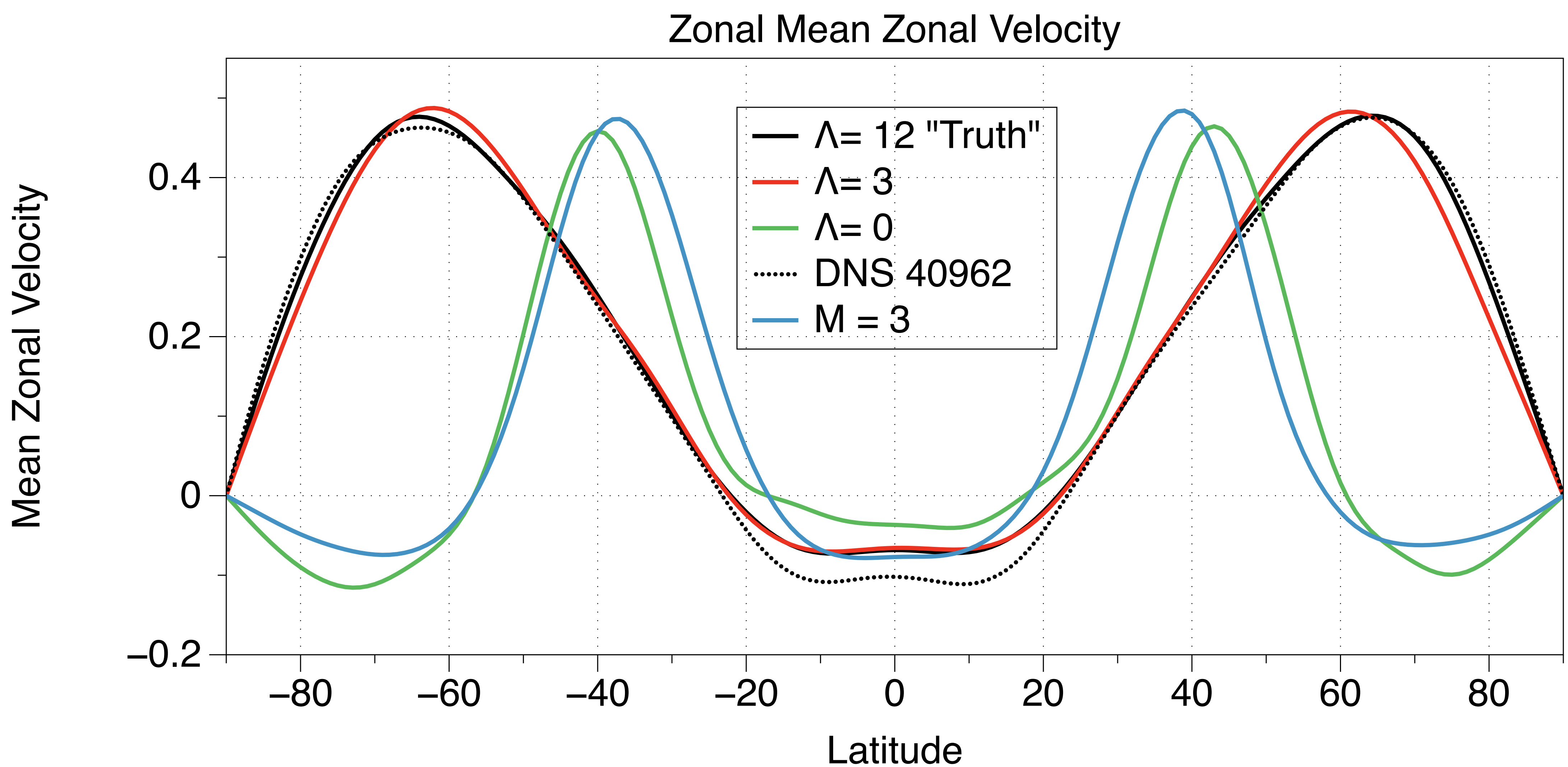
$\Lambda = 0$

Two-layer primitive equations: Relaxation to a prescribed equator-to-pole temperature difference

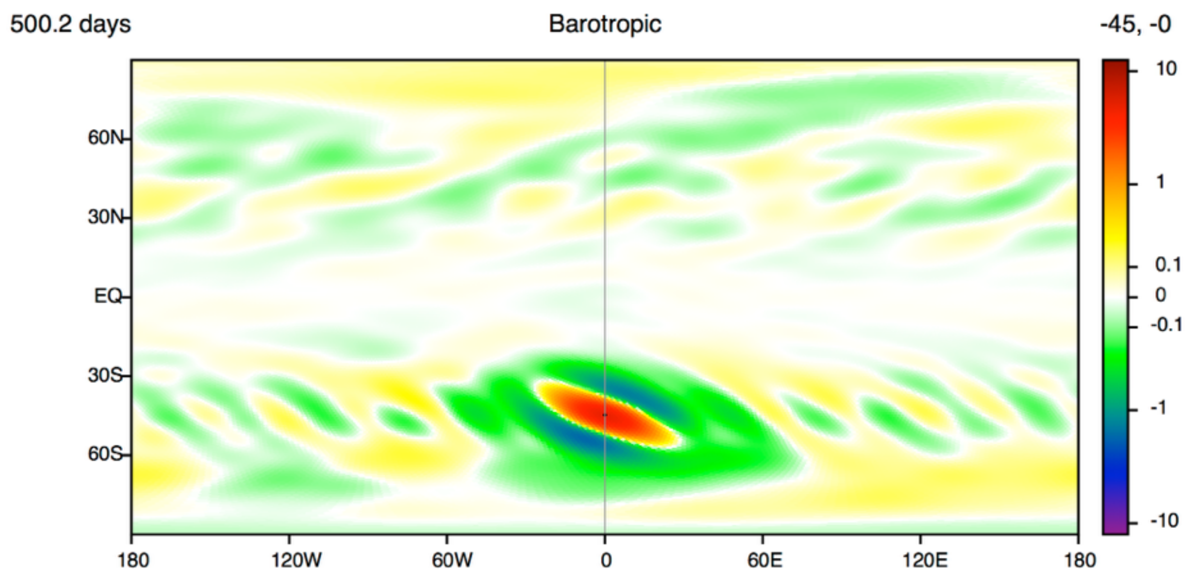



500.2 days
Barotropic
60N
30N
EQ
30S
60S
180
120W
60W
0
60E
120E
180
10
1
0.1
0
-0.1
-1
-10

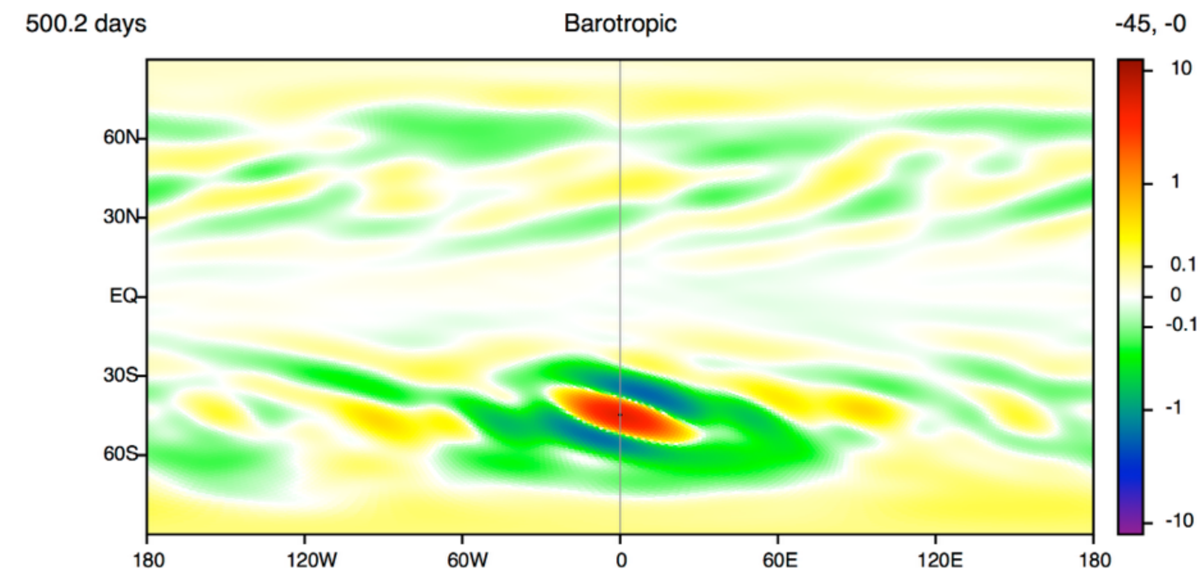

500.2 days
Barotropic
60N
30N
EQ
30S
60S
0
100
200
300
400
500
DAYS
10
1
0.1
0
-0.1
-1
-10


Zonal Mean Zonal Velocity
Mean Zonal Velocity
Latitude
Λ= 12 "Truth"
Λ= 3
Λ= 0
DNS 40962
M = 3
0.4
0.2
0
−0.2
−80
−60
−40
−20
0
20
40
60
80

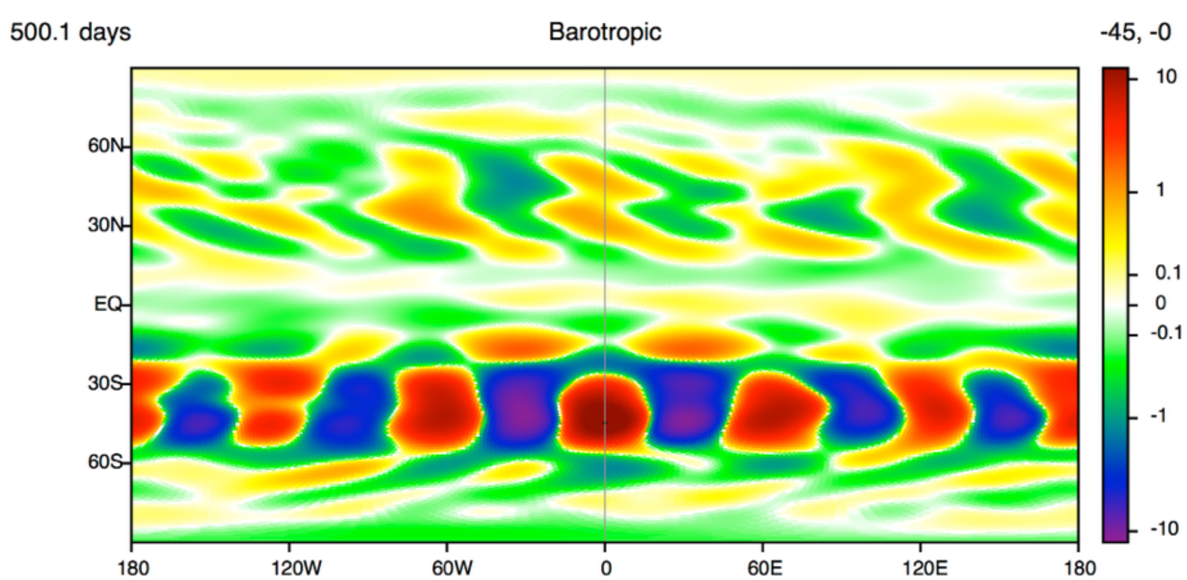
Two-Point Vorticity Correlations



$\Lambda = 12$ "Truth"

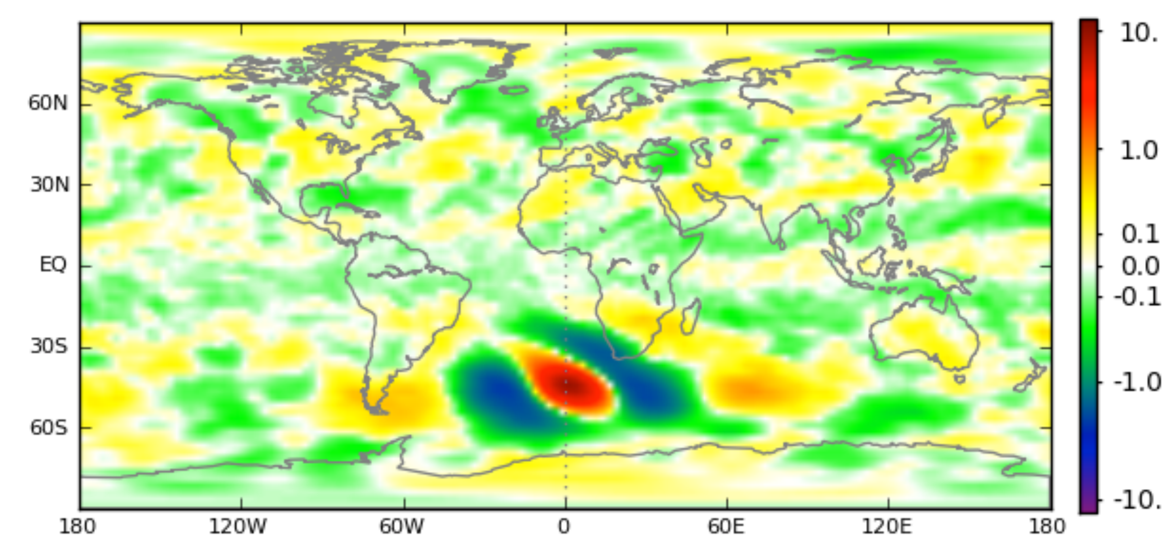


$\Lambda = 3$



$\Lambda = 0$

Observation



(Courtesy F. Sabou)

A Systematic Approximation

Exact in $\Lambda \to \infty$ limit and often accurate for $\Lambda = 3$

A Conservative Approximation

GCE2 conserves total angular momentum, energy, and enstrophy in absence of forcing and damping.

Realizable

GCE2 = Closed under selected triads = Realizable

Parallelism

Well suited for parallel computation: GPUs / OpenCL