Continuous gravitational wave observations to understand nature of compact objects

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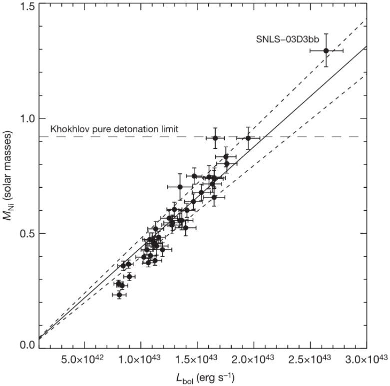
White Dwarfs from Physics to Astrophysics, Nov. 14-17, 2022

Targets of the talk

- Super-Chandrasekhar white dwarfs
- Fast radio bursts
- White dwarf pulsars

Peculiar type Ia supernovae

- Recent observations show some peculiar **SNe Ia** with extremely **high luminosity**.
- Their light curves also show slightly different trend.
- $L \propto M_{\mathrm{WD}} c^2 + mv^2 \implies M_{\mathrm{WD}} \approx 2.1 - 2.8\, M_\odot$.
- **Chandrasekhar mass-limit is violated.**
    - Rotation, magnetic field, modified theory of Einstein's gravity, noncommutative geometry, etc.



Howell *et al.* Nature 443 (2006) 308

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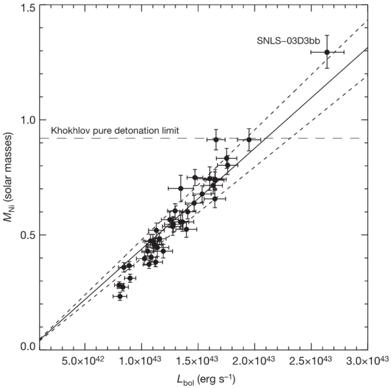

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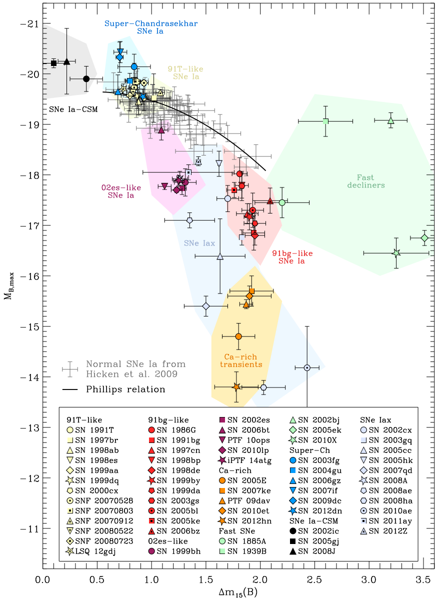
Is it possible to detect them directly?

Peculiar type Ia supernovae



- Peculiar SNe Ia are important as they might affect the standard candle.

S. Taubenberger (2017)

Fast Radio Bursts and WD pulsar

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- Many of these theories incorporate mergers of compact objects.
- Exact masses of some WD pulsars are unknown
  e.g. Mass of AR Scorpii $= [0.81 - 1.29]M_\odot$

Dichotomies/Shortcomings of current observations

1. **Super-Chandrasekhar WDs** are not observed.
2. Progenitor theory of **FRBs** is not known.
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Gravitational wave can be a plausible answer

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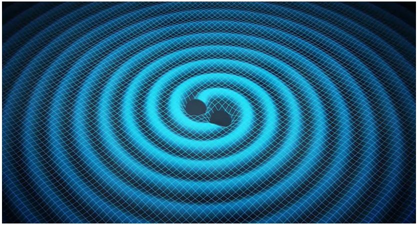

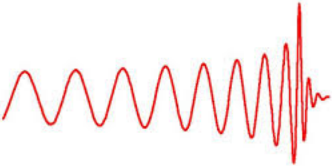

Google Image

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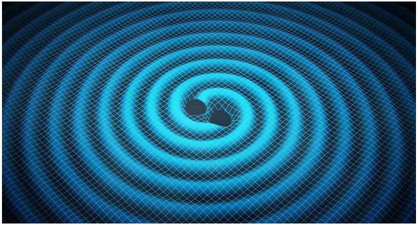

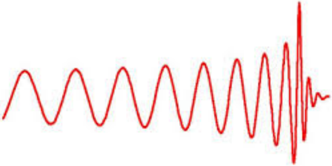

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- Interesting objects, e.g. IMBH with $\mathcal{M} \approx 142\,\mathrm{M}_\odot$ (GW190521), NS or BH with with $\mathcal{M} \approx 2.7\,\mathrm{M}_\odot$ (GW190814) have been detected.

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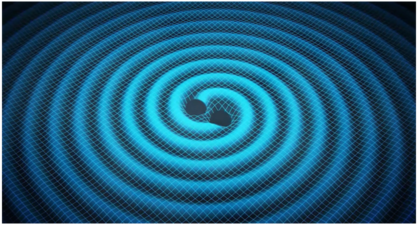

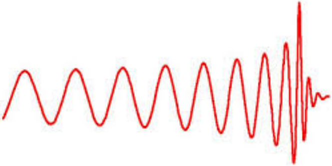

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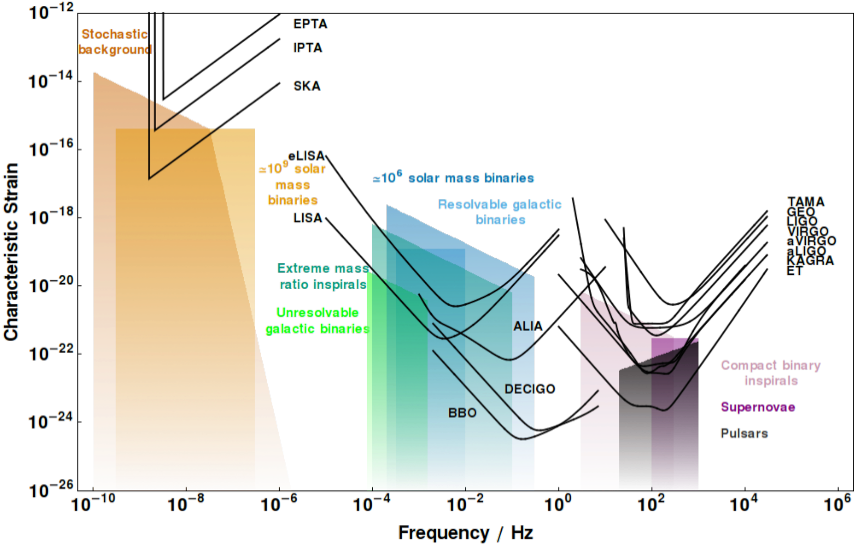
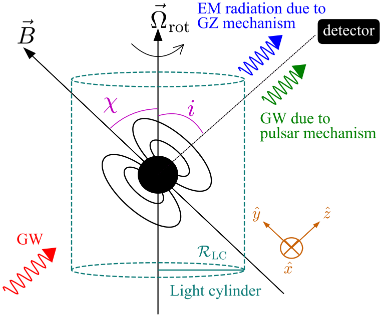
- In future, LIGO/Virgo will be upgraded and around 2035, space-based mission LISA will be launched.

Sensitivity of different GW detectors



http://gwplotter.com/

Magnetized WD/NS



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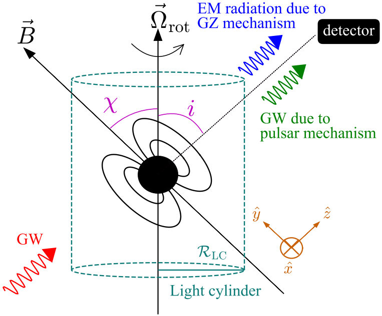
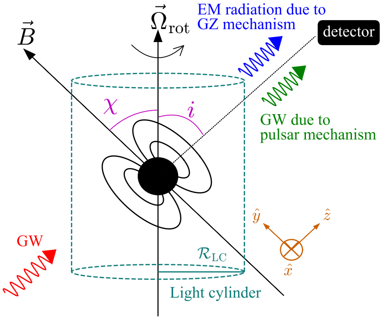


$$\boxed{h_{ij} = \frac{2G}{c^4 r}\ddot{Q}_{ij}}$$

Magnetized WD/NS



$$\boxed{h_{ij} = \frac{2G}{c^4 r}\ddot{Q}_{ij}}$$

$$h_+ = \tilde{A}_{+,1}\cos\left(\Omega_{\rm rot}t\right) + \tilde{A}_{+,2}\cos\left(2\Omega_{\rm rot}t\right)$$
$$h_\times = \tilde{A}_{\times,1}\sin\left(\Omega_{\rm rot}t\right) + \tilde{A}_{\times,2}\sin\left(2\Omega_{\rm rot}t\right)$$

where

$$\tilde{A}_{+,1} = \tilde{h}_0 \sin 2\chi \sin i \cos i,$$
$$\tilde{A}_{+,2} = 2\tilde{h}_0 \sin^2\chi(1+\cos^2 i),$$
$$\tilde{A}_{\times,1} = \tilde{h}_0 \sin 2\chi \sin i,$$
$$\tilde{A}_{\times,2} = 4\tilde{h}_0 \sin^2\chi \cos i,$$

with

$$\tilde{h}_0 = \frac{G}{c^4}\frac{\Omega_{\rm rot}^2(I_3 - I_1)}{r}.$$

M. Maggiore: Gravitational Waves (Vol. 1)

Rotating WDs/NSs

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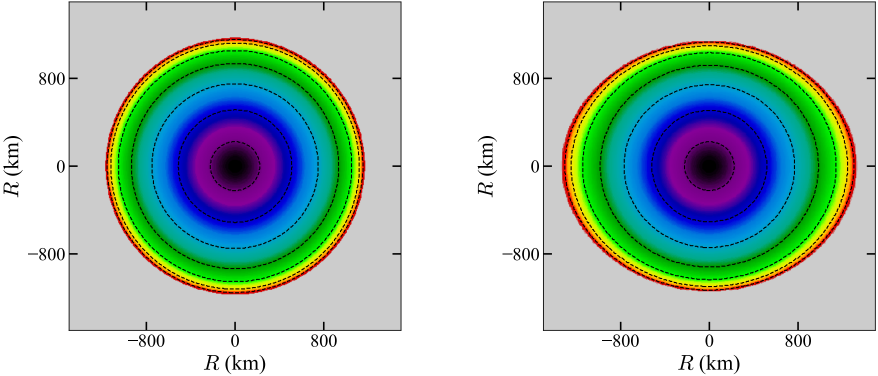
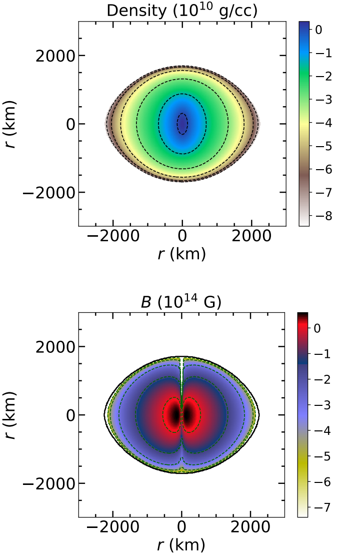
- Ostriker & Hartwick in 1968 showed that rotation alone can increase the mass of a WD up to $\sim 1.8\, M_\odot$.
- Rotation turns a spherical WD to an oblate shaped WD.

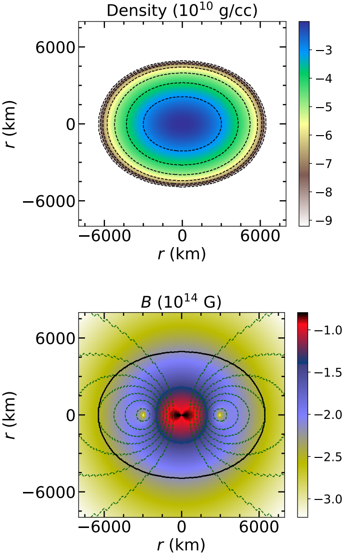


Magnetized WDs

Toroidal magnetic field



Poloidal magnetic field



Kalita & Mukhopadhyay
MNRAS 490 (2019) 2692

Magnetized WDs/NSs

- Rotation $\iff$ oblate.
  Toroidal magnetic field $\iff$ prolate.
  Poloidal magnetic field $\iff$ oblate.

Magnetized WDs/NSs

- Rotation $\Longleftrightarrow$ oblate.
  Toroidal magnetic field $\Longleftrightarrow$ prolate.
  Poloidal magnetic field $\Longleftrightarrow$ oblate.
- *XNS* code is used *developed by Pili, Bucciantini, Del Zanna*.

Magnetized WDs/NSs

- Rotation $\iff$ oblate.
  Toroidal magnetic field $\iff$ prolate.
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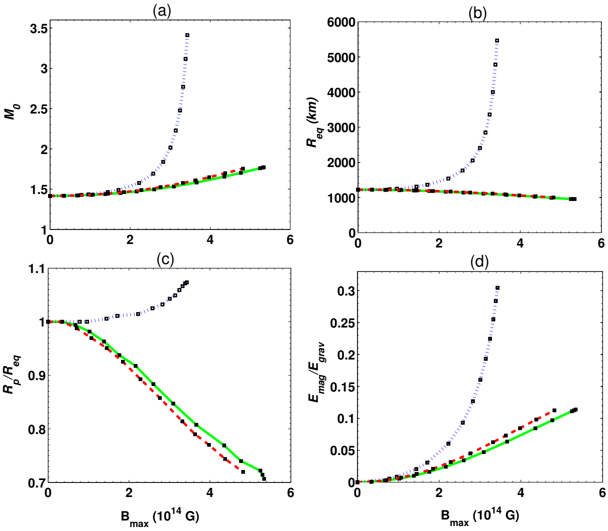
A numerical code for the study of magnetized neutron stars

- **Advantage:** Toroidal/poloidal/mixed magnetic field with uniform/differential rotation.

Magnetized WDs



U. Das & B. Mukhopadhyay, JCAP 05 (2015) 016

EM Dipole and GW quadrupole radiation

- Pulsars emit both EM dipole and GW quadrupole radiations.

$$\begin{aligned} L_{\mathrm{D}} &= \frac{2B_{\mathrm{p}}^2\mathcal{R}_{\mathrm{p}}^6\Omega^4}{3c^3}\left(1+\sin^2\chi\right), \\ L_{\mathrm{GW}} &= \frac{2G}{5c^5}(I_3-I_1)^2\Omega^6\sin^2\chi\left(1+15\sin^2\chi\right), \end{aligned}$$

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$$\frac{\mathrm{d}E}{\mathrm{d}t} = -L_{\mathrm{D}} - L_{\mathrm{GW}}.$$

EM Dipole and GW quadrupole radiation

Energy conservation

$$\frac{\mathrm{d}(\Omega I_{z'z'})}{\mathrm{d}t} = -\frac{2G}{5c^5}\left(I_3 - I_1\right)^2 \Omega^5 \sin^2\chi\left(1 + 15\sin^2\chi\right) - \frac{2B_{\mathrm{p}}^2\mathcal{R}_{\mathrm{p}}^6\Omega^3}{3c^3}\left(1 + \sin^2\chi\right)$$

Angular momentum conservation

$$I_{z'z'}\frac{\mathrm{d}\chi}{\mathrm{d}t} = -\frac{12G}{5c^5}\left(I_3 - I_1\right)^2 \Omega^4 \sin^3\chi\cos\chi - \frac{B_{\mathrm{p}}^2\mathcal{R}_{\mathrm{p}}^6\Omega^2}{3c^3}\sin 2\chi$$

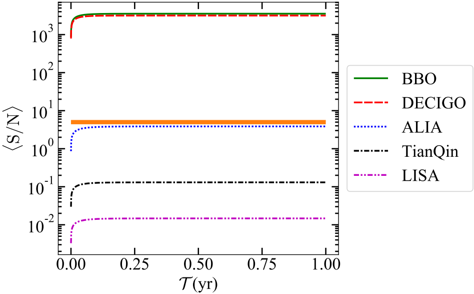
SNR of GWs from magnetized WDs

$$\mathrm{S/N} = \sqrt{\mathrm{S/N}_{\Omega}^2 + \mathrm{S/N}_{2\Omega}^2}\,,$$

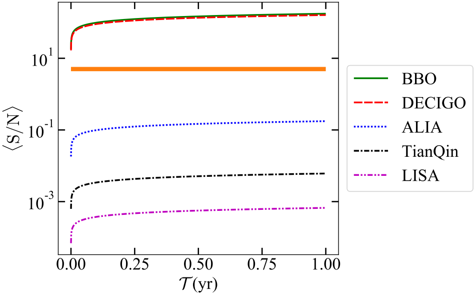
where

$$\langle \mathrm{S/N}_{\Omega}^2 \rangle = \frac{\sin^2\zeta}{100} \frac{h_0^2\, T \sin^2 2\chi}{S_\mathrm{n}(f)}, \quad \langle \mathrm{S/N}_{2\Omega}^2 \rangle = \frac{4\sin^2\zeta}{25} \frac{h_0^2\, T \sin^4\chi}{S_\mathrm{n}(2f)}.$$

GWs from highly magnetized WDs (poloidally dominated)



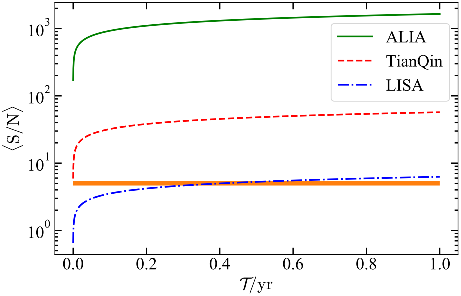
$$B_{\rm p} = 8.9 \times 10^{11}\ \mathrm{G}$$



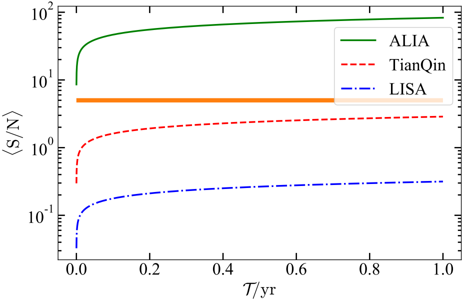
$$B_{\rm p} = 1.4 \times 10^{9}\ \mathrm{G}$$

Kalita *et al.* MNRAS 508 (2021) 842

GWs from highly magnetized WDs (toroidally dominated)



$$B_{\max} = 2.6 \times 10^{14}\ \mathrm{G}$$
$$\mathcal{M} = 1.7\,\mathrm{M}_\odot$$



$$B_{\max} = 1.1 \times 10^{14}\ \mathrm{G}$$
$$\mathcal{M} = 1.4\,\mathrm{M}_\odot$$

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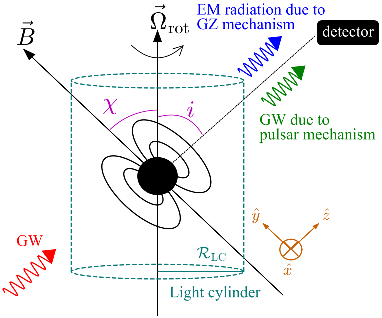
Gertsenshtein-Zel'dovich effect



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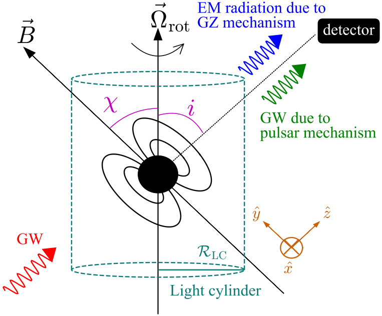

$$h_+ = A_+ e^{i\left(k_g z - \Omega_g t\right)}$$

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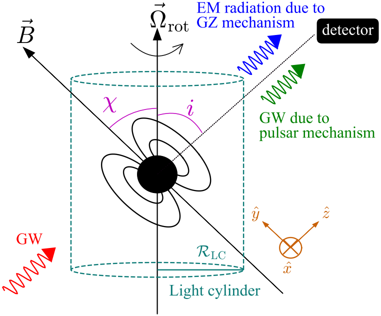

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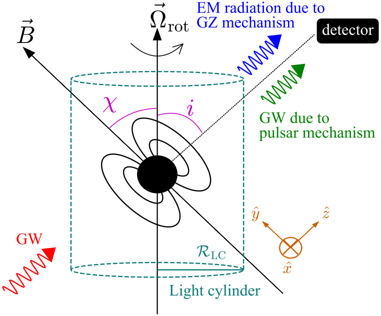

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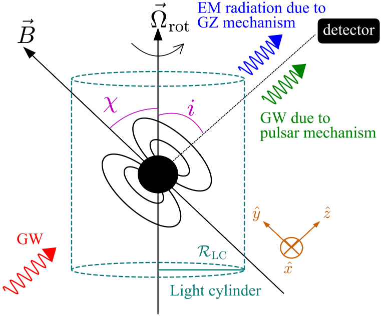
$$\frac{1}{c^2}\frac{\partial^2\tilde{E}_x}{\partial t^2} - \partial_z^2\tilde{E}_x = f_E(z,t)$$

$$\frac{1}{c^2}\frac{\partial^2\tilde{B}_y}{\partial t^2} - \partial_z^2\tilde{B}_y = f_B(z,t)$$

- $f_E, f_B \sim e^{i\left(k_g z - \Omega_\pm t\right)};\ \Omega_\pm = \Omega_g \pm \Omega_{\rm rot}$

Gertsenshtein-Zel'dovich effect



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$$S_z = \frac{A_+^2 \left|B_y^{(0)}\right|^2 c}{128\pi}\left[\sqrt{\frac{24c^2\nu_g^2\alpha_{\rm tot}}{\pi G\left|B_y^{(0)}\right|^2}} - 51 - \frac{6c^2\nu_g\nu_{\rm rot}\alpha_{\rm tot}}{\pi G\left|B_y^{(0)}\right|^2} - 1\right]$$

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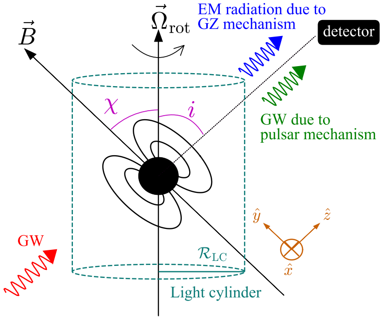
- $f_E, f_B \sim e^{i(k_g z - \Omega_\pm t)}$; $\Omega_\pm = \Omega_g \pm \Omega_{\rm rot}$

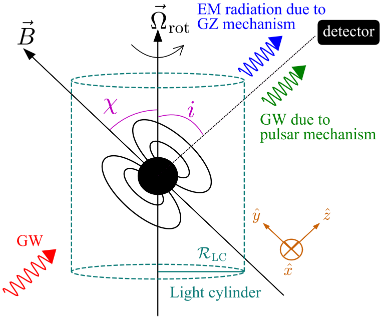
Kushwaha *et al.* **arXiv:2202.00032**

GWs from FRBs with GZ mechanism



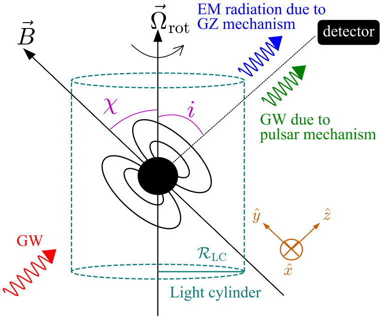
- **FRB 160920**

GWs from FRBs with GZ mechanism



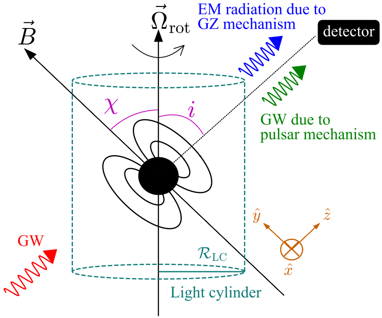
- **FRB 160920**
- Observed at 111 MHz.
- Pulse width $\delta = 5\,\mathrm{s}$.
- Peak flux $= 0.22$ Jy.

GWs from FRBs with GZ mechanism



- **FRB 160920**
- Observed at 111 MHz.
- Pulse width $\delta = 5\,$s.
- Peak flux $= 0.22\,$Jy.
- $\mathcal{R}_{\rm LC} = \frac{\delta c}{2} = 7.49 \times 10^{10}\,\mathrm{cm}.$

GWs from FRBs with GZ mechanism



- $\Omega_{\rm rot} = \frac{c}{\mathcal{R}_{\rm LC}} = 0.4\,{\rm rad\,s^{-1}}$.
- If $A_+ = 10^{-24}$, $\left|B_y^{(0)}\right| = 5.5 \times 10^8\,{\rm G}$.

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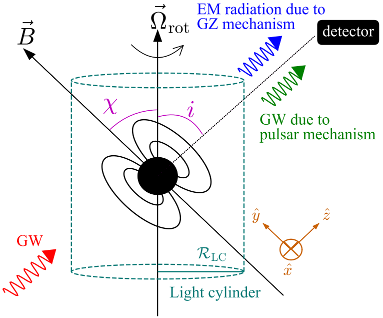

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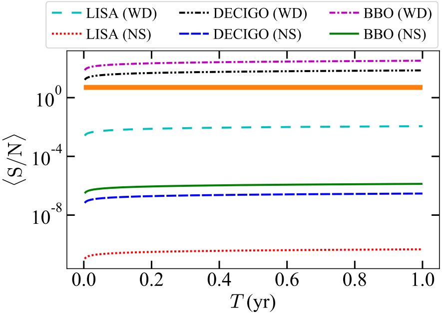
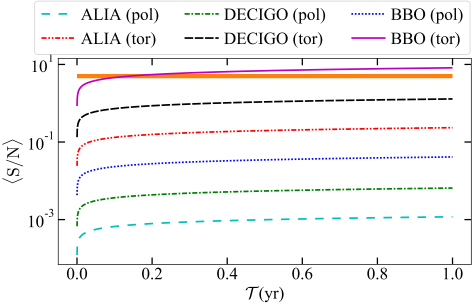
- $\Omega_{\rm rot} = \frac{c}{\mathcal{R}_{\rm LC}} = 0.4\,\mathrm{rad\,s^{-1}}$.
- If $A_+ = 10^{-24}$, $\left|B_y^{(0)}\right| = 5.5 \times 10^8\,\mathrm{G}$.

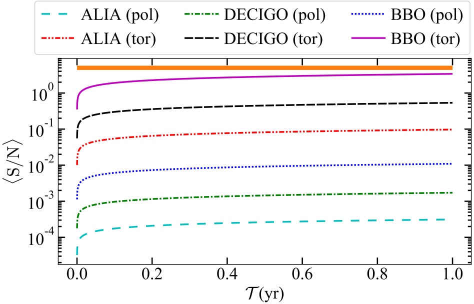


Kalita & Weltman (under review) arXiv:2211.00940

GWs from WD pulsar: AR Scorpii



$\mathcal{M} = 0.81\,\mathrm{M}_\odot$



$\mathcal{M} = 1.29\,\mathrm{M}_\odot$

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Conclusions

- Magnetic fields and rotation can explain the existence of super-Chandrasekhar WDs.
- **GWs may probe the existence of these objects**; thereby restricting the gravity theory.
- Through GWs, we can better understand physics of the compact objects, including FRBs and WD pulsars.
- LISA can only detect highly magnetized WDs within 1 yr of detection period if they are within 100 pc radius.

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