Breathing life into Dead Zones:

On the Goldreich-Schubert-Fricke instability in protoplanetary disks.

Orkan (Matt) Umurhan
SETI Institute
Mountain View, California 94043

and as of November 1 2013 Space Sciences Institute
Nasa Ames Research Center, Moffett Field, California

In Collaboration With: O. Gressel, R. Nelson & R. Yellin-Bergovoy

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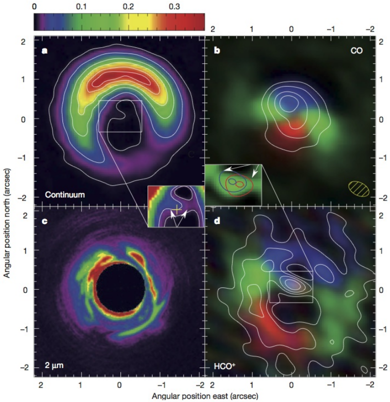
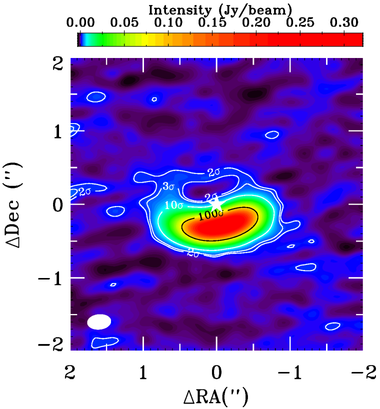
Recent Observational Results-Structures in Disks



Casassus et al (2013)

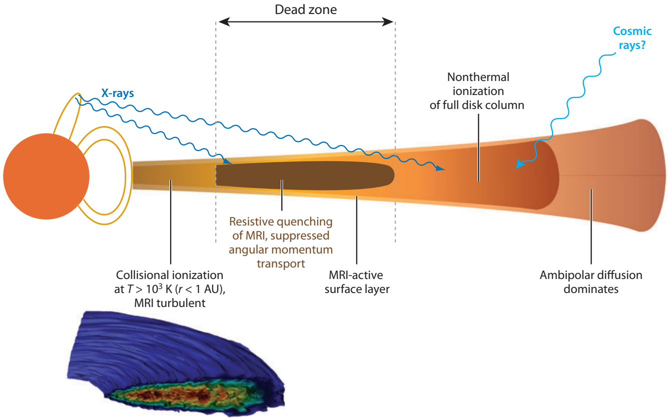
- Continuum emission at 4.65 GHz
- Focus on CO emission (2 micron emission)
- Are source of structures planets?



Van Der Marel et al. (2013)

- 440 μm emission (ALMA)
- $\Leftrightarrow$ mm-sized grains!

Paradigmage - A (controversial) view of protoplanetary disks



Armitage - 2011

Theorists View of a Disk

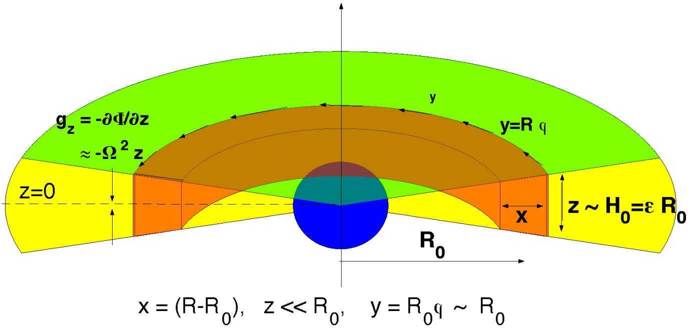


Figure 1: A theoreist's schematic of a pp-disk.

The small parameter: ϵ

- Discussion of “thin-disks" surrounds the disparity of two velocities:

$$\boxed{\epsilon \quad \equiv \quad \frac{H_0}{R_0} = \frac{\text{Vertical Scale Height}}{\text{Radial Length Scale}} = \frac{\text{Local Sound Speed}}{\text{Local Keplerian Speed}}}$$

Another way to think of it:

“the time it takes a sound wave to propagate upwards is the same as one local orbit time"

Barotropic Steady Rotational States

For disks with barotropic equations of state $\boxed{P = P(\rho)}$ only

- Axisymmetric steady configuration $\Longrightarrow$

$$-\Omega^2 R = -\frac{1}{\rho}\frac{\partial P}{\partial R} - \frac{\partial \Phi}{\partial R}, \qquad 0 = -\frac{1}{\rho}\frac{\partial P}{\partial Z} - \frac{\partial \Phi}{\partial Z}$$

(disk – radial momentum balance) (disk – vertical momentum balance)

$\boxed{\text{Disk rotates constantly on cylinders: } \Omega(R, Z) = \Omega(R) \text{ only.}}$

Baroclinic Steady Rotational States

For example, for $\boxed{P = P(\rho, T) = \rho(R,Z)c^2(R)}$

- steady configuration $\Longrightarrow$

$$-\Omega^2 R = -\frac{1}{\rho}\frac{\partial P}{\partial R} - \frac{\partial \Phi}{\partial R},$$
$$0 = -\frac{1}{\rho}\frac{\partial P}{\partial Z} - \frac{\partial \Phi}{\partial Z}$$

$$\boxed{R\frac{\partial \Omega^2}{\partial Z} = \frac{\partial \ln c^2}{\partial R}\frac{\partial \Phi}{\partial Z}}$$

$\boxed{\text{Disk rotation varies with } Z\text{:} \quad \Omega^2(R,Z) \sim \Phi(R,Z).}$

GSF Instability - unstable inertial waves - (nearly incompressible disturbances)

1. Goldreich-Schubert-Fricke (1967/68) [Also Urpin 2003, Arlt & Urpin (2004)]

   – mean rotation not constant on cylinders $\longrightarrow$ $j^2 = R^2\Omega(R,Z)$

   $$\textbf{instability for: } \frac{\partial j}{\partial R} - \frac{\ell_Z}{\ell_R}\frac{\partial j}{\partial Z} < 0 \quad \text{(Solberg-Hoiland)}$$

2. For a “locally isothermal" disk:

   $$T = T_0\left(\frac{R}{R_0}\right)^q \qquad \Longrightarrow \qquad \overline{V}(R,Z) = \overline{V}_{\text{Kep}}(R)\left(1 + \underline{q\left[\frac{H_0^2}{R_0^2}\right]Z^2}\cdots\right)$$

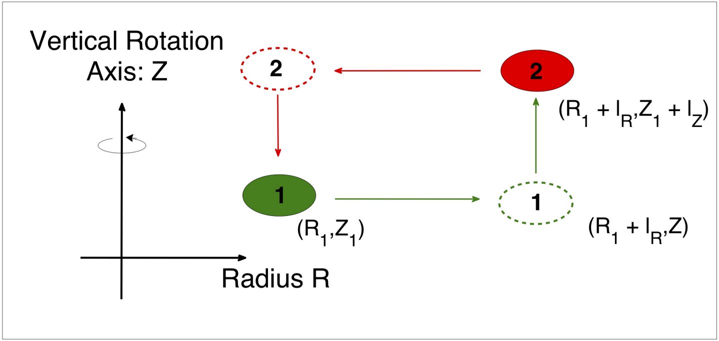
3. cold disks: scale height $\ell_Z = H_0 \ll R_0$ implies (for reference Ω_0 at R_0)

   $$\text{(growth rates)} \sim \frac{3}{2}\Omega_0\frac{H_0}{R_0} \quad \Longleftrightarrow \quad \text{(on radial disturb. length scales)}$$

   $$\ell_R \sim \frac{H_0}{R_0}H_0.$$

$$\boxed{\text{For } H_0/R_0 = 0.05 \Longrightarrow \quad \ell_R \sim 0.01R_0}$$

GSF Mechanism - a relative energy (via parcel interchange) argument



INSTABILITY

→ when total energy of interchanged configuration is less than original state

Method/Parameters/Results

Model Equations

$$\begin{aligned} \partial_t \rho + \nabla \cdot \rho \mathbf{u} &= 0, \\ \partial_t \rho \mathbf{u} + \nabla \cdot \rho \mathbf{u}\mathbf{u} &= -\nabla P - \rho \nabla \Phi, \\ \partial_t T + \mathbf{u} \cdot \nabla T &= -(T - T_{\mathrm{ref}})/\tau_{\mathrm{relax}} \end{aligned}$$

with $P = \rho T$ and $\Phi = -GM/R$
or a proper energy equation

$$\partial_t e + \nabla \cdot e\mathbf{u} = -P\nabla \cdot \mathbf{u} + \mathcal{Q}$$

Code and Setup

- Nirvana and Nirvana-III code
  (spherical coordinates) ($N_r = 1300$ and $N_\theta = 1000$)
- Axisymmetric disturbances -
  $r_{\mathrm{in}}/R_0 = 1 \;\; r_{\mathrm{out}}/R_0 = 2, Z_{\mathrm{max}}/H_0 = 5.$
- Outflow or reflecting conditions -
  (no observed difference in results)
- $T \sim (R/R_0)^q$
- q = -1 (constant $H_0/R_0 = 0.05$ over domain)
- seed with random field in KE

Result

Strong Activity when $\tau_{\mathrm{relax}} \to 0$ and $q \neq 0$.

vertical velocity frames

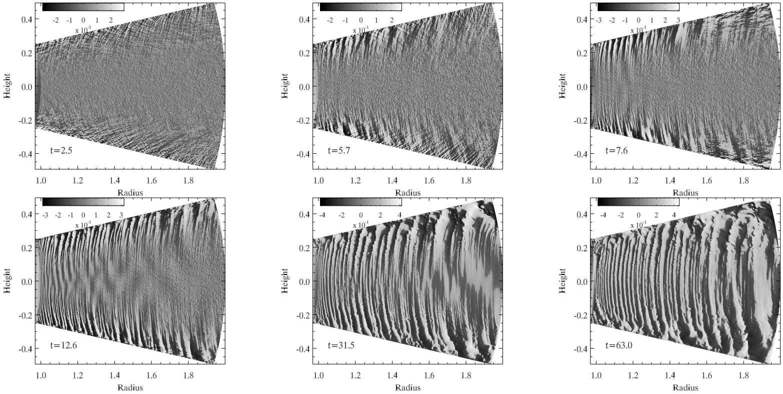


Figure 3. Edge-on contours of the perturbed vertical velocity as a function of R, Z and time for model TR1–0. Note that for clarity, the grey-scale of the image has been stretched by plotting the quantity $\mathrm{sign}(v_Z) \times |v_Z|^{1/4}$. Note that the spectrum bar shows values of $v_Z^{1/4}$.

- Radial Wavelength of dominant growing mode $\sim 0.009R_0$.

features and clues

component KE

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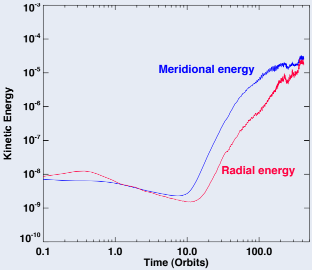


Figure 1. Time evolution of the normalised perturbed kinetic energy in the meridional and radial coordinate directions for model T1R-0 with $p = -1.5$, $q = -1$ and reflecting boundary conditions at the meridional boundaries.

growth rate ~ 0.24 orbit^{-1}

Variation of $\tau_{\rm relax}$

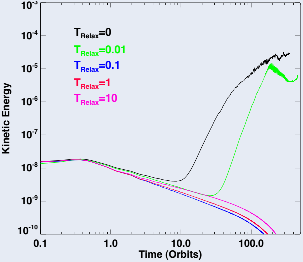
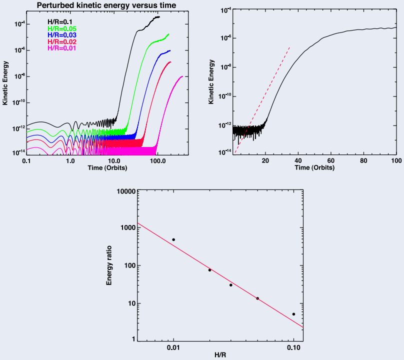


Figure 10. Time evolution of the sum of the (normalised) perturbed radial and meridional kinetic energy in discs where the temperature was initially constant on cylinders, as a function of the thermal relaxation time. Note that only the $\tau_{\rm Relax} = 0$ and 0.01 cases show growth.

- Important Clue: radial velocities dwarfed by vertical velocities

more features and clues

Dependence on $\epsilon = H/R$



- Energy ratios scale as ϵ^2.

Stripped down model exposing instability

- numerically guided asymptotic analysis

$$\text{time}^{-1} \sim \Omega_0 H_0/R_0, \quad \ell_R \sim (H_0/R_0)^2 R_0, \quad \ell_z \sim H_0, \quad \text{and } v_R \ll v_Z, \\ \text{for } \tau_{\text{relax}} \to 0 \quad \text{and} \quad q \neq 0$$

- examined around fiducial radius $R = R_0$ where $T = T_0 \iff c_{s0}^2$

$$\begin{aligned} -2\Omega_0 v &= -c_{s0}^2 \partial_r \ln \rho && \text{Radial Geostrophy!!} \\ \frac{dv}{dt} + \tfrac{1}{2}\Omega_0 v_R + \frac{\partial \bar{V}}{\partial z} w &= 0 && \text{Azimuthal Mom} \\ \frac{dv_Z}{dt} &= -c_{s0}^2 \partial_z \ln \rho && \text{Vertical Mom} \\ \frac{\partial \tilde{\rho} v_R}{\partial r} + \frac{\partial \tilde{\rho} v_Z}{\partial z} &= 0 && \text{Anelastic Eqn.!!} \end{aligned}$$

with $\tilde{\rho} = \exp(-z^2/2H_0^2)$.

linear theory and perturbation analysis

linear perturbations $\rho \to \rho_0 + \rho'$

Inseparable equation in r and z!!

$$\frac{\partial^2}{\partial r^2}\frac{\partial^2 \rho'}{\partial t^2} + \frac{\partial^2 \rho'}{\partial z^2} = \left(1 + q\frac{\partial}{\partial r}\right) z \frac{\partial \rho'}{\partial z}$$

double instability !! ($kq > 1$)

solution modes of the inseparable “form”:

$$\boxed{\rho' = \rho(m,k) \sim \sum_{j=1}^{m} e^{st+jr/s^2} \cos kr\, \mathcal{H}_j(z)}$$

where m are integer indices > 0:

$$s^2 = (m/k^2)\left(-1 \pm \sqrt{1 - q^2k^2}\right)$$

maximal growth rates (our sims: $q = -1, H_0/R_0 = 0.05$)

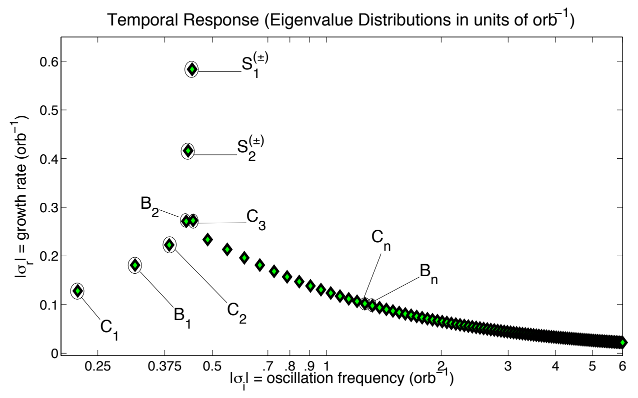
$$\text{radial scale of max growth}: \quad \ell_r = \pi|q|\left(\frac{H_0}{R_0}\right)^2 R_0 \Longrightarrow 0.008R_0$$

$$\text{growth rate of max growth in KE}: \quad s^{(\mathrm{KE})}_{\max} = 2s_{\max} = \sqrt{2m}\pi|q|\left(\frac{H_0}{R_0}\right)\text{orbit}^{-1}$$

$$\Longrightarrow 0.22\sqrt{m}\ \text{orbit}^{-1}$$

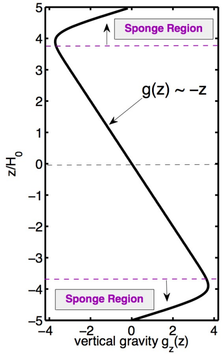
Another variant linear theory calculation:

- Assume separable solution: $\rho \sim e^{st+ikx}\mathcal{H}(z)$. $\sigma(k) \to \pm\Big(|\sigma_r| \pm i|\sigma_i|\Big)$



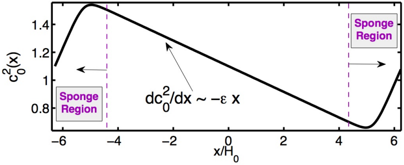
Secondary verification via “Large Shearing Box" experiments

- Large Shearing box equations
- Spectral calculations/ axisymmetric
- $[L_x, L_y] = [10, 10] \iff 512^2$
- Use sponges = 1/10 domain size
- vertical gravity/ temperature periodic
- hyperviscosity ∇^{16}



Additional Results/Insight

- Robust results
- Rightward drift of patterns
- Episodic angular momentum transport



A movie depicting LSB runs with $\varepsilon = 0.2$

LSB run (field quantities)

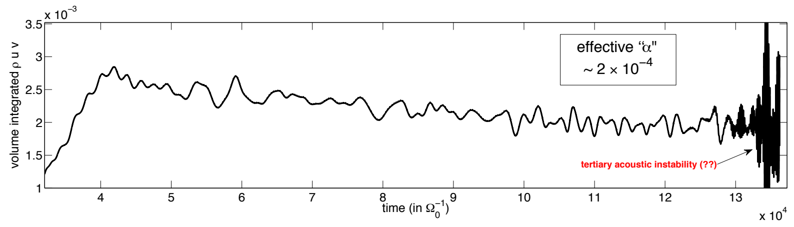
Another movie depicting same LSB run with $\varepsilon = 0.2$: Vorticity and Dilatation

LSB run (vorticity/dilatation)

Another movie depicting same LSB run with $\varepsilon = 0.2$: Energy and Transport

LSB run (energy/transport)

Reynolds Stress Outward Transport



Application to PP-Disks - is this relevant?

Radiative Diffusion Cooling Times $\Longleftrightarrow$ Disturbance length scales ℓ_R and disk position

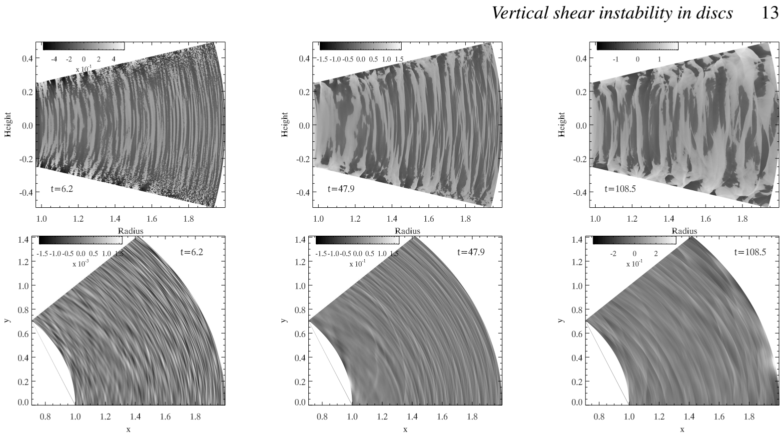
- Using typical protoplanetary disk properties cooling times due to rad. diffusion are

$$\boxed{\tau_{\rm relax}/P_{\rm orb} = 169F^2\left(\kappa_R/{\rm cm}^2/{\rm g}\right)\left(\ell_r/R\right)^2\left(R/20\ {\rm AU}\right)^{-53/14}, \qquad P_{\rm orb} \equiv 2\pi/\Omega_K,}$$

Where $F \leftrightarrow$ disk mass relative to solar disks (e.g. Chiang and Goldreich 1999),
$\kappa_R \leftrightarrow$ mean opacities (near 1cm^2/gm).

- For $\ell_r \sim 0.01R_0$ GSF Operates in disks where $R > 15 - 20AU$

Non-axisymetric numerical experiments:



- vortex production in-plane $\Longrightarrow$

  Outward angular momentum Transportage!

Nonaxisymmetric response - Baroclinic RWI driven by this mechanism?

Transport Properties

- Effective “α"

$$\sim 2 \times 10^{-3}$$

- Distribution around midplane

Next Stages:

- Sort out linear theory of competing instabilities
- Further examine the requirement that thermal relaxation times must be short!
- Verification via other means (see next section)

Transport Map

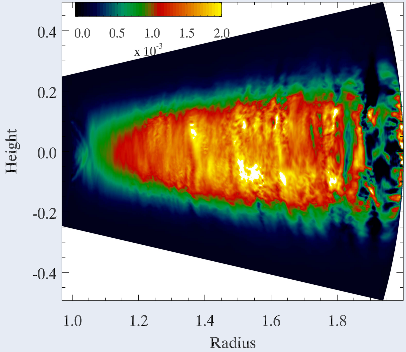


Figure 14. Spatial distribution of the time and horizontally averaged Reynolds stress (normalised by the mean pressure at each radius) for the model T1R–0–3D.

Summary of Part I.

- GSF instability possible in Dead Zones of Disks
- Axisymmetric instability strongest
- Relevant in media/scales with short “effective" cooling times.
- Saturated state - outward drifting vertical jets
- Linear transition: small radial length scales: Radially geostrophic, anelastic dynamics

Motivation

- Axisymmetric saturated state is steady in moving frame
- Simulations show unsteady incomplete vortex roll-up

HYPOTHESIS

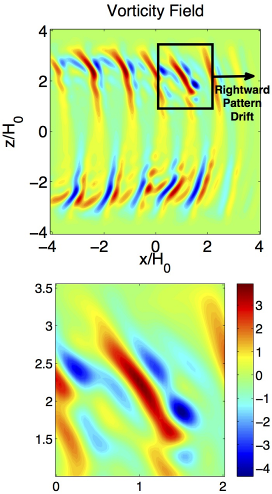
Are the weak secondary rolls due to a variant of one (or more) canonical shear instabilities?

HYPOTHESIS

Can the secondary transition be depicted in a stripped-down model representing the primary instability as an external forcing term?

- Vorticity

$$\Longrightarrow \omega \equiv \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}$$



Proposed model to capture axisymmetric physics at saturation

Simplified Model (by fiat)

Eqns. in rotating frame with b/g shear $\overline{V}_{\text{Kep}} = -\Omega_0 qx$. and constant density:

$$\begin{aligned}
\frac{du}{dt} - 2\Omega_0 v &= -\frac{\partial \Pi}{\partial x}, \\
\frac{dv}{dt} + \Omega_0(2-q)u &= 0 + F_v, \qquad F_v \equiv \frac{1}{\tau}\Big(\mathrm{V} - v\Big) \\
\frac{dw}{dt} &= -\frac{\partial \Pi}{\partial z} + F_w, \qquad F_v \equiv \frac{1}{\tau}\Big(\mathrm{W} - w\Big) \\
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} &= 0.
\end{aligned}$$

Rayleigh Drag Models F_v and F_w $\Longleftrightarrow$ Metaphor for Primary “GSF" Instability

Time scale τ represents growth rate of primary instability, $\mathrm{V}(x)$ and $\mathrm{W}(x)$ denotes the saturation profiles of the instability. These are placed by hand. (H.O.G.)

Analog to Stratified Systems (Leibowicz - 1979)

azimuthally sheared axisymmetric flow

$$\begin{aligned}
\frac{dw}{dt} &= -\frac{\partial \Pi}{\partial z}, \\
\frac{du}{dt} &= -\frac{\partial \Pi}{\partial x} + 2\Omega_0 v, \\
\frac{dv}{dt} &= -\left[\frac{d\mathrm{V}(x)}{dx} + (2-q)\Omega_0\right] u \\
0 &= \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}
\end{aligned}$$

v is the analog variable to entropy

$$\Longleftrightarrow \quad \Longleftrightarrow \quad \Longleftrightarrow \quad \Longleftrightarrow$$

vertically stratified (Boussinesq) fluid

$$\begin{aligned}
\frac{du}{dt} &= -\frac{\partial \Pi}{\partial x}, \\
\frac{dw}{dt} &= -\frac{\partial \Pi}{\partial z} + g\alpha S, \\
\frac{dS}{dt} &= -\frac{d\bar{S}(z)}{dz} w \\
0 &= \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}
\end{aligned}$$

where S is specific entropy.

$u \leftrightarrow w$, $w \leftrightarrow u$, and $v \leftrightarrow S$ in this correspondence.

- $\Longrightarrow$ all theorems of stratified shear flows carry over correspondingly $\Longleftarrow$.
- “Shear in azimuthal velocity akin to vertical entropy gradient"
- Merely a reinterpretation of centrifugally driven flows.

Necessary criteria for instability - I.

In the absence of any additional shear, then if a mode is unstable then it must be true that somewhere in the domain the following happens:

Correspondence b/w Inviscid Rayleigh Criterion and Unstable Stratification

For axisymmetric rotating flow

$$\boxed{\omega_\varepsilon^2 \equiv 2\Omega_0^2 \left[\Omega_0 (2-q) + \frac{d\mathrm{V}}{dx}\right] < 0}$$

Inviscid Rayleigh Criterion

For stratified flow

$$\boxed{N^2 \equiv g\alpha_s \frac{d\bar{S}}{dz} < 0}$$

Criterion for Buoyant Instability

$\Longrightarrow$ No news here.

Necessary criteria for instability - II.

For a flow of the above type also characterised by some additional base shear, then if there is instability it must be true that somewhere in the domain the following is valid

Effective Richardson Criteria and its analog

For axisymmetric rotating flow
With radial shear of vertical flow $W(x)$:

$$\boxed{2\Omega_0^2\left[\Omega_0(2-q)+\frac{dV}{dx}\right] \quad < \quad \frac{1}{4}\left(\frac{dW}{dx}\right)^2}$$

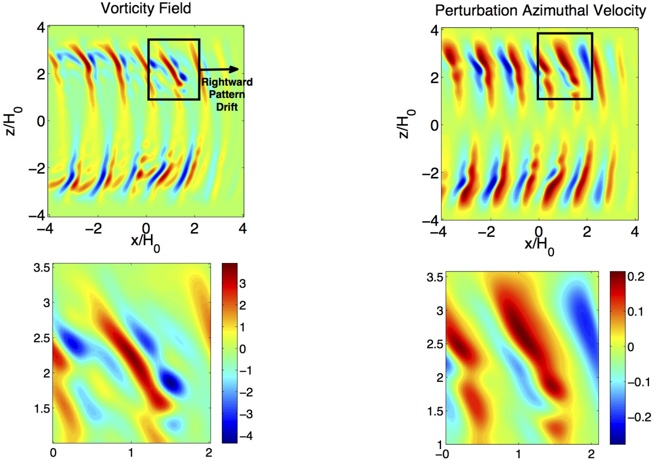
Analog Richardson Criterion

For stratified flow
with vertically sheared
horizontal flow $U(z)$

$$\boxed{N^2 < \frac{1}{4}\left(\frac{dU}{dz}\right)^2}$$

Richardson Criterion

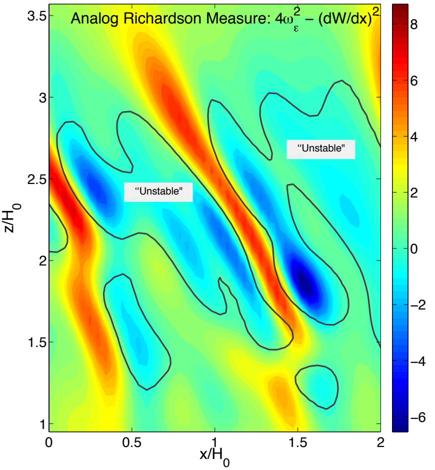
Shearing Sheet Results - Reprise



Minimal Model Configuration

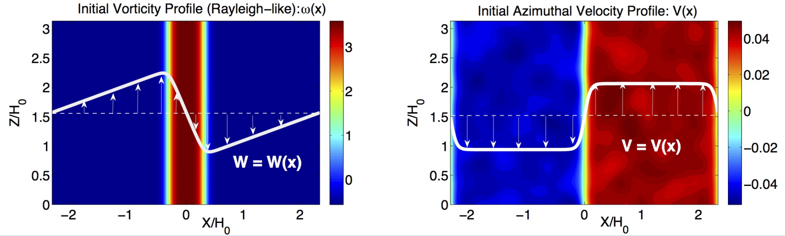
Try to find a simplified setup that captures quality of observed transition.

Richardson Criterion: $8\Omega_0(\Omega_0(2-q)+dV/dx)-(dW/dx)^2<0$



Quasi “Holmboe Type" -I. Setup

Rayleigh profile with stable azimuthal velocity jump



- Stable jump in $V(x)$

  like stable stratification

- Rayleigh Profile
- $\Longrightarrow$ “Holmboe-like" config.

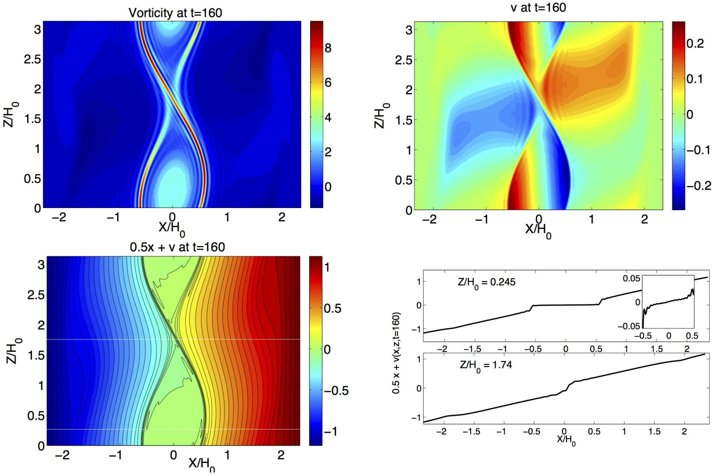
“total $\mathcal{V}$" Lagrangian conserved

$$\frac{d\mathcal{V}}{dt} = 0, \qquad \mathcal{V} \equiv \Omega_0(2-q)x + v(x,t)$$

for $\tau \to \infty$

Quasi “Holmboe Type" - II. Results

- “Short" cooling time results $\tau = 10$.



Vorticity at t=160

v at t=160

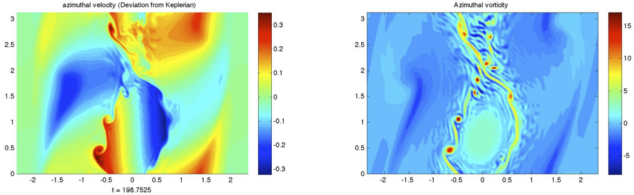
0.5x + v at t=160

Quasi “Holmboe Type" - III. Results

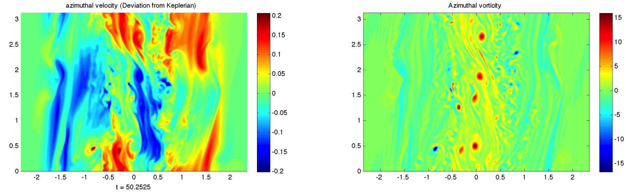
A movie depicting “Holmboe-type" flow at $\tau = 17.5$

Quasi “Holmboe Type" - IV. frames

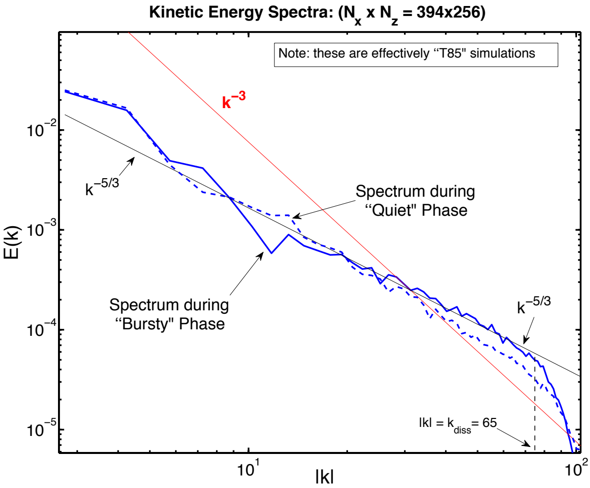
“Quiet"



“Bursty"

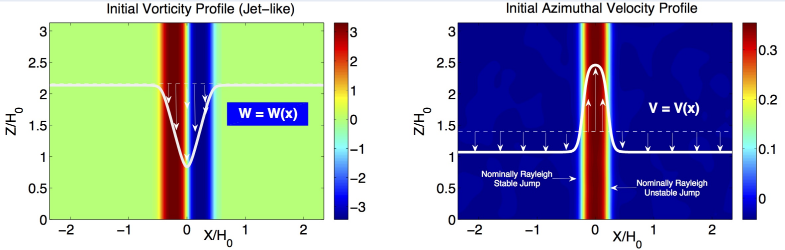


Quasi “Holmboe Type" - IV. Spectra



Jet (“Jetboe") -I. Setup

Jet profile with stable azimuthal velocity jump “The Jetboe"? :)



Jet (“Jetboe") -II. Results

A movie depicting “Jetboe" flow at $\tau = 10$

A Jetboe Experiment

- Interpret instability as a sequence of transitions associated with interacting Rossby Edgewave dynamics.
- Can use the recently developed “Kernal Gravity Wave" perspective to interpret the instability complex here.
- In development now....sorry nothing to show here yet.